Journal of Environmental Engineering and Landscape Management
ISSN 1648–6897 / eISSN 1822-4199
2018 Volume 26 Issue 4: 251–260
https://doi.org/10.3846/jeelm.2018.6122

A GROUP DECISION-MAKING MODEL FOR WASTEWATER TREATMENT PLANS SELECTION BASED ON INTUITIONISTIC FUZZY SETS

Zhexuan ZHOU[1], Yajie DOU[1*], Xiaoxiong ZHANG[1, 2], Danling ZHAO[1], Yuejin TAN[1]

[1] *College of Systems Engineering, National University of Defense Technology, Changsha, Hunan 410073, P. R. China*
[2] *Systems Design Engineering, University of Waterloo, Waterloo, Canada*

Received 22 May 2018; accepted 25 June 2018

Abstract. As the need for environmental protection and resource sustainability has increased in recent times, wastewater treatment has become increasingly important. In this paper, a group decision-making model for plans selection in wastewater treatment is proposed. In order to deal with uncertainties and multiple attributes in wastewater treatment, an intuitionistic fuzzy set is employed to evaluate wastewater treatment plans effectively. A distance measure is defined to obtain an objective weight measuring the expert's judgment. More specifically, experts first use group decision-making on the various plans with an intuitionistic fuzzy set. Meanwhile, Due to the decision-makers psychological behavior, the prospect theory is applied. Next, the various plans are ranked by The Order of Preference by Similarity to Ideal Solution (TOPSIS) method and prospect theory. Finally, an illustrative example of wastewater treatment plans selection is used to verify the proposed model.

Keywords: wastewater treatment plans selection, group decision-making, intuitionistic fuzzy set, TOPSIS, prospect theory.

Introduction

Recently, increasing attention has been paid to environmental protection, which has gained recognition as a critical issue. Wastewater treatment is one of the most important aspects of environmental protection and resource sustainability, which involves people's living environments and the sustainable development of a regional economy. If wastewater treatment is performed correctly, it can turn waste into a useful resource. If not, it could potentially lead to environmental pollution, and even endanger human civilization (Gilcreas, 1966; Arshad et al., 2017; Stenchly, Dao, Lompo, & Buerkert, 2017). Therefore, the selection of an appropriate wastewater treatment plan is a critical part of the scientific treatment of wastewater. As wastewater treatment involves a lot of factors, several experts are invited to evaluate plans to select a more scientific wastewater treatment plan. Thus, wastewater treatment plan selection is a meaningful and challenging subject for research.

Wastewater treatment decision-making is analyzed. Technicians have developed many plans for the selection of a wastewater treatment plans. Experts need to evaluate these plans, and then based on the recommendations made during the group decision-making process, choose the best from amongst multiple plans. The study of environmental protection and water resources plans evaluation in decision making have already yielded recognized results (Chhipi-Shrestha, Hewage, & Sadiq, 2017; Choudhury et al., 2017). Liu and Tay (2004) used state-of-the-art bio-granulation technology for wastewater treatment. Healthcare waste treatment technologies evaluated using a hybrid multi-criteria decision-making model have been established (Liu, You, Lu, & Chen, 2015). An analysis and management of conflicts has been advanced in waste management decision making (Fawcett, 1993; Wiedemann & Femers, 1993). A decision-making framework for wastewater treatment was proposed by Stochastic Dynamic Programming Formulation (Tsai, V. C. P. Chen, Beck, & J. Chen, 2004). This framework included current and emerging technologies for the multi-level liquid and solid lines of a wastewater treatment system. Barber and Stuckey (1999) presented the anaerobic baffled reactor (ABR) application in wastewater treatment. In its conclusion, the development and applicability of the anaerobic baffled reactor for wastewater treatment was well presented.

*Corresponding author. E-mail: *yajiedou_nudt@163.com*

Flexibility of the decision-making approach in the environmental sciences was addressed for pollution prevention and control, and human health assessment (Fisher, 2003). A decision-making method was proposed using scenario-based techniques in case of air pollutant emissions (Kahyaoğlu-Koračin, Bassett, Mouat, & Gertler, 2009). This study's goal was to develop and implement an air quality assessment tool. Then a mathematical technique for multi-criteria decision making was employed in waste management strategies using the Analytic Hierarchy Process (AHP) (Madadian, Amiri, & Abdoli, 2013). Zhang et al. (2015) established an indicator system to reflect the comprehensive risk of environment and health. In detail, the research and assessment result were divided into four types according to risk level in the Jiangsu province of China. Xiao, Yi and Tang (2017) made an assessment of flood prone areas which was of great importance for watershed management. Small wastewater treatment systems sustainability was evaluated by a composite indicator approach (Molinos-Senante, Gómez, Garrido-Baserba, Caballero, & Sala-Garrido, 2014). Maryam, Mohd Bakri, Ali, Latifah, Normala and Jamal (2017) proposed a method of steel-based waste water treatment technology using multi-criteria decision making and fuzzy logic. Karamouz, Rasoulnia, Zahmatkesh, Olyaei and Baghvand (2016) proposed a framework to evaluate the resiliency of Wastewater treatment plans in coastal areas of New York City. A MCDM model based on AHP was implemented to find the best alternative for solving wastewater in Iran (A. Hadipour, Rajaee, V. Hadipour, & Seidirad, 2016). This method can be applied to decision making for other sectors at any industry level from individual plans to national plans.

MCDM is widely used in dealing with decision-making problems (Herrera-Viedma, 2015). The multi-criteria decision-making method originated from the Pareto optimal theory proposed by the scholar Pareto. It became the most common method of decision-making, and often was used with the linear programming model together (Karsak, Sozer, & Alptekin, 2003). It was typical that multiple criteria can evolve from weighting theory to a single criterion. Weights can be obtained using different methodologies such as AHP, ANP, TOPSIS, and so on. Zavadskas, Turskis and Kildienė (2014) made a review of MCDM/MADM methods to s study the strengths and weaknesses of different decision-making methods. Zolfani, Maknoon and Zavadskas (2016) built a novel model with Prospective Multiple Attribute Decision Making (PMADM) to make decision.

Specially, TOPSIS is widely used to address MCDM problem (Zavadskas, Mardani, Turskis, Jusoh, & MD Nor, 2016). TOPSIS was proposed by Hwang and Yoon (1995) in 1981 and this method is widely used to solve the multiple decision-making problems of individual decision-makers. The core idea is that the chosen solution should be as close as possible to the ideal solution, while at the same time as far away as possible from the negative ideal solution. It has many advantages, including it does not strictly control the data distribution and sample size, how many indicators; simple calculation and wide use; it has an intuitive geometric meaning, which make people easy to accept; It is more full use of the original data, which leads less information loss. Recently, Dursun (2016) evaluated wastewater treatment alternatives by using a fuzzy approach with TOPSIS. Akbaş and Bilgen (2017) proposed MCDM methodology to choose the ideal gas fuel by using TOPSIS. Chemical wastewater purification was evaluated in a hybrid MADM analysis with real data (Khodadadi, Zolfani, Yazdani, & Zavadskas, 2017). It is also the reason that this study uses the TOPSIS to select wastewater treatment plan.

As many factors are involved in wastewater treatment plans, unknowns should be considered. The general decision-making method cannot be a more realistic reflection of the plan's uncertainties. An intuitionistic fuzzy set offers a better way to deal with uncertain multi-attribute problems (Mehlawat & Grover, 2018; Rodríguez, Ortega, & Concepción, 2017; Ren, Xu, & Wang, 2017; Khemiri, Elbedouimaktouf, Grabot, & Zouari, 2017; Ye, 2017). Thus, the study introduces the intuitionistic fuzzy set to select a wastewater treatment plans.

Atanassov (1986) proposed the intuitionistic fuzzy set which was a great contribution in decision making. It is an extension of standard fuzzy sets that has developed greatly since its inception (Hashemi & Mousavi, 2013). Analysis is made mainly by this method for energy, environment and sustainability were ranked applying MCDM techniques and approaches (Mardani, Jusoh, Zavadskas, Cavallaro, & Khalifah, 2015). In Ghosh, Roy and Majumder optimized industrial wastewater treatment problem using intuitionistic fuzzy goal geometric programming (Ghosh et al., 2016), the cost involved was minimized by a five-day biochemical oxygen demand removal. A technological and economic based comprehensive efficiency evaluation of wastewater treatment with an intuitionistic fuzzy set was proposed (Jian, Y. Li, Jiang, & L. Li, 2012). The result of this study was that alternatives were ranked and selected clearly. Wibowo and Grandhi (2015) used intuitionistic fuzzy sets to select and evaluate a suitable wastewater treatment technology. Ren and Liang (2017) proposed to develop an intuitionistic fuzzy set (IFS) with group multi-attribute decision analysis (MADA) to determine sustainability sequences of different wastewater treatment processes. A group decision model with an unbalanced linguistic ordered weighted average (IULOWA) operator was established based on intuitionistic fuzzy sets (IFS) (Marin, Valls, Isern, Moreno, & Merigó, 2014). Recently, a new Pythagorean fuzzy aggregation operator was proposed, which induced ordered weighted averaging-weighted average (PFIOWAWA) operator (Zeng, Mu, & Baležentis, 2017). Zeng, Merigó, Palacios-Marqués, Jin and Gu (2016) developed a new approach for intuitionistic fuzzy decision-making with induced aggregation operators and distance measures. Therefore, using intuitionistic fuzzy sets on wastewater treatment plan group decision making is a particularly meaningful study. In the real decision process, the behavior of decision-maker often influents the final decision a lot. Then, the prospect theory is used to deal with this problem.

The main contribution of this paper is to establish an intuitionistic fuzzy set-TOPSIS (IFS-TOPSIS) GDM model to solve the problem of wastewater treatment plans selection. In the model, a distance measure method is proposed to address the unknown expert weight problem. Then wastewater treatment plans selection considers four attributes: economic efficiency, technical performance, management efficiency, and sustainability. A decision group is used to verify the GDM model efficiently. The decision-makers psychological behavior is evaluating by using the prospect theory. In addition, the model can be applied to other environmental aspects such as atmospheric plans and resource sustainability plans. The calculation is simple and easy to achieve using a computer.

The paper is structured as follows: Section 1 is a description about the wastewater treatment problem background and an analysis of the intuitionistic fuzzy set method's results in the literature; Section 2 builds on this and provides a detailed description of wastewater treatment plans selection using a GDM model from intuitionistic fuzzy sets. In addition, wastewater treatment plans ranking with the TOPSIS method is introduced. An illustrative case study in wastewater treatment plans selection with IFS-TOPSIS analysis is conducted in Section 3. Finally, Section 4 concludes the work with a summary discussion.

1. Wastewater treatment plans selection based on intuitionistic fuzzy sets

1.1. Model framework

The aim of this paper is to address the wastewater treatment plans selection from group decision making with intuitionistic fuzzy sets. As shown in Figure 1, A GDM model for wastewater treatment plans selection based on IF-TOPSIS) is proposed that includes three phases: wastewater treatment plans identification, deriving intuitionistic fuzzy set decisions, and ranking the wastewater treatment alternative plans using IF-TOPSIS.

The first phase is to establish an expert's panel for wastewater treatment management organization. Then the decision group evaluates the existing plans of wastewater treatment by language under each attribute.

The second phase aims to obtain the intuitionistic fuzzy decision matrix. In this phase, the language evaluation is transformed to an intuitionistic fuzzy set decision matrix. According to the weight of the plan attributes given by an expert, a weighted intuitionistic fuzzy decision matrix for each expert is calculated.

The third phase is to rank the wastewater treatment alternative plans using IFS-TOPSIS by its expert weight unknown. In detail, the degree to which each plan matches its intuitionistic fuzzy positive and negative ideal solution for each expert is calculated by the TOPSIS method. Then

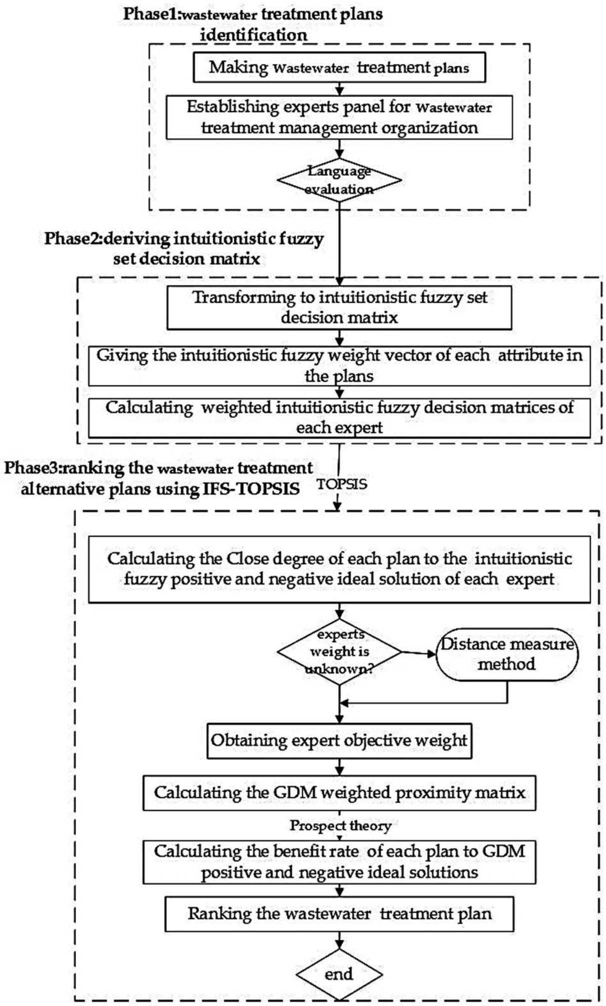

Figure 1. IF-TOPSIS Model for wastewater treatment plans selection

the expert weight is obtained using the distance measure method and the GDM weighted proximity matrices are calculated. Through the TOPSIS method with prospect theory the benefit rate in value of each plan to its corresponding GDM positive and negative ideal solutions can be calculated. Finally, the wastewater treatment alternative plans are ranked, and the best wastewater treatment plan is selected.

Wastewater treatment involves more factors. The wastewater treatment plan evaluation index system can be built based on the existing indicators for wastewater treatment, including: Economic benefit, Technical performance, management benefit, and sustainability. As it shows in Figure 2.

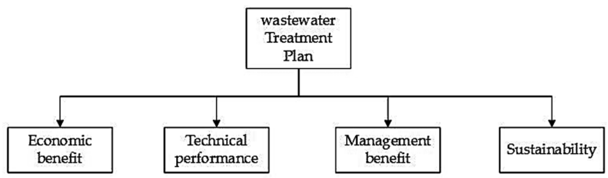

Figure 2. Wastewater treatment plans evaluation index system

1) Economic benefit refers to the benefits and cost of the plan; 2) Technical performance refers to the technical maturity and technical feasibility of implementing the plan; 3) Management benefit refer to the ease of implementation of the plan and the efficiency of the work; 4) Sustainability includes the flexibility of the plan and the development of plan.

1.2. Intuitionistic fuzzy sets

The intuitionistic fuzzy set is an extension of standard fuzzy sets, and an intuitionistic fuzzy set $\tilde{A}$ in X is an object that has been represented (Atanassov, 1986):

$$\tilde{A} = \left\{ x, u_{\tilde{A}}(x), v_{\tilde{A}}(x) \mid x \in X \right\},$$

where the membership $u_{\tilde{A}} \in [0,1]$ and non-membership degree $v_{\tilde{A}} \in [0,1]$, satisfy: $0 \le u_{\tilde{A}} + v_{\tilde{A}} \le 1$ and the hesitancy is π such that $\pi_{\tilde{A}} = 1 - (u_{\tilde{A}} + v_{\tilde{A}})$.

Definition 1 (Atanassov, 1986). Let $\tilde{A}$ and $\tilde{B}$ be two intuitionistic fuzzy sets. Then $\tilde{A} + \tilde{B}$ can be defined:

$$\tilde{A} + \tilde{B} = \left\{ x, u_{\tilde{A}}(x) + u_{\tilde{B}}(x) - u_{\tilde{A}}(x) u_{\tilde{B}}(x), v_{\tilde{A}}(x) v_{\tilde{B}}(x) \mid x \in X \right\}. \quad (1)$$

Definition 2 (Atanassov, 1986). Let $\tilde{A}$ and $\tilde{B}$ be two intuitionistic fuzzy sets. Then $\tilde{A}\tilde{B}$ can be defined:

$$\tilde{A}\tilde{B} = \left\{ x, u_{\tilde{A}}(x) u_{\tilde{B}}(x), v_{\tilde{A}}(x) + v_{\tilde{B}}(x) - v_{\tilde{A}}(x) v_{\tilde{B}}(x) \mid x \in X \right\}. \quad (2)$$

Definition 3 (Li & Nan, 2013). Let $F_{ij} = < u_{ij}, v_{ij} >$ be an intuitionistic fuzzy set matrix and ω the weight of the intuitionistic fuzzy set. Then:

$$\omega = (\omega_1, \omega_2, ..., \omega_m)^T = (< \rho_1, \tau_1 >, < \rho_2, \tau_2 >, ..., < \rho_m, \tau_m >)^T,$$

where the membership $\rho_m \in [0,1]$ and non-membership degree $\tau_m \in [0,1]$, and

$$\begin{aligned} &\bar{F}_{ij} = < \bar{u}_{ij}, \bar{v}_{ij} > = \omega_i F_{ij} = < \rho_m, \tau_m > \\ &< u_{ij}, v_{ij} > = < \rho_m u_{ij}, \tau_m + v_{ij} - \tau_m v_{ij} >, \\ &(i = 1, 2, ..., m;\ j = 1, 2, ..., n), \end{aligned} \quad (3)$$

where $\bar{F}_{ij}$ is called the weighted intuitionistic fuzzy set decision matrix.

Definition 4 (Xing, Xiong, & Liu, 2017). Let $\tilde{A}$ and $\tilde{B}$ be two intuitionistic fuzzy sets. The intuitionistic fuzzy set Euclidean distance can be expressed as:

$$d_1(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{2n} \sum_{j=1}^{n} [(u_{\tilde{A}}(x_j) - u_{\tilde{B}}(x_j))^2 + (v_{\tilde{A}}(x_j) - v_{\tilde{B}}(x_j))^2 + (\pi_{\tilde{A}}(x_j) - \pi_{\tilde{B}}(x_j))^2]}. \quad (4)$$

1.3. Prospect theory

People often deviate from "reason" in the process of decision-making (Kahneman & Tversky, 1979) used a large amount of analysis to explain that the decision-making result often runs counter to the theory of expected utility. Then, the prospect theory was proposed. The prospect theory mainly considers the value function and decision weight, and the prospect value function is as follows:

$$V = \sum_{i=1}^{n} \pi(p_i) v(x_i), \quad (5)$$

where $\pi(p) = \dfrac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}$ is an increasing function of probability, which called the decision weight; and $v(x_i) = \begin{cases} x^\delta, & x \ge 0 \\ -\sigma(-x)^\beta, & x < 0 \end{cases}$ is value function, which is the value of the decision maker's subjective feelings; γ, δ, σ, β are respectively parameters (Tamura, 2005).

1.4. Expert objective weight calculation by distance measure

In the selection of wastewater plans, the most important point in group decision making is the weight of the experts. Because of the plan evaluation process, for unknown reasons, some experts give an evaluation that is more scientific and reasonable, while others may have some deviation from this standard. This requires decision makers to utilize certain scientific methods to determine the weight of the experts.

This study proposes a method of objective expert weight calculation by distance measure. The method first calculates the positive and negative ideal solution from the weighted intuitionistic fuzzy set decision matrix $\bar{F}_{ij}^{k}$. It can be written as:

$$F^{K+} = (< u_1^{k+}, v_1^{k+} >, < u_1^{k+}, v_1^{k+} >, ..., < u_m^{k+}, v_m^{k+} >) \quad (6)$$

and

$$F^{K-} = < u_1^{k-} \ v_1^{k-} > < u_1^{k-} \ v_1^{k-} > \quad < u_m^{k-} \ v_m^{k-} > , \quad (7)$$

where

$u_m^{k+} = \max\limits_{1 \le j \le n} \left\{ \bar{u}_{ij}^{k} \right\}$, $v_m^{k+} = \max\limits_{1 \le j \le n} \left\{ \bar{v}_{ij}^{k} \right\}$, $u_m^{k-} = \min\limits_{1 \le j \le n} \left\{ \bar{u}_{ij}^{k} \right\}$,

and $v_m^{k-} = \min\limits_{1 \le j \le n} \left\{ \bar{v}_{ij}^{k} \right\}$ with $(k = 1, 2, ..., K; i = 1, 2, ..., m)$ as positive and negative ideal solutions, respectively.

This Euclidean distance of intuitionistic fuzzy sets for a positive and negative ideal solution d_2 represents the deviation of experts who are evaluating the plans.

Definition 5. The Euclidean distance of intuitionistic fuzzy sets for positive and negative ideal solutions can be represented as:

$$d_2(F^{K+},F^{K-})=\sqrt{\frac{1}{2n}\sum_{i=1}^{m}[(u_i^{k+}-u_i^{k-})^2+(v_i^{k+}-v_i^{k-})^2+(\pi_i^{k+}-\pi_i^{k-})^2]}. \quad (8)$$

This means that if the Euclidean distance d_2 is large, the deviation is also large. The smaller the deviation, the more that the weight of the expert is considered.

Thus, the expert objective weight can be calculated by distance measure. If there are P experts, then the distance d_2^p can be calculated and the expert objective weight can be derived:

$$\omega^p=\frac{1}{2}(1-d_2^p/(\sum_1^P(d_2^1+d_2^2+\ldots+d_2^p))). \quad (9)$$

1.5. IFS-TOPSIS GDM model with unknown expert weight

In the selection of wastewater treatment, expert group decision making is very important. In group decision making, given n candidates: $x_j(j=1,2,\ldots,n)$, plans constitute plan sets $X=\{x_1,x_2,\ldots,x_n\}$ with K decision makers (experts) $P_k(k=1,2,\ldots,K)$. One decision-making group evaluates each plan with m attributes: $o_i(i=1,2,\ldots,m)$ and attributes set: $O=\{o_1,o_1,\ldots,o_m\}$ (Li & Nan, 2013). Given that the decision makers (experts) $P_k(k=1,2,\ldots,K)$ evaluate the plan sets $x_j\in X$ over attributes $o_i\in O$, the evaluation value can be described by the intuitionistic fuzzy set:

$$F_{ij}^k=<u_{ij}^k,v_{ij}^k>(i=1,2,\ldots,m;j=1,2,\ldots,n),$$

where the membership $u_{ij}^k\in[0,1]$ and non-membership degree $v_{ij}^k\in[0,1]$.

Thus, the intuitionistic fuzzy set decision matrix F^k can be calculated.

The group decision-making model based on an IFS-TOPSIS steps is as follows:

Step 1: The standard intuitionistic fuzzy set decision matrices $\bar{F}^k$ of each decision maker（expert）are calculated by (3)

$$\bar{F}^k=\omega_0F^k, \quad (10)$$

where ω_0 is the weight vector of the attributes $o_i(i=1,2,\ldots,m)$.

Step 2: This positive and negative ideal solution F^{K+}, F^- of standard intuitionistic fuzzy set decision matrix $\bar{F}^k$ can be obtained by (6), (7).

Step 3: The Euclidean distance $D_j^{k^+}$, $D_j^{k^-}$ for an intuitionistic fuzzy set decision matrix $\bar{F}^k$ to its positive and negative ideal solution F^{K+}, F^- can be calculated by (4).

Step 4: The closeness degree p_j^k of plans $x_j\in X$ for the positive and negative ideal solution F^{K+} , F^- to each decision maker（expert）$P_k(k=1,2,\ldots,K)$ can be calculated by:

$$p_j^k=\frac{D_j^{k-}}{D_j^{k^+}+D_j^{k^-}}(k=1,2,\ldots,K;j=1,2,\ldots,n). \quad (11)$$

Step 5: The K decision makers and n plans' proximity matrices F_D can be constructed from the closeness degree p_j^k of each decision maker（expert）$P_k(k=1,2,\ldots,K)$.

$$F_D=(p_1^1,p_2^2,\ldots,p_j^k),(k=1,2,\ldots,K,j=1,2,\ldots,n). \quad (12)$$

Step 6: The weight ω^p of each decision maker (expert) is calculated by (8).

Step 7: The weighted proximity decision matrix $\bar{F}_D$ is calculated by (12)

$$\bar{F}_D=\omega^pF_D. \quad (13)$$

Step 8: This closeness degree of positive and negative ideal solution $\bar{F}_D^{\,+}$, $\bar{F}_D^{\,-}$ can be obtained by (13), (14) from the proximity decision matrix $\bar{F}_D$.

$$\bar{F}_D^{\,+}=\max_{1\le j\le n}\left\{\bar{p}_j^{\,k}\right\}=(\bar{p}_1^{\,+},\bar{p}_2^{\,+},\ldots,\bar{p}_K^{\,+})^T \quad (14)$$

$(k=1,2,\ldots,K)$;

$$\bar{F}_D^{\,-}=\min_{1\le j\le n}\left\{\bar{p}_j^{\,k}\right\}=(\bar{p}_1^{\,-},\bar{p}_2^{\,-},\ldots,\bar{p}_K^{\,-})^T \quad (15)$$

$(k=1,2,\ldots,K)$,

where the degree of positive and negative ideal are respectively $\bar{p}_K^{\,+}$ and $\bar{p}_K^{\,-}$.

Step 9: The Euclidean distance $d_j^{k^+}$, $d_j^{k^-}$ for a proximity decision matrix F_D and its positive and negative ideal solution $F_D{}^+$, $F_D{}^-$ can be calculated by:

$$d_j^{k^+}=\sqrt{\sum_{k=1}^{K}(\bar{p}_j^{\,k}-\bar{p}_j^{\,+})^2}\ \ (j=1,2,\ldots,n); \quad (16)$$

$$d_j^{k^-}=\sqrt{\sum_{k=1}^{K}(\bar{p}_j^{\,k}-\bar{p}_j^{\,-})^2}\ \ (j=1,2,\ldots,n). \quad (17)$$

Step 10: According to the definition of value function in prospect theory and behavioral economics, if positive idealism is used as a reference point, the various schemes are loss relative to positive idealism; on the contrary, if the negative ideal solution is as a reference point, each plan is to benefit. Hence, the new value function of prospect theory can be expressed as:

$$v^-(d_j^{k^+})=-\sigma(d_j^{k^+})^\beta; \quad (18)$$

$$v^+(d_j^{k^-})=(d_j^{k^-})^\delta, \quad (19)$$

where $\delta=\beta=0.88$, $\sigma=2.25$ (Tversky & Kahneman, 1992).

Step 11: calculate the benefit rate C_j of each wastewater treatment plan $x_j \in X(j=1,2,...,n)$:

$$C_j = \left|\frac{v^+(d_j^{k-})}{v^-(d_j^{k+})}\right| = \frac{\sqrt{\sum_{k=1}^{K}(\bar{p}_j^k - \bar{p}_j^-)^2}}{\sqrt{\sum_{k=1}^{K}(\bar{p}_j^k - \bar{p}_j^+)^2}} \tag{20}$$

$(j=1,2,...,n)$.

Through the method of intuitionistic fuzzy set group decision making, the wastewater treatment scheme is selected objectively and scientifically. The wastewater treatment plans are evaluated from four attributes: economic benefit, technical performance, management benefit, and sustainability. In the case where the expert weight in the decision-making group is unknown, the distance between the positive and negative ideal solutions evaluated by each decision maker is used to represent the decision maker's decision deviation. The weight of each decision maker can be obtained by the distance method, and this method is more objective than the previous.

2. Case study

It is considered standard that wastewater treatment management has to select a wastewater treatment plan from the group-decision making evaluation. Three wastewater treatment plans (x_1, x_2, x_3) have been identified as candidates. Three experts (P_1, P_2, P_3) are responsible for the wastewater treatment plans selection problem, who are in the research areas of Wastewater Treatment, Government in environment protected, and Economic investors. The plans are evaluated with four attributes (economic benefit, technical performance, management benefit, and sustainability.). Each expert gives an evaluation for the candidates with the four attributes. The evaluation information is given by each expert as the following with the intuitionistic fuzzy set decision matrices, respectively:

$$F^1 = (u_{ij}^1, u_{ij}^1) = \begin{array}{c} \\ o_1 \\ o_2 \\ o_3 \\ o_4 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{pmatrix} <0.80,0.1> & <0.50,0.30> & <0.72,0.30> \\ <0.75,0.12> & <0.65,0.10> & <0.65,0.20> \\ <0.50,0.30> & <0.45,0.20> & <0.75,0.15> \\ <0.60,0.22> & <0.70,0.30> & <0.55,0.20> \end{pmatrix} \end{array};$$

$$F^2 = (u_{ij}^2, u_{ij}^2) = \begin{array}{c} \\ o_1 \\ o_2 \\ o_3 \\ o_4 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{pmatrix} <0.81,0.11> & <0.60,0.20> & <0.70,0.15> \\ <0.76,0.15> & <0.72,0.18> & <0.65,0.18> \\ <0.70,0.12> & <0.65,0.25> & <0.79,0.10> \\ <0.60,0.25> & <0.58,0.15> & <0.80,0.11> \end{pmatrix} \end{array};$$

$$F^3 = (u_{ij}^3, u_{ij}^3) = \begin{array}{c} \\ o_1 \\ o_2 \\ o_3 \\ o_4 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{pmatrix} <0.78,0.20> & <0.70,0.15> & <0.85,0.10> \\ <0.81,0.11> & <0.65,0.20> & <0.80,0.12> \\ <0.75,0.21> & <0.60,0.30> & <0.73,0.11> \\ <0.68,0.12> & <0.58,0.20> & <0.62,0.30> \end{pmatrix} \end{array}.$$

The experts give the weight vector for the attributes as follows:

$\omega_0^1 = (\rho_i^1, \tau_i^1) =$
$(<0.40,0.30>, <0.35,0.20>, <0.45,0.20>, <0.35,0.25>)$;

$\omega_0^2 = (\rho_i^2, \tau_i^2) =$
$(<0.30,0.25>, <0.40,0.25>, <0.45,0.35>, <0.35,0.20>)$;

$\omega_0^3 = (\rho_i^3, \tau_i^3) =$
$(<0.35,0.20>, <0.25,0.50>, <0.40,0.30>, <0.30,0.55>)$.

The standard intuitionistic fuzzy set decision matrixes $\bar{F}^k$ of each decision maker (expert) can be calculated by (10). (The standard intuitionistic fuzzy set decision matrixes $\bar{F}^k$ are given in Appendix A for convenience)

This positive and negative ideal solution F^{K+}, F^- of a standard intuitionistic fuzzy set decision matrix $\bar{F}^k$ can be obtained by (6), (7) (This positive and negative ideal solution F^{K+}, F^- of a standard intuitionistic fuzzy set decision matrix $\bar{F}^k$ are given in Appendix B for convenience).

The Euclidean distance D_j^{k+}, D_j^{k-} for an intuitionistic fuzzy set decision matrix $\bar{F}^k$ to its positive and negative ideal solution F^{K+}, F^- can be calculated by (4) in Table 1.

Table 1. The Euclidean distance D_j^{k+}, D_j^{k-}

Experts	Euclidean distance D_j^{k+}, D_j^{k-}					
	D_1^{k+}	D_1^{k-}	D_2^{k+}	D_2^{k-}	D_3^{k+}	D_3^{k-}
P_1	0.211	0.1929	0.227	0.117	0.190	0.241
P_2	0.103	0.103	0.132	0.860	0.047	0.133
P_3	0.99	0.109	0.133	0.064	0.86	0.141

The closeness degree p_j^k of plans $x_j \in X$ for the positive and negative ideal solution F^{K+}, F^- of each decision maker (expert) $P_k(k=1,2,...,K)$ can be calculated by (11) in Table 2 (The closeness degree p_j^k are given in Appendix C for convenience).

The K decision makers and n plans' proximity matrices F_D can be constructed by the closeness degree p_j^k of each decision maker (expert) $P_k(k=1,2,...,K)$.

The weight ω^p of each decision maker (expert) is calculated by (9).

$$\omega_D = (0.259, 0.375, 0.366).$$

The weighted proximity decision matrix $\bar{F}_D$ is calculated by (12), (13).

$$\bar{F}_D = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \end{array} \begin{pmatrix} x_1 & x_2 & x_3 \\ 0.124 & 0.088 & 0.145 \\ 0.188 & 0.148 & 0.278 \\ 0.192 & 0.119 & 0.227 \end{pmatrix}.$$

This closeness degree to the positive and negative ideal solutions $\bar{F}_D^{+}$, $\bar{F}_D^{-}$ can be obtained by (14), (15) from close proximity matrix $\bar{F}_D^{k}$.

$$\begin{cases} \bar{F}_D^{+} = (0.145, 0.278, 0.227) \\ \bar{F}_D^{-} = (0.088, 0.188, 0.119) \end{cases}.$$

Then, the benefit rate C_j of wastewater treatment plans $x_j \in X(j = 1, 2, ..., n)$ are calculated by (16), (17), (18), (19) and (20):

$$C_1 = 0.7115,\ C_2 = 0.2248,\ C_3 = 0.3814.$$

The order of wastewater treatment plans is:

$$x_1 \succ x_3 \succ x_2,$$

where plan x_1 is the best.

In this case, three wastewater treatment schemes are evaluated by a decision group with three experts based on the Intuitionistic Fuzzy Set-TOPSIS group decision-making method. Each plan is considered for four attributes (economic benefit, technical performance, management benefit, and sustainability).

In this case, the weight of each expert is unknown in this decision-making group. This study uses distance measures to represent the degree of deviation from expert decision making. Distance measure is a more objective method to determine the weight of each expert. Considering the decision-maker behavior, new value function is built for prospect theory. Hence, wastewater treatment plan selection is completed with the IFS-TOPSIS GDA model. The result of the case study is that wastewater treatment plan is the best wastewater treatment plan is second best, and wastewater treatment plan is the worst.

Discussion and conclusions

Environmental protection and resource sustainability are hot topics and wastewater treatment plans selection is more important than ever before. Because uncertainty and many impact factors need to be considered, the wastewater treatment plans selection is a difficult and meaningful topic for study. There have been several achievements in environmental protection and wastewater treatment decision-making methods. The evaluation of candidates is used more widely by the decision-making group with experts. In general, the weight of each expert in the decision-making group is unknown, and the expert's evaluation of the plan is vague. Based on the existing wastewater treatment plans selection method, this paper proposes a group decision-making model for wastewater treatment plans selection based on IFS-TOPSIS. The proposed distance measurement method is used to determine the expert's objective weight. Considering the psychological factors of decision makers, prospect theory is introduced into this model.

In the illustrative example of wastewater treatment plans selection, four attributes (economic benefit, technical performance, management benefit, and sustainability) are considered. Some significant results of this case study can be summarized as follows:

1) The language evaluation of each expert is transformed into the intuitionistic fuzzy set decision matrices. As a result, their needs are more accurately reflected based on their preference for plan selection.
2) The weight of experts in the decision group is obtained by using the distance measurement method. This method is more objective to gain weight.
3) To deal with the psychological factors of decision-maker, this study redefines the value function of prospect theory. Then three wastewater treatment plans are ranked in TOPSIS with prospect theory.

The main contribution of this paper is selecting the wastewater treatment plans using IFS-TOPSIS. IFS-TOPSIS is a good model which addresses both uncertain information and multiple attributes for the wastewater treatment plans problem well. The next step is to obtain the actual data to select wastewater treatment plans with an IFS-TOPSIS model.

Acknowledgments

We are thankful to the Editor and the reviewers for their valuable comments and detailed suggestions to improve the presentation of the paper. Further, we also acknowledge the support in part by the National Science Foundation of China under grant No. 71690233 and No. 71671186.

References

Akbaş, H., & Bilgen, B. (2017). An integrated fuzzy QFD and TOPSIS methodology for choosing the ideal gas fuel at WWTPs. *Energy*, *125*, 484-497. https://doi.org/10.1016/j.energy.2017.02.153

Arshad, A., Iqbal, J., Siddiq, M., Ali, M., Ali, A., Shabbir, H., Usama, B. N., & ShahbazSaleem, M. (2017). Solar light triggered catalytic performance of graphene-CuO nanocomposite for waste water treatment. *Ceramics International*, *43*(13), 10654-10660. https://doi.org/10.1016/j.ceramint.2017.03.165

Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets & Systems*, *20*(1), 87-96. https://doi.org/10.1016/S0165-0114(86)80034-3

Barber, W. P., & Stuckey, D. C. (1999). The use of the anaerobic baffled reactor (ABR) for wastewater treatment: a review. *Water Research*, *33*(7), 1559-1578. https://doi.org/10.1016/S0043-1354(98)00371-6

Chhipi-Shrestha, G., Hewage, K., & Sadiq, R. (2017). Fit-for-purpose wastewater treatment: Conceptualization to development of decision support tool (I). *Science of the Total Environment*, *607-608*, 600-612. https://doi.org/10.1016/j.scitotenv.2017.06.269

Choudhury, P., Uday, U. S. P., Mahata, N., Tiwari, O., Ray, R., Bandyopadhyay, T., & Bhunia, B. (2017). Performance improvement of microbial fuel cells for waste water treatment along with value addition: A review on past achievements and recent perspectives. *Renewable & Sustainable Energy Reviews*, *79*, 372-389. https://doi.org/10.1016/j.rser.2017.05.098

Dursun, M. (2016). A fuzzy approach for the assessment of wastewater treatment alternatives. *Engineering Letters*, *24*(2), 231-236.

Fawcett, H. (1993). High-level dollars: low-level sense: a critique of present policy for the management of long-lived radioactive waste and discussion of an alternative approach, by A. Makhijani and Scott Saleska, A Report of the Institute for Energy and Environmental Reseal. *Journal of Hazardous Materials*, *33*(3), 330-343. https://doi.org/10.1016/0304-3894(93)85087-U

Fisher, B. (2003). Fuzzy environmental decision-making: applications to air pollution. *Atmospheric Environment*, *37*(14), 1865-1877. https://doi.org/10.1016/S1352-2310(03)00028-1

Gilcreas, F. W. (1966). Standard methods for the examination of water and waste water. *American Journal of Public Health & the Nations Health*, *56*(3), 387-388. https://doi.org/10.2105/AJPH.56.3.387

Ghosh, P., Roy, T. K., & Majumder, C. (2016). Optimization of industrial wastewater treatment using intuitionistic fuzzy goal geometric programming problem. *Fuzzy Information & Engineering*, *8*(3), 329-343. https://doi.org/10.1016/j.fiae.2016.09.002

Hadipour, A., Rajaee, T., Hadipour, V., & Seidirad, S. 2016. Multi-criteria decision-making model for wastewater reuse application: a case study from iran. *Desalination & Water Treatment*, 57(30), 13857-13864. https://doi.org/10.1080/19443994.2015.1060905

Hashemi, H., & Mousavi, S. M. (2013). A compromise ratio method with an application to water resources management: an intuitionistic fuzzy set. *Water Resources Management*, *27*(7), 2029-2051. https://doi.org/10.1007/s11269-013-0271-x

Herrera-Viedma, E. (2015). Fuzzy sets and fuzzy logic in multi-criteria decision making. The 50th anniversary of prof. Lotfi zadeh's theory: introduction. *Technological & Economic Development of Economy*, *21*(5), 677-683. https://doi.org/10.3846/20294913.2015.1084956

Hwang, C. L., & Yoon, K. (1995). *Multiple attribute decision making*. Berlin, Heidelberg: Springer.

Jian, X., Li, Y., Jiang, Z., & Li, L. (2012). The research on technologic economics comprehensive appraisal of wastewater treatment in the paper making factory with intuitionistic fuzzy information. *Advances in Information Sciences & Service Sciences*, *4*(4), 293-299. https://doi.org/10.4156/aiss.vol4.issue4.35

Kahyaoğlu-Koračin, J., Bassett, S. D., Mouat, D. A., & Gertler, A. W. (2009). Application of a scenario-based modeling system to evaluate the air quality impacts of future growth. *Atmospheric Environment*, *43*(5), 1021-1028. https://doi.org/10.1016/j.atmosenv.2008.04.004

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, *47*(2), 263-292. https://doi.org/10.2307/1914185

Karamouz, M., Rasoulnia, E., Zahmatkesh, Z., Olyaei, M. A., & Baghvand, A. (2016). Uncertainty-based flood resiliency evaluation of wastewater treatment plants. *Journal of Hydroinformatics*, *18*(6), 990-1106. https://doi.org/10.2166/hydro.2016.084

Karsak, E. E., Sozer, S., & Alptekin, S. E. (2003). Product planning in quality function deployment using a combined analytic network process and goal programming approach. *Computers & Industrial Engineering*, *44*(1), 171-190. https://doi.org/10.1016/S0360-8352(02)00191-2

Khemiri, R., Elbedouimaktouf, K., Grabot, B., & Zouari, B. (2017). A fuzzy multi-criteria decision-making approach for managing performance and risk in integrated procurement–production planning. *International Journal of Production Research*, *55*(18), 5305-5329. https://doi.org/10.1080/00207543.2017.1308575

Khodadadi, M. R., Zolfani, S. H., Yazdani, M., & Zavadskas, E. K. (2017). A hybrid madm analysis in evaluating process of chemical wastewater purification regarding to advance oxidation processes. *Journal of Environmental Engineering & Landscape Management*, *25*(3), 277-288.

Li, D. F., & Nan, J. X. (2013). Extension of the TOPSIS for multi-attribute group decision making under Atanassov IFS Environments. *International Journal of Fuzzy System Applications*, *1*(4), 47-61. https://doi.org/10.4018/ijfsa.2011100104

Liu, H. C., You, J. X., Lu, C., & Chen, Y. Z. (2015). Evaluating health-care waste treatment technologies using a hybrid multi-criteria decision-making model. *Renewable & Sustainable Energy Reviews*, *41*, 932-942. https://doi.org/10.1016/j.rser.2014.08.061

Liu, Y., & Tay, J. H. (2004). State of the art of bio granulation technology for wastewater treatment. *Biotechnology Advances*, *22*(7), 533-563. https://doi.org/10.1016/j.biotechadv.2004.05.001

Madadian, E., Amiri, L., & Abdoli, M. A. (2013). Application of analytic hierarchy process and multicriteria decision analysis on waste management: A case study in Iran. *Environmental Progress & Sustainable Energy*, *32*(3), 810-817. https://doi.org/10.1002/ep.11695

Mardani, A., Jusoh, A., Zavadskas, E. K., Cavallaro, F., & Khalifah, Z. (2015). Sustainable and renewable energy: an overview of the application of multiple criteria decision making techniques and approaches. *Sustainability*, *7*, 13947-13984. https://doi.org/10.3390/su71013947

Marin, L., Valls, A., Isern, D., Moreno, A., & Merigó, J. M. (2014). Induced unbalanced linguistic ordered weighted average and its application in multi-person decision making. *The Scientific World Journal*, *2014*, ID 642165, 1-19. https://doi.org/10.1155/2014/642165

Maryam, M., Mohd Bakri, I., Ali, T., Latifah, A. M., Normala, H., & Jamal, G. D. (2017). Optimal selection of Iron and Steel wastewater treatment technology using integrated multi-criteria decision-making techniques and fuzzy logic. *Process Safety and Environmental Protection*, *107*, 54-68. https://doi.org/10.1016/j.psep.2017.01.016

Mehlawat, M. K., & Grover, N. (2018). Intuitionistic fuzzy multi-criteria group decision making with an application to critical path selection. *Annals of Operations Research*, 269(1-2), 505-520. https://doi.org/10.1007/s10479-017-2477-4

Molinos-Senante, M., Gómez, T., Garrido-Baserba, M., Caballero, R., & Sala-Garrido, R. (2014). Assessing the sustainability of small wastewater treatment systems: A composite indicator approach. *Science of the Total Environment*, *497-498*, 607-617. https://doi.org/10.1016/j.scitotenv.2014.08.026

Ren, J., & Liang, H. (2017). Multi-criteria group decision-making based sustainability measurement of wastewater treatment processes. *Environmental Impact Assessment Review*, *65*, 91-99. https://doi.org/10.1016/j.eiar.2017.04.008

Ren, Z., Xu, Z., & Wang, H. (2017). Dual hesitant fuzzy VIKOR method for multi-criteria group decision making based on fuzzy measure and new comparison method. *Information Sciences*, *388-389*, 1-16. https://doi.org/10.1016/j.ins.2017.01.024

Rodríguez, A., Ortega, F., & Concepción, R. (2017). An intuitionistic method for the selection of a risk management approach to information technology projects. *Information Sciences*, *375*, 202-218. https://doi.org/10.1016/j.ins.2016.09.053

Stenchly, K., Dao, J., Lompo, D. J., & Buerkert, A. (2017). Effects of waste water irrigation on soil properties and soil fauna of spinach fields in a West African urban vegetable production system. *Environmental Pollution*, *222*, 58-63. https://doi.org/10.1016/j.envpol.2017.01.006

Tamura, H. (2005). Behavioral models for complex decision analysis. *European Journal of Operational Research*, *166*(3), 655-665. https://doi.org/10.1016/j.ejor.2004.03.038

Tsai, J. C. C., Chen, V. C. P., Beck, M. B., & Chen, J. (2004). Stochastic dynamic programming formulation for a wastewater treatment decision-making framework. *Annals of Operations Research*, *132*(1-4), 207-221. https://doi.org/10.1023/B:ANOR.0000045283.86576.62

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: cumulative representation of uncertainty. *Journal of Risk & Uncertainty*, *5*(4), 297-323. https://doi.org/10.1007/BF00122574

Wibowo, S., & Grandhi, S. (2015). Application of the fuzzy approach for the selection of wastewater treatment technologies. In *IEEE 10th Conference on Industrial Electronics and Applications* (pp. 843-848). IEEE. https://doi.org/10.1109/ICIEA.2015.7334228

Wiedemann, P. M., & Femers, S. (1993). Public participation in waste management decision making: Analysis and management of conflicts. *Journal of Hazardous Materials*, *33*(3), 355-368. https://doi.org/10.1016/0304-3894(93)85085-S

Xiao, Y., Yi, S., & Tang, Z. (2017). Integrated flood hazard assessment based on spatial ordered weighted averaging method considering spatial heterogeneity of risk preference. *Science of the Total Environment, 599-600*, 1034-1046. https://doi.org/10.1016/j.scitotenv.2017.04.218

Xing, Z., Xiong, W., & Liu, H. (2017). A Euclidean approach for ranking Atanassov intuitionistic fuzzy values. *IEEE Transactions on Fuzzy Systems*, *99*, 1-1. https://doi.org/10.1109/TFUZZ.2017.2666219

Ye, J. (2017). Intuitionistic fuzzy hybrid arithmetic and geometric aggregation operators for the decision-making of mechanical design schemes. *Applied Intelligence*, *2*, 1-9. https://doi.org/10.1007/s10489-017-0930-3

Zavadskas, E. K., Mardani, A., Turskis, Z., Jusoh, A., & MD Nor, K. (2016). Development of topsis method to solve complicated decision-making problems: an overview on developments from 2000 to 2015. *International Journal of Information Technology & Decision Making*, *15*(03), 645-682. https://doi.org/10.1142/S0219622016300019

Zavadskas, E. K., Turskis, Z., & Kildienė, S. (2014). State of art surveys of overviews on MCDM/MADM methods. *Technological & Economic Development of Economy*, *20*(1), 165-179. https://doi.org/10.3846/20294913.2014.892037

Zeng, S., Merigó, J. M., Palacios-Marqués, D., Jin, H., & Gu, F. (2016). Intuitionistic fuzzy induced ordered weighted averaging distance operator and its application to decision making. *Journal of Intelligent & Fuzzy Systems*, *32*(1), 1-12.

Zeng, S., Mu, Z., & Baležentis, T. (2017). A novel aggregation method for pythagorean fuzzy multiple attribute group decision making. *International Journal of Intelligent Systems*, *33*(3), 573-585. https://doi.org/10.1002/int.21953

Zhang, S., Wei, Z., Liu, W., Yao, L., Suo, W., Xing, J., Huang, B., Jin, D., & Wang, J. (2015). Indicators for environment health risk assessment in the Jiangsu province of china. *International Journal of Environmental Research & Public Health*, *12*(9), 11012-11024. https://doi.org/10.3390/ijerph120911012

Zolfani, S. H., Maknoon, R., & Zavadskas, E. K. (2016). An introduction to prospective multiple attribute decision making (PMADM). *Technological & Economic Development of Economy*, *22*(2), 309-326. https://doi.org/10.3846/20294913.2016.1150363

Appendix A

The standard intuitionistic fuzzy set decision matrixes $\bar{F}^k$ of each decision maker (expert) can be calculated by (9)

$$\bar{F}^1 = (\bar{u}_{ij}^1, \bar{u}_{ij}^1) = \begin{pmatrix} <0.32,0.37> & <0.20,0.51> & <0.29,0.51> \\ <0.26,0.20> & <0.23,0.28> & <0.23,0.34> \\ <0.23,0.44> & <0.20,0.36> & <0.34,0.20> \\ <0.21,0.42> & <0.25,0.48> & <0.19,0.40> \end{pmatrix};$$

$$\bar{F}^2 = (\bar{u}_{ij}^2, \bar{u}_{ij}^2) = \begin{pmatrix} <0.24,0.33> & <0.18,0.40> & <0.21,0.36> \\ <0.30,0.36> & <0.29,0.39> & <0.26,0.39> \\ <0.32,0.43> & <0.29,0.51> & <0.36,0.42> \\ <0.21,0.44> & <0.20,0.36> & <0.28,0.33> \end{pmatrix};$$

$$\bar{F}^3 = (\bar{u}_{ij}^3, \bar{u}_{ij}^3) = \begin{pmatrix} <0.27,0.36> & <0.25,0.32> & <0.30,0.28> \\ <0.20,0.56> & <0.16,0.60> & <0.20,0.56> \\ <0.30,0.45> & <0.24,0.51> & <0.29,0.38> \\ <0.20,0.60> & <0.17,0.64> & <0.19,0.69> \end{pmatrix}.$$

Appendix B

This positive and negative ideal solution F^{K+}, F^{-} of a standard intuitionistic fuzzy set decision matrix $\bar{F}^{k}$ can be obtained by (5), (6).

$F^{1+} = (<0.32, 0.37>, <0.26, 0.2>, <0.34, 0.2>, <0.25, 0.4>)$;

$F^{1-} = (<0.20, 0.51>, <0.23, 0.34>, <0.20, 0.44>, <0.19, 0.48>)$;

$F^{2+} = (<0.24, 0.33>, <0.30, 0.36>, <0.36, 0.42>, <0.28, 0.33>)$;

$F^{2-} = (<0.18, 0.40>, <0.26, 0.39>, <0.29, 0.51>, <0.20, 0.44>)$;

$F^{3+} = (<0.30, 0.28>, <0.20, 0.56>, <0.30, 0.38>, <0.20, 0.60>)$;

$F^{3-} = (<0.25, 0.36, <0.16, 0.60>, <0.24, 0.51>, <0.17, 0.69>)$.

Appendix C

The closeness degree p_j^k of plans $x_j \in X$ for the positive and negative ideal solution F^{K+}, F^{-} of each decision maker (expert) $P_k (k = 1, 2, ..., K)$ can be calculated by (10) in Table 2.

Table 2. The closeness degree p_j^k

Experts	Closeness degree p_j^k		
	p_j^1	p_j^2	p_j^3
P_1	0.478	0.340	0.559
P_2	0.500	0.394	0.739
P_3	0.524	0.324	0.621