STEEL STRUCTURES IN INTERACTION WITH NON-METALLIC MEMBRANES

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Abstract. Behaviour of prestressed textile membranes supported by peripheral non-rigid steel structures and used for various simple temporary and permanent canopies/shelters is described. The investigation relates to structures composed of membranes located between steel arches only. The approaches for global analysis of the structural system and influence of important parameters such as the membrane prestressing and rigidity of the supporting steelwork are discussed. Extensive parametric study concerning possibility of simplified separate analyses of the membranes and supporting steel structures is presented, giving limiting parameters for such approach. Finally, some recommendations for the analysis and realization of prestressed non-metallic membranes and respective steel supporting structures are provided.

Keywords: formfinding, pre-stressing, nonlinear analysis, separate analysis, steel arch, textile membranes.

Introduction

In the last decades the use of non-metallic membranes is widespread due to their enormous lightness and aesthetics required by architects, developers and investors. The world-famous companies (e.g. Base Structures Ltd., FabriTec Structures, TI Tensile structures, Oasis Tension Structures, Mehler Ltd., Archtex s.r.o., Herzog & de Meuron, Vector Foiltec, etc.) are producing countless number of both ordinary and unique tensile structures using fabric or foil membranes.

Textile membranes are becoming required and routine material not only for simple canopies or shelters but also for sophisticated outward load bearing structures. Nowadays a detailed survey of available textile/foil materials suitable for membrane engineering structures is displayed in specialized websites and publications. For example Base Structures Ltd. presents a table mapping the most common external and internal fabrics with basic data on their lifetime, price, joining, fire rating, translucence, possibility of relocation, colour etc.

For common modest use the PVC coated polyester (PES/PVC) seems to be appropriate as an inexpensive variant, giving up to 20 years of lifetime (e.g. Précontraint FERRARI®), joined by welding or sewing. More expensive but longer lifetime provides glass fabric coated by PTFE (Glass/PTFE), with possible replacement of PTFE (Teflon) by silicon rubber or titanium dioxide, and joined by bonding. Rather expensive but excellent material is expanded PTFE coated 2 sides by fluoropolymer film (TENARA®), joined by welding. The membranes may also be thermally insulated using Nanogel Aerogel™ and 2 sides coated with PTFE (in result translucent, with total thickness of approx. 9 mm, see CABOT Cor.). High density polyethylene fabrics (HDPE, coated with LDPE) joined by sewing may also be used, however, with short lifespan up to 10 years. Thin foils (50÷500 μm) are used mainly as inflatable cushions, nowadays predominantly made from ETFE (TEXLON®) or THV materials.

Spectacular structures using textile/foil membranes (e.g. Millennium Dome in London, amphitheatre Memorial dos Povos de Belém do Pará, Metrodome roof in Minnesota, Rosenheim Bahnhof, Allianz Arena Munich, Khan Shatyr Astana, Hajj Terminal Jeddah, Duol domes, etc.) indicate new ways, forms and directions of novel unique structures.

Design of a membrane structure follows general concept (Lewis 2003): i) the predesign of a form ensuring tension within the all membrane area during assembly and loading, requiring a sufficient prestressing: basic shapes are hypar, cone and barrel (Seidel 2009); ii) the decision on boundary conditions (point or continuous and rigid or elastic boundaries) and used peripheral elements (prestressed cables, frame structure, anchor points and cables); iii) form-finding process based usually on the

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force-density method (Formfinder software GmbH 2006–2015); iv) structural analysis of the membrane with supporting structure under required prestressing, dead and live loading – i.e. snow, wind, facility (Foster, Mollaert 2004; Wakefield 1999). The prestressing procedure often determines the design and final geometry of the resulting structure.

Formfinding process based on force-density method with cable network model (Schek 1974; Linkwitz 1999; Gründig *et al.* 2000) enabled to abandon former physical modelling of membrane structures and former simple grid method (Siev, Eidelman 1964). More general approach offered surface stress density method (Maurin, Motro 1998) with isotropic membrane elements and stress distribution control. Recent formfinding efficient methods embody updated reference strategy method (Bonet, Mahaney 2001) and modified updated reference strategy method (Cottrell *et al.* 2009; Capova 2010) which solve the variation task with isogeometric analysis based on non-uniform B-spline base representations (NURBS), using FEM and N-R iteration. In the last decade an intensive effort by Roithmayr and Gründig (Formfinder software GmbH 2006–2015) resulted in software as a user-friendly tool with large project database. The software enables intuitive manipulation with membrane shapes under required stress level in interactive way and export/import to other programmes through DXF/DWG files. Constantly improved versions upgrade the software even for design of supporting elements.

Geometrically and materially non-linear analysis considering imperfections (GMNIA) is required for proper dealing with membranes acting together with a supporting steelwork. Material properties of common textile membranes are under continuous investigations. The material is due to its structure non-homogeneous, orthotropic (warp and fill directions) and non-linear. Such sophisticated formulation based on fabric lattice was proposed (Kato *et al.* 1999), which well corresponded with test data but is considered as too complicated for practical analysis. Gosling (2007), apart from a simplified approach, suggested strain-strain-stress approach using response surfaces linking strains to stresses through three dimensional representations for FE analysis. Recently a non-linear material model for textile membranes based on experimental results was proposed (Galliot, Luchsinger 2009a, 2009b). The model depends on load ratios in warp and fill directions and is described by five parameters: warp and fill Young's modules for 1:1 load ratio, the change in warp and fill Young's modules and the Poisson's ratio. Another new model (Dinh *et al.* 2014) is based on input from uniaxial and biaxial tension test data, which properly simulate relevant nonlinearities, orthotropic behaviour and permanent strains with load cycling. The latest material model (Pargana, Leitao 2015) consists of three nonlinear elements to model yarns and an isotropic plate to model coating. The model is claimed to be simple and credible as confirmed by validations with experimental results.

In accordance with the European recommendations by Tensinet Analysis & Materials working Group (see Foster, Mollaert 2004; Gosling 2007) a simplified elastic approach may be employed using the simple plane stress theory. The supplied test data provides elastic modules for warp and fill directions and corresponding Poisson's ratio, valid for anticlastic type of structures. Recently a comparison of two approaches concerning simplified elastic constants for design of the membranes (Japanese MSAJ and the above European ones) was presented in Uhlemann *et al.* (2015). The European approach was found to be more general and reasonable for PES/PVC material (but still far from satisfactory).

Supporting steelwork as anchor, perimeter, valley and ridge ropes or rather stiff load-bearing structure form an integral part of the resulting membrane structure. Interaction of the membranes with the steel elements requires sophisticated analysis using specialized engineering software (see e.g. Easy Software 2014; ixForten 2015; SOFiSTiK 2014; Rhino Membrane 2015; membrane NDN software 2015, etc.) or other general FEM software packages (Wakefield 1999). With need to model properly the membrane behaviour in large tensile structures using FEM, apart from sophisticated material models the new nonlinear finite elements were developed recently. Novel quadratic membrane element was proposed (Pargana, Leitao 2015) on assumption of flat fabric patterns in their unstressed state and quadratic sliding cable element inside a fabric sleeve. The elements were incorporated into ADAPTIC software and verified in case studies. Assembly method and the prestressing ensuring the full functionality must carefully be taken into account (Seidel 2009).

Nevertheless, simple membrane structures are often designed by non-specialized consultancy offices which do not possess the specialized software. In such situations the prestressed membrane is analyzed separately as isolated one with large deflections and isotropic material behaviour while the resulting reactions are introduced into a supporting steelwork. This paper investigates only the simplest form of the membrane structures called "barrel" (Fig. 1), to assess whether the simplified separate analysis of a membrane stretched between non-rigid steel arches
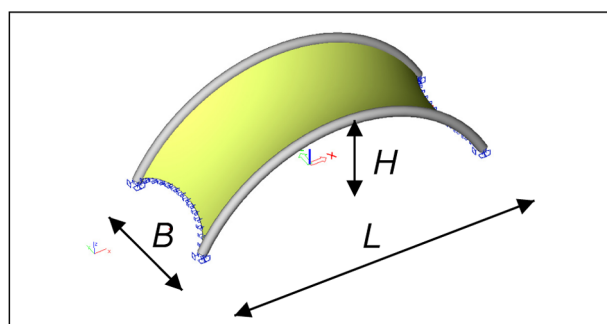

Fig. 1. Geometry of a simple membrane stretched as an anticlastic surface between two steel arches

as a membrane alone (i.e. with rigid supports) and the steel arches (i.e. arches loaded by the relevant membrane response) is reasonable. Separate modelling may only be successful provided the design of the membrane considers geometry and rigidity of the supporting framework. In case the support is not fully rigid, the introduction of real rigidities is necessary, otherwise resultant membrane stresses and deflections are distorted and data provided to the designer of steel structure are incorrect.

Therefore, this study does not model the membranes with all the above mentioned nonlinearities and anisotropy but employs geometrically nonlinear analysis (GNA) with linear isotropic behaviour of a textile membrane, because its principal goal is investigation of the interaction with the supporting steelwork. Such simplification of membrane behaviour is well approaching the Ferrari® Précontraint type of fabric, made of polyester scrim coated both sides with liquid PVC and PVDF topcoat. Due to prestressing of the base fabric before and during the coating operations both warp and fill directions show similar behaviour, elongation and minimum creep. More sophisticated material behaviour of the fabric is used by Svoboda and Machacek (2014), concerning detailed study of the membranes.

1. Analysis of a structure composed of a textile membrane and steel arches

The study follows the general procedure mentioned above. First the membrane shapes were found using FORMFINDER software and transferred via AUTOCAD to the common 3D "frame" SCIA ENGINEER software. The basic shape with dimensions $LxBxH$ as 10000×4000×3000 [mm] according to Figure 2 was found, supported by two steel tubular arches built-in at the supports and made from S235 grade steel. The textile membrane considered was fictitious isotropic material with modulus of elasticity $E = 1000$ MPa, Poisson's ratio $v = 0.25$ and thickness $t = 1$ mm. The analyses cover "membrane alone" (i.e. the membrane with rigid supports along both arches and transverse supports) and "membrane with arches" (i.e. membrane supported by non-rigid arches and rigid transverse supports, the latter always used to reduce the entry data).

The proper prestressing of the membrane to ensure tension throughout the membrane area during assembly and all loadings is required. Within the study the simple planar prestressing through confined deformation $\varepsilon = \varepsilon_x = \varepsilon_y$ was introduced into numerical analysis. In the "membrane alone" $\varepsilon = 0.004$ (giving for $v = 0.25$ unit force 5.333 N/mm), for the "membrane with arches" the value $\varepsilon = 0.0046$ resulted from condition of identical horizontal reaction along arch lines to ensure the same prestressing as in the former due to horizontal deflection of arches. Vertical loading 1 kN/m^2 only was considered in the study, representing snow loading. The arches were designed according to Eurocode 3 (EN 1993) for the above loading. The tubes 324×25 [mm] ensure cross-section utilization of approx. 80%.

Geometrically nonlinear analysis (GNA) with proper boundary shape was performed, arches modelled by beam and the membrane by quadrilateral elements. First a sensitivity of the finite element size was assessed for the case of "membrane alone" through three mesh divisions, with basic lengths 50 mm, 150 mm, 250 mm, Figure 3.

After decreasing size from 150 mm to 50 mm the deflections remained nearly unaffected (see Fig. 4) and stresses increased by 5.1%. After increasing the size from 150 mm to 250 mm the deflections decreased by 0.1% and stresses decreased by 2.6%. The finest mesh gives more appropriate results for detailed stresses in corners/edges. In all the following studies the medium mesh (150 mm) was used and considered adequate even for the parametric studies.

The above analysis was compared with results of another nonlinear solution using common COMSOL MULTIPHYSICS software, and found to be nearly identical.

2. Comparison of results based on full and separate analyses

2.1. The full "membrane with arches" and "membrane alone" analysis

GNA was performed for the directionless (confined) prestressing as defined above, to give the identical horizontal reaction along arch lines for both boundary conditions. The resulting transverse unit forces due to the prestressing are
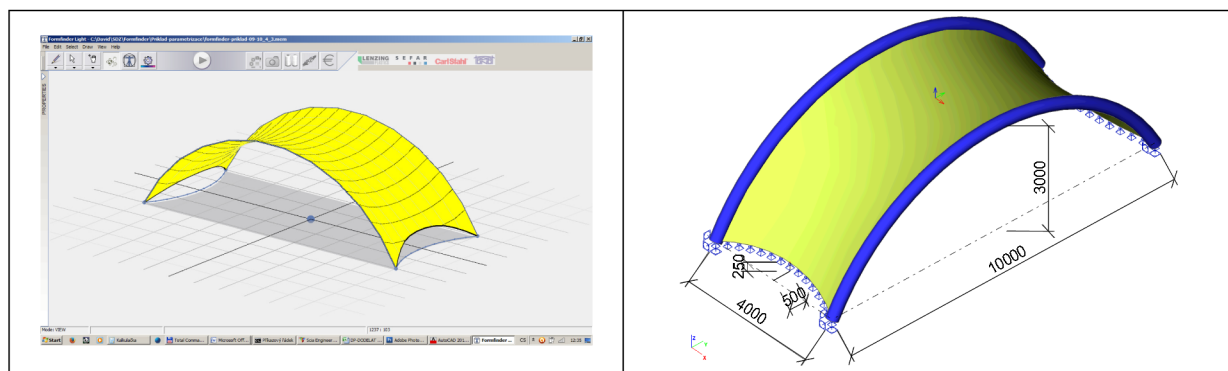

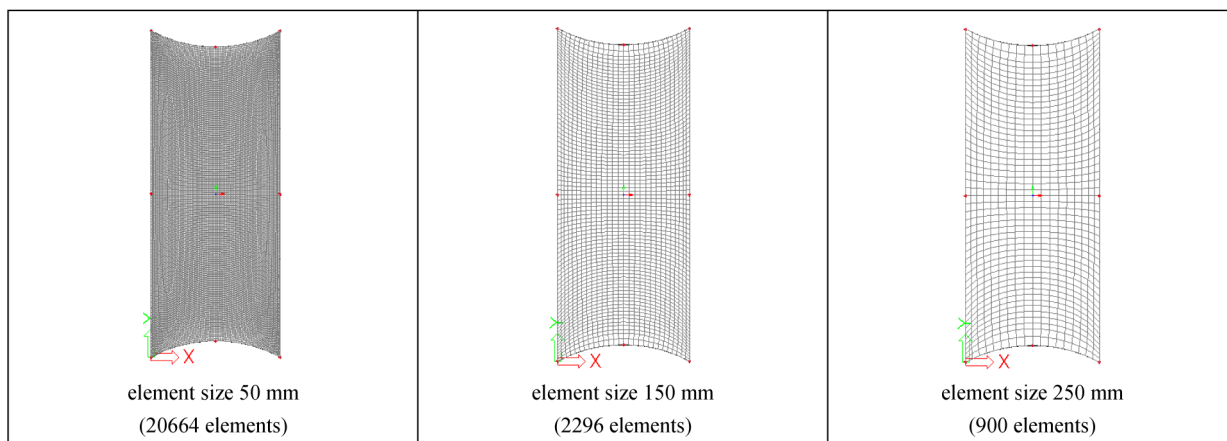Fig. 2. Shape according to FORMFINDER software (left) and investigated geometry (right)

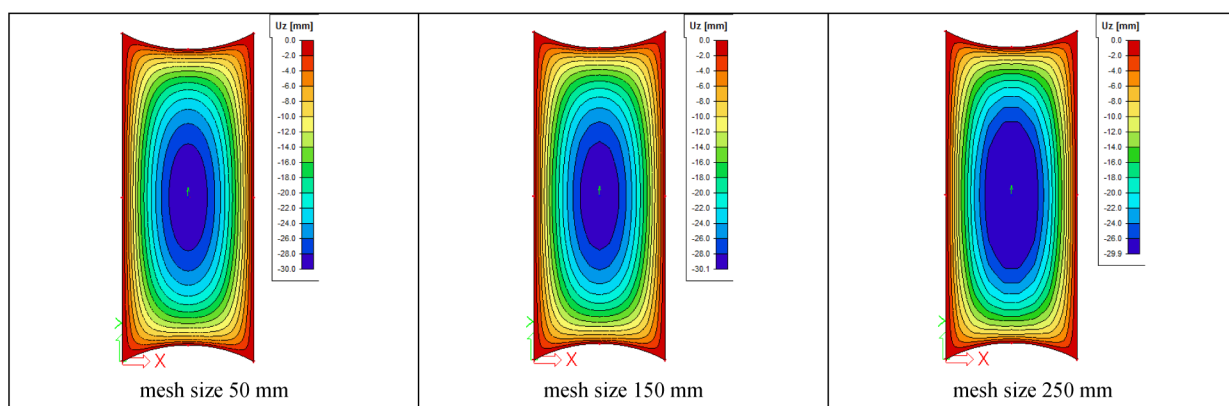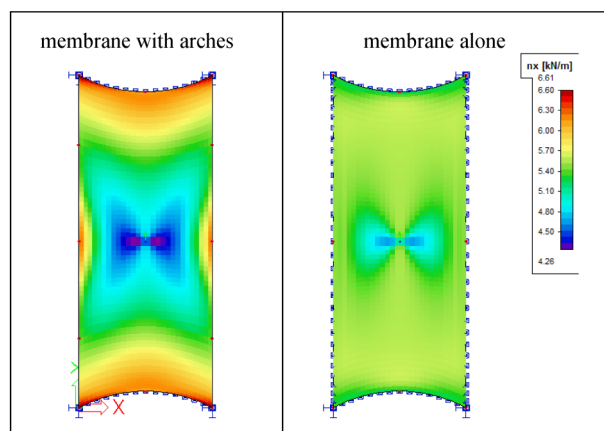Fig. 3. Element divisions of the membrane

Fig. 4. Vertical deflections

shown in Figure 5. In the case of rigid boundary ("membrane alone" analysis) the maximal forces attain just 92% of the maximal forces in the first case ("membrane with arches" analysis).

More marked difference was found for the vertical loading of 1 kN/m² acting together with the prestressing. The comparison of maximal transverse unit forces and membrane deflections is shown in Figure 6. In the case of the "membrane alone (i.e. membrane with fully rigid perimeter supports) the maximal transverse unit force is 11% higher but the maximal membrane deflection attains just 78% in comparison with the "membrane with arches" (i.e. membrane acting together with non-rigid steel arches).

2.2. The "membrane alone" analyses considering elastic supports

It is obvious, that rigidity of supporting steelwork plays important role in determining the final membrane deflection shape, final stresses and required prestressing. Therefore a simulation of the arch flexibility in the membrane direction was examined. Nevertheless, it is not easy to simulate flexible arch tubes by simplified elastic linear supports because the actual value of rigidity along the

Fig. 5. Transverse membrane forces n_x [N/mm] due to prestressing

arches is changing (e.g. in the arch supports is approaching the infinity). Various elastic support rigidities with uniform distribution only along arch lines were tested. The final one, after using a trial-and-error method, resulted from condition of both the same total horizontal reaction and maximal horizontal deflection due to prestressing as in the case of the "membrane with arches". The resulting fictitious uniform elastic rigidity of
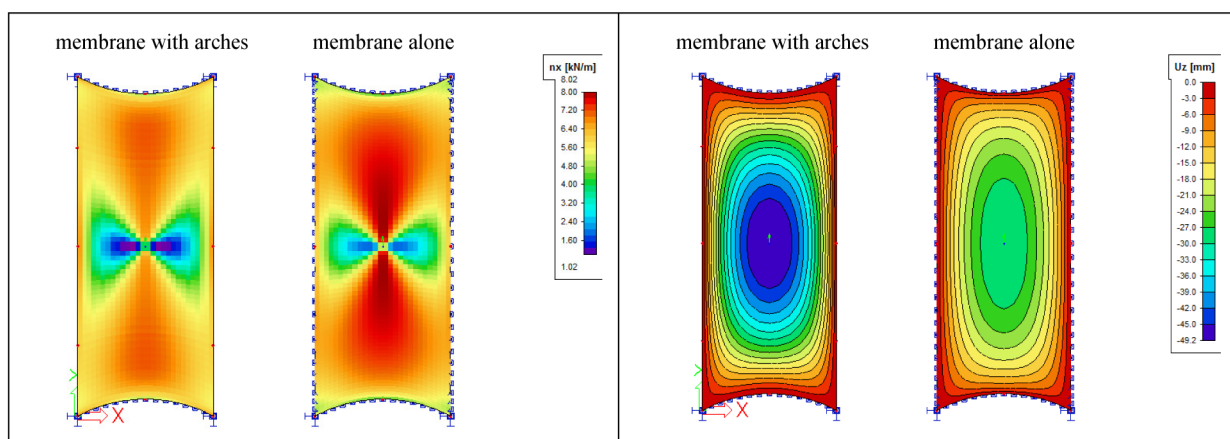
Fig. 6. Transverse membrane forces n_x [N/mm] (left) and vertical deflections u_z [mm] (right)
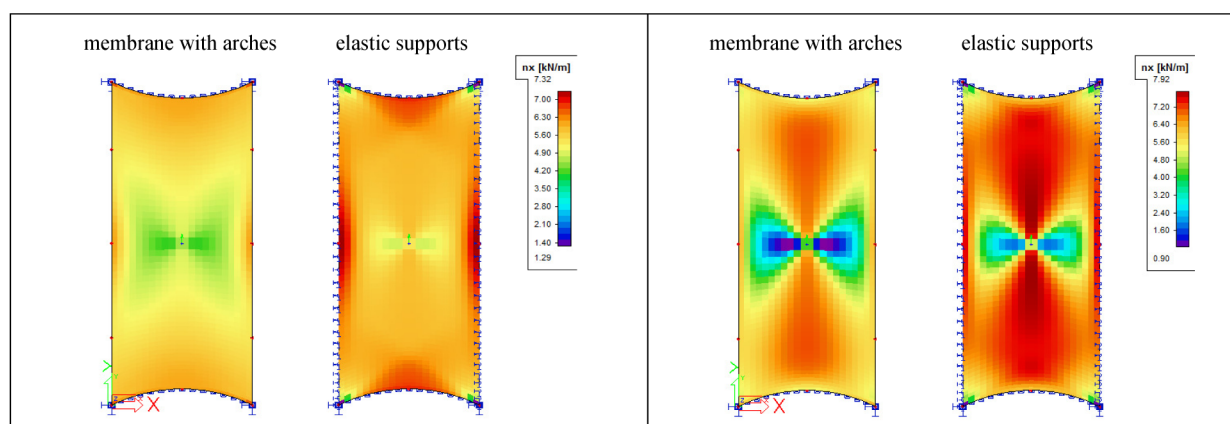

Fig. 7. Transverse membrane forces n_x [N/mm] due to prestressing (left) and due to combined prestressing and vertical loading (right)

1950 N/mm requires prestressing of 148% of the one corresponding to the "membrane alone". Nevertheless, the maximal transverse forces due to both prestressing and combined loading in the membrane with such elastic support are still 11% higher than in "membrane with arches" (Fig. 7).

The same holds for the von Mises equivalent stresses (Fig. 8 left). On the other side the deflections attain only 85% of the "membrane with arches" (Fig. 8 right). Clearly the separate analysis of the membrane on such elastic supports gives rather incorrect results and depends on entry arch and membrane data whether the results are acceptable.

3. Parametrical study

The study concerns the foregoing membrane supported by tube arches according to Figure 1. The FEM elastic nonlinear study (GNA) was performed again with a fictitious membrane (modulus of elasticity E = 1000 MPa, Poisson's ratio v = 0.25, thickness t = 1 mm). The investigation of the full "membrane with arches" and the "membrane alone" is performed for a range of practi-

cal geometrical parameters given in Table 1. In total 27 structures were analyzed which were loaded in the same way as presented above in Chapter 1. It means that planar prestressing by the confined strain ε = 0.4 was used in the "membrane alone" while for the "membrane with arches" the value of ε resulted from condition of identical horizontal reaction along arch lines to ensure the same prestressing as in the former due to horizontal deflection of arches. The only other loading considered was 1 kN/m^2 vertical loading representing snow loading and, therefore, No. 9 in the parametric study corresponds to case studied in the previous chapters. The cross-sections of the tube arches were designed according to Eurocode 3 (EN 1993) for the above loading and give cross-section utilization up to 80%.

A graphical illustration of the studied cases is shown in Figure 9, accentuating the realistic ones.

The boundary conditions are demonstrated again in Figure 10, showing membrane supported along arches either on non-rigid tubes (in case of "membrane with arches") or just simply supported (in case of "membrane alone"). Transverse edges were simply supported in both cases to reduce the entry calculation parameters.
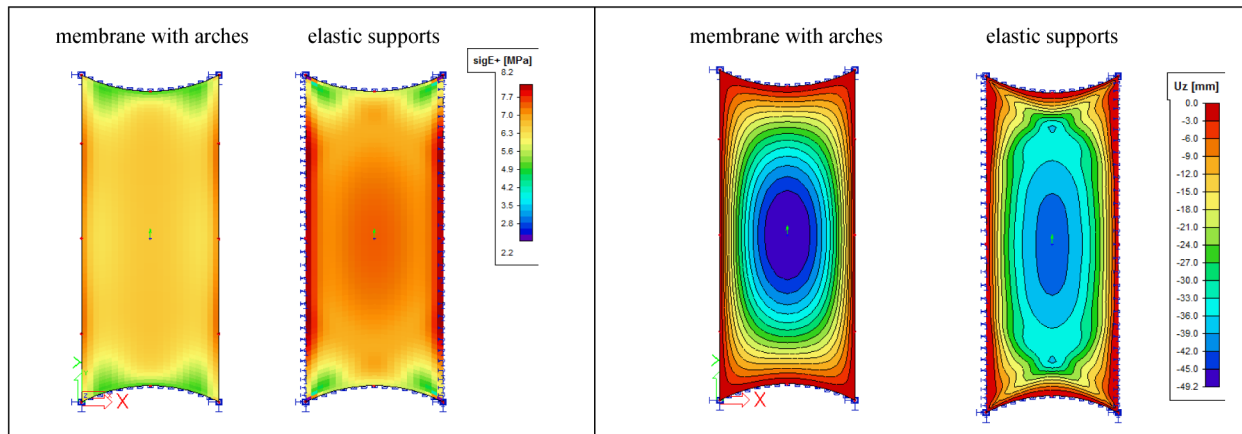
Fig. 8. Equivalent von Mises stresses due to combined prestressing and vertical loading σ [MPa] (left) and vertical deflections under combined loading u_z [mm] (right)

Table 1. Geometrical data of the considered steelwork in accordance with Figure 1

No.	Span L [m]	Width B [m]	Rise H [m]	Tube $D×t$ [mm]	Slenderness $\hat{L}/r$
1	4	2	1	273×16	50.95
2	6	2	1	273×25	73.04
3	6	2	2	273×25	86.77
4	8	2	1	324×20	77.34
5	8	2	2	324×20	86.10
6	8	4	1	324×20	77.34
7	8	4	2	324×20	86.10
8	10	4	2	324×25	104.01
9	10	4	3	324×25	115.46
10	12	4	2	356×22	108.79
11	12	4	3	356×22	117.57
12	12	4	4	356×22	129.23
13	12	6	2	356×22	108.79
14	12	6	3	356×22	117.57
15	12	6	4	356×22	129.23
16	14	4	2	377×20	116.69
17	14	4	3	377×20	123.86
18	14	4	4	377×20	133.48
19	14	6	2	377×20	116.69
20	14	6	3	377×20	123.86
21	14	6	4	377×20	133.48
22	16	6	2	377×25	133.49
23	16	6	3	377×25	139.90
24	16	6	4	377×25	148.61
25	18	6	3	377×28	155.94
26	18	6	4	377×28	163.84
27	18	6	5	377×28	173.68

Using GNA for both prestressing only and combined (prestressing + vertical) loading the values of maximal transverse membrane forces n_x, maximal combined equivalent membrane von Mises stresses σ_e, maximal vertical membrane deflection and maximal transverse arch deflection u_y (in "membrane with arches") were determined. The comparison of results is shown in Table 2 in the form of respective ratios.

The ratios vary from 100 to 271%. For example the distribution of the values for case No. 22 is shown in Figure 11 (transverse membrane forces n_x) and Figure 12 (equivalent membrane stresses σ_e and vertical membrane deflections u_z), in which the ratios concerning prestressing (122%) and deflections (164%) are unacceptable.

From practical design point of view the ratios for arch spans up to 9 m seems to be acceptable, i.e. ≤112% for prestressing and ≤129% for membrane deflections (with the exception of shallow cases No. 4 and 8). Therefore, for such arrangements the interaction of arches with membrane may be neglected (i.e. analysis of the membranes with rigid supports and arches with resulting reactions gives reasonable results). Separated analysis for span above 10 m however, markedly underestimates both vertical membrane and horizontal arch deflections, which negatively influence cutting and prestressing process of membranes. Faulty results give separate analyses for majority of spans above 16 m, where the necessary prestressing for "membrane with arches" in comparison with "membrane alone" should be much higher and in spite of such prestressing an enormous increase of membrane deflection must be expected.

The results may also be related to the arch slenderness (arch length to appropriate tube radius of gyration, see Table 1). For slenderness values 51÷100 the ratios for prestressing ensuring the same total transverse force are in range of 100÷112% and ratios for the membrane vertical deflection between 101÷137%. On the contrary,
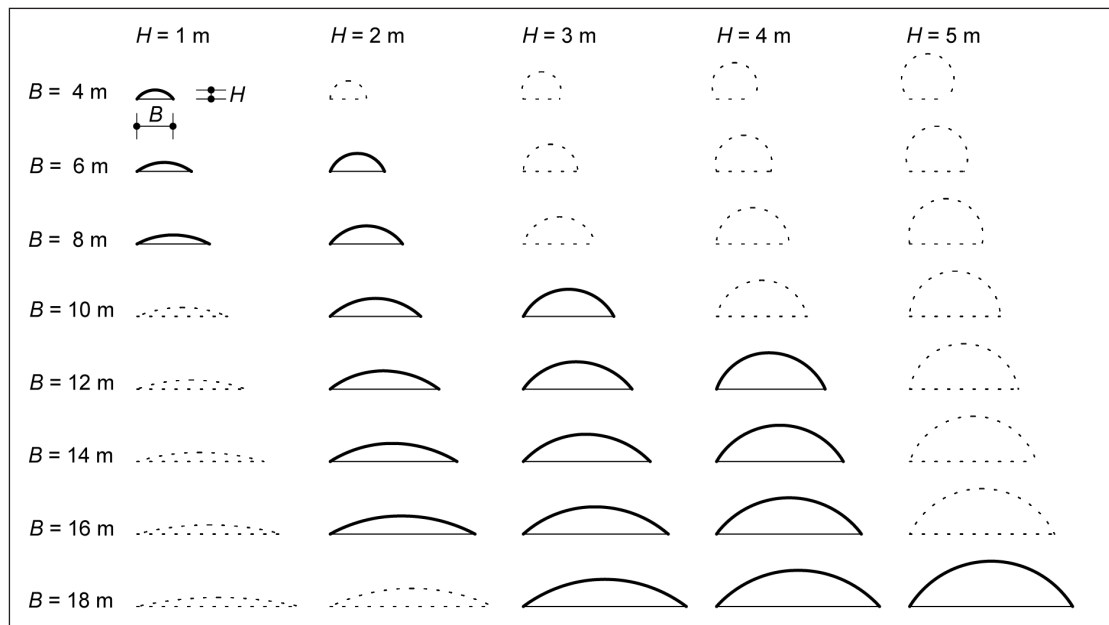
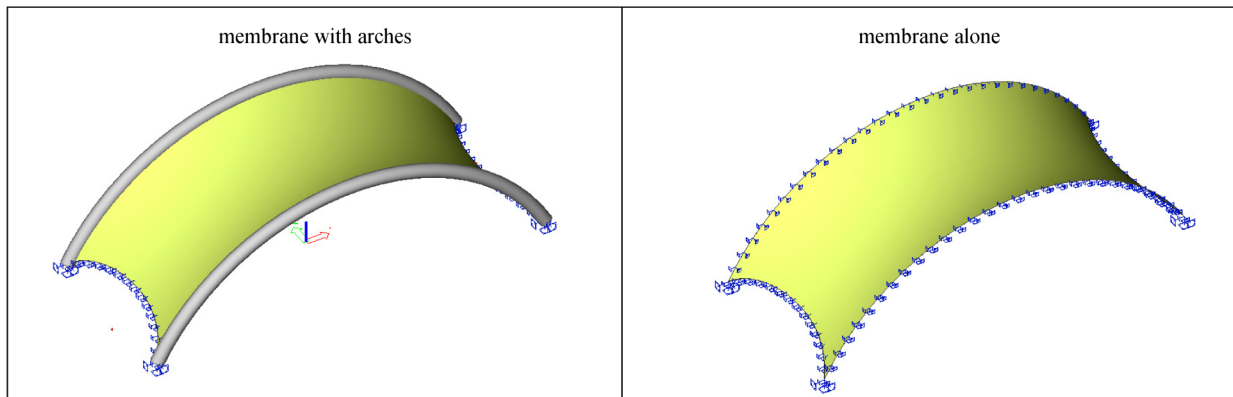Fig. 9. Geometry of arches studied within the parametric study

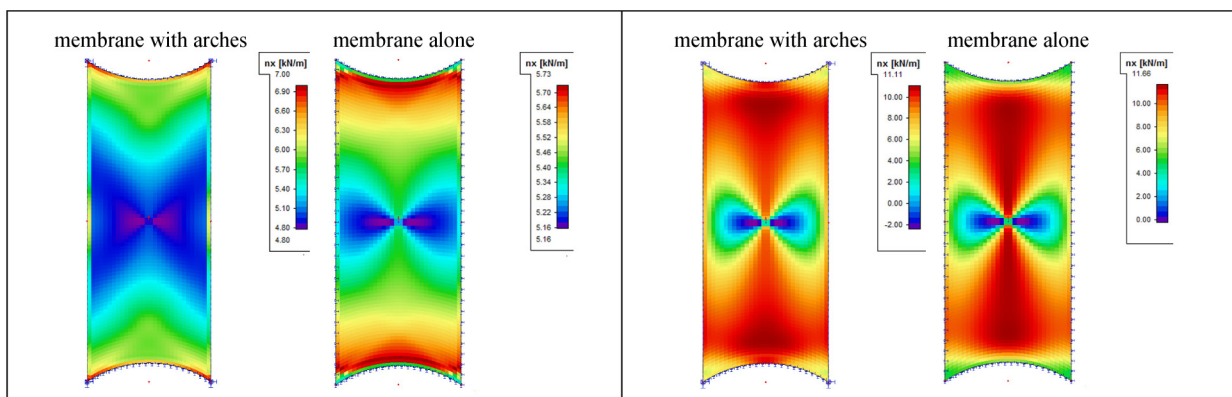Fig. 10. Boundary conditions of arches studied within the parametric study
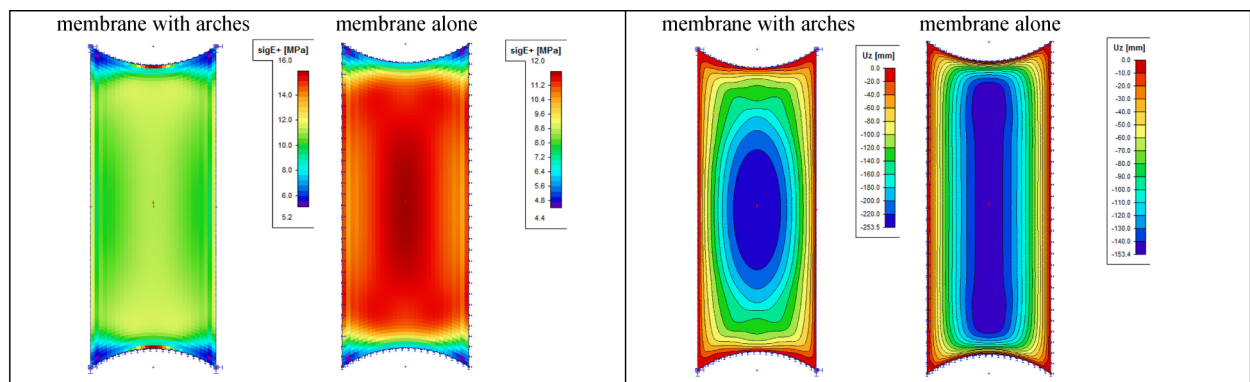

Fig. 11. Transverse membrane forces n_x [N/mm] in the membrane due to prestressing (left) and due to combined prestressing and vertical loading (right)

Table 2. GNA results in the form of ratios "membrane with arches" to "membrane alone"

No.	"membrane with arches" / "membrane alone" ratios in [%]				
	Prestressing: transverse forces $n_{x,max}/n_{x,max}$ [%]	Combined loading: transverse forces $n_{x,max}/n_{x,max}$ [%]	Combined loading: equivalent stresses $\sigma_{e,max}/\sigma_{e,max}$ [%]	Vertical membrane deflections $u_{z,max}/u_{z,max}$ [%]	Transverse arch deflection $u_{y,max}$ [mm]
1	103	102	97	106	0.4
2	100	100	101	103	1.1
3	100	101	90	101	2.2
4	106	101	101	137	2.2
5	112	108	110	112	3.6
6	103	103	104	111	2.6
7	101	101	95	105	3.6
8	103	99	100	141	6.4
9	100	90	92	129	8.7
10	104	98	98	158	9.7
11	109	99	101	179	12.9
12	110	98	93	175	17.6
13	99	96	97	144	10.8
14	101	93	100	153	13.5
15	100	100	89	145	18.8
16	122	86	97	194	15.6
17	114	92	100	196	17.8
18	121	110	109	224	23.4
19	111	97	97	147	17.7
20	102	92	93	166	19.7
21	111	94	102	201	26.0
22	122	96	92	164	24.6
23	112	95	100	180	26.6
24	112	90	98	202	31.8
25	126	94	100	199	36.3
26	128	91	100	231	40.9
27	134	100	104	271	48.8

Fig. 12. Transverse combined equivalent membrane stresses σ_e (left) and vertical membrane deflections u_z (right)

slenderness' 100÷174% give the same ratios in range of 99÷134% for prestressing and 129÷271% for deflections, which are unacceptable.

Conclusions

The correct analysis of textile membranes embedded into a steel structure requires geometrically and materially nonlinear analysis with imperfections (GMNIA), available with specialized software. Nevertheless, the use of simple common "structural frame" software for both the supporting steelwork and a separate geometrically nonlinear analysis of incorporated prestressed membrane is often the choice of designers in rather simple membrane structures.

Using simplified material linear isotropic properties the membranes between arches according to Figure 1 may be designed with common "structural frame" software neglecting joint behaviour (i.e. using design of the membrane alone as an isolated element), provided:

– Geometrically nonlinear analysis (GNA) for both membranes and steelwork is used.
– Prestressing must be used as the basic entry loading.
– The arch span or arch slenderness is limited (approx. to 9 m of the former or to 100 of the latter).
– Introduction of a suitable elastic support provided by the arches may improve the results but is rather a demanding task while a simple uniform distribution of an elastic support along the arches is not sufficient.

Without dispute the above results, the joint modelling (especially using specialized software with proper membrane nonlinear behaviour) of the membrane with actual supporting steelwork is however always preferred. More complicated structures require nonlinear analysis with imperfections (GMNIA) using mentioned relevant input data concerning inherent membrane anisotropic and nonlinear material properties and covering also slacks of cables/rods due to their own weight. Furthermore, methods of assembly are also not covered in this paper, which can substantially affect the resulting geometry and state of stress of the membranes (including the influence of fabric pattern in the unstressed state), necessary rectification and membrane cutting.

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