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Ch. Bucher

1. Introduction

Several recent developments in the analysis of systems with random properties focus on the appropriate description of non-linear effects. Apart from physical non-linearity as introduced by specific material properties (such as yield stress), there is the geometrical non-linearity which is closely related to the question of structural stability. In terms of the practical computation, the issue of stochastic stability breaks down to the determination of the top Lyapunov exponent of the system. For details it is referred to [1]. Confining the analysis to local stability, the system generating the trajectories $x(t)$ can be linearized at the solution $x_0(t)$ whose stability is to be investigated.

The stability of the system will depend on both system properties and loading parameters. Therefore it is to be expected that stability can be influenced by system uncertainty. An appropriate tool for taking into account geometrical non-linearity of complex structures together with the randomness of the describing parameters is the Stochastic Finite Element Method (SFEM). Among various SFE techniques developed so far ([2] - [6]), the methods based on representation of the underlying random fields in the integration points seem to be most suitable [7]. Based on this random field model, a corresponding finite element model can be established. With the aid of eg the Monte Carlo simulation technique, the variability of the response is investigated [8]. The following discussion attempts to cover stability issues under both static and dynamic loading conditions, specifically under wide band random excitation.

2. General Concept

2.1. Linearized Equations of Motion

Most structural systems of engineering significance cannot be described in such relatively simple terms. Discretization must be performed which finally leads to a finite element model. Applying equilibrium conditions in the nodes, a dynamic analysis of geometrically non-linear structures generally requires the solution of the following matrix-vector equation

$$[M]\ddot{x} + [C]\dot{x} + r(x) = f(t). \tag{1}$$

In (1) $[M]$ is the mass matrix, $[C]$ is the damping matrix, x denotes the vector of nodal displacements, $r(x)$ is the vector of restoring forces depending non-linearly on the nodal displacements, and $f(t)$ is the applied load. It cannot be decided a priori whether this load acts additively or multiplicatively. Some investigation is required for that purpose.

Suppose, for simplicity, loading conditions which can be described by a scalar, possibly time-dependent multiplier $\mu(t)$ (load factor) and a constant load configuration vector p

$$f(t) = \mu(t)p. \tag{2}$$

Then the parametric excitation effect can be expressed explicitly by a series expansion in respect of $\mu(t)$ (Taylor series):

$$r(x) = r(x_0) + \left.\frac{\partial r}{\partial \mu}\right|_{\mu=\mu_0}\left(\mu(t) - \mu_0\right) + \frac{1}{2}\left.\frac{\partial^2 r}{\partial \mu^2}\right|_{\mu=\mu_0}\left(\mu(t) - \mu_0\right)^2 + \ldots \tag{3}$$

Here μ_0 is a (fairly arbitrary) reference value for the load factor and x_0 is implicitly defined to be the solution of

$$r(x_0) = f_0 = \mu_0 p. \tag{4}$$

As mentioned above, for the purpose of stability analysis, the system is linearized at the solution whose stability is to be investigated. In the following it is assumed, that the linearization point is defined by the static solution x_0 as implicitly given by (4). Strictly speaking, this assumption is useful for static stability only. In the presence of both external and parametric random excitations the reference solution x_0 whose stability is to be investigated is randomly varying in time. If it can be assumed that the relative effect of the random fluctuations of the external and parametric excitation on the system non-linearity is small as compared to the effect of the statically acting part of the excitation, then it is reasonable to replace the non-linear system by a system linearized at a constant solution. This is the case, eg for wind loads acting on large scale structures where the mean wind load is significantly higher than the random fluctuations caused by atmospheric turbulence. In addition, (3) is frequently linearized in respect of $\mu(t)$ following a similar argument [9].

The so-called tangential equation of motion can then be written as

$$[M]\ddot{y} + [C]\dot{y} + [K]y = = f(t) - f_0 = \left(\mu(t) - \mu_0\right)p \qquad (5)$$

in which y denotes the deviation from the reference solution, i e $y = x - x_0$. The elements of the tangential stiffness matrix become

$$K_{ij} = \left.\frac{\partial r_i(x)}{\partial x_j}\right|_{x = x_0} . \qquad (6)$$

In the above equation (6), the partial derivatives are evaluated at the reference solution $x = x_0$.

For the description of parametric excitation effects, it is convenient to introduce the concept of a so-called geometric stiffness matrix.

This arises from an additional linearization step (3):

$$[K]y = [K_0]y + \left[\frac{\partial K}{\partial \mu}\right]_{\mu = \mu_0} \left(\mu(t) - \mu_0\right)y \qquad (7)$$

with the "linear" stiffness matrix $[K_0]$ and the geometric stiffness matrix $[K_G]$ in the form of

$$[K_0] = [K]\big|_{\mu = \mu_0};\ [K_G] = \left[\frac{\partial K}{\partial \mu}\right]_{\mu = \mu_0} . \qquad (8)$$

This finally leads to an equation of motion which is linear in the state variables and explicitly separates the effects of additive and multiplicative excitations.

$$[M]\ddot{y} + [C]\dot{y} + [K_0]y + [K_G]y\left(\mu(t) - \mu_0\right) = \left(\mu(t) - \mu_0\right)p. \qquad (9)$$

Obviously, a static instability is possible whenever the tangential stiffness matrix becomes singular. This implicitly defines a critical load factor μ_c in terms of (10).

$$\det[K] = \det\left[K\left(\mu_c p\right)\right] = 0. \qquad (10)$$

It is to be noted that together with the linearized equation of motion (9) this corresponds to the widely used approximation of the critical load factor μ_c as solution of the eigenvalue problem

$$\det\left([K_0] - \mu_c [K_G]\right) = 0. \qquad (11)$$

2.2. Dynamic Stochastic Stability

Based on the linearized equation of motion (11), any available method for the stochastic stability analysis of linear systems can be applied. In the present paper, a simplified analysis for the sample stability (almost sure stability) criterion is used. The method is based on stochastic averaging. The details of the approach to be followed have been presented extensively by [10]. This approach requires the projection of the motion onto one mode of vibration which is most likely to become unstable under the influence of random parametric excitation. Here it is assumed that $\mu(t)$ is a wide band stationary random process. Possible stabilisation from the remaining, more stable modes can be accounted for. The resulting sample stability boundaries obtained from this method are asymptotically exact as the system reaches the deterministic stability limit. Therefore, this is an approach well suited for the assessment of the parametric excitation effect near such a deterministic stability limit. In this case, the motion is predominantly governed by one critical mode of vibration ϕ_c as described by

$$\left([K] - \omega_c^2 [M]\right)\phi_c = 0. \qquad (12)$$

This mode shape is assumed to be mass-normalised, so that

$$\phi_c^{\mathrm{T}} [K]\phi_c = \omega_c^2; \quad \phi_c^{\mathrm{T}} [M]\phi_c = 1. \qquad (13)$$

Of particular importance is the parametric excitation effect which is expressed in terms of

$$d = \phi_c^{\mathrm{T}} [K_G] \phi_c. \qquad (14)$$

With the assumption of a modal damping ratio ζ associated with the critical mode ϕ_c and - for the sake of simplicity -neglecting the effect of modal coupling through the parametric excitation, the top Lyapunov exponent λ of the system governing sample stability can be expressed as in [11]

$$\lambda = -2\xi\omega_c + \frac{d^2}{4} S_{\xi\xi}(2\omega_c) \qquad (15)$$

In (15) $S_{\xi\xi}$ $(2\omega_c)$is the power spectral density of the zero mean parametric excitation $\xi(t) = \mu(t) - E[\mu]$ evaluated at twice the frequency of the critical mode. A value of $\lambda < 0$ indicates a system which is asymptotically stable with probability 1. This condition is less stringent than eg moment stability, but it is expected to bear more physical relevance.

3. Applications

3.1. Buckling of Physically Imperfect Beam Structure

A steel structure consisting of beams with tubular cross-sections under static and dynamic load is considered (see Fig 1).

The load *F(t)* acts simultaneously as external and as parametric excitation. The FE-model has 78 degrees of freedom. First of all, a deterministic incremental static analysis to determine the critical load factor is performed. All the loads as indicated in Fig 1 are varied proportionally.

This is achieved by incrementing the load factor μ_0 in small steps, solving for the corresponding displacements, re-calculating the tangent stiffness matrix [*K*] and checking its positive definiteness. As soon as at least one eigenvalue of [*K*] reaches zero, the system has reached a critical load ($\mu_0 = \mu_c$). The smallest eigenvalue of the tangent stiffness matrix is plotted versus the load factor μ_0 in Fig 2. The eigenvector corresponding to the zero eigenvalue at $\mu_0 = \mu_c$ is called buckling shape. For the particular structure under consideration, the buckling shape in reference to the undeformed configuration is shown in Fig 3.

It should be noted that for this structure there is no way of isolating the parametric (destabilizing) contribution of the load *F*(*t*) a priori. Indeed, only a procedure as outlined above can provide the desired answer.

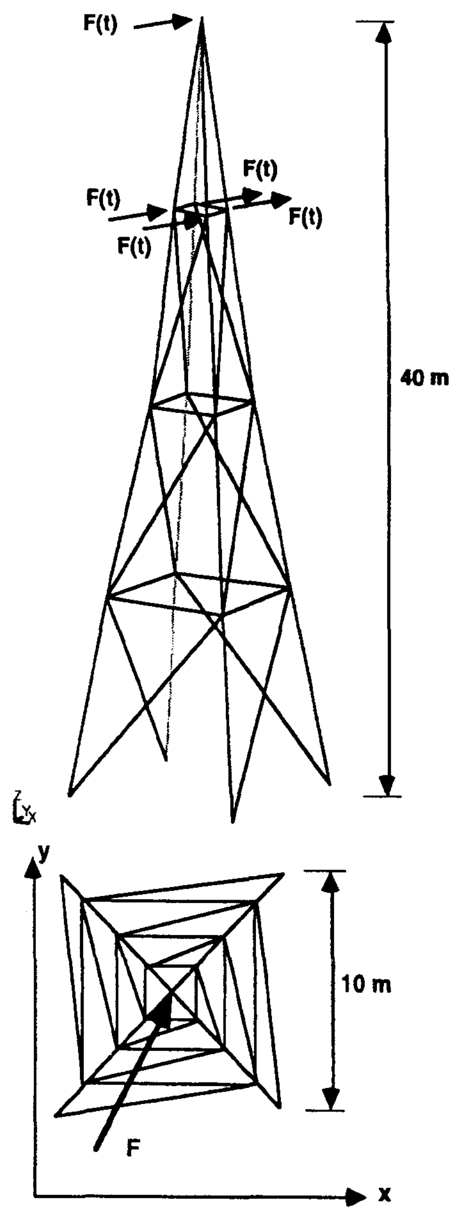

Fig 1. Steel structure with load

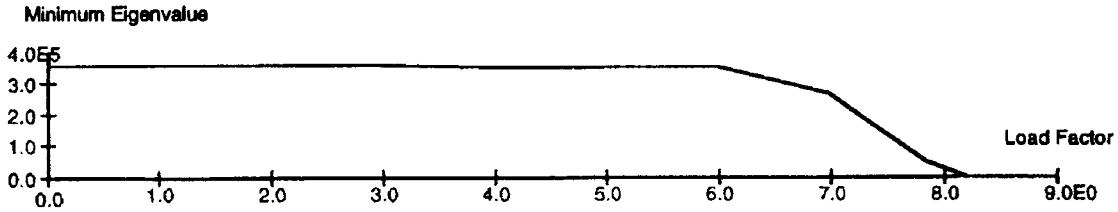

Fig 2. Minimum eigenvalue vs. load factor

In the second step, the elastic modulus $E(s)$ of each vertical bar is assumed to be a homogeneous random field with log-normal probability distribution and an exponential type autocorrelation function

$$C_{EE}(s,r) = \sigma_E^2 \exp\left(-\frac{\|s - r\|}{L_{corr}}\right) \qquad (16)$$

In (16) the symbols s, r denote spatial co-ordinates within the structure, $\sigma_E{}^2$ is the variance of the elastic modulus, and L_{corr} is the correlation length of the random field. The horizontal and diagonal bars are assumed to have deterministic elastic properties. For the calculations, it is assumed that the mean value and the standard deviation of the elastic modulus are $\overline{E} = 2.1 \cdot 10^{11}\, N/m^2$, $\sigma_E = 0.1\overline{E}$, and that its correlation length is $L_{corr} = 20$ m.

The effect of structural randomness on the load carrying capacity (in terms of the critical load factor μ_c) is investigated. This is done by an iterative-incremental method based on a modified Newton-Raphson-iteration. The loads are incremented by changing the load factors μ_0 and at each step calculating the eigenvalues of the tangent stiffness matrix $[K]$. Here the geometric stiffness matrix is not explicitly required.

A Monte-Carlo simulation based on the above assumptions is performed. For each sample, a realisation of the random field $E(r)$ is generated in all integration points and the system matrices are built. The resulting realisations of the critical load factor μ_c are shown as a histogram from 1000 samples in Fig 4. This histogram represents a coefficient of variation of 0.07.

3.2. Motion Stability of Suspension Bridge

A simple bridge model with two mechanical degrees of freedom is considered as investigated in [12]. The model parameters are considered to be deterministic. The effect of random turbulence in the oncoming wind is shown in Fig 5.

This figure indicates an increase of the critical wind speed with increasing level of turbulence as compared to an almost laminar flow in the oncoming wind. The overall effect of turbulence at high levels, however, is destabilizing for the bridge vibration. These statements hold for both second moment stability and almost sure stability criteria.

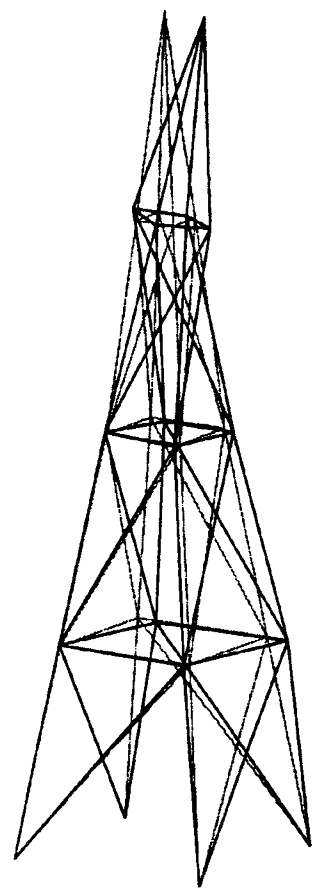

Fig 3. Deterministic buckling shape

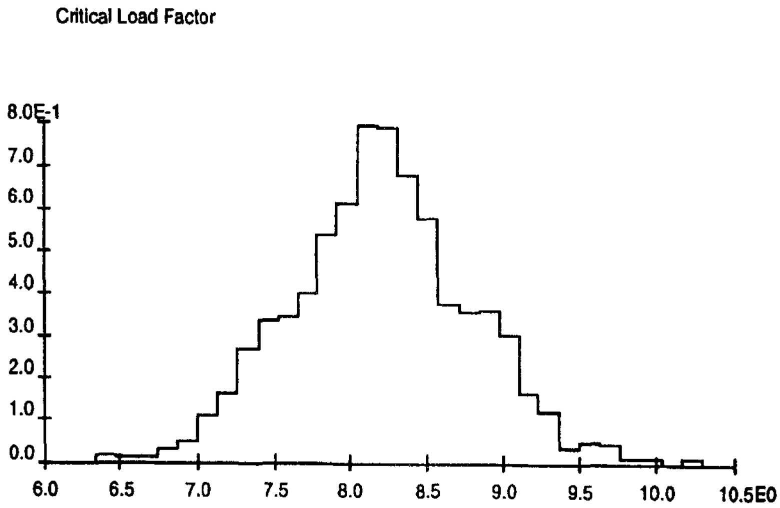

Fig 4. Histogram of critical load factor with random elastic modulus

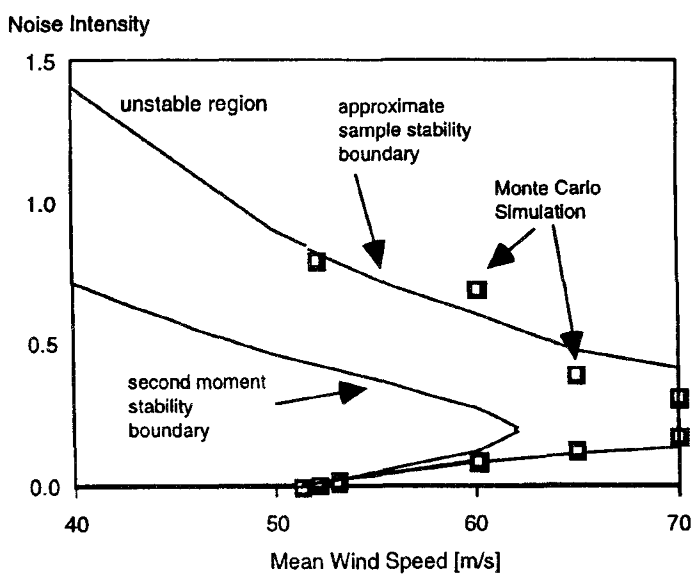

Fig 5. Critical mean wind speed vs. level of turbulence

4. Concluding Remarks

A concept for the stability analysis of geometrically non-linear structures with random properties has been presented. Numerical examples showed how the issues of parametric excitation and system randomness can be covered simultaneously within a Stochastic Finite Element method.

The numerical results indicate that there is a quite considerable effect of material randomness on the load-carrying capacity of geometrically non-linear structures. This statement holds for both static and random dynamic loading.

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STOCHASTINIAI KONSTRUKCIJŲ MECHANIKOS STABILUMO UŽDAVINIAI

Ch. Bucher

S a n t r a u k a

Straipsnyje nagrinėjamas konstrukcijų su atsitiktinėmis savybėmis stabilumas. Tokios konstrukcijos yra veikiamos statinių bei atsitiktinių dinaminių apkrovų. Tiriama priklausomybė tarp konstrukcijos savybių ir parametrinės apkrovos rodiklių. Stochastinių baigtinių elementų idėja taikoma geometriškai netiesinių sistemų analizei.

Stabilumo uždavinys yra nagrinėjamas tiek statinio, tiek dinaminio stabilumo prasme. Išanalizavus apibendrintos diskrečiosios sistemos judėjimo lygtis (1-9), stabilumo uždavinys formuluojamas kaip tikrinių reikšmių uždavinys (10-11) nežinomo apkrovos daugiklio atžvilgiu. Tam tikrais atvejais dinaminio stabilumo uždavinys virsta kritinės virpesių formos stabilumo uždaviniu (12-13).

Stochastinis stabilumo uždavinys yra realizuotas programų paketo pavidalu, o straipsnyje pateikiami išspręsti skaitiniai pavyzdžiai. Pirmajame pavyzdyje nagrinėjamas erdvinės strypinės sistemos statinis stabilumas. Čia strypų tamprumo modulis yra atsitiktinis dydis. Jo atsitiktinės savybės modeliuojamos Monte Karlo metodu. Išnagrinėta kritinės apkrovos rodiklio priklausomybė nuo atsitiktinių veiksnių. Skaičiavimo rezultatai iliustruojami 1-4 paveikslais. Antrame pavyzdyje nagrinėjamas tilto stabilumas veikiant dinaminėms stochastinės prigimties apkrovoms, 5 pav.

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