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STOCHASTIC ANALYSIS OF DEGRADING VIBRATORY SYSTEMS

K. Sobczyk, J. Trębicki

1. Introduction

Temporal variations of the stress field and/or environmental conditions can cause irreversible changes in the characteristics of structural/mechanical systems that may significantly affect their performance. These changes, referred to as degrading (or deterioration) phenomena, are usually not taken into account in the convential analysis of vibratory systems. Such analysis concentrates on the characterisation of the response under various excitations assuming that the systems properties are fixed. As far as the response of dynamical systems to random excitation is concerned, the methods elaborated allow to characterise a stochastic response process in variety of important situations and in the same time they provide the information for the reliability estimation [1], [2].

However, dynamic excitation of engineering systems, including random varying excitation, causes irreversible changes in the material structure and results in decreasing the system ability to carry the intended loading. Damage caused by vibrations manifests itself primarily in the stiffness degradation of the components and systems. There are many examples of engineering systems with stiffness degradation during the vibration process. We mention here three such illustrations.

a) Systems consisting of n brittle fibers in parallel with the same stiffness but independent identically distributed resistances R_i, $i = 1,2, \dots, n$ (Daniels systems). The system responds dynamically to a prescribed random excitation $S(t)$, $t \geq 0$. There is no damage as long as the system displacement $Y(t)$ does not exceed the smallest value y_0. When $Y(t)$ changes in time some damage occurs (failure of some fibers). So, during the motion the system goes through various degradation states. Reliability of Daniels systems is the probability that a specified function of fibers survives in a period T.

b) Elastic-plastic oscillator, ie a vibrating mechanical system in which, in addition to elastic deformation, a plastic deformation occurs due to excursions to plastic domain. For example, the initial stiffness k reduces to $\alpha(L_1)k$ after the first excursion, where L_1 is the length of excursion and α is a non-negative quantity. Assuming that plastic degradation process $D(t)$ is scalar and equal to the sum of all plastic partial deformations in the interval $(0,t)$ we have a plastically degrading oscillator.

c) Vibrating elastic (mechanical/structural) component with fatigue process taking place in it. It can be modelled as a simple oscillator with mass m, damping c and time dependent stiffness $q(A(t))$ where $A(t)$ is the length of a fatigue crack, and $q(x)$ characterises the dependence of stiffness on the crack length. In what follows this example will be discussed in more details.

In the last years an increasing amount of research efforts has been directed to stochastic modelling of various deterioration (or degradation) processes in mechanical/structural components. Because of the practical importance of fatigue damage and fracture in various engineering structures, stochastic models of fatigue accumulation have been a subject of special interest ([3] and references therein). It should be underlined, however, that though the fatigue process is inherently associated with vibrations of mechanical/structural systems the research in random vibration theory, and in modelling of fatigue has been conducted without a proper mutual coupling. Stochastic analysis of mechanical/structural dynamic systems has been focused on the characterisation of the response (and its unsafe states, eg instability regions, first-passage probabilities), whereas the analysis of fatigue deterioration has been concentrated on the fatigue crack growth analysis assuming that the characteristics of the response (eg stresses) are given.

It is clear that a more adequate approach should account for the joint (coupled) treatment of both the system dynamics and fatigue accumulation (eg fatigue crack growth). Such an analysis allows to account for the effect of stiffness degradation during the vibration process on the response and, in the same time, gives the actual stress values for estimation of fatigue [4], [5], [6]. In this paper we discuss the basic features of the problem formulation and the approach to a coupled analysis of the response-degradation via a sequential characterisation of the stiffness degradation process.

2. Coupled analysis of the response-degradation process

In general, the coupled response-degradation problem for non-linear vibratory systems with random excitation can be formulated in the following form

$$\ddot{Y}(t) + F[\dot{Y}(t), Y(t), D(t), X(t,\gamma)] = 0, \qquad (1)$$

$$Q[\dot{D}(t), D(t), Y(t), \dot{Y}(t)] = 0, \qquad (2)$$

$$Y(t_0) = Y_0, \quad \dot{Y}(t_0) = Y_{1,0}, \quad D(t_0) = D_0, \qquad (3)$$

where $Y(t)$ is an unknown response process, $D(t)$ is a degradation process, $F[.]$ is the given function of indicated variables satisfying the appropriate conditions for the existence and uniqueness of the solution, $X(t,\gamma)$ is the given stochastic process characterising the excitation; $\gamma \in \Gamma$, and Γ is the space of elementary events in the basic scheme (Γ, B, P) of probability theory, $Q[.]$ symbolises the relationship between degradation and response process (its specific mathematical form depends on the particular situation) and Y_0, $Y_{1,0}$, D_0 are given initial values of the response and degradation, respectively.

An important special class of problem (1)-(3) is obtained if relationship (2) takes the form of differential equation, that is

$$\dot{D}(t) = G[D(t), Y(t), \dot{Y}(t)], \qquad (4)$$

where G is the appropriate function specifying the evolution of degradation; its mathematical form is inferred from empirical data, or it is derived from the analysis of the physics of the process. In equation (4) the degradation rate $\dot{D}(t)$ may depend on the actual values of $Y(t), \dot{Y}(t)$, but it can also depend on some functionals of $Y(t), \dot{Y}(t)$; for example, on the integral of $Y(\tau)$, $\tau \in [t_0, t]$. In fatigue degradation problem with $D(t)$ interpreted as a "normalised" crack size, the most common evolution equation is the Paris equation which includes not $Y(t)$ itself, but the stress range $\Delta S = S_{max} - S_{min}$ which is related to $Y_{max} - Y_{min}$.

Another special class of problems characterised generally by equations (1)-(3) is identified if functional relationship (2) does not include $\dot{D}(t)$, and $D(t)$ depends on some statistical characteristics of the response process $Y(t)$; a good example is a vibrating systems in which a degradation process depends on the time which the response $Y(t)$ spends above some critical level y^*. This might be the case of an elastic-plastic oscillatory system with $D(t)$ interpreted as accumulated plastic deformation governed by the plastic excursions of the response $Y(t)$ into plastic domain. Formally, the situations which we have in mind can be characterised by the equations

$$\ddot{Y}(t) + F[\dot{Y}(t), Y(t), D(t), X(t,\gamma)] = 0, \qquad D(t) = D_0 + \sum_{i=1}^{N(t)} \eta_i(\gamma), \qquad (5)$$

where $\eta_i(\gamma) = \Delta D_i(\gamma)$ are random variables characterising the "elementary" degradations associated with the specific degradation process; the magnitude of $\eta_i(\gamma)$ depend on characteristics of the process $Y(t)$ above a fixed level y^*. Process $N(t)$ is a stochastic counting process characterising a number of degrading events in the interval $(t_0, t]$. Other possible situations governed by equations (1)–(3) are discussed in [6].

3. Random vibration with stiffness degradation

Let us consider now the response-degradation problem when a stiffness degradation of elastic component of vibrating system is due to the fatigue accumulation (cf. Fig 1). In such a situation, in order to formulate the equation for a degradation $D(t)$ we make use of the Paris-Erdogan equation for fatigue crack size A

$$\frac{dA}{dN} = C(\Delta K)^m, \quad \Delta K > 0, \qquad (6)$$

where ΔK is the stress intensity factor range [3], C and m are empirical constants.

As it is known, the stress intensity factor K can be interpreted as a quantity which characterises the stress distribution around the crack tip. In general, it can be represented in the form

$$K = B(A) S \sqrt{\pi A}, \qquad (7)$$

where A is the crack length, S describes the far-field stress resulting from the response process $Y(t)$ and $B(A)$ accounts for the geometry of the crack and the specimen. To make the further analysis easier it is convenient to deal not with A directly but with the non-linear transformation $\psi(A)$ of A defined as

$$\psi(A) = \int_{A_0}^{A} \frac{\mathrm{d}x}{B^m(x)(\sqrt{\pi x})^m}, \tag{8}$$

where A_0 is the initial crack size. Let us denote by ψ^* the value of $\psi(A)$ for the critical crack length $A = A^*$, and define the degradation measure D as

$$D = \frac{\psi(A)}{\psi^*}, \quad \psi^* = \psi(A^*), \quad D \in [D_0, 1]. \tag{9}$$

Of course,

$$\begin{aligned} dD &= \frac{1}{\psi^*} d\psi(A) = \frac{1}{\psi^*} \frac{dA}{B^m(A)(\sqrt{\pi A})^m} = \\ &= \frac{1}{\psi^*} C(\Delta S)^m dN. \end{aligned} \tag{10}$$

Therefore, the evolution equation for the fatigue crack induced degradation $D(t)$ defined by (9) takes the form

$$\frac{\mathrm{d}D}{\mathrm{d}N} = \frac{1}{\psi^*} C(\Delta S_Y)^m, \tag{11}$$

where ΔS_Y is the stress range generated by the response process $Y(\tau)$.

Using the degradation measure D defined in (9) the equation (11) indicates that the increment of D in one equivalent cycle can be taken as

$$\Delta D_i = \frac{1}{\psi^*} C(\Delta S_i)^m, \tag{12}$$

where ΔS_i is the stress range in the i-th cycle.

In order to account for the cumulative nature of the degradation process and its randomness, let us represent $D(\tau)$ in the form of a sequence of random variables $D_N(\gamma) =: D(t_N)$, $N = 0,1,\ldots,N^*$, where $D_N(\gamma)$ characterises the state of the degradation random process after N cycles. Therefore

$$D_N(\gamma) = \sum_{i=1}^{N} \Delta D_i(\gamma), \tag{13}$$

$$\Delta D_i(\gamma) = D_i(\gamma) - D_{i-1}(\gamma). \tag{14}$$

The coupled computational response-degradation model has the form

$$\ddot{Y}(\tau) + 2\zeta\dot{Y}(\tau) + q(D_{N-1}(\gamma))Y(\tau) = \xi_1(\tau,\gamma), \tag{15}$$

$$D_N(\gamma) = D_{N-1}(\gamma) + \Delta D_N(\gamma), \tag{16}$$

where $\Delta D_N(\gamma)$ denotes the increment of the degradation process during N-th cycle. It is defined by formula (3.14) in which ΔS_N is the stress range in N-th cycle. Assuming that the degradation starts when response $Y(t)$ is in its stationary state and that the response is a narrow-band process $(2\zeta << 1)$ we approximate $\Delta Y_i = Y_{max,i} - Y_{min,i}$ by two times the amplitude H_i of the $Y(t)$, ie $2H_i$. Therefore, the stress range ΔS_i in the i-th cycle is

$$\Delta S_i = 2H_i \frac{E}{l_0}, \tag{17}$$

where l_0 is the length of the elastic element (cf. Fig 1) and E is its Young modulus.

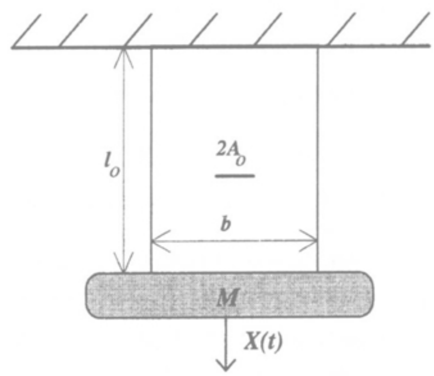

Fig 1. Diagram of a vibratory system with stiffness degradation due to fatigue crack propagation

Finally, the increment ΔD_N of the degradation process occurring in (13) has the form

$$\Delta D_N(\gamma) = C_1 H_N^m(\gamma), \tag{18}$$

where constant $C_1 = C\, 2^m E^m \sigma_y^m / l_o^m \psi^*$ is obtained during the transformation from dimensional to non-dimensional system.

Equations (15), (16) along with (12) and (17), (18) constitute a complete sequential model for characterisation of the response-degradation process $[Y(t), D(t)]$ in discretised time instants (cycles) $N = 0,1,\ldots,N^*$. Because the degradation process is slow in comparison to the response itself and the degradation process D starts when the system (15) reached its stationary state for initial stiffness $q(D_{N=0})$ generated by deterministic or random value of the initial damage measure $D_{N=0} = D_0$ we take the distribution of the amplitude H_N given D_{N-1} as being the Rayleigh distribution. In this model the response after N cycles is affected by the stiffness degradation state after $N-1$ cycles, whereas the degradation process after N cycles depends on the response amplitude H_N at cycle N, given D_{N-1}.

4. Numerical results

The probabilistic characteristics of the response-degradation process $[Y_N, D_N]$ (where $Y_N = H_N$ and H_N is the amplitude of the process Y at cycle N) can be obtained via the conditioning method presented in detail in [6]. The effectiveness of the method has been checked for the specific crack and specimen geometry, and for $\zeta = 0.01$, $C_1 = 4.7015799 \cdot 10^{-7}$, m=3. The stiffness degradation is the monotonically decreasing function [4] of the crack length A. Figures 2 and 3 illustrate the evolution of the mean values and standard deviations of the degradation measure D for non-degraded and degraded system.

In Fig 2 we see that in the case of degraded system the mean value of the degradation measure D has non-linear characteristic in comparison to its linear characteristic in the case of non-degraded system. The difference in the number of cycles required to reach the assumed critical level $D^* = 0.98$ is about 25%. Fig 3 visualises the comparison between standard deviations of the degradation measure for considered systems. A significant growth of standard deviation of degradation measure in degraded system is observed. It is due to "non-linear" behaviour of the degradation measure in the case of the degraded system.

Fig 4 shows the probability density functions of the degradation measure for different number of the response cycles N. This figure indicates also that stiffness degradation should play an important role in reliability analysis of the system. For example, for fixed level D^*=0.8 and N=140 thousands of cycles we have probability of failure $P_F = 1 - P(D < D^*) \approx 0.05$ for non-degraded system (see dashed probability densities) and $P_F = 1 - P(D < D^*) \approx 0.45$ for degraded system (see continuous curves). The non-degraded system is understood here as the system whose stiffness degradation is not taken into account.

Fig 5 shows the probability density function of the response amplitude process both in the case of the system with non-degraded stiffness (dashed line) and system with degraded stiffness. The distortion of the density due to stiffness degradation is clearly visible for larger values of the number of cycles.

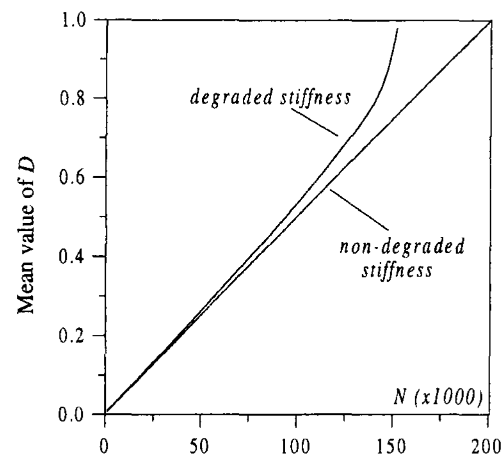

Fig 2. Mean value of the degradation measure D versus number of cycles for the system with degraded and non-degraded stiffness, respectively

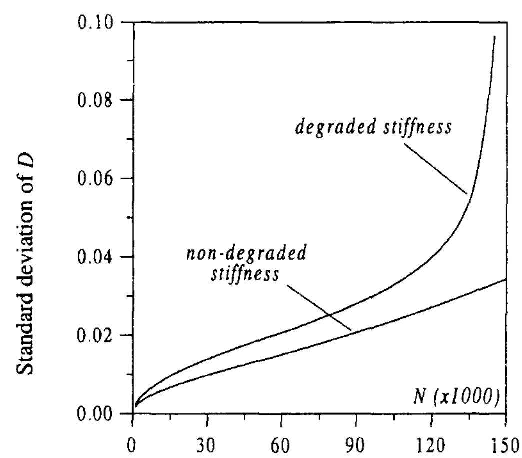

Fig 3. Standard deviation of the degradation measure D versus number of cycles

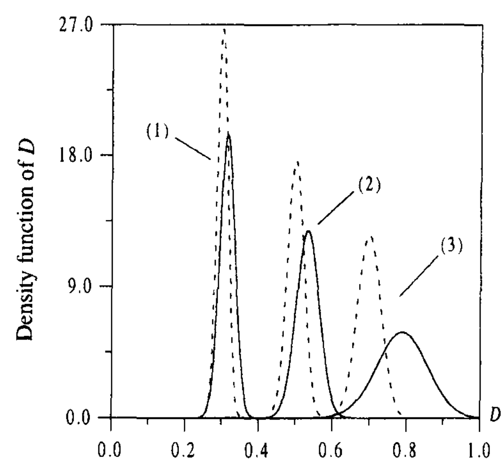

Fig 4. Probability densities of the degradation measure D for the system with non-degraded stiffness (dashed line) and degraded stiffness (continuous line) at different number of cycles: (1) – N = 60, (2) – N = 100, (3)- N = 140 (x1000)

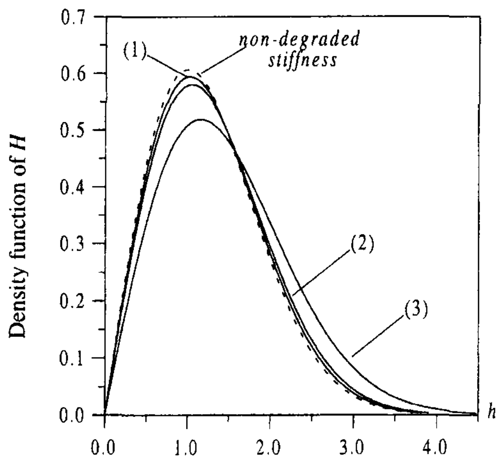

Fig 5. Probability density of the response amplitude for different number of cycles: (1)- $N = 60$, (2)- $N = 100$, (3)- $N = 140$ (x1000)

5. Conclusions

In the paper we showed how the response-degradation problems for randomly vibrating systems can be formulated and analysed effectively.

Such a couple formulation makes it possible to account for the effect of stiffness degradation (during the vibration process) on the response and simultaneously it yields the actual stress values for the characterisation of the evaluation of degradation. The numerical calculations provide quantitative and graphical information on the response and degradation process.

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PAŽEISTŲ VIBRACINIŲ SISTEMŲ STOCHASTINĖ ANALIZĖ

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S a n t r a u k a

Laikini įtempių bei aplinkos sąlygų kitimai gali lemti negrįžtamus konstrukcinių/mechaninių sistemų savybių pokyčius. Šie pokyčiai, vadinami pažeidimais, paprastai nėra įvertinami vibracinių sistemų skaičiavimais. Atliekant tokius skaičiavimus laikoma, kad sistemų, paveiktų įvairių poveikių, savybės yra nekintamos.

Šiame straipsnyje šalia tradicinio sistemų dinaminių efektų modeliavimo yra vertinami pažeidimai (pavyzdžiui, mikroplyšio didėjimas). Tokia analizė leidžia įvertinti ne tik sistemos standumo mažėjimą vibracinio proceso metu, bet ir leidžia nustatyti įtempius. Skaitiniai pavyzdžiai pateikia kiekybinę ir grafinę informaciją apie vibracinių sistemų būklę bei pažeidimų procesus.

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