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DETERMINATION OF OPTIMUM QUANTITY OF BITUMEN IN ASPHALT CONCRETE MIXTURES

Kęstutis Vislavičius

Dept of Strength Materials, Vilnius Gediminas Technical University, Saulėtekio al. 11, LT-2040 Vilnius, Lithuania. E-mail: vislavicius@fm.vtu.lt

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Abstract. The problem of calculating the quantity of a required bitumen is formulated as follows: from certain mineral materials the grading curves and narrow fraction bitumen receptivities which are known to determine the composition of an asphalt concrete mixture that satisfies grading limitations of mineral materials and contains minimum quantity of a required bitumen. Mathematical analogues of the problem are presented.

A personal computer programme for calculating the minimum quantity of a required bitumen is prepared, and some example problems are solved. An analysis of the results is presented.

Keywords: heterogeneous material, optimization, linear programming, bitumen receptivity, mathematical modelling.

1. Introduction

Asphalt concrete is a heterogeneous material. It consists of endless number of mineral particles, bitumen and pores. In their turn, mineral particles are located in the volume of material anyhow and are characterized by their size, shape and chemical nature. The main aim of an asphalt concrete designer is to get such a ratio of mineral particles, bitumen and pores that corresponds to the best physical and mechanical properties of asphalt concrete. It is a difficult and not solved yet problem. The most popular "ideal" grading curves method in use [1–4] is only a compromise between the scientific view on asphalt concrete as a heterogeneous material model and engineering goals. According to this method, a mineral part of asphalt concrete mixture is sufficiently homogeneous if the decrease of diameter of mineral particles of a narrow fraction by ξ times, corresponds to the decrease of a quantity of the same fraction by ζ times. Coefficients ξ and ζ are found experimentally. Taking into account the fact that real shape of mineral particles is different, not one but two limit grading curves are drawn. Thus, when the "ideal" grading curves method is used, the properties of mineral part of asphalt concrete mixture depends both on the number of sieves (fractions) and the distance between points of grading curve of asphalt concrete designed and middle points of limit curves. The bitumen quantity is established separately taking into account the porosity of framework of mineral party or using bitumen receptivities.

It is evident, the analysis of an asphalt concrete using heterogeneous material model gives an opportunity to forecast physical and mechanical properties of an asphalt concrete mixture, or an asphalt concrete more exactly. It is known [5] that for description of materials of similar composition differential equations are used. However, the discrete element method [6-10] is the best way for complex investigation of asphalt concrete composition. One of the main advantages of this method in comparison with that of the "ideal" grading curves is the possibility to evaluate the reciprocal forces between mineral particles and bitumen, peculiarities of mixing, consolidating the mixture as well as to investigate segregation, ie to "observe" the process of asphalt concrete mixing or asphalt concrete formation.

As the discrete element method demands large computer resources and its putting into practice is limited, the "ideal" grading curves method is used at present. In the article the problem of optimal quantity of bitumen in an asphalt concrete mixture is investigated by this method.

2. Peculiarities of bitumen quantity optimization

The bitumen quantity in an asphalt concrete mixture depends on the bitumen grade, the composition of mineral materials as well as on the size, shape and chemical nature of mineral particles. Thus the optimization of the bitumen quantity is a complex problem. First, many different parameters must be taken into consideration; secondly, the defined quantity of the bitumen must both be minimum (it is especially important as bitumen is the most expensive component) and guarantee the best properties of an asphalt concrete. There are three ways to get an asphalt concrete mixture with minimum quantity of the bitumen: a) to improve the technology of manufacture; b) to choose materials with minimum bitumen receptivity [11]; c) to determine the optimal composition of mineral part of asphalt mixture in respect of bitumen. In the article the third way is considered only.

For minimization of bitumen quantity mathematical programming is proposed. This method is successfully used in designing asphalt concrete mixtures both for minimal price and for mineral part with homogeneous composition [12, 13].

In practice, the establishment of optimal quantity of bitumen consists of two stages. Firstly, optimal quantity of bitumen is obtained by using different numerical methods; secondly, the obtained quantity is defined more exactly by experiment. In the article, the method only for calculating optimal quantity of bitumen is proposed.

3. Formulation of problem

The problem of calculating the quantity of a required bitumen is formulated as follows: from certain mineral materials the grading curves and narrow fraction bitumen receptivities which are known to determine the composition of an asphalt concrete mixture that satisfies grading limitations of mineral materials and contains minimum quantity of a required bitumen.

It is necessary to note that the bitumen quantity in an asphalt concrete mixture may be optimal in two aspects: a) because it corresponds to the best parameters of physical and mechanical properties of an asphalt concrete; b) because the quantity of it (optimal according to the first aspect) is minimal. To avoid the ambiguity, the bitumen quantity that guarantees high parameters of an asphalt concrete is considered required. It is evident that the optimum of this quantity of required bitumen depends on precision of establishing of bitumen receptivities of mineral fractions (remember, the bitumen receptivity of mineral material fraction is the bitumen percentage which corresponds to the best parameters of an asphalt concrete). In the article, the optimal quantity of bitumen will be reasonable as a minimum quantity of required bitumen. It will be determined by changing the grading of mineral part of a mixture.

4. Mathematical analogue

Suppose we have to design a certain asphalt concrete mixture, which consists of m mineral materials and contains a minimum of required bitumen. Grading of all mineral materials is known, ie we know coefficients α_{jk} indicating the quantity of fraction k in mineral material j ($j = 1, 2,\dots, m$, $k = 1, 2,\dots, r$; where r is number of fractions). Quantities of bitumen receptivities of mineral fractions are known too, ie we know coefficients β_{jk} indicating the quantity of required bitumen of fraction k of mineral material j (in percent). The problem will be solved if quantities of all fractions of each mineral material are known, ie the coefficients x_{jk} will be obtained. These coefficients indicate the quantity of fraction k of mineral material j of asphalt concrete mixture designed in parts of the unit.

Thus all known and unknown values are defined. It is necessary to determine the object function and conditions which have to be satisfied by asphalt concrete mixture designed.

The product $\alpha_{jk} \cdot x_{jk}$ denotes the quantity of fraction k of mineral material j for the designed mixture. Let's multiply this quantity by the corresponding bitumen receptivity. The result is the quantity of the required bitumen of fraction k of mineral material j: $\frac{1}{100}\beta_{jk} \cdot \alpha_{jk} \cdot x_{jk}$. Hence, the expression $\frac{1}{100}\sum_{k=1}^{r}\beta_{jk} \cdot \alpha_{jk} \cdot x_{jk}$ denotes the quantity of required bitumen of mineral material j, like expression $\frac{1}{100}\sum_{j=1}^{m}\sum_{k=1}^{r}\beta_{jk} \cdot \alpha_{jk} \cdot x_{jk}$ denotes the quantity of required bitumen of all mineral materials. However, this quantity must be minimum. Consequently, the object function of the problem acquires such an expression:

$$\frac{1}{100}\sum_{j=1}^{m}\sum_{k=1}^{r}\beta_{jk} \cdot \alpha_{jk} \cdot x_{jk} \rightarrow \min . \tag{1}$$

Suppose the designer has mineral materials with certain fractions (not separate mineral material fractions). This is the most common case in practice. Then the object function (1) can be expressed by values related not to a fraction but to a mineral material:

$$\sum_{j=1}^{m} B_j \cdot x_j \rightarrow \min . \tag{2}$$

Here $B_j = \frac{1}{100}\sum_{k=1}^{r}\beta_{jk} \cdot \alpha_{jk}$ is a coefficient which indicates the quantity of required bitumen of mineral material j in percent, $x_j = \sum_{k=1}^{r} x_{jk}$ is the quantity of mineral material j of an asphalt concrete mixture in parts of the unit.

It's evident, the required bitumen optimisation problem can be solved by using basic mathematical analogue presented in [13]. In our case $c_j = B_j$, ie the weight multiplier of mineral material j equals the quantity of required bitumen.

Thus the mathematical analogue of the problem assumes the following expression:

$$\left.\begin{array}{lr}
\sum_{j=1}^{m} B_j \cdot x_j \rightarrow \min, & (a) \\
b_{\min,i} \le \sum_{j=1}^{m} a_{ij} \cdot x_j \le b_{\max,i},\ i = 1, 2, \dots, n, & (b) \\
\sum_{j=1}^{m} x_j = 1, & (c) \\
x_j \ge 0, & (d)
\end{array}\right\}, \tag{3}$$

where a_{ij} is the quantity of mineral material j in percent that pass through sieve i; $b_{\min,i}$, $b_{\max,i}$ are limited quantities of a mineral part of asphalt concrete mixture in percent that can be passed through sieve i; n is the number of sieves;

All conditions in the mathematical analogue (3) are necessary. If we want to give the problem a perfect mathematical expression, we have to include one additional inequality:

$$\sum_{j=1}^{m} d_{jv} \cdot x_j \le h_v, \quad v = 1, 2, ..., s, \tag{4}$$

where d_{vj} is the coefficient of additional inequality v corresponding to mineral material j; h_v is the limit value of additional inequality v; s is number of additional inequalities. The inequalities (4) mathematically express technological limitations (there may be none of them). For example, these inequalities can be used for limitation of a maximum or minimum quantity of certain mineral material in a mixture.

The final expression of mathematical analogue for determining optimal quantity of required bitumen is as follows:

$$\left.\begin{array}{lr} \sum_{j=1}^{m} B_j \cdot x_j \to \min, & (a) \\ b_{\min,i} \le \sum_{j=1}^{m} a_{ij} \cdot x_j \le b_{\max,i}, \quad i = 1, 2, ..., n, & (b) \\ \sum_{j=1}^{m} d_{vj} \cdot x_j \le h_v, \quad v = 1, 2, ..., s, & (c) \\ \sum_{j=1}^{m} x_j = 1, & (d) \\ x_j \ge 0, & (e) \end{array}\right\}. \tag{5}$$

It should be kept in mind that the quantity of required bitumen can be calculated using a mathematical analogue presented in [13]. Then the cheapest or the most homogeneous asphalt concrete mixture with a certain quantity of required bitumen is obtained. If mathematical analogues (3) or (5) are used, the asphalt concrete mixture having a minimum quantity of required bitumen but with a certain price and composition is obtained.

5. Numerical experiment

Four mineral materials are given: granite crushed stone, granite sifting, natural sand and mineral filler. Mineral material grading curve coefficients are presented in 2, 3, 4, 5 columns of Table 1. Bitumen receptivities are presented in Table 2. It is necessary to design an asphalt concrete mixture 0/16-V [14] for the upper layer of pavement. The limit curve coefficients of asphalt concrete mixture designed are presented in 6 and 7 columns of Table 2.

Primary data are illustrated in the figure. The figure presents grading curves of mineral materials and limit curves of the mineral part of asphalt concrete designed. The dashed region denotes admissible solutions. Thus the problem to be solved may be formulated as follows: from all admissible asphalt concrete mixtures, ie from asphalt concrete mixtures grading curves lying in the dashed region to choose the one that guarantees the minimum quantity of required bitumen.

Table 1. Coefficients of grading curves of the mineral materials and of limit curves of the mineral part of asphalt concrete mixture designed

Sieve size (mm)	Fallings through sieves (mass %)				Limit curves	
	Crushed stone	Sifting	Sand	Min. filler	Min	Max
1	2	3	4	5	6	7
0,09	0,4	8,9	1,6	90,2	6	10
0,25	0,5	22,2	13,2	98,6	10	25
0,71	0,6	39,6	56,6	100,0	16	36
2	0,8	61,0	78,6	100,0	26	45
5	1,0	94,0	87,2	100,0	45	64
8	1,5	99,9	95,2	100,0	57	75
11,2	42,9	100,0	100,0	100,0	70	86
16	100,0	100,0	100,0	100,0	85	100

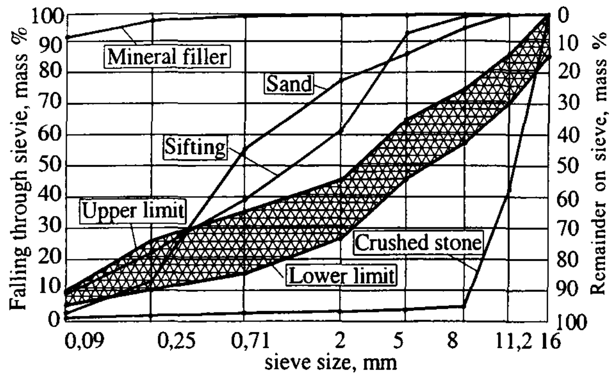
Table 2. Bitumen receptivity of mineral materials

Narrow fraction (mm)	Bitumen receptivity (%)			
	Crushed stone	Sifting	Sand	Min. filler
Less 0,09	15,2	15,2	12,0	20,5
0,9–0,25	7,9	7,9	6,4	15,7
0,25–0,71	6,2	6,2	4,6	15,0
0,71–2	5,8	5,8	4,2	13,9
2–5	5,6	5,6	3,3	*
5–8	5,3	5,3	3,0	*
8–11,2	5,1	5,1	2,6	*
11,2–16	4,7	4,7	2,2	*
More 16	4,5	4,5	2,0	*

* No data

The results are presented in Table 3. In the first row there is the solution of the mathematical analogue (3). Hence, we can affirm that it is not possible to obtain asphalt concrete mixture with the quantity of required bitumen less than 5,7%. It is an optimal solution in respect of bitumen. In the second row of Table 3 there is the solution obtained by mathematical analogue (5). In this case there was included the condition which requires that mineral filler in the asphalt concrete mixture must be equal to or larger than 5,0% [14]. It is evident the quantity of required bitumen becomes larger.

For comparison, some problems using mathematical analogues presented in [13] were solved (the same primary data were used). The minimum price problem was solved accepting the following prices of one ton of mineral materials: granite crushed stone 28,00 Lt, granite sifting 26,00 Lt, natural sand 18,00 Lt, mineral filler 139,00 Lt. The pouring densities of these mineral mate-



Mineral materials grading curves and limit curves of the mineral part of asphalt concrete mixture designed

Table 3. Results

No	Quantity of min. material (%)				Quant. of bitum. (%)	Corr. coeff.	Price (Lt)
	Crushed stone	Sifting	Sand	Min. filler			
1.	41,6	12,5	41,4	4.5	5,7	0,977	19.35
2.	41,6	12.9	40.5	5,0	5,8	0,977	19,79
3.	41,7	12,8	40,5	5,0	5.8	0,977	19,71
4.	41,0	20,7	33,2	5,0	5,9	0,982	19,86
5.	40,4	28,6	25,9	5.0	6,1	0,986	20.01
6.	39,8	36,5	18,5	5,0	6,3	0,989	20,16
7.	39,2	44,4	11,2	5,0	6,5	0,992	20,32

rials are correspondingly equal to: $1{,}32\,t/m^3$, $1{,}76\,t/m^3$, $1{,}49\,t/m^3$ and $1{,}70\,t/m^3$. The results are presented in row 3 of Table 3. Thus the price of one ton of the cheapest asphalt concrete is equal to 19,71 Lt. To obtain the most heterogeneous mixture the mathematical analogue presented in [13] was used too. In this case the limit curves step by step were close one to the other. Four solutions were obtained. All of them are presented in 4–7 rows of Table 3. In the 7th row there is the solution of the most homogeneous mixture. For comparison, the prices and coefficients of correlation are calculated for the first two mixtures (Table 3). Note the price of asphalt concrete mixture with a minimum of the required bitumen (the first row) is less than 19,71 Lt, because it was designed without any technological limitations. The examples presented show a large potential of mathematical modelling of asphalt concrete mixture when mathematical analogues presented in this article and in [13] are used. Then it is possible to obtain the mixture with a minimum quantity of required bitumen, the mixture with a minimum price and the mixture with the most homogeneous composition. In addition, when optimal asphalt concrete mixture is determined according to one or another weight multiplier, it is possible to model solutions by including some technological or other limitations.

6. Conclusions

1. In Lithuania for manufacturing asphalt concrete mixture not a very large number of mineral materials is used. That is why for all of them the bitumen receptivities should be determined. Then using mathematical analogues (3) or (5) it would be possible to calculate a minimum quantity of bitumen in asphalt concrete mixture and at the same time to decrease both a price of the mixture and volume of expensive experiments.

2. Mathematical analogue presented together with mathematical analogues in [13] can be used for preparation of algorithm for asphalt concrete mixture mathematical modelling. Such a software would be a good tool for designing the cheapest, the most homogeneous, the optimal in respect of the required bitumen and other asphalt concrete mixtures.

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