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# STATISTICAL ANALYSIS OF DESIGN CODES CALCULATION METHODS FOR PUNCHING SHEER RESISTANCE IN COLUMN-TO-SLAB CONNECTIONS

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**Abstract**. This paper analyses the compliance of the design codes calculation methods for punching shear resistance in reinforced concrete slabs STR 2.05.05:2005, E DIN 1045-1, ENV 1992-1-1 EC 2, prEN 1992-1 [Final draft] EC 2, Model Code CEB-FIP 1990, BS 8110, ACI 318-99 to the experimental data. It has been analysed whether the difference in the results of the mean punching shear resistance received according to these methods and through experiments is statistically significant, when the level of significance value is 0,05. To analyse the significance of the difference of the means Student *t* test was used. An analysis was carried out to find out which methods show the least different resistance results from the experimental data. According to this analysis, a classification of methods was made. Student *t* test was applied to analyse in which methods the ratio between the punching shear resistance results obtained and the punching shear resistance results received through experiment is statistically insignificant. The level of significance value considered was 0,05.

It has been determined that almost in all cases the difference between the punching shear resistance results received experimentally and theoretically is statistically significant. It has also been found out that generally the punching shear resistance can be calculated by applying the prEN 1992-1 [Final draft] EC 2 method. The best method to describe the punching shear resistance in minimally reinforced slabs is ACI 318. The worst results are obtained by applying ENV 1992-1-1 EC 2 and E DIN 1045-1 methods.

**Keywords:** punching shear resistance of concrete slabs, design codes of concrete slabs, statistical analysis of punching shear.

#### 1. Introduction

Most works [1–4] analyse the impact of the main parameters on the punching shear resistance in reinforced concrete column-to-slab connections under axial forces.

These parameters are: punching shear resistance  $f_c^n$ , reinforcement ratios  $\rho$ , effective depth of the slab d, column geometry (transverse section c and form). Calculation methods for punching shear resistance of reinforced concrete constructions provided in design codes of different countries and international codes differ as well as the results obtained through these calculations.

Some works [1] provide a comparison of design codes punching shear resistance calculation methods. However, no statistical analysis of the compliance of the latest edition design codes calculation methods to the experimental data was found in the existing literature. Therefore, this work provides a statistical analysis of the Model Code CEB-FIP 1990 [5], E DIN 1045-1 [6], prEN 1992-1 [Final draft] EC 2 [7], ENV 1992-1-1 EC 2 [8], BS 8110 [9], ACI 318-99 [10] and STR 2.05.05 [11] calculation methods as well as experimental data. Further in this article the STR 2.05.05:2005 method is abbreviated to STR, E DIN 1045-1 to DIN, ENV 1992-1-1 EC 2 to EC2, prEN 1992-1 (Final draft) EC 2 to EC2Dr, Model Code CEB-FIP 1990 to MC, BS 8110 to BS and

ACI 318-99 to ACI. Experimental data used were provided in [12]. It was analysed in which methods the difference between the mean punching shear resistance calculated theoretically and obtained experimentally is significant statistically. Also, it was examined which methods give the most precise calculation of the punching shear resistance.

# 2. Design codes

Punching shear of slabs under axial forces in column-to-slab connection occurs when a punching cone is formed. The area of the punching cone makes a 26,6° to 45° angle to the horizontal column face [2, 4]. Based on this failure mechanism, design codes of different countries and international design codes suggest to use a half empirical critical section method to calculate the punching shear resistance in a slab. This method is based on the assumption that the slab fails when there is a vertical section at a certain distance from the column face which extends to the whole perimeter of the column-toslab connection. The perimeter of this section on the slab surface is called critical perimeter (u). The punching shear in slab occurs when punching shear strength in critical section exceed the punching shear resistance of the concrete.

The distance of the critical section from the column face as well as the geometry of the critical perimeter differ in design codes of different countries and international codes (Fig 1). Concrete punching shear strength dependence on approximation of cylindrical compressive strength also differs. These quantities are not precise in reinforced concrete theory. The values of these quantities provided in codes are empirical, based on experimental results [13].

The main code parameters of calculating punching shear in slabs are provided in Table 1. In this Table  $f_c$  – cylindrical compressive strength concrete,  $f_{cu} = 1,25f_c$  – cubical compressive strength in concrete. As one can see from Table 1, all methods approximate punching shear resistance in concrete by function  $f_c^n$ , only exponent quantities are different. Differently from other methods, ACI 318 does not evaluate the impact of the longitudinal reinforcement and the scale factor on the punching shear strength. ACI 318 admits that the maximal punching shear strengths in a slab 0,5d from the column surface are of constant size and direct distribution. Other calculation codes evaluate the non-linear distribution of tangent stresses in column-to-slab connection by increasing the distance of the critical section from the column surface.

Further we concisely present different punching shear calculation methods when axial forces are located centrally.

The punching strength by MC, DIN, EC2Dr and STR methods may be calculated by the following expressions:

$$V = \xi k (100 \rho f_c)^{1/3} ud, \qquad (1)$$

where  $\xi = 0.12$  by MC 90,  $\xi = 0.14$  by DIN,  $\xi = 0.18$  by EC2Dr and STR,  $u = 2(c_1 + c_2) + 4\pi d$  by MC and EC2Dr,  $u = 2(c_1 + c_2) + 3\pi d$  by DIN and STR, k – values are provided in Table 1. The punching strength according to EC2 method may be calculated by the following formulas:

$$V = \tau_R k \left( 1, 2 + 40\rho \right) ud , \qquad (2)$$

where  $\tau_R$  – concrete shear strength (MPa) [8],  $u = 2(c_1 + c_2) + 3\pi d$ . The k – value is given in Table 1. The punching strength by BS 8110 methods is as follows:

$$V = \xi (100\rho)^{1/3} k^{0.25} (f_c/25)^{1/3} ud, \qquad (3)$$

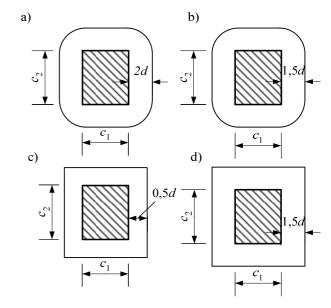
where  $\xi = 0.12$ ,  $u = 2(c_1 + c_2) + 12d$ . The resistance of punching shear force by ACI is as follows:

$$V = (2 + 4/\beta)\sqrt{f_c} ud \text{ (kips)}, \tag{4}$$

$$V = \left(\frac{\alpha_s d}{u} + 2\right) \sqrt{f_c} u d \text{ (kips)}, \tag{5}$$

$$V = ud\sqrt[4]{f_c} \text{ (kips)}, \tag{6}$$

where  $u = 2(c_1 + c_2) + 4d$ ,  $\beta$  is the ratio the long side to the short side of the concentration load (or columns),  $\alpha_s$  is 40 for interior column.



**Fig 1.** Critical perimeters of interior column: a – by MC, EC2Dr; b – by DIN, STR, EC2; c – by ACI; d – by BS

#### 3. Statistical analysis of data

Due to the spread of the punching shear resistance data obtained through experiments, theoretically obtained values will always differ from the experimental results. However, this difference can be statistically insignificant. If the spread of the experimental data is only achieved because of an accidental error, as it is known, the bigger the number of experiments performed, the closer the mean of the accidental error to zero. Then the proximity of the theoretical method to the experimental data can be compared by applying the difference:

Table 1. Expressions of the main punching shear parameters in calculation codes

Parameters	MC DIN EC2Dr			EC2	BS	ACI 318	STR
Shear resistance	$\sqrt[3]{f_c}$				$\sqrt[3]{f_{cu}}$	$\sqrt{f_c}$	$\sqrt[3]{f_c}$
Reinforcement ratios	∛ρ			$1,2+40\rho$	∛ρ	_	∛ρ
Scale factor k	$1 + \sqrt{200/d}$			1,6-d	<sup>4</sup> √400/d	-	$1+\sqrt{200/d}$
Critical section	2 <i>d</i> 1,5 <i>d</i> 2 <i>d</i>			1,5 <i>d</i>	1,5 <i>d</i>	0,5 <i>d</i>	1,5 <i>d</i>

$$\frac{1}{n} \sum_{i=1}^{n} V_{calc,i} - \frac{1}{n} \sum_{i=1}^{n} V_{exp,i} = \overline{V_{calc}} - \overline{V_{exp}} = \Delta V , \quad (7)$$

where  $V_{exp}$  and  $V_{calc}$  are experimentally obtained and theoretically calculated values of punching shear resistance,  $\overline{V_{exp}}$  and  $\overline{V_{calc}}$  — the means of the experimentally obtained and theoretically calculated values of punching shear resistance. The smaller  $\Delta V$ , the closer the theoretically obtained values to the experimental data. If  $n \to \infty$ , then  $\Delta V \to 0$ . In reality, the number of experimental data is always limited, that is why to verify the equality of the means hypothesis  $H_0$  is put against a competing hypothesis  $H_1$ :

$$\begin{cases} H_0: \overline{V_{calc}} = \overline{V_{exp}}, \\ H_1: \overline{V_{calc}} \neq \overline{V_{exp}}, \end{cases}$$
 (8)

where

$$\begin{split} &\overline{V_{calc}} \in \left\{ \overline{V_{STR}}, \overline{V_{DIN}}, \overline{V_{EC2}}, \overline{V_{EC2Dr}}, \overline{V_{MC}}, \overline{V_{BS}}, \overline{V_{ACI}} \right\}, \\ &\overline{V_{STR}}, \ \ \overline{V_{DIN}}, \ \ \overline{V_{EC2}}, \ \ \overline{V_{EC2Dr}}, \ \ \overline{V_{MC}}, \ \ \overline{V_{BS}}, \ \ \overline{V_{ACI}} - \\ &\text{means of punching forces calculated according to STR,} \\ &\text{DIN, EC2, EC2Dr, MC, BS and ACI [5-11] methods.} \end{split}$$

It is important to analyse which methods allow more precise calculations of punching shear resistance. The test of accuracy is the  $V_{calc}$  / $V_{exp}$  ratio mean  $\overline{V_{calc}}$  / $V_{exp}$ . The closer this ratio to 1, the more precisely allows the theoretical method to calculate the punching shear. According to the closeness of the obtained  $\overline{V_{calc}}$  / $V_{exp}$  values to 1, theoretical calculation methods can be ranged into giving the best and the worst calculations of the punching shear resistance.

# Sampling

Samples of 7 different slabs were chosen from [12] for statistical analysis. Parameters of slabs and characteristics of materials are given in Table 2. In this Table, c – measurements of transverse column section (m),  $\rho$  – relative area of tensile reinforcement (%), d – the useful height of reinforced concrete slabs (m),  $f_v$  – reinforcement yield point (MPa),  $f_c$  – compressive cylinder strength of concrete (MPa). Literature source [12] offers the estimate of the average compressive strength  $f_c$  in concrete of each slab; however, it does not provide the estimation of standard deviation and the number of tested samples. Without these data, it is not possible to evaluate the influence of distribution of compressive strength in concrete of each slab on the sheer strength  $V_{calc}$  that is calculated according to the theoretical model. Therefore, it is agreed that, for the purpose of further analysis, the compressive strength of concrete of the  $i^{th}$  slab is equal to the estimation of the average of the compressive strength of this slab provided in [12].

According to STR, DIN, EC2, EC2Dr, MC, BS and ACI methods, if we use parameters of each sample, it is possible to calculate the theoretical punching shear resistance of a slab  $V_{calc}$ . Concrete strength  $f_c$  in each slab is different. As for the formulas (1)–(6), the analyzed formulas evaluate the impact of  $f_c$  on the punching shear

strength. That is why the theoretical punching shear strength values  $V_{calc}$  of a certain sample from a certain slab calculated with different  $f_c$  values are compared to the experimental punching shear strength  $V_{exp}$  of the same sample from the same slab. Since additionally to theoretical and experimental punching shear strength values we also analyze  $\overline{V_{calc}/V_{exp}}$ , in each sample additionally to  $V_{exp}$  and  $V_{calc}$  variables, we shall have  $V_{calc}/V_{exp}$  variables

The data normality is verified by applying the Shapiro-Wilk *W* test. As shown in [14], this test is the best to verify the normality of the data.

The main statistical variable estimates: the minimal and the maximal values, the mean, standard deviation as well as the values of estimation test of hypotheses on the normality of data are given in Table 2. In this Table W – the obtained Shapiro-Wilk test values, and P – Shapiro-Wilk test P values. The W was calculated according to the method described in [15] and P values were taken from [15]. As shown in Table 2, the P values of the W with all variables except for  $f_c$  and  $V_{calc}$  of sample 1 and for  $V_{MC}$  of sample 4 is higher than the usually applied significance level  $\alpha = 0.05$ . That is why the theoretically and experimentally obtained punching shear values  $V_{exp}$  and  $V_{calc}$ , except in sample 1, do not contradict the hypothesis that the data are distributed in a normal distribution.

# Verification of the hypothesis about the equality of the means obtained experimentally and theoretically

When data are in normal distribution (8), we can apply the Student t test for independent samples when general set variances are unequal [16] to verify the hypothesis. First of all, the t statistics is calculated applying the formula [16]:

$$t = \frac{\overline{V_{\text{exp}}} - \overline{V_{calc}}}{\sqrt{S_{\text{exp}}^2 / n + S_{calc}^2 / n}},$$
 (9)

where

$$\begin{split} S_{calc} \in & \left\{ S_{STR}, S_{DIN}, S_{EC2}, S_{EC2Dr}, S_{MC}, S_{BS}, S_{ACI} \right\} - \\ & V_{STR} - V_{BS} \text{ standard sample deviations (Table 2).} \end{split}$$

Hypothesis  $H_0$  is rejected if  $|t| > t_{\alpha/2}(k)$ . Here  $t_{\alpha/2}(k)$  is the critical value of  $\alpha/2$  level in Student distribution with k degree of freedom. It is supposed that  $\alpha = 0.05$ . Results of verifying (8) hypothesis is given in Table 3.

Calculation degree of freedom k, which is the smallest whole number satisfying the condition:

$$k \le \left(S_{\text{exp}}^2 / n + S_{calc}^2 / n\right)^2 / \left(S_{\text{exp}}^4 / n^3 + S_{calc}^4 / n^3\right).$$
 (10)

As shown in Table 3, the difference of experimentally and theoretically obtained means of punching shear forces, except for sample 3  $\overline{V_{EC2Dr}} = \overline{V_{exp}}$  is statistically significant. Therefore, generally we can make a conclusion that we cannot get an accurate calculation of punching shear force by applying the mentioned methods.

Table 2. The main statistical sample estimates and the values of verification test for hypotheses about the normality of data

Sample	Slab pa- rameters	Sample size	Sample variables	Minimal value (MN) (for $f_c$ – (MPa))	Maximal value (MN) (for $f_c$ – (MPa))	Mean (MN) (for $f_c$ – (MPa))	Standard deviation (MN) (for $f_c$ – (MPa))	W	P
1	2	3	4	5	6	7	8	9	10
			$f_c$	2,090·10 <sup>1</sup>	2,910·10 <sup>1</sup>	2,580·10 <sup>1</sup>	2,247	0,781	0,000
			$V_{ m exp}$	5,600·10 <sup>-2</sup>	8,940·10 <sup>-2</sup>	7,328·10 <sup>-2</sup>	9,844 · 10 - 3	0,966	0,661
	22		$V_{ m STR}$	5,230·10 <sup>-2</sup>	5,840·10 <sup>-2</sup>	5,605·10 <sup>-2</sup>	1,660·10 <sup>-3</sup>	0,893	0,031
	4		$V_{ m DIN}$	4,070·10 <sup>-2</sup>	4,540·10 <sup>-2</sup>	4,361·10 <sup>-2</sup>	1,292·10 <sup>-3</sup>	0,889	0,026
	:361		$V_{\mathrm{EC2}}$	3,960·10 <sup>-2</sup>	4,420 10-2	4,243 · 10 <sup>-2</sup>	1,253·10 <sup>-3</sup>	0,893	0,031
	$f_{y=0}$		$V_{ m EC2Dr}$	6,130 10-2	6,840 10 <sup>-2</sup>	6,570 10-2	1,948 10 <sup>-3</sup>	0,892	0,029
	$c$ =0,015; $p$ =2,53; $d$ =0,046; $f_y$ =361–467		$V_{ m MC}$	4,100·10 <sup>-2</sup>	4,600 · 10 - 2	4,380 10-2	1,361 · 10 <sup>-3</sup>	0,916	0,086
1	<i>t</i> =0,	20	$V_{ m BS}$	5,500 10-2	6,200 · 10 <sup>-2</sup>	5,930 10-2	1,895 · 10 <sup>-3</sup>	0,877	0,016
	3;6		$V_{ m ACI}$	4,100·10 <sup>-2</sup>	4,900·10 <sup>-2</sup>	4,580 · 10 <sup>-2</sup>	2,093·10 <sup>-3</sup>	0,874	0,014
	=2,5		$V_{\rm STR}/V_{\rm exp}$	6,359 10-1	9,656·10 <sup>-1</sup>	7,772·10 <sup>-1</sup>	9,726·10 <sup>-2</sup>	0,963	0,601
	.; Ε		$V_{ m DIN}/V_{ m exp}$	4,946 10-1	7,510·10 <sup>-1</sup>	6,045 10-1	7,565·10 <sup>-2</sup>	0,963	0,600
	,015		$V_{\rm EC2}/V_{\rm exp}$	4,813·10 <sup>-1</sup>	7,308 10-1	5,882·10 <sup>-1</sup>	7,362·10 <sup>-2</sup>	0,963	0,600
	0		$V_{\rm EC2Dr}/V_{\rm exp}$	7,451 10-1	1,131	9,106·10 <sup>-1</sup>	1,140 10-1	0,963	0,601
			V <sub>MC</sub> /V <sub>exp</sub>	4,970 10-1	7,540 10-1	6,071 10-1	7,594 10-2	0,962	0,594
			$V_{\rm BS}/V_{\rm exp}$	6,710·10 <sup>-1</sup>	1,019	8,205·10 <sup>-1</sup>	1,027 10-1	0,962	0,603
-			V <sub>ACI</sub> /V <sub>exp</sub>	5,180·10 <sup>-1</sup> 2,050·10 <sup>1</sup>	8,030·10 <sup>-1</sup> 2,540·10 <sup>1</sup>	6,358·10 <sup>-1</sup> 2,344·10 <sup>1</sup>	7,689·10 <sup>-2</sup> 1,666·10 <sup>1</sup>	0,972 0,956	0,803
			$\frac{f_c}{V_{\rm exp}}$	3,114·10 <sup>-1</sup>	3,923·10 <sup>-1</sup>	3,622·10 <sup>-1</sup>	2,793·10 <sup>-2</sup>	0,930	0,776
			$V_{\rm exp}$ $V_{\rm STR}$	2,394 10 <sup>-1</sup>	2,571·10 <sup>-1</sup>	2,502·10 <sup>-1</sup>	6,012·10 <sup>-3</sup>	0,923	0,710
	83		V <sub>DIN</sub>	1,862 10 1	2,000 10-1	1,946·10 <sup>-1</sup>	4,680·10 <sup>-3</sup>	0,950	0,710
	99-4		$V_{\mathrm{EC2}}$	1,700·10 <sup>-1</sup>	1,826·10 <sup>-1</sup>	1,777·10 <sup>-1</sup>	4,267·10 <sup>-3</sup>	0,952	0,723
	)=3 <sup>6</sup>		$V_{\rm EC2Dr}$	2,804 10 <sup>-1</sup>	3,012·10 <sup>-1</sup>	2,931·10 <sup>-1</sup>	7,047 10 <sup>-3</sup>	0,952	0,730
	4; f		V <sub>MC</sub>	1,870·10 <sup>-1</sup>	2,010·10 <sup>-1</sup>	1,954 10-1	4,627 · 10 <sup>-3</sup>	0,959	0,802
	0,11		$V_{ m BS}$	2,390·10 <sup>-1</sup>	2,560·10 <sup>-1</sup>	2,495·10 <sup>-1</sup>	5,806·10 <sup>-3</sup>	0,944	0,655
2	; <i>d</i> =	8	$V_{ m ACI}$	2,520·10 <sup>-1</sup>	2,810·10 <sup>-1</sup>	2,696·10 <sup>-1</sup>	9,812·10 <sup>-3</sup>	0,953	0,737
	$c$ =0,287; $p$ =1,06; $d$ =0,114; $f_y$ =399-483		$V_{\rm STR}/V_{\rm exp}$	6,419 10 <sup>-1</sup>	7,688·10 <sup>-1</sup>	6,937 10-1	4,581·10 <sup>-2</sup>	0,869	0,147
	ρ=1		$V_{ m DIN}/V_{ m exp}$	4,993 10-1	5,980·10 <sup>-1</sup>	5,396·10 <sup>-1</sup>	3,564 10-2	0,869	0,146
	87;		$V_{\rm EC2}/V_{\rm exp}$	4,558 10 1	5,459 10-1	4,926 10-1	3,253 10 <sup>-2</sup>	0,869	0,147
	=0,2		$V_{ m EC2Dr}/V_{ m exp}$	7,519 10-1	9,005 10-1	8,126 10-1	5,364 10-2	0,869	0,147
	Ü		V <sub>MC</sub> /V <sub>exp</sub>	5,010·10 <sup>-1</sup>	6,000·10 <sup>-1</sup>	5,418·10 <sup>-1</sup>	3,572·10 <sup>-2</sup>	0,870	0,151
			$V_{\rm BS}/V_{\rm exp}$	6,400 · 10 - 1	7,670·10 <sup>-1</sup>	6,919·10 <sup>-1</sup>	4,571 · 10 - 2	0,871	0,153
			$V_{ m ACI}/V_{ m exp}$	6,910-10-1	8,170·10 <sup>-1</sup>	7,471 · 10 - 1	4,542 · 10 - 2	0,902	0,303
			$f_c$	2,340·10 <sup>1</sup>	2,840·10 <sup>1</sup>	2,565·10 <sup>1</sup>	1,626	0,924	0,281
			$V_{\rm exp}$	2,455·10 <sup>-1</sup>	3,714·10 <sup>-1</sup>	3,092 · 10 - 1	3,670·10 <sup>-2</sup>	0,930	0,341
		13	$V_{ m STR}$	2,571·10 <sup>-1</sup>	2,742·10 <sup>-1</sup>	2,650·10 <sup>-1</sup>	5,569·10 <sup>-3</sup>	0,927	0,312
	8		$V_{ m DIN}$	2,000 · 10 - 1	2,133·10 <sup>-1</sup>	2,061 · 10 <sup>-1</sup>	4,328·10 <sup>-3</sup>	0,929	0,327
	=37		$V_{\mathrm{EC2}}$	1,816·10 <sup>-1</sup>	1,937·10 <sup>-1</sup>	1,872·10 <sup>-1</sup>	3,927·10 <sup>-3</sup>	0,929	0,328
	<b>4</b> ; <i>f</i>		$V_{ m EC2Dr}$	3,011 10-1	3,212·10 <sup>-1</sup>	3,104·10 <sup>-1</sup>	6,520·10 <sup>-3</sup>	0,929	0,336
	),11		$V_{ m MC}$	2,010·10 <sup>-1</sup>	2,140·10 <sup>-1</sup>	2,068·10 <sup>-1</sup>	4,318 10 <sup>-3</sup>	0,916	0,224
2	<i>d</i> =(		$V_{ m BS}$	2,560·10 <sup>-1</sup>	2,730·10 <sup>-1</sup>	2,643 · 10 <sup>-1</sup>	5,663·10 <sup>-3</sup>	0,923	0,278
3	,15;	13	$V_{ m ACI}$	2,700·10 <sup>-1</sup>	2,970 10-1	2,824 10-1	8,732 · 10 <sup>-3</sup>	0,930	0,339
	<u>=</u> 1		$V_{ m STR}/V_{ m exp}$	7,179 10-1	1,117	8,701 10-1	1,212 10-1	0,883	0,078
	87; 1		$V_{ m DIN}/V_{ m exp}$	5,583·10 <sup>-1</sup>	8,688 10-1	6,767·10 <sup>-1</sup>	9,430 10-2	0,883	0,078
	0,28		$V_{\rm EC2}/V_{\rm exp}$	5,071·10 <sup>-1</sup>	7,890 10-1	6,146·10 <sup>-1</sup>	8,562 10-2	0,883	0,078
	f.		$V_{ m EC2Dr}/V_{ m exp}$	8,409 10-1	1,308	1,019·10	1,420 · 10 - 1	0,883	0,078
			$V_{ m MC}/V_{ m exp}$	5,610·10 <sup>-1</sup>	8,720·10 <sup>-1</sup>	6,793 · 10 <sup>-1</sup>	9,465 10-2	0,882	0,076
			$V_{ m BS}/V_{ m exp}$	7,160·10 <sup>-1</sup>	1,114	8,677·10 <sup>-1</sup>	1,208 · 10 <sup>-1</sup>	0,883	0,077
			$V_{\rm ACI}/V_{\rm exp}$	7,670 10-1	1,210	9,272 10-1	1,355 10-1	0,879	0,068

Continuation of Table 2

	1 2		1 4	<del></del>			Ι ο		10
1	2	3	4	5	6	7	8	9	10
			$f_c$	2,640·101	3,100·101	2,908·10 <sup>1</sup>	1,693·10 <sup>1</sup>	0,878	0,261
			$V_{\rm exp}$	8,180·10 <sup>-2</sup>	9,390·10 <sup>-2</sup>	8,575·10 <sup>-2</sup>	5,278·10 <sup>-3</sup>	0,794	0,052
	300		$V_{ m STR}$	5,180·10 <sup>-2</sup>	5,470·10 <sup>-2</sup>	5,347 10 <sup>-2</sup>	1,060·10 <sup>-3</sup>	0,886	0,296
	c=0,115; p=0,722; <i>d</i> =0,057; <i>f</i> <sub>3</sub> =300		$V_{ m DIN}$	4,030·10 <sup>-2</sup>	4,250 10-2	4,162·10 <sup>-2</sup>	8,329·10 <sup>-4</sup>	0,856	0,177
	7; J		$V_{\rm EC2}$	3,980 10-2	4,200·10 <sup>-2</sup>	4,107·10 <sup>-2</sup>	8,066·10 <sup>-4</sup>	0,883	0,285
	,05		$V_{\rm EC2Dr}$	6,160·10 <sup>-2</sup>	6,500·10 <sup>-2</sup>	6,363 · 10 <sup>-2</sup>	1,268·10 <sup>-3</sup>	0,864	0,205
	0=#		$V_{ m MC}$	4,100·10 <sup>-2</sup>	4,300·10 <sup>-2</sup>	4,250·10 <sup>-2</sup>	8,367·10 <sup>-4</sup>	0,701	0,006
4	2; 6	6	$V_{\mathrm{BS}}$	5,500·10 <sup>-2</sup>	5,800·10 <sup>-2</sup>	5,667·10 <sup>-2</sup>	1,033·10 <sup>-3</sup>	0,915	0,473
	,72		V <sub>ACI</sub>	6,200 10-2	6,700 · 10 - 2	6,500·10 <sup>-2</sup>	2,000·10 <sup>-3</sup>	0,823	0,094
	O Q		$V_{\rm STR}/V_{\rm exp}$	5,821·10 <sup>-1</sup>	6,595·10 <sup>-1</sup>	6,253 · 10 - 1	3,013·10 <sup>-2</sup>	0,900	0,373
	Σ; β		$V_{\rm DIN}/V_{\rm exp}$	4,528 10 <sup>-1</sup>	5,129·10 <sup>-1</sup>	4,863 · 10 <sup>-1</sup>	2,341 · 10 - 2	0,900	0,373
	11.		$V_{\rm EC2}/V_{\rm exp}$	4,472 · 10 -1	5,066·10 <sup>-1</sup>	4,804 · 10 <sup>-1</sup>	2,313·10 <sup>-2</sup>	0,899	0,371
	0		$V_{\rm EC2Dr}/V_{\rm exp}$	6,924·10 <sup>-1</sup>	7,844·10 <sup>-1</sup>	7,438 10-1	3,582·10 <sup>-2</sup>	0,899	0,371
	0		V <sub>MC</sub> /V <sub>exp</sub>	4,620 10 <sup>-1</sup>	5,230·10 <sup>-1</sup>	4,958·10 <sup>-1</sup>	2,363·10 <sup>-2</sup>	0,904	0,397
			$V_{\rm BS}/V_{\rm exp}$	6,140 10-1	6,960·10 <sup>-1</sup>	6,595 10-1	3,192·10 <sup>-2</sup>	0,901	0,383
			$V_{\rm ACI}/V_{\rm exp}$	7,140·10 <sup>-1</sup>	8,040·10 <sup>-1</sup>	7,587·10 <sup>-1</sup>	3,415·10 <sup>-2</sup>	0,950	0,739
			$f_c$	2,630·10 <sup>1</sup>	3,130·10 <sup>1</sup>	2,920·10 <sup>-1</sup>	1,815·10 <sup>1</sup>	0,950	0,743
			$V_{\rm exp}$	9,960 10-2	1,254 10 <sup>-1</sup>	1,130·10 <sup>-1</sup>	8,486·10 <sup>-3</sup>	0,961	0,829
	300		$V_{\rm STR}$	6,780·10 <sup>-2</sup>	7,190·10 <sup>-2</sup>	7,018·10 <sup>-2</sup>	1,491 10 <sup>-3</sup>	0,950 0,944	0,742
	$f_{y=1}$		$V_{\rm DIN}$	5,270·10 <sup>-2</sup>	5,590·10 <sup>-2</sup>	5,460·10 <sup>-2</sup>	1,152·10 <sup>-3</sup>		0,691
	57;		$V_{\rm EC2}$	4,940 10 <sup>-2</sup>	5,230·10 <sup>-2</sup>	5,112·10 <sup>-2</sup>	1,065 10 <sup>-3</sup>	0,942	0,674
	),0,		V <sub>EC2Dr</sub>	8,070·10 <sup>-2</sup> 5,400·10 <sup>-2</sup>	8,550·10 <sup>-2</sup>	8,352·10 <sup>-2</sup> 5,567·10 <sup>-2</sup>	1,747·10 <sup>-3</sup> 1,033·10 <sup>-3</sup>	0,946	0,709 0,473
	<i>d</i> =(		V <sub>MC</sub>	7,200 10-2	5,700·10 <sup>-2</sup> 7,600·10 <sup>-2</sup>	7,417·10	1,472 10 <sup>-3</sup>	0,915 0,958	0,804
5	c=0,115; p=1,625; d=0,057; f <sub>y</sub> =300	6	V <sub>BS</sub>	6,200 10	6,700·10	6,500·10 <sup>-2</sup>	1,789·10 <sup>-3</sup>		
	,62		V <sub>ACI</sub>	5,731·10 <sup>-1</sup>	6,808·10	6,234·10 <sup>-1</sup>	3,782·10 <sup>-2</sup>	0,933 0,988	0,607 0,984
	Ξ		$V_{\rm STR}/V_{\rm exp}$	4,457·10	5,295·10 <sup>-1</sup>	4,849·10	2,943·10 <sup>-2</sup>	0,988	0,984
	5; 1		$V_{\rm DIN}/V_{\rm exp}$	4,437.10	4,959·10	4,541 10	2,756·10 <sup>-2</sup>	0,988	0,984
	,11		$V_{\rm EC2}/V_{\rm exp}$	6,816 10 <sup>-1</sup>	8,098·10 <sup>-1</sup>	7,415·10	4,501·10 <sup>-2</sup>	0,988	0,984
	OII		$V_{\rm EC2Dr}/V_{\rm exp}$	4,540·10	5,400·10	4,943 10 <sup>-1</sup>	3,018·10 <sup>-2</sup>	0,988	0,984
			$V_{ m MC}/V_{ m exp}$ $V_{ m BS}/V_{ m exp}$	6,050 10	7,180·10	6,578·10	3,974·10 <sup>-2</sup>	0,987	0,982
			$V_{\rm ACI}/V_{\rm exp}$	5,370·10 <sup>-1</sup>	6,200·10 <sup>-1</sup>	5,772·10 <sup>-1</sup>	3,138·10 <sup>-2</sup>	0,963	0,845
			$f_c$	2,120·10 <sup>1</sup>	2,870·10 <sup>1</sup>	2,489·10 <sup>1</sup>	$2,195 \cdot 10^{1}$	0,963	0,721
	10		$V_{\rm exp}$	1,837	2,309	2,114	1,382·10 <sup>-1</sup>	0,948	0,461
	515		$V_{ m STR}$	1,065	1,178	1,122	3,334 10 <sup>-2</sup>	0,957	0,613
	1		$V_{\rm DIN}$	8,280·10 <sup>-1</sup>	9,160·10 <sup>-1</sup>	8,728·10 <sup>-1</sup>	2,593·10 <sup>-2</sup>	0,957	0,614
	306		$V_{\rm EC2}$	8,528·10 <sup>-1</sup>	9,434·10 <sup>-1</sup>	8,989·10 <sup>-1</sup>	2,671 10 <sup>-2</sup>	0,957	0,610
	f, = (	<i>d</i> =0,356; <i>f</i> <sub>y</sub> =309 – 515	$V_{\rm EC2Dr}$	1,314	1,45	1,385	4,115·10 <sup>-2</sup>	0,957	0,613
	56;		$V_{ m MC}$	8,760·10 <sup>-1</sup>	9,690·10 <sup>-1</sup>	9,233·10 <sup>-1</sup>	2,738·10 <sup>-2</sup>	0,957	0,607
	),35		$V_{ m BS}$	1,181	1,307	1,245	3,708·10 <sup>-2</sup>	0,958	0,626
6	<i>d</i> =(	16	$V_{ m ACI}$	1,551	1,804	1,679	7,449·10 <sup>-2</sup>	0,959	0,642
	37;		$V_{\rm STR}/V_{\rm exp}$	4,675·10 <sup>-1</sup>	6,297·10 <sup>-1</sup>	5,329·10 <sup>-1</sup>	3,919·10 <sup>-2</sup>	0,956	0,590
	5,0=		$V_{\rm DIN}/V_{\rm exp}$	3,636·10 <sup>-1</sup>	4,898·10 <sup>-1</sup>	4,145 10-1	3,048·10 <sup>-2</sup>	0,956	0,589
	 ۾		$V_{\rm EC2}/V_{\rm exp}$	3,745·10 <sup>-1</sup>	5,045 · 10 <sup>-1</sup>	4,269·10 <sup>-1</sup>	3,139·10 <sup>-2</sup>	0,956	0,586
	102	c=0,402; p=0,37;	$V_{\rm EC2Dr}/V_{\rm exp}$	5,768·10 <sup>-1</sup>	7,771·10 <sup>-1</sup>	6,576·10 <sup>-1</sup>	4,837·10 <sup>-2</sup>	0,956	0,590
	6,0		$V_{\rm MC}/V_{\rm exp}$	3,850·10 <sup>-1</sup>	5,180·10 <sup>-1</sup>	4,386·10 <sup>-1</sup>	3,219·10 <sup>-2</sup>	0,955	0,581
	<u>"</u>	U	$V_{\rm BS}/V_{\rm exp}$	5,190·10 <sup>-1</sup>	6,990·10 <sup>-1</sup>	5,914 10 <sup>-1</sup>	4,340·10 <sup>-2</sup>	0,955	0,570
			$V_{\rm ACI}/V_{\rm exp}$	6,810·10 <sup>-1</sup>	9,560·10 <sup>-1</sup>	7,973·10 <sup>-1</sup>	6,466 10-2	0,930	0,244
			$f_c$	2,210·10 <sup>1</sup>	2,980·10 <sup>1</sup>	2,526·10 <sup>1</sup>	1,838·10 <sup>1</sup>	0,973	0,601
			$V_{\rm exp}$	1,668	2,669	2,202	2,136·10 <sup>-1</sup>	0,973	0,598
			$V_{ m STR}$	1,099	1,214	1,148	2,784 10-2	0,973	0,610
7		31	$V_{ m DIN}$	8,544 10 <sup>-1</sup>	9,440 · 10 - 1	8,929·10 <sup>-1</sup>	2,164·10 <sup>-2</sup>	0,973	0,618
/			$V_{\mathrm{EC2}}$	8,698·10 <sup>-1</sup>	9,610·10 <sup>-1</sup>	9,090·10 <sup>-1</sup>	2,204 · 10 <sup>-2</sup>	0,973	0,617
			$V_{ m EC2Dr}$	1,35	1,498	1,417	3,436 10-2	0,973	0,616
			$V_{ m MC}$	9,040·10 <sup>-1</sup>	9,980·10 <sup>-1</sup>	9,445 10 <sup>-1</sup>	2,287·10 <sup>-2</sup>	0,973	0,603
			$V_{ m BS}$	1,219	1,347	1,274	3,098 10-2	0,973	0,608
			$V_{ m ACI}$	1,583	1,838	1,691	6,155 10-2	0,974	0,624
	39; 19 –		$V_{\rm STR}/V_{\rm exp}$	4,470·10 <sup>-1</sup>	6,626·10 <sup>-1</sup>	5,258 · 10 -1	5,015·10 <sup>-2</sup>	0,954	0,195
	=0,		$V_{ m DIN}/V_{ m exp}$	3,477 10-1	5,153·10 <sup>-1</sup>	4,090·10 <sup>-1</sup>	3,900·10 <sup>-2</sup>	0,954	0,195
7	;; p; ; f;=	31	$V_{\rm EC2}/V_{\rm exp}$	3,539 10-1	5,246 10-1	4,163 10-1	3,970·10 <sup>-2</sup>	0,954	0,197
,	402 356 5	J1	$V_{\rm EC2Dr}/V_{\rm exp}$	5,516·10 <sup>-1</sup>	8,176·10 <sup>-1</sup>	6,488·10 <sup>-1</sup>	6,188·10 <sup>-2</sup>	0,954	0,196
	0,4		$V_{ m MC}/V_{ m exp}$	3,680 · 10 -1	5,450·10 <sup>-1</sup>	4,326·10 <sup>-1</sup>	4,134 10-2	0,953	0,190
						1			
	c=0,402; p=0,39; d=0,356; f <sub>y</sub> =309 – 515		$V_{ m BS}/V_{ m exp}$ $V_{ m ACI}/V_{ m exp}$	4,960·10 <sup>-1</sup> 6,470·10 <sup>-1</sup>	7,350·10 <sup>-1</sup> 9,580·10 <sup>-1</sup>	5,833·10 <sup>-1</sup> 7,745·10 <sup>-1</sup>	5,560·10 <sup>-2</sup> 7,367·10 <sup>-2</sup>	0,954 0,966	0,197 0,406

**Table 3.** Results of verifying hypothesis (8)

Sample number	Sample variables	t	$t_{\alpha/2}$	$H_0$	
number	$V_{ m STR}$	-11,090	2,306		
		-16,742	2,365	-	
	$V_{ m DIN}$	-10,742 -18,477	2,365		
2	V <sub>EC2</sub>	-6,792		rejected	
2	$V_{\rm EC2Dr}$		2,306	- Ĝ.	
	V <sub>MC</sub>	-16,670	2,365		
	$V_{\mathrm{BS}}$	-11,177	2,306		
	$V_{ m ACI}$	-8,848	2,262		
	$V_{\rm STR}$	-4,293	2,160	_	
	$V_{ m DIN}$	-10,055	2,179	4 _	
_	$V_{\mathrm{EC2}}$	-11,918	2,179	rejected	
3	$V_{\mathrm{EC2Dr}}$	0,116	2,160	jec	
	$V_{ m MC}$	-9,983	2,179	T.e.	
	$V_{ m BS}$	-4,355	2,160	_	
	$V_{ m ACI}$	-2,560	2,160		
	$V_{ m STR}$	-14,689	2,571		
	$V_{ m DIN}$	-20,231	2,571		
	$V_{ m EC2}$	-20,499	2,571	eq	
4	$V_{\mathrm{EC2Dr}}$	-9,980	2,447	rejected	
	$V_{ m MC}$	-19,824	2,571	rej.	
	$V_{ m BS}$	-13,246	2,571		
	$V_{ m ACI}$	-9,005	2,447		
	$V_{ m STR}$	-12,178	2,571		
	$V_{ m DIN}$	-16,709	2,571		
	$V_{\mathrm{EC2}}$	-17,729	2,571	700	
5	$V_{\rm EC2Dr}$	-8,340	2,571	rejected	
	$V_{ m MC}$	-16,433	2,571	rēj.	
	$V_{\mathrm{BS}}$	-11,049	2,571		
	$V_{ m ACI}$	-13,562	2,571		
	$V_{\rm STR}$	-27,910	2,110		
	$V_{ m DIN}$	-35,311	2,120		
	$V_{\mathrm{EC2}}$	-34,532	2,120	<del>ੂ</del>	
6	$V_{\rm EC2Dr}$	-20,234	2,101	rejected	
O	$V_{ m MC}$	-33,809	2,120	· ge	
	$V_{ m BS}$	-24,295	2,110	┪  ̄	
	$V_{\rm ACI}$	-11,100	2,069	1	
		-27,260	2,040	+	
	V <sub>STR</sub>	-33,967	2,040	-	
	$V_{ m DIN}$	-33,543	2,040	┨╼	
7	$V_{\rm EC2}$			rejected	
/	$V_{\rm EC2Dr}$	-20,226	2,037	eje.  -	
	$V_{ m MC}$	-32,609	2,040	- i	
	$V_{ m BS}$	-23,961	2,040		
	$V_{ m ACI}$	-12,798 2,030	2,030		

# Analysis of the accuracy of calculation methods

Further this article analyses which methods allow to make the most accurate calculations of punching shear force.  $\overline{V_{calc}/V_{exp}}$  ratios as well as error bands of these rations are given in Table 4. The confidence intervals of means are calculated by using t test. The value of significance level is 0,05. The ratio difference calculated with different methods:

$$\overline{V_{calc} / V_{exp}} - \overline{V_{calc1} / V_{exp}} \;, \tag{11}$$
 here  $\overline{V_{calc1}} \in \left\{ \overline{V_{STR}}, \overline{V_{DIN}}, \overline{V_{EC2}}, \overline{V_{EC2Dr}}, \overline{V_{MC}}, \overline{V_{BS}}, \overline{V_{ACI}} \right\}$  and  $\overline{V_{calc}} \neq \overline{V_{calc1}}$  can be statistically insignificant. That is why to verify the significance of the difference  $H_0$ 

hypothesis is put against a competing hypothesis  $H_1$ :

**Table 4.** Values of  $\overline{V_{calc}/V_{exp}}$  ratios

				ſ
Sample	Sample	$\overline{V_{calc}/V_{exp}}$	$V_{calc}$ / $V_{exp}$	$V_{calc}$ / $V_{exp}$
number	variables	values	the lowest limit	the highest limit
	$V_{\rm STR}/V_{\rm exp}$	0,777	0,223	0,728
	$V_{\rm DIN}/V_{\rm exp}$	0,604	0,396	0,567
	$V_{\rm EC2}/V_{\rm exp}$	0,588	0,412	0,551
1	$V_{\rm EC2Dr}/V_{\rm exp}$	0,911	0,089	0,854
•	$V_{\rm MC}/V_{\rm exp}$	0,607	0,393	0,569
	$V_{\rm BS}/V_{\rm exp}$	0,821	0,180	0,769
	$V_{\rm ACI}/V_{\rm exp}$	0,636	0,364	0,597
	$V_{\rm STR}/V_{\rm exp}$	0,694	0,657	0,730
	$V_{\rm DIN}/V_{\rm exp}$	0,540	0,511	0,568
	$V_{\rm EC2}/V_{\rm exp}$	0,493	0,467	0,518
2	$V_{\rm EC2Dr}/V_{\rm exp}$	0,813	0,770	0,855
-	$V_{\rm MC}/V_{\rm exp}$	0,542	0,513	0,570
	$V_{\rm BS}/V_{\rm exp}$	0,692	0,656	0,728
	$V_{\rm ACI}/V_{\rm exp}$	0,747	0,711	0,783
	$V_{\rm STR}/V_{\rm exp}$	0,870	0,795	0,765
	$V_{\rm DIN}/V_{\rm exp}$	0,677	0,618	0,735
	$V_{\rm EC2}/V_{\rm exp}$	0,615	0,561	0,668
3	$V_{\rm EC2}V_{\rm exp}$	1,019	0,931	1,107
3	$V_{\rm MC}/V_{\rm exp}$	0,679	0,620	0,738
	$V_{\rm BS}/V_{\rm exp}$	0,868	0,793	0,738
	$V_{\rm ACI}/V_{\rm exp}$	0,927	0,793	1,011
	$V_{\rm STR}/V_{\rm exp}$	0,625	0,598	0,653
	$V_{\text{DIN}}/V_{\text{exp}}$	0,486	0,465	0,508
	$V_{\rm EC2}/V_{\rm exp}$	0,480	0,459	0,502
4	$V_{\rm EC2}V_{\rm exp}$ $V_{\rm EC2Dr}/V_{\rm exp}$	0,744	0,711	0,777
7	$V_{\rm MC}/V_{\rm exp}$	0,496	0,474	0,517
	$V_{\rm BS}/V_{\rm exp}$	0,660	0,630	0,689
	$V_{\rm ACI}/V_{\rm exp}$	0,759	0,727	0,790
	$V_{\rm STR}/V_{\rm exp}$	0,623	0,589	0,658
	$V_{\rm DIN}/V_{\rm exp}$	0,485	0,458	0,512
	$V_{\rm EC2}/V_{\rm exp}$	0,454	0,429	0,479
5	$V_{\rm EC2Dr}/V_{\rm exp}$	0,742	0,700	0,783
3	$V_{\rm MC}/V_{\rm exp}$	0,494	0,467	0,783
	$V_{\rm BS}/V_{\rm exp}$	0,658	0,621	0,694
	$V_{\rm ACI}/V_{\rm exp}$	0,577	0,548	0,606
	$V_{\rm STR}/V_{\rm exp}$	0,533	0,511	0,555
	$V_{\rm DIN}/V_{\rm exp}$	0,414	0,397	0,432
	$V_{\rm EC2}/V_{\rm exp}$	0,427	0,409	0,444
7	$V_{\rm EC2Dr}/V_{\rm exp}$	0,658	0,631	0,685
•	$V_{\rm MC}/V_{\rm exp}$	0,439	0,421	0,457
	$V_{\rm BS}/V_{\rm exp}$	0,591	0,567	0,616
	$V_{\rm ACI}/V_{\rm exp}$	0,797	0,761	0,833
	$V_{\rm STR}/V_{\rm exp}$	0,526	0,506	0,546
	$V_{\rm DIN}/V_{\rm exp}$	0,409	0,393	0,425
	$V_{\rm EC2}/V_{\rm exp}$	0,416	0,400	0,432
8	$V_{\rm EC2Dr}/V_{\rm exp}$	0,649	0,624	0,674
5	$V_{\rm MC}/V_{\rm exp}$	0,433	0,416	0,449
	$V_{\rm BS}/V_{\rm exp}$	0,583	0,561	0,606
	$V_{\rm ACI}/V_{\rm exp}$	0,774	0,745	0,804
	· ACP ' exp	٠,,,,	0,, 10	0,001

$$\begin{cases} H_0: \overline{V_{calc}/V_{exp}} = \overline{V_{calc1}/V_{exp}}, \\ H_1: \overline{V_{calc}/V_{exp}} \neq \overline{V_{calc1}/V_{exp}}, \end{cases}$$
(12)

Verification of (12) hypothesis is done similarly as for hypothesis (8) applying (9) and (10) formulas, only instead of  $\overline{V_{\rm exp}}$  and  $\overline{V_{calc}}$  we use  $\overline{V_{calc}/V_{exp}}$  and

 $\overline{V_{calc1}/V_{exp}}$ , and instead of  $S_{\rm exp}$  and  $S_{calc}$  we use  $S_{calc/\exp}$  and  $S_{calc1/\exp}$ . Here  $\left\{S_{calc/\exp}, S_{calc1/\exp}\right\} \in \left\{S_{STR/\exp}, S_{EC2Dr/\exp}, S_{MC/\exp}, S_{BS/\exp}, S_{ACI/\exp}\right\}$ ,  $S_{\rm STR/\exp} - S_{\rm ACI/\exp}$  are estimates of standard  $V_{calc}/V_{exp}$  deviations given in Table 2. Due to abundant data t values and  $t_{\alpha/2}(k)$  critical value in verifying the hypothesis (12) are not provided. Table 5 provides the final results of the verifying hypothesis (12).

This Table also shows the theoretical methods used to calculate the punching shear force ranged by the proximity of the obtained punching shear values to the experimentally received punching shear values. First in a row are the methods where calculated punching shear force is the least different from the punching shear force obtained experimentally.

Column 3 in Table 5 shows the methods adequate to ranges 1, 2 etc. Column 4 shows the methods where the mean of the ratio between the theoretical punching shear values and the experimental punching shear values is insignificantly different from the mean of the ratio of the theoretical and experimental punching shear values of the method given in column 3, ie here hypothesis (12)  $H_0$  is in force.

As the results given in Table 5 show almost in all cases the punching shear force calculated by the EC2Dr method is the closest to the results obtained experimentally.

The  $\overline{V_{EC2Dr}/V_{exp}}$   $-\overline{V_{calc}/V_{exp}}$  difference in samples 1, 2, 5 is statistically significant in all methods except for ECDr. The  $\overline{V_{EC2Dr}/V_{exp}}$   $-\overline{V_{ACI}/V_{exp}}$  difference in samples 3 and 4 is statistically insignificant. Therefore, in this case we can state that ACI and EC2Dr methods similarly accurately calculate the punching shear force in respect to experimental results.

Punching shear force calculated by the ACI method is the closest to the experimental punching shear results obtained in samples 6 and 7. Besides, the  $\overline{V_{ACI}/V_{exp}}$  –  $\overline{V_{calc}/V_{exp}}$  difference is statistically significant when  $V_{calc}$  is calculated applying all the methods except ACI. These samples are special because reinforcement of slabs is minimal. It is known that in a minimally reinforced slab punching shear cone is 45°, which corresponds to the punching shear angle in ACI method. In limited reinforcement the shear force taken over by the longitudinal reinforcement is not big. Most part of the shear force is taken over by the concrete which is in the area of the punching shear cone. Therefore, absence of evaluation of reinforcement ratio  $\rho$  in ACI method does not cause a significant calculation error.

In this case, therefore, experimental results confirm the theoretical presumptions. This allows to make a conclusion that ACI method is the best to calculate the punching shear force in slabs. It is possible to notice from the data provided in Table 5 that, when the reinforcement percentage is high calculation results of punching shear

**Table 5.** Results of verifying hypothesis (12)

Sample Range of calculation		Calculation method	Methods when (6) H <sub>0</sub> hypothesis is
	method		accepted
1	2	3	4
	1	EC2Dr	CTD
	2	BS	STR
	3	STR	BS BRY EGG MG
1	4	ACI	DIN; EC2; MC
	5	MC	DIN; EC2; ACI
	6	DIN	EC2; MC; ACI
	7	EC2	DIN; MC; ACI
	1	EC2Dr	
	2	ACI	7.0
	3	STR	BS
2	4	BS	STR
	5	MC	DIN
	6	DIN	MC
	7	EC2	
	1	EC2Dr	ACI
	2	ACI	EC2Dr; STR; BS
3	3	STR	BS; ACI
3	4	BS	ACI; STR
	5	MC	DIN; EC2
	6	DIN	MC; EC2
	7	EC2	DIN; MC
	1	ACI	EC2Dr
	2	EC2Dr	ACI
	3	BS	STR
4	4	STR	BS
	5	MC	DIN; EC2
	6	DIN	MC; EC2
	7	EC2	MC; DIN
	1	EC2Dr	OTED
	2	BS	STR
5	3 4	STR	BS
3	5	ACI MC	DIN
	6	DIN	EC2; MC
	7	EC2	DIN
	1	ACI	DIN
	2	EC2Dr	
	3	BS	
6	4	STR	
	5	MC	EC2
	6	EC2	MC; DIN
	7	DIN	EC2
	1	ACI	-
	2	EC2Dr	
	3	BS	
7	4	STR	
	5	MC	EC2
	6	EC2	MC; DIN
	7	DIN	EC2

strength by applying the ACI method are less correct than applying other methods if compared to experimental results. This is clearly seen from comparison of samples 1 and 5  $\overline{V_{ACI}/V_{exp}} - \overline{V_{calc}/V_{exp}}$  (the Table 5). When the amount of reinforcement is approximately 1 %, the ACI

and the ECDr methods are equally good to calculate the punching shear force.

Analysis of the results given in Table 5 also clearly shows that when reinforcement is strong, ie samples 1 and 5 in the second position, according to  $\overline{V_{calc}/V_{exp}}$  proximity to 1, is  $V_{calc}$  values calculated by the BS method, and the third in a row is the STR method. Since  $\overline{V_{BS}/V_{exp}} - \overline{V_{calc}/V_{exp}}$  and  $\overline{V_{STR}/V_{exp}} - \overline{V_{calc}/V_{exp}}$  differences are statistically insignificant, it is possible to state that, when reinforcement is bigger than 1,6 %, BS and STR methods are second in a row to make an accurate calculation of punching shear. When reinforcement is small, better results than ACI and EC2Dr are obtained by applying the BS method, which is clearly seen from samples 6 and 7 (Table 5).

Punching shear values obtained using EC2 and DIN methods are the most different from the experimental results. This is clearly seen in Table 5.

### 4. Conclusions

Generally, the difference of the punching shear force in slabs calculated applying STR; DIN; EC2; EC2Dr; MC; BS; ACI methods from the punching shear force in slabs obtained experimentally is statistically significant. This shows that none of the analysed methods allows an accurate calculation of punching shear force.

Generally, the method allowing the most accurate calculation of punching shear force is EC2Dr method.

When reinforcement is minimal, less than 0.5%, ACI is the best to make an accurate calculation of punching shear force.

When reinforcement is  $\rho \ge 1,6$  %, BS and STR methods are second in a row to calculate punching shear accurately.

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# KOLONOS-PLOKŠTĖS JUNGTIES BE SKERSINIO ARMAVIMO VEIKIANT SUTELKTAJAI APKROVAI PRASPAUDŽIAMOJO STIPRIO NORMATYVINIŲ SKAIČIAVIMO METODIKŲ STATISTINĖ ANALIZĖ

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Santrauka

Darbe nagrinėjamas gelžbetoninių plokščių praspaudžiamojo stiprio skaičiavimo normatyvinių metodikų STR 2.05.05:2005, E DIN 1045-1, ENV 1992-1-1 EC 2, prEN 1992-1 [Final draft] EC 2, Model Code CEB-FIP 1990, BS 8110, AC 318 atitikimas eksperimentiniams duomenims. Išanalizuota, ar pagal šias metodikas apskaičiuotų ir eksperimentiškai nustatytų praspaudžiamojo vidutinių stiprio reikšmių skirtumas statistiškai reikšmingas, kai reikšmingumo lygmuo yra 0,05. Vidurkių skirtumo reikšmingumo analizei naudotas Stjudento t kriterijus. Išnagrinėta, pagal kurias metodikas apskaičiuotos stiprio reikšmės mažiausiai skiriasi nuo eksperimentinių duomenų. Tuo remiantis metodikoms priskirti rangai. Taikant Stjudento t kriterijų, išanalizuota, pagal kurias metodikas apskaičiuotų praspaudžiamojo stiprio reikšmių santykis su eksperimentiškai nustatytomis praspaudžiamojo stiprio reikšmėmis statistiškai nereikšmingas. Reikšmingumo lygmuo imtas 0,05.

Nustatyta, kad beveik visais atvejais skirtumas tarp eksperimentiškai ir teoriškai apskaičiuotų praspaudžiamojo stiprio reikšmių statistiškai reikšmingas. Taip pat nustatyta, kad tiksliausiai praspaudžiamąjį stiprį galima apskaičiuoti pagal prEN 1992-1 [Final draft] EC 2 metodiką.

**Reikšminiai žodžiai**: gelžbetoninių plokščių praspaudžiamasis stipris, gelžbetoninių plokščių projektavimo normos, praspaudžiamojo stiprio statistinė analizė.

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