JOURNAL OF CIVIL ENGINEERING AND MANAGEMENT
2009
15(1): 77–86

DYNAMIC ANALYSIS OF STRUCTURES WITH MULTIPLE TUNED MASS DAMPERS

Roman Lewandowski[1], **Justyna Grzymisławska**[2]
Institute of Structural Engineering, Poznan University of Technology, ul. Piotrowo 5, 60-965 Poznań, Poland
E-mail: [1]*roman.lewandowski@put.poznan.pl;* [2]*justyna.grzymislawska@put.poznan.pl*

Received 16 May 2008, accepted 20 Nov 2008

Abstract. It is the purpose of this paper to analyse the possibility of reducing the vibrations of frame building structures with the help of multiple tuned mass dampers. Structures exposed to strong winds are considered. Excitation forces, which are functions of wind velocity fluctuations, are treated as random forces. The spectral density functions of wind velocity fluctuations are assumed as proposed by Davenport. The correlation theory of random vibration is used and the root mean squares of displacements and accelerations are determined. Several remarks, concerning the effectiveness of multiple tuned or mass dampers, are formulated from the results of calculation.

Keywords: reduction of vibration, random vibration, multiple-tuned mass dampers (MTMD).

1. Introduction

Mass dampers have been used for reducing the vibrations of structures for many years (McNamara 1977). They have been successfully used in reducing the vibrations of building structures subjected to strong winds and seismic excitations (Xu *et al.* 1992). Principally, tuned mass dampers (TMD) installed on top floors have been studied. They have been designed in such a way that they are tuned to the fundamental mode of vibration. In the paper by Warburton (1982), a method for optimization of various types of excitation forces was presented. The formulae given by him there have often been used to design TMD parameters, when reduction of dynamical displacements and/or accelerations is required. Reduction of accelerations is important due to undesired influences, exerted not only upon the building structure, but also on people inside. The problems of TMD analysis and designing are still present in scientific papers. For example, in his excellent paper Krenk (2005) derived a new formula for the TMD optimal damping coefficient. Moreover, in paper (Leung *et al*. 2008) used the particle swarm optimization method to optimise the TDM parameters in the case of non-stationary excited structures. Optimization of TMD parameters is also the subject of paper (Singh *et al.* 2002).

In the 90's, studies on the application of multiple tuned mass dampers (MTMD) for one-degree of freedom systems were started (Xu, Igusa 1992; Igusa, Xu 1994). It has been proved that MTMD with distributed natural frequencies are more effective than TMD. The studies of MTMD were also developed in (Kareem, Klime 1995; Jangid 1995). Later on, structures subjected to seismic loads, treated as a multi degree of freedom structures and with the MTMD on them were analysed in (Chen, Wu 2001). The MTMD were designed in such a way that they are tuned to several modes of structure vibration. The number of dampers depends on the number of vibration modes for which dampers are tuned. The performance of multiple mass dampers under both wind and seismic excitation is analysed by Kareem and Kline (1995).

The effectiveness and robustness of a particular version of MTMD, called "the multiple dual tuned mass dampers", is analysed in the paper (Han, Li 2006). The problem of determination of optimum properties of MTMD is considered in the papers (Li, Qu 2006; Li 2002). Spatial structures with MTMD are analysed in (Guo, Chen 2007).

Moreover, the possibilities of using the so-called active and semi-active versions of TMD are also considered in a number of papers (Han, Li 2006; Li, Han 2007; Li, Zhu 2007; Lin *et al.* 2005).

The practical application of TMD on an extremely high telecommunication tower is described in a paper (Ghorbani-Tanha *et al.* 2008).

Up to now, reduction of vibration of structures with MTMD caused by earthquake forces are mainly investigated. The analysis of such type of structures under wind loads are rare (Kareem, Kline 1995) and the dynamic behaviour of structures with MTMD are not fully understand. For this reasons, in the present paper, the possibility to reduce the vibration of a frame structure with the help of MTMD is analysed. The presented description of the structure with MTMD exploits a particular form of the motion equations to simplify the numerical algorithm of the applied method of solution. The structure is under the effect of dynamic forces caused by wind pressure. Wind velocities are treated as random and ergodic processes. The spectral density functions of wind velocity fluctuations are assumed as proposed by Davenport.

ISSN 1392–3730 print / ISSN 1822–3605 online
DOI: 10.3846/1392-3730.2009.15.77-86

Some calculations were made for a 20-story building and on this basis the effectiveness of MTMD was estimated. The effects of detuning of structure parameters are also presented. In this case, the reduction of accelerations of structures with MTMD is noticeably greater than the structure without or with TMD.

2. Designing of multiple tuned mass dampers (MTMD)

The aim of designing MTMD is to tune damper parameters to the modal parameters of selected modes of vibration. It means that the natural damper frequency (or a group of dampers) ω_d must be close to the natural frequency of a selected vibration mode of structure ω_s $(\omega_d \approx \omega_s)$. Moreover, the damping factor of the damper must be appropriately chosen.

The optimal parameters of such a damper (or group of dampers) can be determined from the formulae given in a paper (Warburton 1982). The optimal frequency ratio is determined from:

$$\frac{\omega_d^2}{\omega_s^2} = \frac{2+\mu}{2(1+\mu)^2}, \tag{1}$$

where

$$\mu = \frac{m_d}{M_s}, \quad \omega_s^2 = \frac{K_s}{M_s}, \quad \omega_d^2 = \frac{k_d}{m_d}. \tag{2}$$

Here M_s and K_s is the modal mass of the structure and the modal stiffness of the s-th mode of vibration, respectively.

If only a single damper is tuned to the s-th mode of vibration with frequency ω_s, then m_d is the mass of the damper, and k_d is the stiffness coefficient of the damper. However, if a group of dampers are designed to tune to the frequency ω_s, then m_d and k_d denote the mass and the stiffness coefficients of the selected damper of this group, respectively.

Assuming that the mass ratio μ is known, the damper frequency ω_d and the damper stiffness coefficient k_d can be obtained from the above formulae.

If excitation forces acting on a structure, have a random character and can be treated as white-noise excitation, the optimal value of non-dimensional damping coefficient is determined from the formula (McNamara 1977; Warburton 1982):

$$\gamma_{opt} = \sqrt{\frac{\mu(4+3\mu)}{8(1+\mu)(2+\mu)}}. \tag{3}$$

The value of the damping coefficient c_d can be calculated from the relation

$$c_d = 2\gamma_{opt}\omega_d m_d. \tag{4}$$

Using the above formulae, the parameters of MTMD can be determined.

3. Equation of motion

The building structure is treated as a discrete, linear elastic system. The frame in Fig. 1 is the model of the building structure. The mass of it is concentrated at the level of building floors and the beams of the frame are infinitely stiff. Horizontal displacements of floors are the dynamic degrees of freedom. The fluctuations of wind velocity forces are a load to the frame, and these forces are applied at the building floor levels (Fig. 1).

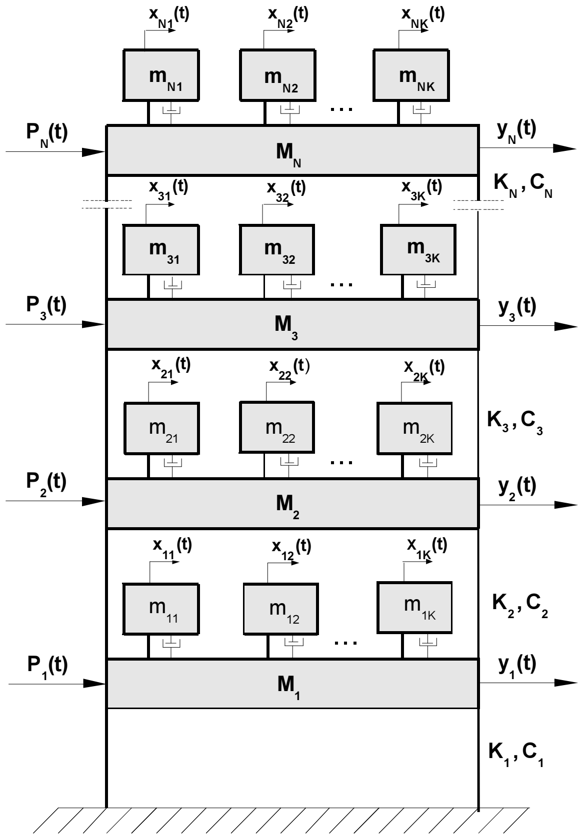

Fig. 1. The model of structure with MTMD

A set of mass dampers are mounted on the structure. A model configuration of dampers is also shown in Fig. 1, while in Fig. 2 the scheme of a typical mass damper is presented. The concept of a group of dampers is introduced in this paper. Each group of dampers consists of a few dampers. Each damper in a particular group of dampers can be installed on different floors and may have different mass, stiffness and damping parameters. However, all dampers in the group are designed in such a way that they are tuned to a particular mode of vibration. A special notation described below and concerning dampers is introduced. The symbols $x_{ij}(t)$, m_{ij}, k_{ij} and c_{ij} denote, respectively, the damper displacement, damper mass, stiffness and damping factor of the damper which belongs to the j-th group and is located on the i-th floor (Fig. 2).

The equation of motion of the system shown in Fig. 1 and briefly described above can be written in the following form:

$$\tilde{\mathbf{M}}\ddot{\mathbf{q}}(t) + \tilde{\mathbf{C}}\dot{\mathbf{q}}(t) + \tilde{\mathbf{K}}\mathbf{q}(t) = \tilde{\mathbf{P}}(t), \tag{5}$$

where $\tilde{\mathbf{M}}$, $\tilde{\mathbf{C}}$, $\tilde{\mathbf{K}}$ are the global matrices of mass, damping and stiffness of the considered system (i.e. the structure and MTMD), respectively, $\mathbf{q}(t) = col(\mathbf{y}(t), \mathbf{x}(t))$ is the vector of displacements of the system, $\mathbf{y}(t)$ – the vector of horizontal displacements of frame, and $\mathbf{x}(t)$ – the vector of horizontal displacements of dampers. Moreover, $\widetilde{\mathbf{P}}(t) = col(\mathbf{P}(t),\ \mathbf{0})$ and $\mathbf{P}(t)$ is the vector of excitation forces acting upon the structure.

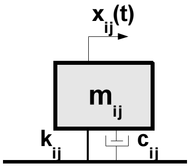

Fig. 2. Diagram of damper

The theory presented below could be applied to the non-proportionally damped structures. However, in the paper we assume that the structure is proportionally damped, i.e. the damping matrix of the structure is in the following form: $\mathbf{C} = \alpha\, \mathbf{M} + \kappa\, \mathbf{K}$.

The $\widetilde{\mathbf{M}}$ matrix of the system is in the following form (Fig. 1):

$$\widetilde{\mathbf{M}} = \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{m} \end{bmatrix} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{22} \end{bmatrix}, \tag{6a}$$

where

$$\mathbf{M} = diag\left[M_1, M_2, M_3, ..., M_N\right],$$

$$\mathbf{m} = diag[m_{11}, m_{12}, ..., m_{1K}, m_{21}, m_{22,} ...$$
$$...m_{2K}, m_{31}, m_{32}, ...m_{3K}, ...m_{N1}, m_{N2}, ...m_{NK}].$$

In the above formula, $\mathbf{M}$ is the mass matrix of the structure and $\mathbf{m}$ – the mass matrix of the dampers. The symbol m_{ij} denotes the mass damper of the j -th group located on the i -th floor.

The stiffness matrix $\widetilde{\mathbf{K}}$ of the considered system can also be shown in the block form written below:

$$\widetilde{\mathbf{K}} = \begin{bmatrix} \mathbf{K} + \mathbf{k}_1 & \mathbf{k}^* \\ \mathbf{k}^{*T} & \mathbf{k} \end{bmatrix} = \begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} \end{bmatrix}, \tag{6b}$$

where $\mathbf{K}$ is the stiffness matrix of the structure

$$\mathbf{K} = \begin{bmatrix} K_1 + K_2 & -K_2 & 0 & 0 \\ -K_2 & K_2 + K_3 & -K_3 & 0 \\ 0 & -K_3 & ... & -K_N \\ 0 & 0 & -K_N & K_N \end{bmatrix}.$$

The block matrices $\mathbf{k}_1$ and $\mathbf{k}^*$ are in the following form:

$$\mathbf{k}_1 = diag[k_{11} + k_{12} + ... + k_{1K}, k_{21} + k_{22} + ... + k_{2K},$$
$$k_{31} + k_{32} + ... + k_{3K}, ..., k_{N1} + k_{N2} + ... + k_{NK}]$$

$$\mathbf{k}^* = \left[\begin{array}{cccccccc} -k_{11} & -k_{12} & ... & -k_{1K} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -k_{21} & -k_{22} & ... & k_{2K} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right.$$
$$\left.\begin{array}{ccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -k_{31} & -k_{32} & ... & -k_{3K} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & ... & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -k_{N1} & -k_{N2} & ... & -k_{NK} \end{array}\right].$$

As mentioned above, the symbol k_{ij} denotes the stiffness coefficient of a damper of the j-th group which is located on the i-th floor (Fig. 2).

The $\mathbf{k}$ block of the matrix $\widetilde{\mathbf{K}}$ is the diagonal matrix and in the following form:

$$\mathbf{k} = diag[k_{11}, k_{12}, ..., k_{1K}, k_{21}, k_{22,} ...$$
$$...k_{2K}, k_{31}, k_{32}, ...k_{3K}, ... \quad .$$
$$...k_{N1}, k_{N2}, ...k_{NK}].$$

The damping matrix of the system $\widetilde{\mathbf{C}}$ is in a form similar to that of the stiffness matrix $\widetilde{\mathbf{K}}$. The specific blocks of this matrix are defined below:

$$\widetilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C} + \mathbf{c}_1 & \mathbf{c}^* \\ \mathbf{c}^{*T} & \mathbf{c} \end{bmatrix} = \begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{C}_{22} \end{bmatrix}, \tag{6c}$$

$$\mathbf{C} = \begin{bmatrix} \overline{C}_1 + \overline{C}_2 & -C_2 & 0 & 0 \\ -C_2 & \overline{C}_2 + \overline{C}_3 & -C_3 & 0 \\ 0 & -C_3 & ... & -C_N \\ 0 & 0 & -C_N & \overline{C}_N \end{bmatrix},$$

$$\mathbf{c}_1 = diag[c_{11} + c_{12} + ... + c_{1K}, c_{21} + c_{22} + ... + c_{2K},$$
$$c_{31} + c_{32} + ... + c_{3K}, ..., c_{N1} + c_{N2} + ...$$
$$... + c_{NK}]$$

$$\mathbf{c}^* = \left[\begin{array}{cccccccc} -c_{11} & -c_{12} & ... & -c_{1K} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -c_{21} & -c_{22} & ... & c_{2K} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right.$$
$$\left.\begin{array}{ccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -c_{31} & -c_{32} & ... & -c_{3K} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & ... & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -c_{N1} & -c_{N2} & ... & -c_{NK} \end{array}\right],$$

where $\overline{C}_i = \alpha\ M_i + \kappa\, K_i$ and $C_i = \kappa\, K_i$.

In the above formulae the symbol c_{ij} denotes the damping coefficient of the damper of the j -th group which is located on the i -th floor (Fig. 2). The block $\mathbf{c}$ of the $\widetilde{\mathbf{C}}$ matrix is the diagonal matrix and has the following form:

$$\mathbf{c} = diag[c_{11}, c_{12}, ..., c_{1K}, c_{21}, c_{22,} ...$$
$$...c_{2K}, c_{31}, c_{32}, ...c_{3K}, ...$$
$$...c_{N1}, c_{N2}, ...c_{NK}]$$

Taking into account that the matrices of mass, stiffness and damping are in the form (6), the equation of motion (5) can be rewritten in the following block matrix form:

$$\mathbf{M}_{11}\ddot{\mathbf{y}}(t) + \mathbf{C}_{11}\dot{\mathbf{y}}(t) + \mathbf{C}_{12}\dot{\mathbf{x}}(t) + \mathbf{K}_{11}\mathbf{y}(t) + \mathbf{K}_{12}\mathbf{x}(t) = \mathbf{P}(t), \tag{7a}$$

$$\mathbf{M}_{22}\ddot{\mathbf{x}}(t) + \mathbf{C}_{21}\dot{\mathbf{y}}(t) + \mathbf{C}_{22}\dot{\mathbf{x}}(t) + \mathbf{K}_{21}\mathbf{y}(t) + \mathbf{K}_{22}\mathbf{x}(t) = \mathbf{0}. \tag{7b}$$

4. Modelling of wind loads

Wind speed acting on a structure consists of along-wind and cross-wind components, and it varies randomly in time and space (Simiu, Scanlan 1996; Dyrbye, Hansen 1999; Holmes 1997). A complete wind velocity field should be modelled as a two-dimensional, multivariate stochastic process. Usually, the wind speed is treated as a stationary Gaussian stochastic process (Simiu, Scanlan 1996; Dyrbye, Hansen 1999). The wind speed $U(z,t)$ is assumed to be the sum of a steady part $U(z)$ and a superimposed random fluctuation of wind velocities $u(z,t)$, i.e.

$$U(z,t) = U(z) + u(z,t) \ .$$

The random fluctuation of wind velocity $u(z,t)$ is a zero-mean stationary Gaussian process with a known correlation function.

The along wind speed described above is a stochastic process that is continuous in space and time. When high buildings are considered it is necessary to introduce some simplifications, to replace the continuous space and time random function $u(z,t)$ with a set of functions $u_i(t)$ which depend on time only. The building is divided into *N* section along its height. It is assumed that the wind speed does not vary along the section. The typical midpoint of the section is chosen at a structure storey level. It means that wind velocity fluctuations $u(z,t)$ can be replaced by a set of zero-mean stationary processes $u_i(t)$, where $i=1,\ 2,\ ...,N$. Thus, the wind force in the midpoint of an arbitrarily chosen structure section can be described in the following way:

$$P_i(t) = C_A A \rho U_i X^2 u_i(t) \ , \tag{8}$$

where C_A is the aerodynamic drag coefficient, A – the wind-exposed area and the ρ symbol denotes the air density.

The admittance function X describes the influence of the building on wind pressure forces and it is always that $X \le 1$. According to Holmes (1997), the admittance function is connected with a correlation coefficient Φ, which is used to determine the matrix elements of the spectral density function. It is troublesome to determine the admittance function. Moreover, this value is unknown in many cases. Therefore, quite often, and also in this paper, it is assumed that $X = 1$.

For multi-degree-of-freedom systems the correlation matrix of the fluctuations of wind velocities is formulated as:

$$\mathbf{R}_u(\tau) = E[\mathbf{u}\,\mathbf{u}^T] \ , \tag{9}$$

where $\mathbf{u} = col\{u_1(t), u_2(t),..., u_i(t),...., u_N(t)\}$ is the vector of the fluctuations of wind velocity and the symbol $E[\cdot]$ denotes the expected value of $[\cdot]$.

Using the Fourier transform, the following expression of the spectral density function of wind velocity fluctuations is obtained:

$$\mathbf{S}_u(\lambda) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathbf{R}_u(\tau) e^{-i\lambda\tau} d\tau \ . \tag{10}$$

In this paper, the spectral density function proposed by Davenport (Simiu, Scanlan 1996; Dyrbye, Hansen 1999; Holmes 1997) is used. The elements of the matrix $\mathbf{S}_u(\lambda)$ are calculated from the formula:

$$S_{lk}^u = \sqrt{S_{ll}^u(\lambda) S_{kk}^u(\lambda)}\, e^{-\Phi} = \sqrt{S_u(\lambda, z_l) S_u(\lambda, z_k)}\, e^{-\Phi} \ . \tag{11}$$

where $S_u(\lambda, z_l)$ and $S_u(\lambda, z_k)$ are the elements taken from the main diagonal of the $\mathbf{S}_u(\lambda)$ matrix. They are calculated with the help of the spectral density function for the particular stories.

The diagonal elements of the matrix spectral density function of wind velocity fluctuations $S_u(\lambda, z_i)$ are calculated using the spectral density function as proposed by Davenport (Simiu, Scanlan 1996; Dtrbye, Hansen 1999; Holmes 1997)

$$S_u(n) = \frac{4u_*^2 f^2(n)}{n[1 + f^2(n)]^{4/3}} \ , \tag{12}$$

where

$$f(n) = \frac{1200n}{U(10)} \ , \tag{13}$$

and n denotes frequency in Hz.

The mean wind velocity acting at the level of the *i*-th floor can be calculated from formula:

$$U_i(z_i) = 2{,}5u_* \ln\left(\frac{z_i}{z_0}\right) , \tag{14}$$

where

$$u_* = U(10)\sqrt{k} \ . \tag{15}$$

In relationships (14) and (15), $U(10)$ is the mean wind velocity at the altitude of 10 m, k – the coefficient depended on type of area, z_0 – the roughness length and the symbol z_i denotes the altitude of the *i*-th floor over ground.

The Φ symbol denotes the correlation coefficient, which takes into consideration spatial correlations of the fluctuations of wind velocity. According to monographs (Li, Zhu 2007; Dyrbye, Hansen 1999), this coefficient can be determined from the formula:

$$\Phi = \frac{2\lambda C_z |z_l - z_k|}{U(z_l) + U(z_k)} \ , \tag{16}$$

where λ is the force frequency, whereas C_z is the empirical constant. The symbol $U(z_l)$ is the mean wind velocity at the level of the *l*-th story. If it is assumed that the fluctuations of wind velocity are totally correlated, then $e^{-\Phi} = 1$ while, if the correlation is disregarded, the matrix $\mathbf{S}_u(\lambda)$ is the diagonal one.

The correlation matrix of the forces excited by the wind pressure can be written in the following form:

$$\mathbf{R}_p(\tau) = E[\mathbf{P}\,\mathbf{P}^T] \ , \tag{17}$$

where $\mathbf{P} = col\{P_1(t), P_2(t),..., P_i(t),...., P_N(t)\}$ is the vector of wind forces acting upon the structure.

Using relationships (8) and (9), the elements $R_{P_iP_j}$ of the $\mathbf{R}_p(\tau)$ matrix can be written as

$$R_{P_iP_j}(\tau) = (C_A A \rho X)^2 U_i U_j R_{u_i u_j}(\tau). \quad (18)$$

The spectral density matrix of excitation forces $\mathbf{S}_p(\lambda)$ and the correlation matrix $\mathbf{R}_p(\tau)$ are interrelated in such a way that

$$\mathbf{S}_p(\lambda) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathbf{R}_p(\tau) e^{-i\lambda\tau} d\tau,$$

which means that the element $S_{P_iP_j}$ of the $\mathbf{S}_p(\lambda)$ matrix can be written in the following form:

$$S_{P_iP_j}(\tau) = (C_A A \rho X)^2 U_i U_j S_{u_i u_j}(\tau). \quad (19)$$

This is the relationship between the spectral density matrix of wind loads acting upon the structure and the spectral density matrix of the fluctuations of wind velocity.

5. Solution to equation of motion

A solution to the equation of motion (5) that fulfills the precondition: $t = 0$, $\dot{\mathbf{q}}(t) = \mathbf{0}$, $\mathbf{q}(t) = \mathbf{0}$, can be written in the following form:

$$\mathbf{q}(t) = \int_0^t \mathbf{h}(t-\tau)\,\widetilde{\mathbf{P}}(\tau)\,d\tau, \quad (20)$$

where the symbol $\mathbf{h}(t-\tau)$ denotes the matrix of impulse transfer function and $\widetilde{\mathbf{P}}(\tau) = col(\mathbf{P}(\tau), \mathbf{0})$ is the vector of excitation forces appearing in Eq (5).

Because random loads acting upon the structure are stationary processes, then also dynamic responses of the system are a stationary process. Thus, the correlation matrix of the structure responses can be written as:

$$\mathbf{R}_q(t_1, t_2) = E[\mathbf{q}(t_1), \mathbf{q}^T(t_2)]. \quad (21)$$

By substituting (20) into (21), we obtain:

$$\mathbf{R}_q(t_1, t_2) = \int_0^{t_1}\int_0^{t_2} \mathbf{h}(t_1-\tau_1) E[\widetilde{\mathbf{P}}(\tau_1)\widetilde{\mathbf{P}}^T(\tau_2)]\,\mathbf{h}^T(t_2-\tau_2)\,d\tau_1 d\tau_2. \quad (22)$$

Taking into account that

$$\mathbf{R}_{\widetilde{p}}(\tau) = \int_{-\infty}^{+\infty} \mathbf{S}_{\widetilde{p}}(\lambda)\, e^{i\lambda\tau} d\lambda, \quad (23)$$

$$E[\widetilde{\mathbf{P}}(t), \widetilde{\mathbf{P}}(t-\tau)] = \int_{-\infty}^{+\infty} \mathbf{S}_{\widetilde{p}}(\lambda) e^{i\lambda\tau} d\tau. \quad (24)$$

Eq (22) can be rewritten in the form:

$$\mathbf{R}_q(\tau) = \int_{-\infty}^{+\infty} \widetilde{\mathbf{H}}(\lambda)\,\mathbf{S}_{\widetilde{p}}(\lambda)\,\overline{\widetilde{\mathbf{H}}}^T(\lambda)\, e^{i\lambda\tau} d\tau, \quad (25)$$

where $\overline{\widetilde{\mathbf{H}}}$ is the matrix conjugate to the $\widetilde{\mathbf{H}}$ matrix defined below

$$\widetilde{\mathbf{H}}(\lambda) = \left(\widetilde{\mathbf{K}} - \lambda^2\widetilde{\mathbf{M}} + i\lambda\widetilde{\mathbf{C}}\right)^{-1}. \quad (26)$$

Moreover,

$$\mathbf{S}_{\widetilde{p}}(\lambda) = \begin{bmatrix} \mathbf{S}_P(\lambda) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad (27)$$

where the $\mathbf{S}_P(\lambda)$ is a matrix of which the elements are given by formula (19).

After inserting $\tau = 0$ into Eq (25) the correlation matrix of displacements $\mathbf{R}_q(0)$ is obtained and, on this basis, the root mean square of displacements can be determined from

$$\mathbf{R}_q(0) = \int_{-\infty}^{+\infty} \mathbf{S}_q(\lambda) d\lambda, \quad (28)$$

where

$$\mathbf{S}_q(\lambda) = \begin{bmatrix} \mathbf{S}_{yy} & \mathbf{S}_{yx} \\ \mathbf{S}_{xy} & \mathbf{S}_{xx} \end{bmatrix} = \widetilde{\mathbf{H}}(\lambda)\mathbf{S}_{\widetilde{p}}(\lambda)\overline{\widetilde{\mathbf{H}}}^T(\lambda). \quad (29)$$

The integral appearing in Eq (29) can be calculated numerically.

The calculation of the $\mathbf{S}_q(\lambda)$ matrix is substantially simplified if we take into account the structure of the $\mathbf{S}_{\widetilde{p}}(\lambda)$ and $\widetilde{\mathbf{H}}(\lambda)$ matrices.

After writing the $\widetilde{\mathbf{H}}(\lambda)$ matrix in the following block form:

$$\widetilde{\mathbf{H}}(\lambda) = \begin{bmatrix} \mathbf{H}_{11}(\lambda) & \mathbf{H}_{12}(\lambda) \\ \mathbf{H}_{21}(\lambda) & \mathbf{H}_{22}(\lambda) \end{bmatrix} \quad (30)$$

and introducing Eq (27) and (30) into (29) we obtain:

$$\mathbf{S}_{yy}(\lambda) = \mathbf{H}_{11}(\lambda)\mathbf{S}_p(\lambda)\overline{\mathbf{H}}_{11}^T(\lambda), \quad (31)$$

$$\mathbf{S}_{xx}(\lambda) = \mathbf{H}_{21}(\lambda)\mathbf{S}_p(\lambda)\overline{\mathbf{H}}_{21}^T(\lambda), \quad (32)$$

$$\mathbf{S}_{yx}(\lambda) = \mathbf{H}_{11}(\lambda)\mathbf{S}_p(\lambda)\overline{\mathbf{H}}_{21}^T(\lambda), \quad (33)$$

$$\mathbf{S}_{xy}(\lambda) = \mathbf{H}_{21}(\lambda)\mathbf{S}_p(\lambda)\overline{\mathbf{H}}_{11}^T(\lambda). \quad (34)$$

It is easy to observe that the root mean square of structure displacements and the root mean square of dampers displacements can be calculated from the following relationships:

$$\mathbf{R}_y(0) = \int_{-\infty}^{+\infty} \mathbf{S}_{yy}(\lambda) d\lambda, \quad (35)$$

$$\mathbf{R}_x(0) = \int_{-\infty}^{+\infty} \mathbf{S}_{xx}(\lambda) d\lambda, \quad (36)$$

respectively. It means that, in fact, only the $\mathbf{S}_{yy}(\lambda)$ and $\mathbf{S}_{xx}(\lambda)$ matrices must be calculated.

Now, the matrices $\mathbf{H}_{11}(\lambda)$ and $\mathbf{H}_{21}(\lambda)$, which are blocks of matrix $\widetilde{\mathbf{H}}(\lambda)$, must be determined. This can be done by assuming the excitation and the solution to Eq (7) in the form:

$$\mathbf{P}(t) = \mathbf{I}\exp(i\lambda t), \quad (37)$$

$$\mathbf{y}(t) = \mathbf{H}_{11}(\lambda)\exp(i\lambda t), \quad (38)$$

$$\mathbf{x}(t) = \mathbf{H}_{21}(\lambda)\exp(i\lambda t). \quad (39)$$

where $\mathbf{I}$ denotes the identity matrix.

After introducing Eq (37–39) into Eq (7) we obtain:

$$(\mathbf{K}_{11} - \lambda^2\mathbf{M}_{11} + i\lambda\mathbf{C}_{11})\,\mathbf{H}_{11} + (\mathbf{K}_{12} + i\lambda\mathbf{C}_{12})\,\mathbf{H}_{21} = \mathbf{I}, \quad (40)$$

$$(\mathbf{K}_{22} - \lambda^2\mathbf{M}_{22} + i\lambda\mathbf{C}_{22})\,\mathbf{H}_{21} + (\mathbf{K}_{21} + i\lambda\mathbf{C}_{21})\,\mathbf{H}_{11} = \mathbf{0}\,. \tag{41}$$

From Eq (41) it follows that

$$\mathbf{H}_{21} = \mathbf{G}(\lambda)\,(\mathbf{K}_{21} + i\lambda\mathbf{C}_{21})\,\mathbf{H}_{11}\,, \tag{42}$$

where the matrix

$$\mathbf{G} = -(\mathbf{K}_{22} - \lambda^2\mathbf{M}_{22} + i\lambda\,\mathbf{C}_{22})^{-1}\,, \tag{43}$$

is easy to calculate because the matrices $\mathbf{K}_{22}$, $\mathbf{M}_{22}$ and $\mathbf{C}_{22}$ are diagonal.

After introducing Eq (42) into (40) we obtain

$$\mathbf{H}_{11} = \left[(\mathbf{K}_{11} - \lambda^2\mathbf{M}_{11} + i\lambda\mathbf{C}_{11}) + (\mathbf{K}_{12} + i\lambda\mathbf{C}_{12})\mathbf{G}(\lambda)\,(\mathbf{K}_{21} + i\lambda\mathbf{C}_{21})\right]^{-1}. \tag{44}$$

6. Results of exemplary calculations

In this section, the results of dynamic analysis of the exemplary structure with MTMD are discussed. Additionally, for comparison, results for the structure with only one tuned mass damper (TMD), which is tuned to the first vibration mode of structure will be presented. The above-mentioned TMD is located on the top floor of the structure.

The building parameters were calculated on the basis of paper (Spencer *et al.*) and they are given in Table 1.

Table 1. Main parameters of structure

Story	Mass [kg]	Stiffness [N/m]
1	2.83×10^5	3.31×10^8
2–4	2.76×10^5	1.06×10^9
4–7	2.76×10^5	6.79×10^8
8–10	2.76×10^5	6.79×10^8
11–13	2.76×10^5	5.84×10^8
14–16	2.76×10^5	3.86×10^8
17–19	2.76×10^5	3.47×10^8
20	2.92×10^5	2.29×10^8

Damper parameters were designed using Formulae (1–4) and assuming that these parameters tune dampers to the structure's first three modes of vibration. The shapes of the first, second and third mode of vibration are shown in Fig. 3. In this case, it has been assumed that 3 groups of dampers are installed on the structure. Each group of dampers consists of one damper only. All dampers are located on the top floor. The damper parameters and their locations on the structures are given in Table 2. The total mass of MTMD is nearly equal (by 4.4% smaller) to the mass of TMD.

Moreover, the values of non-dimensional damping coefficients of the first and second vibration modes are equal to 1% of critical damping.

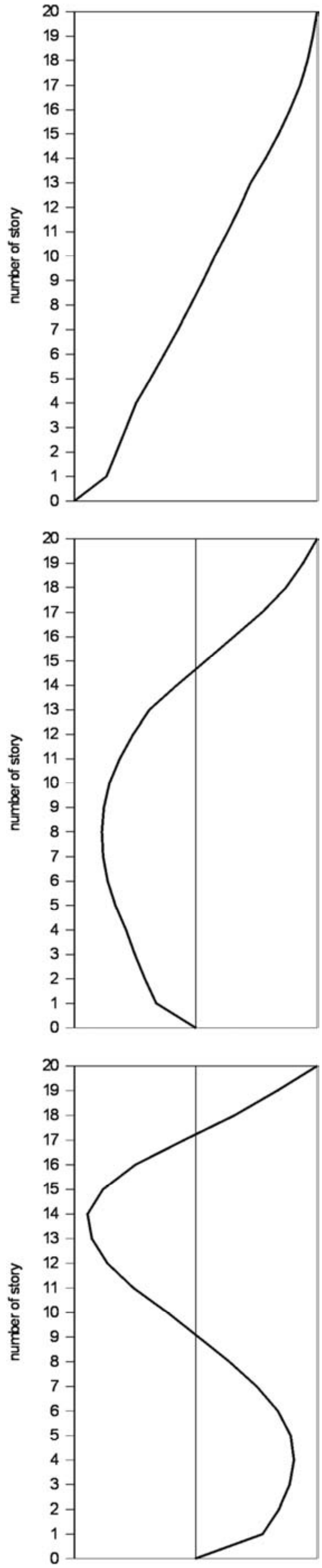

Fig. 3. First 3 vibration modes of structure

The following values of parameters appearing in Relationships (25–28) are chosen: $z_0 = 0{,}3$, $\rho = 1.226\ \text{kg/m}^3$, $U(10) = 30\ \text{m/s}^2$, $k = 12 \cdot 10^{-3}$.

Table 2. Parameters of dampers

Number of mode/ placement	Mass [kg]	Stiffness [N/m]
TMD		
1/20	36214	472468
MTMD		
1/20	18107	238870
2/20	7956	722685
3/20	8550	2182386

Because the dynamic response of the structure is a stationary and ergodic random process, the root mean square of freely chosen displacement q_i and acceleration $\ddot{q}_i$ could be calculated from the following formulae:

$$\sigma_{q_i}^2 = \int_{-\infty}^{+\infty} S_{ii}^q(\lambda)d\lambda\,,\ \sigma_{\ddot{q}_i}^2 = \int_{-\infty}^{+\infty} \lambda^4 S_{ii}^q(\lambda)d\lambda\,, \qquad (45)$$

where S_{ii}^q is the diagonal element of the $\mathbf{S}_q(\lambda)$ matrix.

Using the above formulae, an analysis of the structure without dampers, with installed conventional TMD and with MTMD was made. The results of the analysis are shown in Figs 4 and 5. In Fig. 4 the root mean square of structure displacements is shown. It has been observed that displacements reduction with MTMD installed is a little smaller than in the case of TMD installed on the structure. Compared with the structure without dampers, the maximum reduction of root mean square of structure displacements (top floor) is 30% for TMD and 25% for MTMD, respectively.

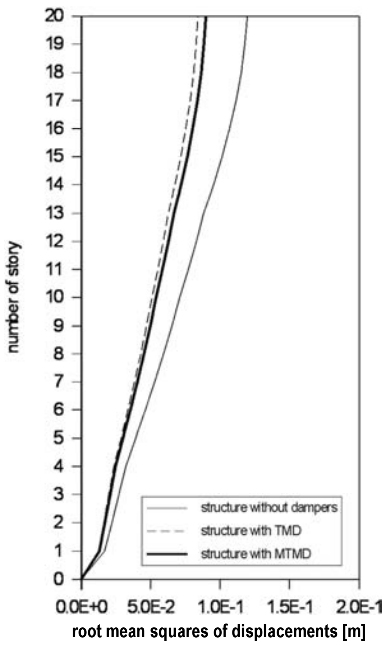

Fig. 4. Root mean squares of displacements

As it was mentioned previously, the results concerning accelerations were elaborated (Fig. 5). It has been observed that, when using MTMD acceleration, reduction is bigger only below the 11th floor than when using TMD. Above the 11th floor, the observed reduction of accelerations is smaller, compared with TMD. The maximum root mean squares of acceleration (top floor) are almost equal. The total sum of root mean square of acceleration is 38% for MTMD and 40% for TMD, compared with the structure without dampers.

The sensitivity of both TMD and MTDM with respect to change of structure parameters is also investigated. Calculations are made for a structure for which the values of all masses and all stiffness coefficients change by $\pm 10\%$, but the parameters of TMD and MTMD are kept constant. The above-mentioned changes of structure parameters reflect some possible uncertainties connected, for example, with determining the properties of structural material and/or with errors which are introduced when the theoretical model of structure is chosen. All these irregularities lead to the so-called detuning of dampers.

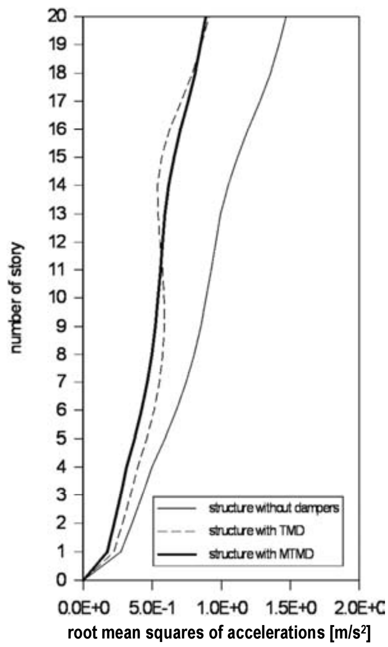

Fig. 5. Root mean squares of accelerations

Figs 6–8 illustrate the effects of such detuning of dampers for structures of which the stiffness increases by 10%. In Fig. 6, the resonance curves are presented. The thin solid line shows results for structures without dampers, the dashed line shows the response curve for the structure with TMD, while the thick line presents results for structures with MTMD. In a similar way, in Figs 7, 8, the root mean squares of displacements and accelerations are presented, respectively. It is obvious that now MTMD reduce both displacements and accelerations to a greater extent than TMD. Similar trends are observed when the structure stiffness decreases and when the mass of structures increases or decreases. The quantitative information concerning the effectiveness of TMD and MTMD concerning the effects of detuning of structure parameters is given in Table 3.

Table 3. Reduction effects for structures with changed parameters

Reduction of top displacement – mass changes			
	Original structure	+10% M_S	–10% M_S
TMD	18 %	20%	24%
MTMD	16 %	26%	27%
Reduction of top displacement – stiffness changes			
	Original structure	+10% K_S	–10% K_S
TMD	18%	24%	20%
MTMD	16%	27%	26%
Reduction of top acceleration – mass changes			
	Original structure	+10% M_S	–10% M_S
TMD	28%	26%	37%
MTMD	23%	32%	45%
Reduction of top acceleration – stiffness changes			
	Original structure	+10% K_S	–10% K_S
TMD	28%	37%	33%
MTMD	23%	43%	42%

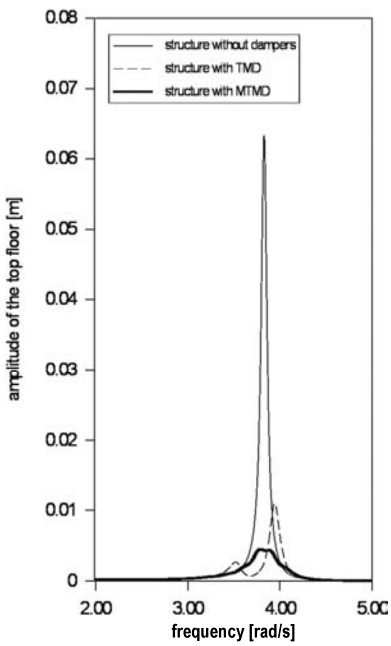

Fig. 6. The response curve of top of structure – structure with changed stiffness

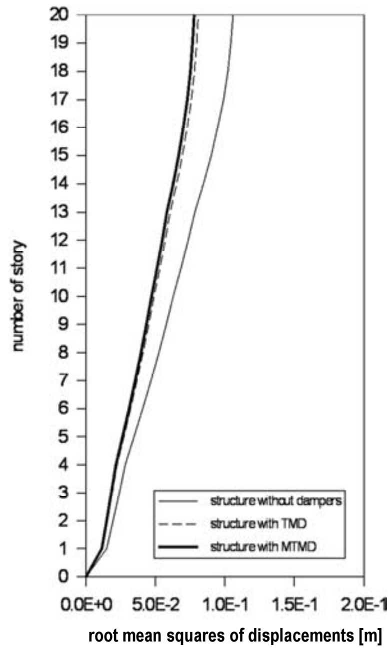

Fig. 7. Root mean squares of displacements – structures with changed stiffness

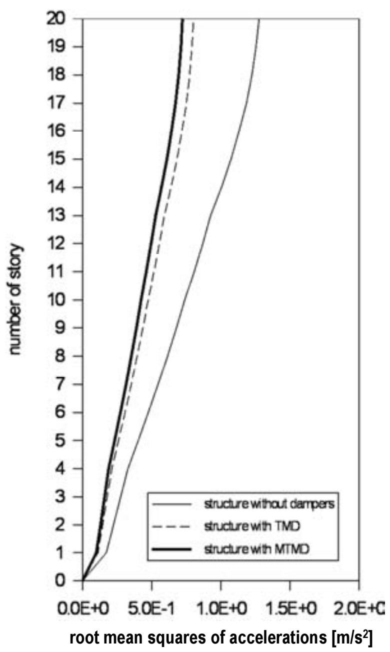

Fig. 8. Root mean squares of accelerations – structure with changed stiffness

7. Concluding remarks

The analysis of vibrations of a building structure with MTMD installed, which are tuned to selected modes of vibration, has been studied in this paper. The root mean squares of displacement and accelerations of a structure with MTMD were determined. These calculations were compared with the root mean squares of displacement and acceleration of the same structure with conventional TMD installed.

The following conclusions could be formulated from the results of calculations:

- In the case where parameters of structures are not exactly known, MTMD reduce better both dynamics displacements and accelerations of structures than do TMD. This is the main advantage of MTMD.
- MTMD reduce both displacements and accelerations of structures to a similar extent.
- MTMD reduce accelerations on lower floors of structures to a greater extent, compared with TMD.

These are the first results of calculation and, therefore, the above conclusions cannot be treated as definitive. Generally speaking, the effectiveness of MTMD and TMD are similar. However, MTMD are smaller than conventional TMD and they occupy a much smaller space for installation.

Moreover, the acceleration reduction of structures with MTMD is noticeably greater in comparison with structures without or with TMD and when the values of structure parameters are not exactly known. The problem of detuning the dampers parameters needs further investigations. In particular, parameters of structures must be regarded as the random quantities.

Acknowledgments

The authors acknowledge the financial support received from the Poznan University of Technology (Grant No. BW. 11-008/08) is connection with this work.

References

Chen, G.; Wu, J. 2001. Optimal placement of multiple tune mass dampers for seismic structures, *Journal of Structural Engineering* 127: 1054–1062.

Chmielewski, T. 1982. *Metody probabilistyczne w dynamice konstrukcji.* Wyd. Wyższej Szkoły Inżynierskiej w Opolu.

Dyrbye, C.; Hansen, S. O. 1999. *Wind Loads on Structures*. 3rd edition. John Wiley and Sons, Inc.

Ghorbani-Tanha, A. K.; Noorzad, A.; Rahimian, M. 2008. Mitigation of wind-induced motion of Milad Tower by tuned mass damper, *The Structural Design of Tall and Special Buildings* (in press).

Guo, Y. Q.; Chen, W. Q. 2007. Dynamic analysis of space structures with multiple tuned mass dampers, *Engineering Structures* 29: 3390–3403.

Han, B. K.; Li, C. X. 2006. Evaluation of multiple dual tuned mass dampers for structures under harmonic ground excitation, *International Journal of Structural Stability and Dynamics* 6: 59–75.

Han, B.; Li, C. 2006. Seismic response of controlled structures with active multiple tuned mass dampers, *Earthquake Engineering and Engineering Vibration* 5: 205–213.

Holmes, J. D. 1997. Equivalent time averaging in wind engineering, *Journal of Wind Engineering and Industrial Aerodynamics* 72: 411–419.

Igusa, T.; Xu, K. 1994. Vibration control using multiple tuned mass damper, *Journal of Sound and Vibration* 175: 491–503.

Jangid, R. S. 1995. Dynamic characteristic of structures with multiple tuned mass dampers, *Structural Engineering Mechanics* 3: 497–509.

Kareem, A.; Kline, S. 1995. Performance of multiple tuned mass dampers under random loading, *Journal of Structural Engineering* 121: 348–361.

Krenk, K. 2005. Frequency analysis of the tuned mass dampers, *Journal of Applied Mechanics, Transactions of ASME* 72: 936–942.

Leung, A. Y. T.; Zhang, H.; Cheng, C. C.; Lee, Y. Y. 2008. Particle swarm optimization of TMD by non-stationary base excitation during earthquake, *Earthquake Engineering and Structural Dynamics* (in press).

Li, C.; Han, B. 2007. Control strategy of the lever-type active multiple mass dampers for structures, *Wind and Structures* 10: 301–314.

Li, C.; Zhu, B. 2007. Investigation of response of systems with active multiple tuned mass dampers, *Structural Control and Health Monitoring* 14: 1138–1154

Lin, P. Y.; Chung, L. L.; Loh, Ch. 2005. Semiactive control of building structures with semiactive tuned mass dampers, *Computer-Aided Civil and Infrastructure Engineering* 20: 35–51.

Li, C.; Qu, W. 2006. Optimum properties of multiple tuned mass dampers for reduction of translational and torsional response of structures subject to ground acceleration, *Engineering Structures* 28: 472–494.

Li, C. 2002. Optimum multiple tuned mass dampers for structures under the ground acceleration based on DDMF and ADMF, *Earthquake Engineering and Structural Dynamics* 31: 897–919.

McNamara, R. J. 1977. Tuned mass dampers for buildings, *Journal of the Structural Engineering. Division Proc*., ASCE 105: 1785–1798.

Singh, M. P.; Singh, S.; Moreschi, L. M. 2002. Tuned mass dampers for response control of torsional buildings, *Earthquake Engineering and Structural Dynamics* 31: 749–769.

Spencer, B. F. Jr.; Christenson, R. E.; Dyke, S. J. 1999. *Next Generation Benchmark Problem for Seismically Excited Buildings*. Available from Internet: <http:/cee.uiuc.edu/sstl/papers/NGbench.pdf>.

Simiu, E.; Scanlan, R. 1996. *Wind effects on structures, Fundamentals and applications to design*. John Wiley and Sons, Inc.

Warburton, G. B. 1982. Optimum absorber parameters for various combinations of response and excitation parameters, *Earthquake Engineering and Structural Dynamics* 10: 381–401.

Xu, Y. L.; Kwok, K. C. S.; Samali, B. 1992. Control of wind-induced tall building vibration by tuned mass dampers, *Journal of Wind Engineering and Industrial Aerodynamics* 22: 833–854.

Xu, K.; Igusa, T. 1992. Dynamic characteristic of multiple substructures with closely-spaced frequencies, *Earthquake Engineering and Structural Dynamics* 21: 1059–1070.

KONSTRUKCIJŲ SU KELIAIS MASĖS SLOPINTUVAIS DINAMINĖ ANALIZĖ

R. Lewandowski, J. Grzymisławska

Santrauka

Nagrinėjama galimybė sumažinti stipraus vėjo veikiamų rėminių pastatų konstrukcijų svyravimus, taikant masės slopintuvų sistemą. Vibracijas sukeliančios jėgos, priklausančios nuo vėjo greičio svyravimų, laikomos atsitiktiniais dydžiais. Vėjo greičio svyravimo spektro tankio funkcijos nagrinėjamos Davenport metodu. Atlikus atsitiktinių vibracijų regresinę analizę, nustatytos poslinkių ir pagreičių vidutinės kvadratinės paklaidos. Remiantis skaičiavimo rezultatais, padarytos masės slopintuvų efektyvumo išvados.

Reikšminiai žodžiai: svyravimų mažinimas, atsitiktiniai svyravimai, masės slopintuvai.

Roman LEWANDOWSKI. Professor at the Poznan University of Technology, Faculty of Civil and Environmental Engineering, Poznan, Poland. Member of the Polish Society for Theoretical and Applied Mechanics and Polish Association for Computational Mechanics. Main research interests: non-linear vibration of structures, reduction of vibration by means of passive, active and semi-active methods.

Justyna GRZYMISŁAWSKA. Teaching and research assistant at the Poznan University of Technology, Faculty of Civil and Environmental Engineering, Poznan, Poland. She teaches the strength of materials. Main research interests: passive, active and semi-active methods of reduction of structure vibration.