JOURNAL OF CIVIL ENGINEERING AND MANAGEMENT
ISSN 1392-3730 print/ISSN 1822-3605 online
2011 Volume 17(4): 540–557
http://dx.doi.org/10.3846/13923730.2011.619325

OPTIMUM MULTIPLE TUNED MASS DAMPERS FOR THE WIND EXCITED BENCHMARK BUILDING

Veeranagouda B. Patil[1], **Radhey Shyam Jangid**[2]

Department of Civil Engineering, I.I.T. Bombay, Powai, Mumbai -400076, India
E-mails: [1]vbpatil.iitb@gmail.com (corresponding author); [2]rsjangid@civil.iitb.ac.in

Received 24 Jun. 2010; accepted 07 Dec. 2010

Abstract. The performance of multiple tuned mass dampers (MTMD) installed at the top floor of the wind excited benchmark building under across wind loads is investigated. The performance of MTMD is compared with that of single tuned mass damper (TMD) having same total mass. The governing equations of motion of the building with MTMD/ TMD are solved by employing state space formulation. Initially, the TMD is installed at the top floor of the benchmark building and the optimum parameters of the damper for the minimization of various performance criteria of the building are obtained for different mass ratios. Later on, the MTMD is installed at the top floor of the building and the optimum parameters are obtained for the minimization of various performance criteria of the building for different mass ratios and number of dampers. As it is easier to maintain the same stiffness of dampers, the stiffness of each damper in MTMD is maintained as constant. From the study, it is found that the MTMDs are quite effective and robust in the vibration control of the benchmark building.

Keywords: benchmark building, optimum parameters, MTMD, passive control, wind load, tall building.

1. Introduction

Passive control devices dissipate energy due to the motion of the structure (Housner *et al.* 1997) itself. Matsagar and Jangid (2005) studied the vibration control of adjacent buildings connected by visco-elastic dampers subjected to earthquake ground motion. Tuned Mass Damper (TMD) is a classical engineering device consisting of a mass, a spring and a viscous damper attached to a vibrating main system in order to attenuate any undesirable vibration. The natural frequency of the damper system is tuned to a frequency near to the natural frequency of the main system, the vibration of the main system causes the damper to vibrate in resonance, and as a result, the vibration energy is dissipated through the damping in the tuned mass damper. The main disadvantage of a single TMD is its sensitivity of the effectiveness to the error in the natural frequency of the structure and/ or that in the damping ratio of the TMD. This is due to following reasons. Errors in predicting or identifying the natural frequency of the structure and the errors in fabricating TMD are ineviTable to some degree. Therefore, in practical design, the optimum values of parameters of TMD are not maintained. The damping of the TMD is intentionally made higher than the optimal value such that TMDs become less sensitive to tuning errors. This results in increase in the mass of the TMD to meet the design requirement. All these uncertainties can be reduced by use of MTMD. Use of MTMD has been proposed to increase the robustness of the vibration control system to various uncertainties in the structures and/ or TMD. The basic configuration of MTMD consists of large number of small oscillators whose natural frequencies are distributed around the natural frequency of the controlled mode of the structure. It is now well established that an optimal MTMD is more effective and robust than optimal TMD (Li 2000). Ayorinde and Warburton (1980) extended the application of MTMDs to civil engineering structures. Iwanami and Seto (1984) had shown that two TMDs are more effective than single TMD. However, the improvement on the effectiveness was not significant. Xu and Igusa (1992) proposed use of multiple sub-oscillators with closely spaced frequencies. Multiple tuned mass dampers with distributed natural frequencies were also studied by Yamaguchi and Harnpornchai (1993). The optimum parameters of MTMD installed on an undamped SDOF subjected to harmonic base excitation is studied by Jangid (1999). Lewandowski and Grzymisławska (2009) studied the possibility of reduction of vibrations of a multi-storey frame with MTMDs tuned to different modes.

Due to advancement in the technology of materials, taller and slender buildings could be designed and constructed. However, such buildings pose the problems of excessive vibrations due to dynamic loads like earthquake or wind. To mitigate the seismic and wind effects on the high rise buildings, various structural control systems/ devices are being developed in the field of Civil Engineering. Therefore, it is felt necessary to compare the results of different control systems when implemented on the same structural model subjected to same loads. Thus, the concept of benchmark building comes into picture. Therefore, based on realistic problems two structural control problems have been selected- one for earthquake and another for wind excitations (Yang *et al.* 2004). The wind excited benchmark building is tall and slender and hence it is wind sensitive. Wind tunnel tests (Samali *et al.* 2004a) for the 76-storey building model have already been conducted at the University of Sydney; and the results of across-wind data for a duration of 3600 s are also provided for the analysis of the benchmark problem. Performance of various dampers like tuned liquid column dampers (Min *et al.* 2005), liquid column vibration absorbers (Samali *et al.* 2004b), hybrid viscous-tuned liquid column damper (Kim, Adeli 2005), variable stiffness tuned mass damper (Varadarajan, Nagarajaiah 2004) on the benchmark building have been studied. The above literature review has revealed that the performance of MTMD on the wind excited benchmark building is not studied so far. Hence, it will be interesting to install MTMD on the slender building.

The present study evaluates the performance of MTMDs on the wind excited benchmark building. The specific objectives of the present study may be summarized as: (i) to compare the performance criteria of the wind excited benchmark building installed with MTMD with those of the uncontrolled building; (ii) to obtain the optimum parameters of TMD for the minimization of various performance criteria of the benchmark building; (iii) to compare the performance criteria of the wind excited benchmark building installed with MTMD with those of the building with TMD; (iv) to obtain the optimum parameters of MTMD for the minimization of various performance criteria of the benchmark building and (v) to study the variation of the performance criteria with the number of dampers in MTMD.

2. Benchmark building

Generally, tall buildings are flexible and hence, they experience excessive wind induced vibrations. The wind excited benchmark building considered for the study is 306 m high and 42×42 m in plan. Therefore, the aspect ratio (height to width ratio) is 7.3 and hence, the building is wind sensitive. The typical story height is 3.9 m except the first floor which has a height of 10 m and stories 38 to 40 and 74 to 76 which have a height of 4.5 m. As the rotational degrees of freedom have been removed by the static condensation procedure, only translational degrees of freedom, one at each floor of the building is remaining. The building is modeled as a cantilever beam (Bernoulli-Euler beam). The detailed description of the benchmark building and its model can be found in Yang *et al.* (2004). The front view of the wind excited benchmark building installed with MTMD at the top floor is shown in Fig. 1. In this figure, the heights of various floors of the building and configuration of MTMD can be seen. The building is proposed for an office tower at Melbourne, Australia.

3. Governing equations of motion

The wind excited benchmark building is supported by N number of TMDs with different dynamic characteristics as shown in Fig. 1. The parameters of the j^{th} TMD are mass m_j, damping c_j and stiffness k_j. Natural frequencies of the MTMD are uniformly distributed around their average frequency. The natural frequency, ω_j (*i.e.* $\sqrt{k_j/m_j}$) of the j^{th} TMD is expressed by:

$$\omega_j = \omega_{\mathrm{T}}\left[1+\left(j-\frac{\mathrm{N}+1}{2}\right)\frac{\beta}{\mathrm{N}-1}\right]; \quad (1)$$

$$\omega_{\mathrm{T}} = \sum_{\varphi=1}^{\mathrm{N}} \omega_{\varphi} / \mathrm{N}; \quad (2)$$

$$\beta = \frac{\omega_N - \omega_1}{\omega_T}, \quad (3)$$

where: ω_T is the average frequency of all MTMD, β is the non-dimensional frequency band-width of the MTMD system.

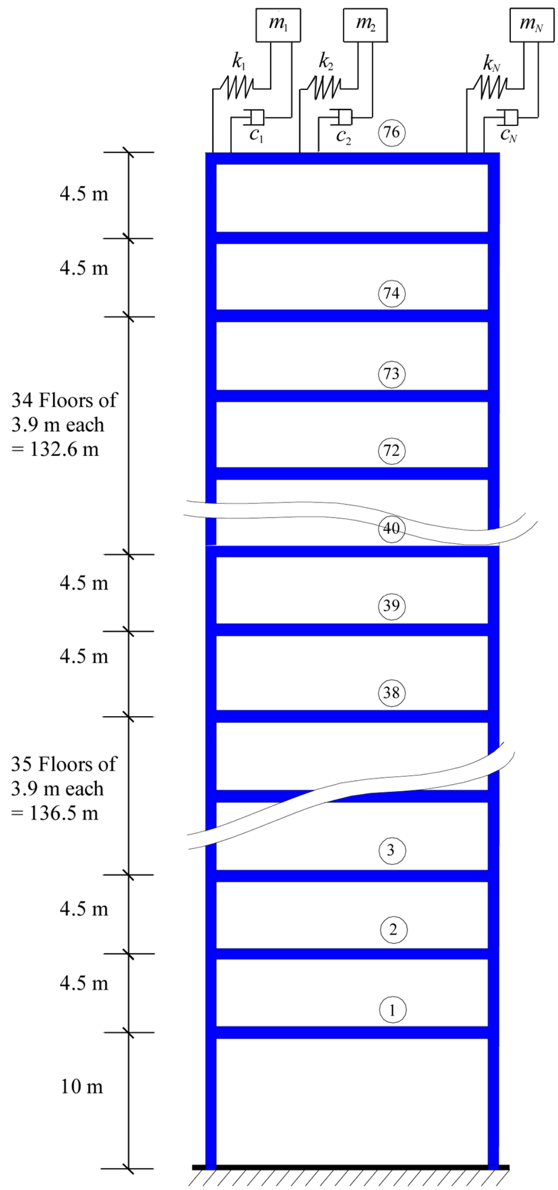

Fig. 1. Wind excited benchmark building with MTMD

As suggested by Xu and Igusa (1992) that the manufacturing of the MTMD with uniform stiffness is simpler than that with the varying stiffness. As a result, the distribution of natural frequencies of the MTMD is obtained by keeping the stiffness constant but varying the mass of each TMD (i.e., $k_1 = k_2 = \ldots k_N = k_T$).

The damping constant of the j^{th} TMD is expressed as:

$$c_j = 2 m_j \zeta_T \omega_j\,, \tag{4}$$

where ζ_T is the damping ratio kept constant for all the MTMD.

Total mass of the MTMD system is expressed by the mass ratio defined as:

$$\mu = \frac{\sum_{j=1}^{N} m_j}{m_s}\,, \tag{5}$$

where μ is the mass ratio of the MTMD system and m_s is the mass of the structure.

Tuning frequency ratio of the MTMD system is expressed by:

$$f = \frac{\omega_T}{\omega_s}\,, \tag{6}$$

where ω_s is the fundamental frequency of the main system.

For the wind excited benchmark building along with MTMD, the governing equations of motion are obtained by considering the equilibrium of forces at the location of each DOF during wind excitations. Therefore, the governing equations of motion for the controlled building structure model subjected to wind excitations can be written as:

$$\mathbf{M}_{\mathrm{d}}\ddot{\mathbf{x}} + \mathbf{C}_{\mathrm{d}}\dot{\mathbf{x}} + \mathbf{K}_{\mathrm{d}}\mathbf{x} = \mathbf{F}_{\mathrm{d}}\,; \tag{7}$$

$$\mathbf{M}_{\mathrm{d}} = \left[\begin{array}{c:c} \mathbf{M} & \mathbf{0} \\ \hdashline \mathbf{0} & \mathbf{m}_{\mathrm{T}} \end{array}\right]; \tag{8}$$

$$\mathbf{m}_{\mathrm{T}} = \mathrm{diag}\,[m_1\; m_2\; \ldots\; m_n]; \tag{9}$$

$$\mathbf{K}_{\mathrm{d}} = \left[\begin{array}{ccccc|ccccc}
K_{11} & K_{12} & . & . & K_{1m} & & & & & \\
K_{21} & K_{22} & . & . & K_{2m} & & & \mathbf{0}_{(m-1)\times(N)} & & \\
. & & & & & & & & & \\
K_{m1} & . & & . & K_{mm}+\sum_{j=1}^{N} k_j & -k_1 & -k_2 & . & . & -k_N \\
\hline
 & & & & & k_1 & 0 & . & . & 0 \\
 & & & & & & k_2 & . & . & 0 \\
 & & \mathbf{0}_{(N)\times(m-1)} & & & & & . & . & . \\
 & & & & & & & & . & \\
 & & & & & sym & & & & k_N
\end{array}\right] \tag{10}$$

$$\mathbf{C}_{\mathrm{d}} = \left[\begin{array}{ccccc|ccccc}
C_{11} & C_{12} & . & . & C_{1m} & & & & & \\
C_{21} & C_{22} & . & . & C_{2m} & & & \mathbf{0}_{(m-1)\times(N)} & & \\
. & & & & & & & & & \\
C_{m1} & . & & . & C_{mm}+\sum_{j=1}^{N} c_j & -c_1 & -c_2 & . & . & -c_N \\
\hline
 & & & & & c_1 & 0 & . & . & 0 \\
 & & & & & & c_2 & . & . & 0 \\
 & & \mathbf{0}_{(N)\times(m-1)} & & & & & . & . & . \\
 & & & & & & & & . & \\
 & & & & & sym & & & & c_N
\end{array}\right], \tag{11}$$

where the mass matrix **M,** stiffness matrix **K,** and damping matrix **C** each of the order of (76×76) are constructed for the finite element model of the uncontrolled building and provided for the analysis. The mass matrix of the building installed with MTMD/TMD ($\mathbf{M}_{\mathrm{d}}$) is constructed by appending the masses of dampers in MTMD diagonally to the mass matrix (**M**) as given in Eq. (8). Similarly, the stiffness matrix ($\mathbf{K}_{\mathrm{d}}$) and the damping matrix ($\mathbf{C}_{\mathrm{d}}$) of the structure with MTMD/TMD are constructed by expressing the set of equations of motion in the matrix form (refer Eqs 10 and 11). The first five natural frequencies of the uncontrolled structure (i.e. without dampers) are calculated as 0.1600, 0.7651, 1.9921, 3.7899 and 6.3945 Hz. $\mathbf{x}$ is displacement vector of order $(m+N)$ where m is the degree of freedom (DOF) of the building, N is the number of dampers in MTMD, $\dot{\mathbf{x}}$ and $\ddot{\mathbf{x}}$ the first and second time derivatives and $\mathbf{F}_{\mathrm{d}}$ is the wind load vector of order $(m+N)$. The first m elements are the wind loads at the m floors of the building and remaining N elements are zeros as there is no wind load on dampers.

The set of Eqs (7) is expressed as a set of first order differential equations as:

$$\dot{\mathbf{z}} = \mathbf{A}\,\mathbf{z} + \mathbf{E}\,\mathbf{F}_{\mathrm{d}}\,, \tag{12}$$

where: $\mathbf{z}$ is the state vector of structure along with MTMD/TMD, and contains displacement and velocity of each floor and also of dampers in MTMD/TMD; $\mathbf{A}$ denotes the system matrix composed of mass matrix of the structure with MTMD/TMD, damping and stiffness matrices; and $\mathbf{E}$ represents the distributing matrices for excitation.

The Eq. (12) is discretized in the time domain and excitation force is assumed to be constant within any time interval and can be written into a discrete-time form (Lu 2004):

$$\mathbf{z}[k+1] = \mathbf{A}_{\mathrm{d}}\mathbf{z}[k] + \mathbf{E}_{\mathrm{d}}\mathbf{F}_{\mathrm{d}}[k]\,, \tag{13}$$

where:

$$\mathbf{A}_{\mathrm{d}} = e^{\mathbf{A}\Delta t}\,. \tag{14}$$

It represents the discrete time system matrix with Δt as the time interval.

The system matrix $\mathbf{A}$ is defined as follows:

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}_d^{-1}\mathbf{K}_d & -\mathbf{M}_d^{-1}\mathbf{C}_d \end{bmatrix}, \quad (15)$$

whereas, coefficient matrix $\mathbf{E}_d$ is given by:

$$\mathbf{E}_d = \mathbf{A}^{-1}(\mathbf{A}_d - \mathbf{I})\mathbf{E} \ , \quad (16)$$

where:

$$\mathbf{E} = \left[\frac{\mathbf{0}}{\mathbf{M}_d^{-1}} \right]. \quad (17)$$

4. Numerical study

The mass matrix and the stiffness matrix, each of the order of (76×76) are constructed from the finite element model (FEM) of the building. The damping ratio $\zeta = 1\%$ is assumed for the first five modes to construct the damping matrix of the order (76×76) using Rayleigh's approach (Yang *et al.* 2004). The first five natural frequencies of the uncontrolled structure are calculated as 0.1600, 0.7651, 1.9921, 3.7899 and 6.3945 Hz. Thus the wind excited benchmark building is completely characterized by the parameters. The performance of TMD installed on the wind excited benchmark building is studied. To know the effectiveness and robustness of MTMD, the response of the wind excited benchmark building by using MTMD is also investigated. The detailed description of the wind tunnel tests conducted at the University of Sydney is given in Samali *et al.* (2004 a, b) and the time histories of across wind loads are available at the website (SSTL 2002).

To facilitate the direct comparison and to evaluate the capabilities of various protective devices and algorithms, a set of 12 performance criteria are proposed. These performance criteria are defined in Yang *et al.* (2004). The performance criteria J_1 to J_4 are defined to measure the reduction in RMS response quantities of the wind excited benchmark building, evaluated by normalizing the response quantities by the corresponding response quantities of the uncontrolled building; J_7 to J_{10} are based on the peak responses calculated by normalizing the peak response quantities by the corresponding peak response quantities of the uncontrolled building. Among the 12 criteria, only eight criteria J_1 to J_4 and J_7 to J_{10} are used in this work, because the other four criteria (J_5, J_6, J_{11}, and J_{12}) represent the performance of the actuator.

In the first part of the study, TMD is installed at the top of the building and optimum parameters such as tuning frequency ratio, damping ratio for various mass ratios are obtained by numerical procedure. In the second part of the study, the performance of MTMD installed at the top of the building is studied and the optimum parameters are obtained. In the present study, the performance of dampers is studied only up-to the duration of 900 s.

4.1. TMD at the top of the benchmark building

In this part of the study, the performance criteria obtained with TMD installed at the top of the building are studied. The tuning frequency ratio, damping ratio and mass ratio are the critical parameters in the design of TMD. To know the optimum parameters of TMD, the mass ratio of TMD are varied as 0.16%, 0.33%, 0.5%, 0.66%, 0.82%, 1%, 1.16% and 1.33%. The frequency ratio is varied from 0.1 to 1.00 with an increment of 0.1 and the damping ratio is varied from 0.01 to 0.1 with an increment of 0.01. Then the minimization of the performance criterion J_1 is carried out. The optimum parameters obtained by the minimization of J_1 (listed in Table 1) are used to know the variation of the criteria J_2, J_3 and J_4. Later on, each performance criterion is minimized to know the most optimum parameters.

4.1.1. Optimal tuning frequency ratio

In this section, optimal tuning frequency ratios are obtained for the various values of mass ratios. The mass ratio (μ) is the ratio of the TMD to the mass of the structure. For an undamped system with a TMD, closed form solutions for optimum tuning frequency ratio and damping ratio can be obtained with regard to mass ratio (Soong, Dargush 1997). But the optimal parameters of a TMD for damped system like the benchmark building can not be given in the closed form, and they can be determined only by numerical methods.

Variation of performance criteria J_2 to J_4 with tuning frequency ratio for different values of mass ratio are presented in Fig. 2. The damping ratio that minimizes J_1 is selected. For this purpose, the damping ratio of damper (listed in Table 1 corresponding to the column 'J_1') obtained by the minimization of the performance criterion J_1 for a value of mass ratio is selected, to get the optimum tuning frequency ratio for that mass ratio. And the stiffness of TMD is calculated based on the mass of TMD for the corresponding mass ratio and the angular frequency of damper required for the corresponding frequency ratio. It can be seen from the Fig. 2, that the optimal value of tuning frequency ratio that minimizes performance criteria is close to 1, and it approaches 1 as the mass ratio decreases. Performance is improved and the sensitivity of performance criteria to frequency ratio is reduced with increasing μ. However the performance improvement becomes negligible if μ>0.82%.

4.1.2. Optimal damping ratio

Variation of performance criteria with damping ratio for various values of mass ratio are presented in Fig. 3. Tuning frequency ratio that minimizes J_1 is selected. And the stiffness of TMD is calculated based on the mass of TMD for the corresponding mass ratio and the angular frequency of damper required for the corresponding frequency ratio. From this Figure, it is can be seen that the optimum value of damping ratio increases with mass ratio. Performance is improved with the increasing mass ratio and the sensitivity of the performance criteria to damping ratio is also reduced with increase in mass ratio.

4.1.3. Optimum parameters of TMD

Optimum parameters of TMD for the minimization of various performance criteria are presented in Table 1.

Table 1. Variations of optimum parameters of TMD with mass ratio for the minimization of the performance criteria

Mass of TMD (Ton)		Performance criterion optimized							
		J_1	J_2	J_3	J_4	J_7	J_8	J_9	J_{10}
250 (μ = 0.16%)	f^{opt}	0.99	0.99	0.98	0.98	0.99	1.00	1.00	1.00
	ζ_T^{opt}	0.03	0.03	0.03	0.03	0.03	0.02	0.01	0.01
	J^{opt}	0.5028	0.5001	0.6163	0.6181	0.4911	0.5087	0.6915	0.6916
500 (μ = 0.33%)	f^{opt}	0.98	0.98	0.98	0.98	0.97	0.98	0.97	0.97
	ζ_T^{opt}	0.04	0.04	0.04	0.04	0.06	0.07	0.02	0.02
	J^{opt}	0.4353	0.4320	0.5715	0.5735	0.4434	0.4702	0.6072	0.6135
750 (μ = 0.5%)	f^{opt}	0.98	0.98	0.97	0.97	0.94	0.95	0.94	0.95
	ζ_T^{opt}	0.06	0.06	0.05	0.05	0.09	0.09	0.05	0.04
	J^{opt}	0.4066	0.4029	0.5537	0.5559	0.4280	0.4546	0.5987	0.6002
1000 (μ = 0.66%)	f^{opt}	0.97	0.97	0.96	0.96	0.92	0.94	0.94	0.94
	ζ_T^{opt}	0.07	0.07	0.07	0.07	0.10	0.10	0.06	0.05
	J^{opt}	0.3883	0.3846	0.5430	0.5453	0.4175	0.4479	0.5789	0.5864
1250 (μ = 0.82%)	f^{opt}	0.97	0.97	0.95	0.96	0.92	0.94	0.94	0.94
	ζ_T^{opt}	0.08	0.08	0.08	0.08	0.10	0.10	0.06	0.06
	J^{opt}	0.3741	0.3702	0.5354	0.5377	0.4175	0.4438	0.5770	0.5852
1500 (μ = 1%)	f^{opt}	0.97	0.97	0.95	0.95	0.91	0.93	0.94	0.93
	ζ_T^{opt}	0.10	0.10	0.09	0.09	0.10	0.10	0.06	0.07
	J^{opt}	0.3619	0.3579	0.5295	0.5318	0.4126	0.4353	0.5834	0.5868
1750 (μ = 1.16%)	f^{opt}	0.97	0.97	0.94	0.95	0.91	0.91	0.92	0.92
	ζ_T^{opt}	0.10	0.10	0.10	0.10	0.10	0.10	0.05	0.06
	J^{opt}	0.3515	0.3474	0.5249	0.5272	0.4118	0.4256	0.5864	0.5902
2000 (μ = 1.32%)	f^{opt}	0.97	0.97	0.94	0.94	0.91	0.91	0.91	0.91
	ζ_T^{opt}	0.10	0.10	0.10	0.10	0.10	0.10	0.07	0.07
	J^{opt}	0.3434	0.3391	0.5249	0.5234	0.4157	0.4193	0.5872	0.5889

From the Table 1, it is seen that when the performance criterion J_1 is minimized, optimum tuning frequency ratio decreases with increase in mass ratio whereas optimum damping ratio (ζ_T^{opt}) increases. The optimum value of J_1 decreases with increase in the mass ratio. Similar trend is seen when the criteria J_2, J_3 and J_4 are optimized. From the Table, it is implied that while minimizing J_7, (with different mass ratios) the mass ratio of 1.16% gives the minimum value of J_7, where as while minimizing J_9 and J_{10}, the mass ratio of 0.82 gives the minimum values. However, there is no much difference in the values of J_7 for μ = 0.82 and μ = 1.16%. Hence, the mass ratio of the TMD may be maintained as 0.82. It can be also noticed that there is no much difference in the optimum values obtained by the minimization of the criteria J_1 to J_4 for μ = 0.82 and higher values. Therefore the optimum parameters that are obtained by the minimization of J_1 with μ = 0.82 may be maintained.

4.2. MTMD at the top of the benchmark building

MTMD offers the advantages of portability and ease of installation (because of the reduced size of an individual damper), which makes it attractive not only for new installation, but also for temporary use during construction or for retrofitting existing structures. The design parameters of MTMDs are the number of dampers, the frequency band width, tuning frequency ratio, mass ratio and damping ratio. It is reported that tuning the frequencies of every TMD to the fundamental mode in the MTMD is more effective than tuning it to different modes (Kareem, Kline 1995). Accordingly, the frequency of the MTMD is tuned to the fundamental mode in this study. Initially, the performance indices for RMS responses are considered by minimizing J_1, similarly to the case for determining the optimal parameters of a single TMD. And later on, each of the eight performance criteria is minimized and the most optimum parameters are obtained.

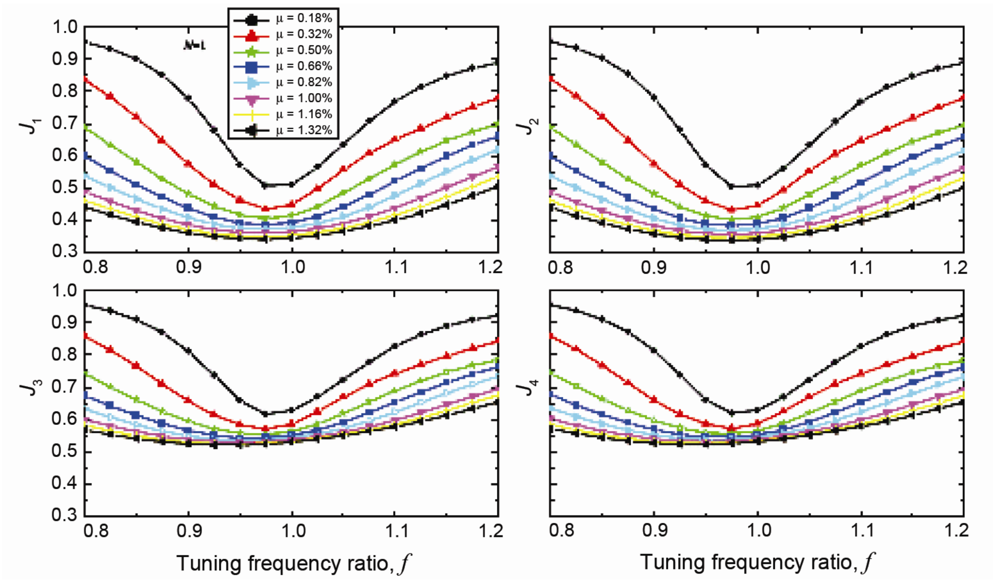

Fig. 2. Variation of performance criteria with tuning frequency ratio of TMD for different values of mass ratio

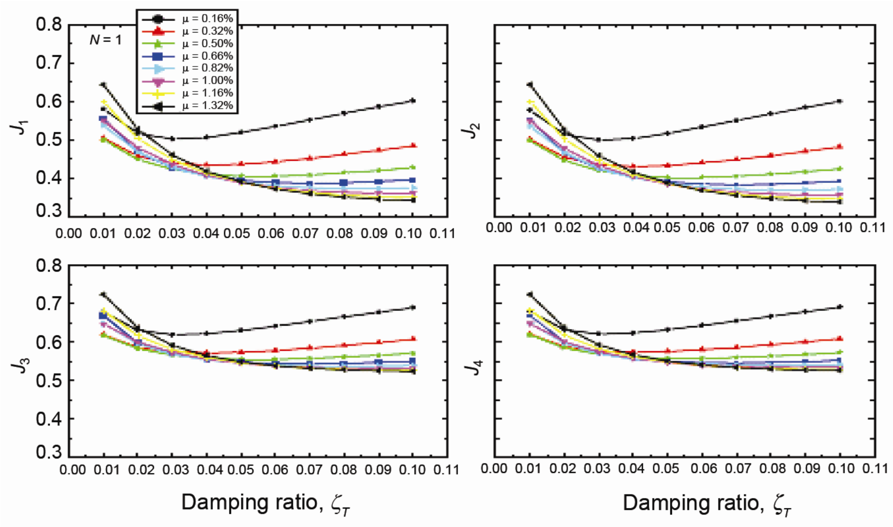

Fig. 3. Variation of performance criteria with damping ratio of TMD for different values of mass ratio

4.2.1. Optimal number of dampers

The optimal number of dampers, N, should be determined to consider the control performance and the constructional efficiency. Fig. 4 shows the variation of performance criteria with number of dampers for various mass ratios of MTMD. Other parameters like frequency band width, damping ratio and central tuning frequency are maintained as those required to minimize the performance criteria J_1. It is observed that larger mass ratio results in better control performance. Further, it can be seen that the performances are generally enhanced when 5 dampers are used in MTMD compared to that of a single TMD. However, increasing N over 5 does not provide significant response reduction. Accordingly, $N = 5$ will be used in the subsequent sections.

4.2.2. Optimal frequency band width

Frequency band width is one of the important parameters of MTMD which depends on ω_1 and ω_N where ω_1 and ω_N are the frequencies of first and N^{th} TMD, respectively. The variation of performance criteria with frequency band width (β) is shown in Fig. 5. The frequency ratio and damping ratio are maintained that are required for minimization of J_1. It is seen from the figure that for the lower values of mass ratios performance criteria increase with (β), whereas there exists an optimum value of (β) for the higher mass ratios. It is also observed that with the increase in mass ratio the optimum value of β increases.

4.2.3. Optimal tuning frequency ratio

The variation of performance criteria with tuning frequency ratio is shown in Fig. 6. Number of dampers is

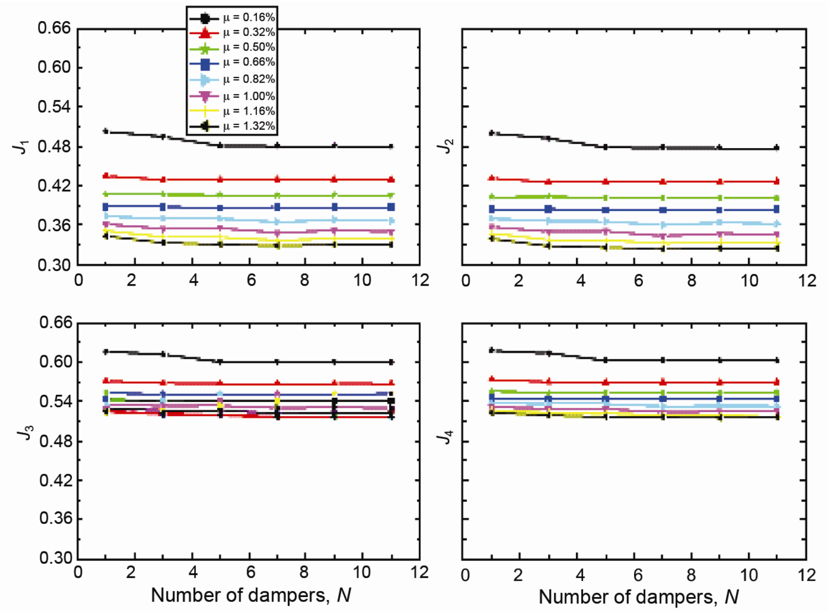

Fig. 4. Variation of performance criteria with number of dampers in MTMD for various values mass ratios

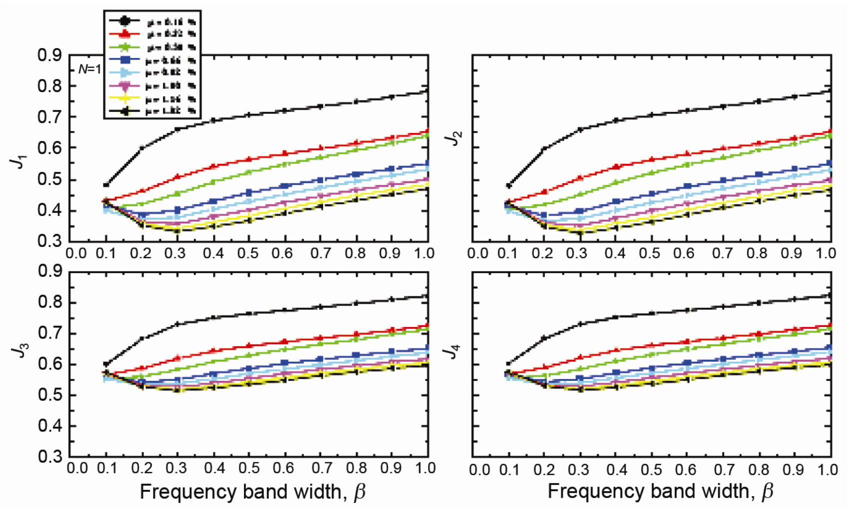

Fig. 5. Variation of performance criteria with frequency band width of MTMD for different values of mass ratio

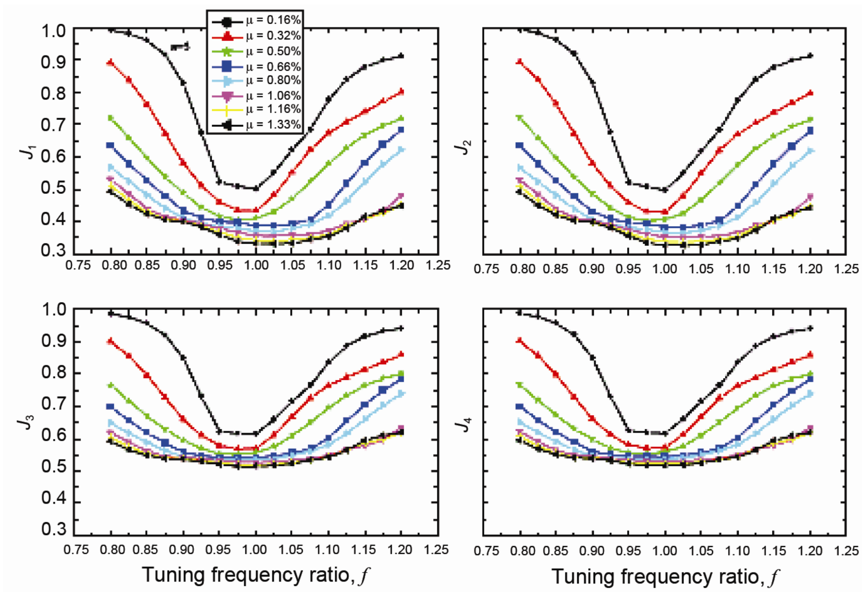

Fig. 6. Variation of performance criteria with tuning frequency ratio of MTMD for different values of mass ratio

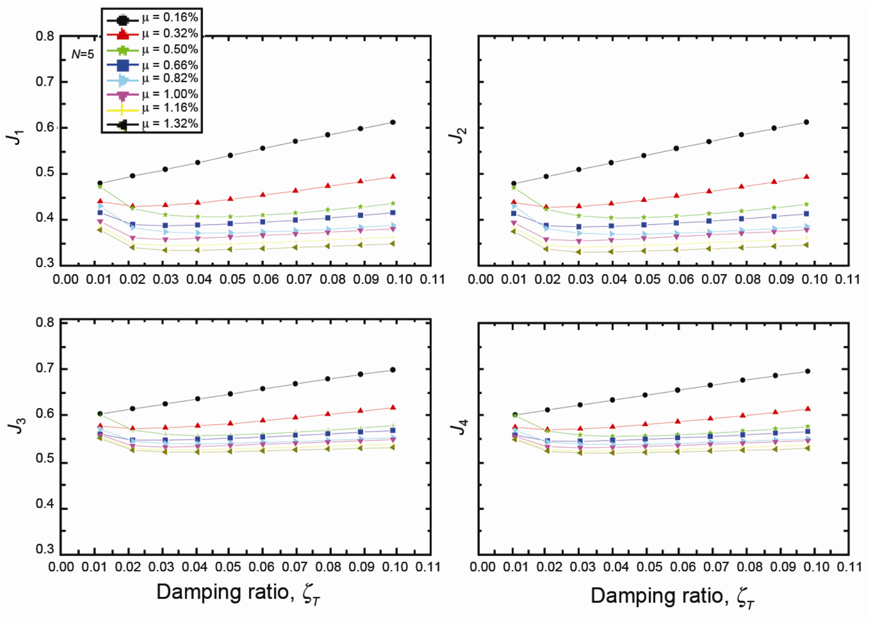

Fig. 7. Variation of performance criteria with damping ratio of MTMD for different values of mass ratio

maintained as 5. Damping ratio is maintained that required for minimizing the criteria J_1. Similar to single TMD, it can be seen that optimum tuning frequency is close to 1 and it becomes closer to 1 with decrease in mass ratio. The control performance can be enhanced by increasing mass ratio and sensitivity can be reduced with increase in mass ratio.

4.2.4. Optimal damping ratio

The variation of performance criteria with damping ratio is shown in Fig. 7. Other parameters like frequency band width and frequency ratio are maintained those required for the minimization of J_1. The number of dampers in MTMD is maintained as 5. From the Figure it is depicted that there exists an optimum damping ratio for the performance criteria.

4.2.5. Robustness of MTMD

Robustness of MTMDs is also the most important features of damping devices. Since the value of f is critical in the control performance of TMDs, the variation of the frequency of the structure due to the measurement or calculation error and the variation of mass or stiffness may cause significant performance deterioration. For these reasons, the MTMDs have been proposed for enhancing the robustness of TMDs. In this section, the robustness of TMDs is discussed by evaluating the control performance when uncertainty exists in the stiffness of the structure.

A comparison of the performance criteria of a single TMD and MTMD is given in Table 2. Although the performance of the MTMD is almost equivalent to that of a single TMD, when there exists no uncertainty (i.e., $\Delta\mathbf{K} = 0$), the MTMD shows superior performance to the single TMD when stiffness uncertainty exists (i.e., $\Delta\mathbf{K} = \pm 15\%$) with the exception of J_1 and J_2. As expected, robustness could be guaranteed by using an MTMD.

Table 2. Performance criteria of TMD and MTMD

Performance criteria minimized	μ = 1%					
	ΔK = +15%		ΔK = 0%		ΔK = –15%	
	$N = 1$	$N = 5$	$N = 1$	$N = 5$	$N = 1$	$N = 5$
J_1	0.3542	0.3600	0.3619	0.3558	0.3855	0.3649
J_2	0.3504	0.3566	0.3579	0.3511	0.3806	0.3596
J_3	0.4565	0.4503	0.5295	0.5267	0.6499	0.6313
J_4	0.4583	0.4526	0.5318	0.5289	0.6524	0.6340
J_7	0.4170	0.4021	0.4126	0.3790	0.4260	0.3995
J_8	0.4470	0.4140	0.4353	0.4056	0.4384	0.4151
J_9	0.5627	0.5496	0.5834	0.5714	0.6955	0.6861
J_{10}	0.5698	0.5547	0.5868	0.5778	0.7035	0.6879

4.2.6. Optimum parameters of MTMD

In this section, intensive numerical simulations have been carried out by minimizing J_1 to J_4 and J_7 to J_{10} separately, to know the optimum parameters of MTMD. The corresponding optimum parameters obtained by the minimization of each performance criteria are presented in Tables 3 to 10.

From discussions in the above sections, it is clear that increasing the number of dampers beyond 5 will not provide any further reduction in response. The sensitivity of the performance criteria beyond the mass ratio (μ) = 1% is sufficiently low.

From Table 3, it can be seen that while minimizing J_1, for μ = 1% and $N = 5$, the optimum damping ratio is 0.03 whereas the corresponding value of ζ_T for TMD is 0.1. Similar trend is seen for other values of mass ratio also. From Tables 4–10, it is can also be observed that same trend is continued while minimizing other performance criteria also (except for few cases of mass ratio for

Table 3. Variations of optimum parameters with the number of dampers in MTMD for the minimization of the performance criterion J_1

Mass of MTMD/TMD (Ton)	N	1	3	5	7	9	11
250 (μ = 0.16%)	β^{opt}	--------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	0.99	1	0.99	0.99	0.99	0.99
	ζ_T^{opt}	0.03	0.01	0.01	0.01	0.01	0.01
	J_1^{opt}	0.5028	0.4956	0.4813	0.4807	0.4807	0.4806
500 (μ = 0.33%)	β^{opt}	--------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	0.98	1.0	0.99	0.99	0.99	0.99
	ζ_T^{opt}	0.04	0.02	0.02	0.02	0.02	0.02
	J_1^{opt}	0.4353	0.4294	0.4298	0.4297	0.4298	0.4299
750 (μ = 0.5%)	β^{opt}	--------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	0.98	0.98	0.98	0.98	0.98	0.98
	ζ_T^{opt}	0.06	0.04	0.04	0.04	0.05	0.05
	J_1^{opt}	0.4066	0.4069	0.4058	0.4058	0.4058	0.4057
1000 (μ = 0.66%)	β^{opt}	--------	0.1	0.2	0.2	0.2	0.2
	f^{opt}	0.97	0.98	1	1	1	1
	ζ_T^{opt}	0.07	0.06	0.03	0.04	0.04	0.04
	J_1^{opt}	0.3883	0.3884	0.3862	0.3878	0.3879	0.3879
1250 (μ = 0.82%)	β^{opt}	--------	0.2	0.2	0.3	0.3	0.3
	f^{opt}	0.97	1	1	1	1	1
	ζ_T^{opt}	0.08	0.04	0.04	0.02	0.02	0.02
	J_1^{opt}	0.3741	0.3703	0.3701	0.3648	0.3684	0.3666
1500 (μ = 1%)	β^{opt}	--------	0.2	0.3	0.3	0.3	0.3
	f^{opt}	0.97	1	1	1	1	1
	ζ_T^{opt}	0.1	0.05	0.03	0.02	0.02	0.02
	J_1^{opt}	0.3619	0.3549	0.3558	0.3485	0.3522	0.3505
1750 (μ = 1.16%)	β^{opt}	--------	0.2	0.3	0.3	0.3	0.3
	f^{opt}	0.97	1	1	1	1	1
	ζ_T^{opt}	0.1	0.05	0.03	0.02	0.03	0.03
	J_1^{opt}	0.3515	0.3425	0.3418	0.3371	0.3396	0.3396
2000 (μ = 1.32%)	β^{opt}	--------	0.2	0.3	0.3	0.3	0.3
	f^{opt}	0.97	1	1	1	1	1
	ζ_T^{opt}	0.1	0.06	0.03	0.03	0.03	0.03
	J_1^{opt}	0.3434	0.3329	0.3310	0.3286	0.3298	0.3298

J_9 and J_{10}). Thus, in general the optimum damping ratio of MTMD system is found to be less than that of TMD.

From Table 3, for the minimization of the performance criterion J_1, the optimum frequency band width, frequency ratio and damping ratio are 0.3, 1.00 and 0.03 respectively, when the number of dampers is maintained as 5 and the mass ratio is 1.00%. Exactly the same values of optimum parameters are obtained for the minimization of the criteria J_2, J_3 and J_4 also. On the other hand, when the criteria J_7 and J_8 are minimized these optimum parameters are 0.4, 1.00 and 0.02 respectively. The dimensionless quantities J_7 and J_8 are based on the peak acceleration quantities of the selected floors. These values are also near the optimum parameters obtained by the minimization of the criteria J_1 to J_4. And, by the minimization of the criteria J_9 and J_{10}, the optimum parameters are 0.1, 0.93 and 0.05, respectively. The dimensionless quantities J_9 and J_{10} are based on peak displacement quantities of the selected floors. However, the maximum peak acceleration quantity for the human comfort level is 0.02 g (20 cm/s^2) and the maximum peak displacement quantity is H/500 (i.e., 61.2 cm). The peak acceleration and the peak displacement quantities of the top floor are 12.59 cm/s^2 and 22.26 cm respectively, when the optimum parameters are maintained to minimize the criteria J_1 to J_4. Thus, the optimum parameters of 0.3, 1.00 and 0.03 may be maintained for the maximum advantage with 5 dampers in MTMD with a mass ratio of 1% (i.e., total mass of MTMD as 1500 Ton, i.e. of 300 Ton each).

Table 4. Variations of optimum parameters with the number of dampers in MTMD for the minimization of the performance criterion J_2

Mass of MTMD/TMD (Ton)	N	1	3	5	7	9	11
250 (μ = 0.16%)	β^{opt}	------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	0.99	1	0.99	0.99	0.99	0.99
	ζ_T^{opt}	0.03	0.01	0.01	0.01	0.01	0.01
	J_2^{opt}	0.5001	0.4928	0.4787	0.4781	0.4780	0.4779
500 (μ = 0.33%)	β^{opt}	------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	0.98	1	0.99	0.99	0.99	0.99
	ζ_T^{opt}	0.04	0.02	0.02	0.02	0.02	0.02
	J_2^{opt}	0.4320	0.4258	0.4264	0.4262	0.4263	0.4265
750 (μ = 0.5%)	β^{opt}	------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	0.98	0.98	0.98	0.98	0.98	0.98
	ζ_T^{opt}	0.06	0.04	0.04	0.04	0.05	0.05
	J_2^{opt}	0.4029	0.4034	0.4022	0.4022	0.4022	0.4021
1000 (μ = 0.66%)	β^{opt}	------	0.1	0.2	0.2	0.2	0.2
	f^{opt}	0.9700	0.98	1	1	1	1
	ζ_T^{opt}	0.0700	0.06	0.03	0.04	0.04	0.04
	J_2^{opt}	0.3846	0.3846	0.3823	0.3839	0.3840	0.3841
1250 (μ = 0.82%)	β^{opt}	------	0.2	0.2	0.3	0.3	0.3
	f^{opt}	0.97	1	1	1	1	1
	ζ_T^{opt}	0.08	0.04	0.04	0.02	0.02	0.02
	J_2^{opt}	0.3702	0.3660	0.3659	0.3603	0.3642	0.3624
1500 (μ = 1%)	β^{opt}	------	0.2	0.3	0.3	0.3	0.3
	f^{opt}	0.9700	1	1	1	1	1
	ζ_T^{opt}	0.1000	0.05	0.03	0.02	0.02	0.02
	J_2^{opt}	0.3579	0.3504	0.3511	0.3437	0.3477	0.3460
1750 (μ = 1.16%)	β^{opt}	------	0.2	0.3	0.3	0.3	0.3
	f^{opt}	0.97	1	1	1	1	1
	ζ_T^{opt}	0.1	0.05	0.03	0.02	0.03	0.03
	J_2^{opt}	0.3474	0.3378	0.3369	0.3320	0.3348	0.3348
2000 (μ = 1.32%)	β^{opt}	------	0.2	0.3	0.3	0.3	0.3
	f^{opt}	0.9700	1	1	1	1	1
	ζ_T^{opt}	0.1000	0.05	0.03	0.03	0.03	0.03
	J_2^{opt}	0.3391	0.3281	0.3259	0.3235	0.3249	0.3249

Table 5. Variations of optimum parameters with the number of dampers in MTMD for the minimization of the performance criterion J_3

Mass of MTMD/TMD (Ton)	N	1	3	5	7	9	11
250 (μ = 0.16%)	β^{opt}	------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	0.9800	1	0.99	0.99	0.99	0.99
	ζ_T^{opt}	0.0300	0.01	0.01	0.01	0.01	0.01
	J_3^{opt}	0.6163	0.6120	0.6008	0.6006	0.6006	0.6006
500 (μ = 0.33%)	β^{opt}	------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	0.9800	1	0.99	0.99	0.98	0.98
	ζ_T^{opt}	0.0400	0.02	0.02	0.02	0.03	0.03
	J_3^{opt}	0.5715	0.5683	0.5677	0.5677	0.5676	0.5675
750 (μ = 0.5%)	β^{opt}	------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	0.9700	0.96	0.97	0.97	0.97	0.97
	ζ_T^{opt}	0.0500	0.03	0.04	0.04	0.04	0.04
	J_3^{opt}	0.5537	0.5510	0.5513	0.5515	0.5516	0.5517
1000 (μ = 0.66%)	β^{opt}	------	0.1	0.1	0.1	0.1	0.3
	f^{opt}	0.9600	0.96	0.96	0.96	0.96	0.98
	ζ_T^{opt}	0.0700	0.05	0.05	0.05	0.05	0.01
	J_3^{opt}	0.5430	0.5409	0.5412	0.5414	0.5415	0.5412
1250 (μ = 0.82%)	β^{opt}	------	0.1	0.2	0.3	0.3	0.3
	f^{opt}	0.95	0.95	0.99	1	0.98	0.98
	ζ_T^{opt}	0.08	0.06	0.04	0.02	0.02	0.01
	J_3^{opt}	0.5354	0.5340	0.5342	0.5301	0.5315	0.5304
1500 (μ = 1%)	β^{opt}	------	0.2	0.3	0.3	0.3	0.3
	f^{opt}	0.9500	0.99	1	1	0.98	0.99
	ζ_T^{opt}	0.0900	0.05	0.03	0.02	0.02	0.02
	J_3^{opt}	0.5295	0.5272	0.5267	0.5219	0.5227	0.5230
1750 (μ = 1.16%)	β^{opt}	------	0.2	0.3	0.3	0.3	0.3
	f^{opt}	0.94	0.99	0.99	1	0.98	0.98
	ζ_T^{opt}	0.1	0.05	0.03	0.02	0.02	0.02
	J_3^{opt}	0.5249	0.5215	0.5194	0.5170	0.5168	0.5168
2000 (μ = 1.32%)	β^{opt}	------	0.3	0.3	0.3	0.3	0.3
	f^{opt}	0.94	0.98	0.99	1	0.98	0.98
	ζ_T^{opt}	0.1000	0.04	0.03	0.03	0.02	0.02
	J_3^{opt}	0.5249	0.5215	0.5194	0.5170	0.5168	0.5168

Table 6. Variations of optimum parameters with the number of dampers in MTMD for the minimization of the performance criterion J_4

Mass of MTMD/TMD (Ton)	N	1	3	5	7	9	11
250 (μ = 0.16%)	β^{opt}	------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	0.98	1	0.99	0.99	0.99	0.99
	ζ_T^{opt}	0.03	0.01	0.01	0.01	0.01	0.01
	J_4^{opt}	0.6181	0.6137	0.6027	0.6025	0.6025	0.6025
500 (μ = 0.33%)	β^{opt}	------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	0.98	1	0.99	0.99	0.98	0.98
	ζ_T^{opt}	0.04	0.02	0.02	0.02	0.03	0.03
	J_4^{opt}	0.5735	0.5702	0.5697	0.5697	0.5697	0.5696
750 (μ = 0.5%)	β^{opt}	------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	0.97	0.96	0.97	0.97	0.97	0.97
	ζ_T^{opt}	0.05	0.03	0.04	0.04	0.04	0.04
	J_4^{opt}	0.5559	0.5534	0.5535	0.5537	0.5538	0.5539
1000 (μ = 0.66%)	β^{opt}	------	0.1	0.1	0.1	0.1	0.3
	f^{opt}	0.96	0.96	0.96	0.96	0.96	0.98
	ζ_T^{opt}	0.07	0.05	0.05	0.05	0.05	0.01
	J_4^{opt}	0.5453	0.5433	0.5435	0.5437	0.5439	0.5436
1250 (μ = 0.82%)	β^{opt}	------	0.1	0.2	0.3	0.3	0.3
	f^{opt}	0.9600	0.96	0.99	1	0.98	0.98
	ζ_T^{opt}	0.0800	0.06	0.04	0.02	0.02	0.01
	J_4^{opt}	0.5377	0.5364	0.5364	0.5324	0.5339	0.5329
1500 (μ = 1%)	β^{opt}	------	0.2	0.3	0.3	0.3	0.3
	f^{opt}	0.95	0.99	1	1	0.98	0.99
	ζ_T^{opt}	0.09	0.05	0.03	0.02	0.02	0.02
	J_4^{opt}	0.5318	0.5295	0.5289	0.5242	0.5252	0.5254
1750 (μ = 1.16%)	β^{opt}	------	0.2	0.3	0.3	0.3	0.3
	f^{opt}	0.95	0.99	0.99	1	0.98	0.98
	ζ_T^{opt}	0.1	0.05	0.03	0.02	0.02	0.02
	J_4^{opt}	0.5272	0.5237	0.5218	0.5192	0.5194	0.5194
2000 (μ = 1.32%)	β^{opt}	------	0.3	0.3	0.3	0.3	0.3
	f^{opt}	0.9400	0.98	0.99	1	0.98	0.98
	ζ_T^{opt}	0.1000	0.04	0.03	0.03	0.02	0.02
	J_4^{opt}	0.5234	0.5187	0.5166	0.5160	0.5155	0.5155

Table 7. Variations of optimum parameters with the number of dampers in MTMD for the minimization of the performance criterion J_7

Mass of MTMD/TMD (Ton)	N	1	3	5	7	9	11
250 (μ = 0.16%)	β^{opt}	------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	0.99	0.99	0.99	0.99	0.99	0.99
	ζ_T^{opt}	0.03	0.01	0.01	0.01	0.01	0.01
	J_7^{opt}	0.4911	0.5149	0.4964	0.4915	0.4924	0.4903
500 (μ = 0.33%)	β^{opt}	------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	0.97	0.98	0.96	0.97	0.97	0.97
	ζ_T^{opt}	0.06	0.02	0.04	0.04	0.04	0.04
	J_7^{opt}	0.4434	0.4433	0.4388	0.4396	0.4401	0.4403
750 (μ = 0.5%)	β^{opt}	------	0.1	0.2	0.2	0.1	0.2
	f^{opt}	0.94	0.94	0.99	0.94	0.94	0.94
	ζ_T^{opt}	0.09	0.07	0.02	0.04	0.08	0.05
	J_7^{opt}	0.4280	0.4228	0.4203	0.4257	0.4248	0.4225
1000 (μ = 0.66%)	β^{opt}	------	0.1	0.3	0.3	0.1	0.1
	f^{opt}	0.92	0.92	0.99	0.99	0.91	0.91
	ζ_T^{opt}	0.10	0.1	0.02	0.02	0.1	0.1
	J_7^{opt}	0.4175	0.4145	0.4099	0.4120	0.4132	0.4132
1250 (μ = 0.82%)	β^{opt}	------	0.2	0.3	0.4	0.4	0.2
	f^{opt}	0.92	0.91	0.99	1	1	0.91
	ζ_T^{opt}	0.1	0.09	0.01	0.01	0.02	0.1
	J_7^{opt}	0.4175	0.4107	0.3945	0.3897	0.4017	0.4108
1500 (μ = 1%)	β^{opt}	------	0.4	0.4	0.4	0.4	0.5
	f^{opt}	0.91	0.97	1	1	0.99	0.95
	ζ_T^{opt}	0.1	0.06	0.02	0.02	0.02	0.02
	J_7^{opt}	0.4126	0.4062	0.3790	0.3744	0.3931	0.3990
1750 (μ = 1.16%)	β^{opt}	------	0.4	0.4	0.4	0.5	0.5
	f^{opt}	0.91	0.97	1	1	1	1
	ζ_T^{opt}	0.1	0.08	0.02	0.02	0.02	0.02
	J_7^{opt}	0.4118	0.4026	0.3763	0.3769	0.3753	0.3753
2000 (μ = 1.32%)	β^{opt}	------	0.5	0.5	0.5	0.5	0.5
	f^{opt}	0.91	0.96	1	1	1	1
	ζ_T^{opt}	0.10	0.07	0.02	0.02	0.02	0.02
	J_7^{opt}	0.4157	0.3983	0.3703	0.3749	0.3655	0.3655

Table 8. Variations of optimum parameters with the number of dampers in MTMD for the minimization of the performance criterion J_8

Mass of MTMD/TMD (Ton)	N	1	3	5	7	9	11
250 (μ = 0.16%)	β^{opt}	------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	1	0.99	0.99	0.99	0.99	0.99
	ζ_T^{opt}	0.02	0.02	0.01	0.01	0.01	0.01
	J_8^{opt}	0.5087	0.5141	0.4760	0.4832	0.4808	0.4794
500 (μ = 0.33%)	β^{opt}	------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	0.98	0.98	0.98	0.98	0.98	0.98
	ζ_T^{opt}	0.07	0.04	0.04	0.04	0.04	0.04
	J_8^{opt}	0.4702	0.4632	0.4634	0.4642	0.4646	0.4648
750 (μ = 0.5%)	β^{opt}	------	0.2	0.2	0.2	0.2	0.2
	f^{opt}	0.95	1	0.95	0.95	0.95	0.95
	ζ_T^{opt}	0.09	0.02	0.06	0.06	0.07	0.07
	J_8^{opt}	0.4546	0.4425	0.4503	0.4498	0.4500	0.4499
1000 (μ = 0.66%)	β^{opt}	------	0.3	0.3	0.2	0.2	0.2
	f^{opt}	0.94	1	0.99	0.93	0.93	0.93
	ζ_T^{opt}	0.10	0.03	0.02	0.09	0.09	0.09
	J_8^{opt}	0.4479	0.4344	0.4351	0.4389	0.4387	0.4388
1250 (μ = 0.82%)	β^{opt}	------	0.3	0.4	0.4	0.4	0.4
	f^{opt}	0.94	1	0.99	1	1	1
	ζ_T^{opt}	0.10	0.05	0.03	0.02	0.03	0.03
	J_8^{opt}	0.4438	0.4292	0.4230	0.4299	0.4281	0.4314
1500 (μ = 1%)	β^{opt}	------	0.4	0.5	0.4	0.5	0.5
	f^{opt}	0.93	1	1	1	1	1
	ζ_T^{opt}	0.10	0.06	0.02	0.03	0.02	0.03
	J_8^{opt}	0.4353	0.4249	0.4056	0.4202	0.4217	0.4233
1750 (μ = 1.16%)	β^{opt}	------	0.4	0.5	0.5	0.5	0.5
	f^{opt}	0.91	1	1	0.99	1	1
	ζ_T^{opt}	0.1	0.08	0.02	0.02	0.01	0.01
	J_8^{opt}	0.4256	0.4183	0.3943	0.4087	0.4081	0.4081
2000 (μ = 1.32%)	β^{opt}	------	0.4	0.5	0.5	0.5	0.5
	f^{opt}	0.91	1	1	0.99	1	1
	ζ_T^{opt}	0.1000	0.08	0.02	0.02	0.01	0.01
	J_8^{opt}	0.4193	0.4135	0.3917	0.4046	0.3996	0.3996

Table 9. Variations of optimum parameters with the number of dampers in MTMD for the minimization of the performance criterion J_9

Mass of MTMD/TMD (Ton)	N	1	3	5	7	9	11
250 (μ = 0.16%)	β^{opt}	------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	1.00	0.97	0.98	0.98	0.98	0.98
	ζ_T^{opt}	0.01	0.01	0.01	0.01	0.01	0.01
	J_9^{opt}	0.6915	0.6765	0.6743	0.6598	0.6636	0.6645
500 (μ = 0.33%)	β^{opt}	------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	0.97	0.96	0.96	0.96	0.96	0.96
	ζ_T^{opt}	0.02	0.02	0.02	0.02	0.02	0.02
	J_9^{opt}	0.6072	0.5957	0.5944	0.5978	0.5989	0.5996
750 (μ = 0.5%)	β^{opt}	------	0.1	0.1	0.2	0.1	0.1
	f^{opt}	0.94	0.96	0.95	0.96	0.95	0.95
	ζ_T^{opt}	0.05	0.02	0.03	0.01	0.04	0.04
	J_9^{opt}	0.5987	0.5746	0.5772	0.5820	0.5848	0.5847
1000 (μ = 0.66%)	β^{opt}	------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	0.94	0.95	0.95	0.94	0.94	0.94
	ζ_T^{opt}	0.06	0.03	0.03	0.05	0.05	0.05
	J_9^{opt}	0.5789	0.5714	0.5747	0.5745	0.5742	0.5740
1250 (μ = 0.82%)	β^{opt}	------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	0.94	0.94	0.94	0.94	0.94	0.94
	ζ_T^{opt}	0.06	0.04	0.04	0.04	0.04	0.05
	J_9^{opt}	0.5770	0.5717	0.5695	0.5688	0.5699	0.5696
1500 (μ = 1%)	β^{opt}	------	0.3	0.1	0.1	0.1	0.1
	f^{opt}	0.94	1	0.93	0.93	0.93	0.93
	ζ_T^{opt}	0.06	0.02	0.05	0.05	0.05	0.05
	J_9^{opt}	0.5834	0.5724	0.5714	0.5713	0.5713	0.5713
1750 (μ = 1.16%)	β^{opt}	------	0.4	0.1	0.1	0.1	0.1
	f^{opt}	0.92	1	0.91	0.92	0.92	0.92
	ζ_T^{opt}	0.05	0.02	0.07	0.06	0.06	0.06
	J_9^{opt}	0.5864	0.5546	0.5774	0.5754	0.5740	0.5740
2000 (μ = 1.32%)	β^{opt}	------	0.4	0.1	0.1	0.1	0.1
	f^{opt}	0.91	1	0.91	0.91	0.91	0.91
	ζ_T^{opt}	0.07	0.02	0.06	0.06	0.06	0.06
	J_9^{opt}	0.5872	0.5736	0.5887	0.5861	0.5848	0.5848

Table 10. Variations of optimum parameters with the number of dampers in MTMD for the minimization of the performance criterion J_{10}

Mass of MTMD/TMD (Ton)	N	1	3	5	7	9	11
250 (μ = 0.16%)	β^{opt}	-------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	1.00	0.97	0.98	0.98	0.98	0.98
	ζ_T^{opt}	0.01	0.01	0.01	0.01	0.01	0.01
	J_{10}^{opt}	0.6916	0.6849	0.6797	0.6668	0.6706	0.6715
500 (μ = 0.33%)	β^{opt}	-------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	0.97	0.96	0.96	0.96	0.96	0.96
	ζ_T^{opt}	0.02	0.02	0.02	0.02	0.02	0.02
	J_{10}^{opt}	0.6135	0.6012	0.5999	0.6034	0.6047	0.6055
750 (μ = 0.5%)	β^{opt}	-------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	0.95	0.96	0.95	0.95	0.95	0.95
	ζ_T^{opt}	0.04	0.02	0.03	0.03	0.03	0.03
	J_{10}^{opt}	0.6002	0.5816	0.5829	0.5844	0.5858	0.5869
1000 (μ = 0.66%)	β^{opt}	-------	0.1	0.1	0.1	0.1	0.1
	f^{opt}	0.94	0.95	0.95	0.95	0.94	0.94
	ζ_T^{opt}	0.0500	0.03	0.03	0.03	0.05	0.05
	J_{10}^{opt}	0.5864	0.5787	0.5773	0.5799	0.5812	0.5810
1250 (μ = 0.82%)	β^{opt}	-------	0.1	0.1	0.1	0.1000	0.1
	f^{opt}	0.94	0.94	0.94	0.9400	0.94	0.94
	ζ_T^{opt}	0.06	0.04	0.04	0.04	0.04	0.04
	J_{10}^{opt}	0.5852	0.5786	0.5765	0.5758	0.5757	0.5760
1500 (μ = 1%)	β^{opt}	-------	0.5	0.1	0.1	0.1	0.1
	f^{opt}	0.93	0.99	0.93	0.93	0.93	0.93
	ζ_T^{opt}	0.07	0.02	0.05	0.05	0.05	0.05
	J_{10}^{opt}	0.5868	0.5751	0.5778	0.5777	0.5777	0.5777
1750 (μ = 1.16%)	β^{opt}	-------	0.4	0.1	0.1	0.1	0.1
	f^{opt}	0.92	1	0.91	0.91	0.92	0.92
	ζ_T^{opt}	0.06	0.02	0.07	0.07	0.06	0.06
	J_{10}^{opt}	0.5902	0.5623	0.5852	0.5844	0.5832	0.5832
2000 (μ = 1.32%)	β^{opt}	-------	0.4	0.5	0.1	0.1	0.1
	f^{opt}	0.91	1	1	0.91	0.91	0.91
	ζ_T^{opt}	0.07	0.02	0.02	0.06	0.06	0.06
	J_{10}^{opt}	0.5889	0.5817	0.5941	0.5953	0.5940	0.5940

5. Conclusions

Numerical study of wind excited benchmark building with TMD/MTMD at the top floor of the benchmark building is carried out under the deterministic across wind load. Optimum parameters of TMD are obtained by numerical procedure. The robustness of MTMD is investigated. The effects of design parameters such as mass ratio, damping ratio, number of dampers, tuning frequency and frequency band width is investigated. From the trends of the numerical results of the present study, the following conclusions may be drawn:

1. It is found that the optimal value of tuning frequency ratio is close to 1, and the value becomes closer to 1 as the mass ratio decreases in cases of both TMD and MTMD.

2. The sensitivity of performance to the tuning frequency ratio is reduced with increasing mass ratio in cases of both TMD and MTMD.

3. The optimum value of damping ratio increases with mass ratio for TMD.

4. In general the optimum damping ratio of MTMD system is found to be less than that of TMD.

5. There exists an optimum damping ratio of MTMD for the performance criteria.

6. The sensitivity of the performance to the damping ratio of TMD decreases with increase in mass ratio.

7. Looking at the overall performance of TMD on the wind excited benchmark building, the optimum parameters that are obtained by the minimization of J_1 may be maintained with μ=0.82.

8. Increasing number of dampers (N) in MTMD beyond 5 does not provide significant response reduction.

9. For the lower values of mass ratio the performance criteria increase with frequency band width (β). However, for higher values of mass ratio there exists an optimum value of β. With the increase in the mass ratio the optimum value of β increases.

10. The design parameters like frequency band width, frequency ratio and damping ratio may be maintained as 0.3, 1.0 and 0.03, respectively with 5 dampers and the mass ratio of 1% for MTMD on the benchmark building.

11. It is found that the performance of an MTMD is almost equivalent to that of a single TMD when there exists no uncertainty in stiffness, whereas an MTMD shows superior performance to a single TMD when stiffness uncertainty exists.

References

Ayorinde, E. O.; Warburton, G. B. 1980. Minimizing structural vibrations with absorbers, *Earthquake Engineering & Structural Dynamics* 8(3): 219–236. doi:10.1002/eqe.4290080303

Housner, G. W.; Bergman, L. A.; Caughey, T. K.; Chassiakos, A. G.; Claus, R. O.; Masri, S. F.; Skelton, R. E.; Soong, T. T.; Spencer, B. F.; Yao, J. T. P. 1997. Structural control: Past, Present and Future, *Journal of Engineering Mechanics* ASCE 123(9): 897–971. doi:0.1061/(ASCE)0733-9399(1997)123:9(897)

Iwanami, K.; Seto, K. 1984. Optimum design of dual tuned mass dampers and their effectiveness, in *Proc. of the Japan Society of Mechanical Engineering* 50: 44–52.

Jangid, R. S. 1999. Optimum multiple tuned mass dampers for base excited undamped system, *Earthquake Engineering & Structural Dynamics* 28(9): 1041–1049. doi:10.1002/(SICI)1096-9845(199909)28:9<1041::AID-EQE853>3.0.CO;2-E

Kareem, A.; Kline, S. 1995. Performance of multiple mass dampers under random loading, *Journal of Structural Engineering* ASCE 121(2): 348–361. doi:10.1061/(ASCE)0733-9445(1995)121:2(348)

Kim, H.; Adeli, H. 2005. Wind-induced motion control of 76-storey benchmark building using the hybrid damper-TLCD system, *Journal of Structural Engineering* ASCE 131(12): 1794–1802.

Lewandowski, R.; Grzymisławska, J. 2009. Analysis of structures with multiple tuned mass dampers, *Journal of Civil Engineering and Management* 15(1): 77–86. doi:10.3846/1392-3730.2009.15.77-86

Li, C. 2000. Performance of multiple tuned mass dampers for attenuating undesirable oscillations of structures under the ground acceleration, *Earthquake Engineering & Structural Dynamics* 29(9): 1405–1421. doi:10.1002/1096-9845(200009)29:9<1405::AID-EQE976>3.0.CO;2-4

Lu, L.-Y. 2004. Predictive control of seismic structures with semi-active friction dampers, *Earthquake Engineering & Structural Dynamics* 33(5): 647–668. doi:10.1002/eqe.371

Matsagar, V. A.; Jangid, R. S. 2005. Visco-elastic damper connected adjacent structures involving seismic isolation, *Journal of Civil Engineering and Management* 11(4): 309–322.

Min, K.-W.; Kim, H.-S.; Lee, S.-H.; Kim, H.; Ahn, S. K. 2005. Performance evaluation of tuned liquid column dampers for response control of a 76-storey benchmark building, *Engineering Structures* 27(7): 1101–1112. doi:10.1016/j.engstruct.2005.02.008

Samali, B.; Kwok, K. C. S.; Wood, G. S.; Yang, J. N. 2004a. Wind tunnel tests for wind-excited benchmark building, *Journal of Engineering Mechanics* ASCE 130(4): 447–450. doi:10.1061/(ASCE)0733-9399(2004)130:4(447)

Samali, B.; Mayol, E.; Kwock, K. C. S.; Mack, A.; Hitchcock, P. 2004b. Vibration Control of the Wind-Excited 76-Storey Benchmark Building by Liquid Column Vibration Absorber, *Journal of Engineering Mechanics* ASCE 130(4): 478–485. doi:10.1061/(ASCE)0733-9399(2004)130:4(478)

Smart Stuctures Technology Laboratory (SSTL). 2002. *Structural control: Benchmark comparisons* [accessed 14 May 2010]. Available from Internet: <http://www.nd.edu/~quake/benh.html>.

Soong, T. T.; Dargush, G. F. 1997. *Passive energy dissipation systems in structural engineering*. John Wiley & Sons, Chichester. 368 p.

Varadarajan, N.; Nagarajaiah, S. 2004. Wind response control of building with variable stiffness tuned mass damper using empirical mode decomposition/Hilbert transform, *Journal of Engineering Mechanics* ASCE 130(4): 451–458. doi:10.1061/(ASCE)0733-9399(2004)130:4(451)

Xu, K.; Igusa, T. 1992. Dynamic characteristics of multiple substructures with closely spaced frequencies, *Earthquake Engineering & Structural Dynamics* 21(12): 1059–1070. doi:10.1002/eqe.4290211203

Yamaguchi, H.; Harnpornachai, N. 1993. Fundamental characteristics of multiple tuned mass dampers for suppressing harmonically forced oscillations, *Earthquake Engineering & Structural Dynamics* 22(1): 51–62. doi:10.1002/eqe.4290220105

Yang, J. N.; Agrawal, A. K.; Samali, B.; Wu, J.-C. 2004. Benchmark problem for response control of wind-excited tall buildings, *Journal of Engineering Mechanics* ASCE 130(4): 437–446. doi:10.1061/(ASCE)0733-9399(2004)130:4(437)

OPTIMALŪS MASĖS SLOPINTUVAI VĖJO VEIKIAMUOSE AUKŠTYBINIUOSE PASTATUOSE

V. B. Patil, R. S. Jangid

Santrauka

Straipsnyje tiriamas kelių masės slopintuvų (KMS), įrengtų aukštybinio pastato, kurį veikia vėjo apkrovos, viršutiniame aukšte, poveikis konstrukcijai. Šis KMS poveikis lyginamas su vieno masės slopintuvo (VMS) poveikiu, teigiant, kad abiem atvejais suminės masės reikšmė yra ta pati. Pagal KMS ir VMS sudarytos judėjimo lygtys išspręstos pritaikius erdvinio būvio formuluotę. Iš pradžių VMS įrengiamas viršutiniame pastato aukšte ir šiam atvejui suskaičiuojami optimalūs slopintuvo parametrai, minimizuojant įvairius darbo kriterijus ir įvertinant skirtingus masės koeficientus. Po to KMS įrengiami viršutiniame pastato aukšte ir optimalūs parametrai apskaičiuojami šiam atvejui, įvertinant skirtingus masės koeficientus ir skirtingą slopintuvų skaičių. Kiekvieno slopintuvo standumas KMS atveju nekinta. Daroma išvada, kad KMS įrengimas – gana efektyvi ir veiksminga priemonė siekiant išvengti vibracijų aukštybiniuose pastatuose.

Reikšminiai žodžiai: aukštybinis pastatas, optimalūs parametrai, KMS, pasyvioji kontrolė, vėjo apkrova, aukštas pastatas.

Veeranagouda B. PATIL. Professor in B.V.B. College of Engineering and Technology, Hubli, Karnataka State, India. obtained his Ph.D. degree in Civil Engineering Department, I.I.T. Bombay, Mumbai, India. He is a Life member of Indian concrete Institute, India. He is also a life member of Institute of Engineers, India.

Radhey Shyam JANGID. Professor at the Department of Civil Engineering at Indian Institute of Technology Bombay, India. He received B.E. (Hons.) in Civil Engineering from the University of Jodhpur, India in 1989, and his M.Tech. and Ph.D. in Structural Engineering from Indian Institute of Technology Delhi, India in 1991 and 1993, respectively. His research interest includes the aseismic design of structures using active and passive control devices and the dynamic analysis of non-classically damped systems. His recent research contribution includes the base isolation for near-fault motions and its application to the bridges and tanks, multiple tuned mass dampers for vibration control and active control of torsionally coupled structures.