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LIMITED DEFORMABILITY DESIGN OF HIGH-STRENGTH CONCRETE BEAMS IN LOW TO MODERATE SEISMICITY REGIONS

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Abstract. In the design of reinforced concrete (RC) beams located in low-moderate seismicity regions, adequate flexural deformability apart from flexural strength to cater for the imposed seismic demand should be designed. As per the existing RC design codes, this is achieved by restricting the maximum neutral axis depth or tension steel ratio, or limiting the minimum confining steel. However, these deemed-to-satisfy rules were derived many years ago based on normal-strength concrete and steel, which would impair the deformability when applied directly to RC beams made of high-strength materials. To resolve the problem, a new design method based on a prescribed deformability is advocated. In this study, the authors proposed that instead of complying with the deemed-to-satisfy rules, a consistent deformability derived based on the design requirements of Eurocode 2 should be provided to all RC beams located in low-moderate seismicity regions. Using the theoretical formulas developed previously by the authors, two different sets of design values expressed in terms of maximum tension steel ratio and neutral axis to beam effective depths for different concrete and steel yield strengths are evaluated. Finally, simplified guidelines for designing RC beams satisfying the proposed deformability requirement are developed for practical design application.

Keywords: beams, confinement, curvature, deformability, design formulas, high-strength concrete, high-strength steel, low-moderate seismicity, reinforced concrete, rotation capacity.

1. Introduction

In the traditional design of reinforced concrete (RC) beams not located in seismic regions, more attention has been put on the design of sufficient flexural strength than flexural deformability. The provision of flexural deformability only relies on some empirical deemed-to-satisfy rules that control the maximum tension steel area or neutral axis depth. This is understandable because the deformability demand in non-seismic region is not very large and that provided by the existing deemed-to-satisfy rules has been proven to be sufficient for RC beams made of normal-strength concrete (NSC) and normal-strength steel (NSS) (Park and Ruitong 1988; Kwan *et al*. 2006). However, for RC beams located in regions of low to moderate seismicity, the design for sufficient deformability to cater for the imposed seismic demand (Kuang and Atanda 2005; Djebbar and Chikh 2007; Seifi *et al*. 2008; Tsang *et al*. 2009) is as crucial as the design of sufficient flexural strength. Furthermore, the deformability provided to these beams should be larger than that provided to those in non-seismic regions. Therefore, the existing empirical deemed-to-satisfy rules should not be applied for designing RC beams located in low to moderate seismicity regions.

Apart from the above, the existing deemed-to-satisfy rules are not be able to provide a consistent deformability to RC beams made of high-strength concrete (HSC) and/or high-strength steel (HSS). Except in Eurocode 2 (ECS 2004) that a set of more stringent requirements are specified for HSC beams, these empirical rules are not dependent on concrete and steel yield strength. However, as reported in a series of theoretical studies conducted on flexural deformability carried out previously by the authors (Ho *et al*. 2010a,b; Zhou *et al*. 2010), it is evident that at a given tension steel ratio or neutral axis depth, the deformability of RC beams varies significantly with the concrete and steel yield strength. Therefore, it is apparent that the deformability provided to RC beams made of HSC and/or HSS as per the deemed-to-satisfy rules would be smaller than that provided to RC beams made of NSC and NSS. More critically, the deformability would decrease to an unacceptably low level if these rules are adopted for HSC beams. Considering nowadays that the adoption of HSC and HSS, which reduces the amount of construction materials under the same design load and hence lower the embodied energy and carbon level in the structures (Bilodeau and Malhotra 2000; Kosior-Kazberuk and Lelusz 2007; Scrivener and Kirkpatrick 2008; Xu *et al*. 2008), are getting more popular in tall buildings construction, the existing empirical rules for deformability design of RC beams in low to moderate seismicity regions should be revised to incorporate the adoption of HSC and/or HSS.

From performance-based design point of view, adequate flexural deformability design would prevent the beams from immediate collapse under earthquake attack (Vaidogas 2005; Zareian *et al*. 2010). During an earthquake attack, RC beams with sufficient deformability could resist the required seismic deflection without suffering severe inelastic damage and collapse (Wu *et al.* 2004). The enormous energy imposed by earthquake can subsequently be dissipated by redistributing moment to other parts of the beams through formation of plastic hinges (Bae and Bayrak 2008). To achieve this purpose, the reinforcement within the critical regions (Pam and Ho 2009) should be designed carefully such that a certain level of deformability would be provided and plastic hinges can be formed successfully (Ho and Pam 2003; Havaei and Keramati 2011; Yan and Au 2010; Ho 2011). The deformability could also be increased by providing sufficient confining pressure to the concrete core within critical region in the following ways by: (1) confining the concrete member using circular or rectangular hollow steel tube (Ellobody and Young 2006; Kuranovas and Kvedaras 2007; Šalna and Marčiukaitis 2007; Szmigiera 2007; Soundararajan and Shanmuhasundaram 2008; Kuranovas *et al*. 2009); (2) using external steel plate (Su *et al.* 2009); (3) wrapping the concrete member with fibre reinforced polymer (Kamiński and Trapko 2006; Valivonis and Skuturna 2007; Benzaid *et al*. 2008; Lam and Teng 2009; Wu and Wei 2010). These methods are commonly adopted in the design of low to medium rise buildings. For very tall building structures, the huge amount of energy induced by earthquake can in addition be dissipated by installing dampers (Matsagar and Jangid 2005; Lewandowski and Grzymislawska 2009; Chen and Han 2010) and adopting base isolation (Takewaki and Fujita 2009).

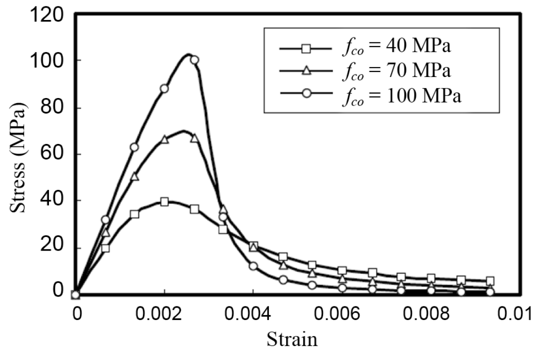
To evaluate the deformability of RC beams, the authors have carried out a series of theoretical studies to investigate the critical factors affecting their deformability (Ho *et al*. 2010a; Zhou *et al*. 2010). In these studies, it was proposed to use the "normalised rotation capacity" – defined as the product of ultimate beam curvature and effective depth – to evaluate the deformability of RC beams. Based on the results, it was found that the deformability of RC beams increases as the degree of reinforcement decreases or confining pressure increases. The use of HSC would decrease the deformability at a constant degree of reinforcement, but increase the deformability at a constant tension steel ratio. On the other hand, the use of HSS would decrease the deformability at a constant tension steel ratio, but it increases the deformability at a constant degree of reinforcement. Furthermore, the addition of confining steel would always increase the deformability of RC beams. Based on the results obtained, a formula for direct evaluation of deformability of RC beams based on the above parameters was developed. In a separate study the authors have also investigated the effects of these parameters on the limits of both flexural strength and deformability that can be achieved by a given beam section. From the results, it was found that for a given concrete strength and confining pressure, there is a maximum and minimum limits of flexural strength and deformability that can be achieved simultaneously. Moreover, for a given pair of concrete strength and deformability, there is a maximum allowable limit of the degree of reinforcement or tension steel ratio, beyond which the deformability can never be achieved apart from increasing the confining pressure or beam size.

In this paper, the deformability required for designing RC beams located in low to moderate seismicity regions will be derived based on the design requirements of Eurocode 2 (ECS 2004). The flexural design of RC beams possessing this deformability is named by the authors as the "Limited Deformability Design". As per Eurocode 2, the derived deformability will have sufficient rotation capacity at ultimate limit state for the formation of plastic hinge and hence allow moment redistribution to occur. Based on this, the maximum degree of reinforcement and tension steel ratio for designing RC beams with the prescribed deformability would be derived for different concrete and steel yield strength (Zhou *et al*. 2010). Lastly, for practical design application, a set of simplified design guidelines that depend on concrete and steel yield strength are developed for limited deformability design.

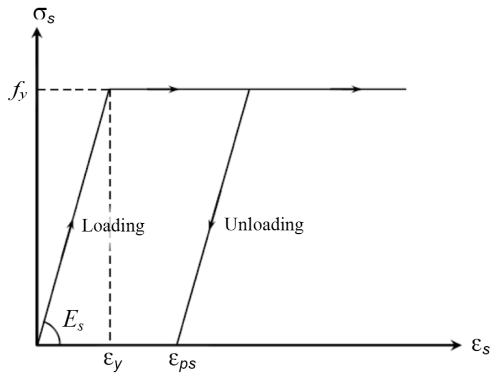
2. Nonlinear moment-curvature analysis

The deformability of RC beams is studied using the method of nonlinear moment-curvature analysis developed previously by the authors Pam *et al.* (2001) and Ho *et al.* (2003). The stress-strain curves of concrete as per Attard and Setunge (1996) were adopted while that of steel reinforcement follows the model given by Eurocode 2 (ECS 2004) but with the stress-path dependence incorporated to take account of the unloading properties. The unloading path is having the same initial elastic modulus until it reaches zero steel stress. The stress-strain curves of concrete and steel are shown in Fig. 1.

There were five assumptions made in the analysis: (1) plane sections before bending remain plane after bending; (2) the tensile strength of the concrete may be neglected; (3) there is no relative slip between concrete and steel reinforcement; (4) the concrete core is confined while the concrete cover is unconfined; (5) the confining pressure provided to the concrete core by confinement is assumed to be constant throughout the concrete compression zone. Assumptions (1) to (4) are commonly accepted and have been adopted by various researchers (Park *et al.* 2007; Au *et al*. 2009; Bai and Au 2009; Lam *et al*. 2009; Kwak and Kim 2010). Assumption (5) is not exact but however a fairly reasonable assumption in the sense that: (i) at small concrete strains, the variation of confining pressure would not have significant effect on the confined concrete stress (Attard and Setunge 1996); (ii) when the extreme fibre of confined concrete reaches about 0.003–0.004 before concrete cover spalls off entirely, there will be some variations of confining pressure within the concrete compression zone due to strain gradient. However, as this happens within a narrow range of concrete strain, the differences in the confined concrete compressive force and moment capacity of column are not significant; (iii) after the concrete cover had spalled off completely at large concrete strain, the Poisson's ratio of concrete increases abruptly that causes the confining steel to yield. The confining pressure becomes a constant equal to $0.5\ k_e \rho_s f_{ys}$, where k_e is the confinement effectiveness factor

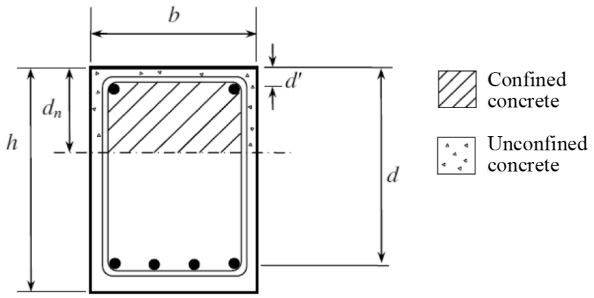


a) Stress-strain curves of concrete



b) Stress-strain curve of steel with stress-path dependence considered

Fig. 1. Stress-strain curves of concrete and steel reinforcement



Section properties

- b = 300 mm
- h = 600 mm
- d' = 50 mm
- d = 550 mm
- f_{co} = 40 to 100 MPa
- f_y = 400 to 800 MPa
- f_r = 0 to 4 MPa
- $\rho_t = A_{st}/bd$ = 0.4 to 2 ρ_b
- $\rho_c = A_{sc}/bd$ = 0 to 2%

Fig. 2. Beam sections analysed

(Mander *et al.* 1988), ρ_s and f_{ys} are respectively the volumetric ratio and yield strength of confining steel. In the analysis, the moment-curvature curve of the beam section is analysed by applying prescribed curvatures incrementally starting from zero. At a prescribed curvature, the stresses developed in the concrete and the steel are determined from their stress-strain curves. Then, the neutral axis depth and resisting moment are evaluated from equilibrium conditions. The above procedure is repeated until the resisting moment has increased to the peak and then decreased to below 80% of the peak moment. Fig. 2 describes a typical beam sections adopted in the nonlinear moment-curvature analysis.

3. Parametric study for deformability

3.1. Definition of deformability

In this study, the flexural deformability of beam sections are expressed in terms of normalised rotation capacity θ_{pl} defined as follows (Ho *et al.* 2010a):

$$\theta_{pl} = \phi_u d, \tag{1}$$

where ϕ_u is the ultimate curvature, d is the effective depth. The ultimate curvature is taken as the curvature when the resisting moment has dropped to 0.8 M_p after reaching M_p, where M_p is the peak moment. The value of θ_{pl} represents the rotation capacity of beam with plastic hinge length ℓ_p equal to its effective depth. For concrete beams subjected to pure flexure, the plastic hinge length remains relatively constant at about 0.4d (Mendis 2001) to 0.5d (Standards New Zealand 2006). Therefore, it is fairly reasonable to use the proposed normalised rotation capacity to compare the deformability of different RC beams.

A comprehensive parametric study on the effects of various factors on the normalised rotation capacity has been conducted previously (Ho *et al.* 2010a). The studied factors are: (1) degree of reinforcement – which measures the degree of beam section being under- or over-reinforced (Eq. 4); (2) concrete strength; (3) steel area ratios – which is defined as the tension or compression steel area divided by the effective area of beam section (i.e. breadth × effective depth); (4) steel yield strength; and (5) confining pressure. The beam sections analysed was shown in Fig. 2. The concrete strength f_{co} was varied from 40 to 100 MPa, the confining pressure f_r evaluated using the method recommended by Mander *et al.* (1988) was varied from 0 to 4 MPa, the tension steel ratio ρ_t was varied from 0.4 to 2 times the balanced steel ratio, the compression steel ratio ρ_c was varied from 0 to 2%, and the tension f_{yt} and compression f_{yc} steel yield strength were varied from 400 to 800 MPa.

3.2. Definition of balanced steel ratio

The balanced steel ratio of a beam section provides the area of tension steel which causes the steel with maximum tensile stress to yield during failure. It is defined as $\rho_{bo} = A_{sb}/bd$, where A_{sb} is the balanced steel area. For beam section containing tension steel area less than the balanced steel area, the steel will yield during failure and the section is under-reinforced. Otherwise, the steel will not yield during failure and the section in over-reinforced. For beam sections containing compression steel ratio ρ_c, the balanced steel ratio ρ_b is given by:

$$\rho_b = \rho_{bo} + (f_{yc} / f_{yt})\rho_c. \tag{2}$$

The values of ρ_{bo} for various concrete strengths and confining pressure are listed in Table 1 for different yield strength of tension steel (Ho *et al.* 2003).

Table 1. Balanced steel ratios ρ_{bo} for different tension steel yield strength

Balanced steel ratios ρ_{bo}(%) for f_{yt} = 400 MPa					
f_{co} (MPa)	f_r = 0 MPa	f_r = 1 MPa	f_r = 2 MPa	f_r = 3 MPa	f_r = 4 MPa
40	4.74	5.98	6.90	7.73	8.56
50	5.63	6.91	7.86	8.78	9.60
60	6.46	7.79	8.77	9.70	10.59
70	7.29	8.62	9.61	10.54	11.50
80	8.06	9.38	10.37	11.35	12.29
90	8.77	10.11	11.13	12.11	13.03
100	9.42	10.80	11.82	12.78	13.76
Balanced steel ratios ρ_{bo}(%) for f_{yt} = 600 MPa					
f_{co} (MPa)	f_r = 0 MPa	f_r = 1 MPa	f_r = 2 MPa	f_r = 3 MPa	f_r = 4 MPa
40	2.74	3.60	4.23	4.83	5.37
50	3.23	4.12	4.78	5.40	6.00
60	3.69	4.61	5.29	5.93	6.55
70	4.13	5.06	5.76	6.41	7.04
80	4.56	5.50	6.19	6.85	7.49
90	4.94	5.90	6.59	7.28	7.91
100	5.29	6.27	6.97	7.67	8.29
Balanced steel ratios ρ_{bo}(%) for f_{yt} = 800 MPa					
f_{co} (MPa)	f_r = 0 MPa	f_r = 1 MPa	f_r = 2 MPa	f_r = 3 MPa	f_r = 4 MPa
40	1.82	2.48	2.96	3.42	3.84
50	2.13	2.82	3.33	3.80	4.25
60	2.43	3.14	3.66	4.14	4.61
70	2.70	3.43	3.96	4.45	4.93
80	2.97	3.69	4.22	4.75	5.21
90	3.22	3.95	4.50	5.00	5.49
100	3.44	4.19	4.74	5.22	5.74

The following empirical equation was also derived for easy practical design application:

$$\rho_{bo} = 0.005\left(f_{co}\right)^{0.58}\left(1+1.2f_r\right)^{0.3}\left(f_{yt}/460\right)^{-1.35}. \quad (3)$$

All strengths are in MPa, 400 MPa $\le f_{yt} \le$ 800 MPa and $0 \le f_r \le 4$ MPa.

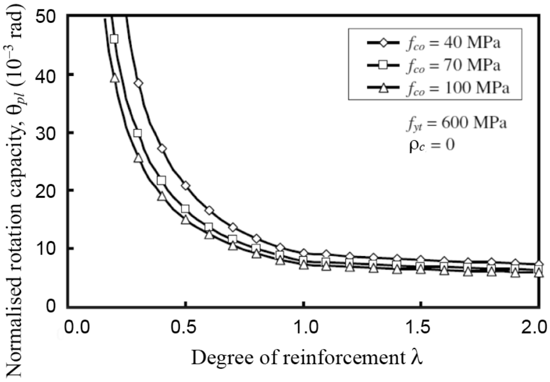
3.3. Effects of degree of reinforcement, concrete and steel yield strength

The degree of reinforcement λ, which accounts for the degree of section being under- or over-reinforced, is expressed in Eq. (4):

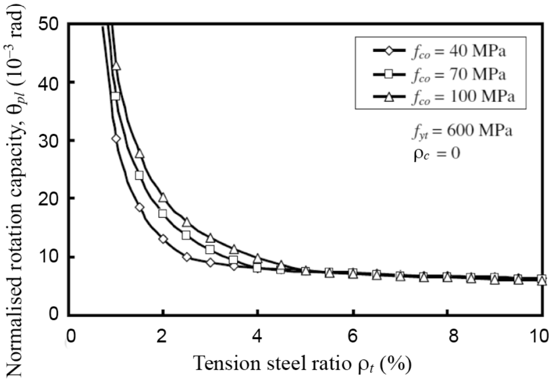
$$\lambda = \frac{f_{yt}\rho_t - f_{yc}\rho_c}{f_{yt}\rho_{bo}}. \quad (4)$$

By definition, the beam section is classified as under-reinforced, balanced and over-reinforced sections when λ is less than, equal to and larger than 1.0 respectively. To investigate the effects of λ on the deformability of RC beams, the normalised rotation capacity θ_{pl} is plotted against λ in Fig. 3a for different concrete strength. It could be seen that at a constant concrete strength, the deformability decreases as λ increases until reaching λ = 1.0. After that, the deformability remains relatively constant. On the other hand, it can be seen from Fig. 3a that at a constant λ, the deformability decreases as the concrete strength increases. However, if HSC is used at the same tension steel ratio ρ_t, it is evident from Fig. 3b that the deformability increases as concrete strength increases albeit that HSC is less deformable *per se*. This is because the balanced steel ratio increases as concrete strength increases, and hence for a given ρ_t, λ decreases and the deformability increases.

To investigate the effects of steel yield strengths on the deformability of RC beams, θ_{pl} is plotted against λ and ρ_t in Figs 4a and 4b respectively for different tension steel yield strength f_{yt}. On the other hand, Figs 5a and 5b plot the θ_{pl} against λ and ρ_t for different compression steel yield strength f_{yc}. Generally, it is observed from Fig. 4 that at a constant λ, the deformability increases as the tension steel yield strength increases, notwithstanding that HSS is less deformable *per se*. However, it decreases as the tension steel yield strength increases at a given ρ_t. From Figs 5a and 5b, it is seen that the deformability increases only very slightly as the compression steel yield strength increases at constant λ. Nonetheless, the deformability increases significantly as the compression steel yield strength increases at constant ρ_t.

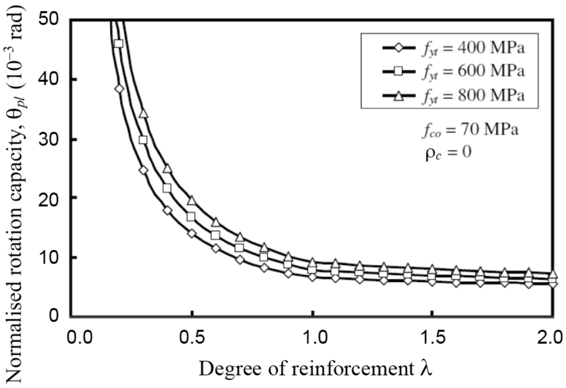


a) θ_{pl} against λ

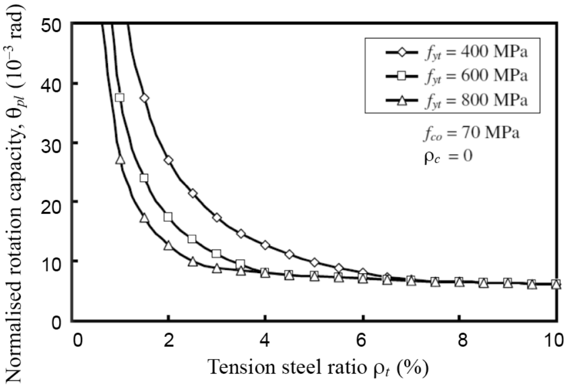


b) θ_{pl} against ρ_t

Fig. 3. Variation of deformability with degree of reinforcement and tension steel ratio

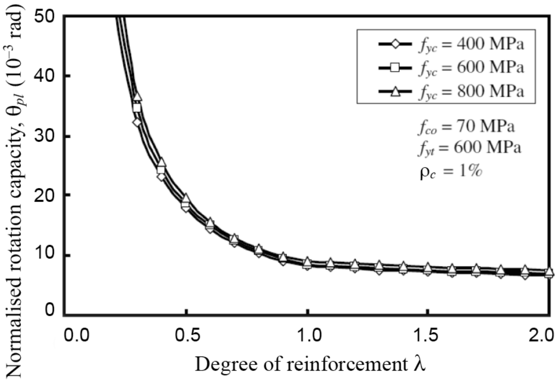


a) θ_{pl} against λ

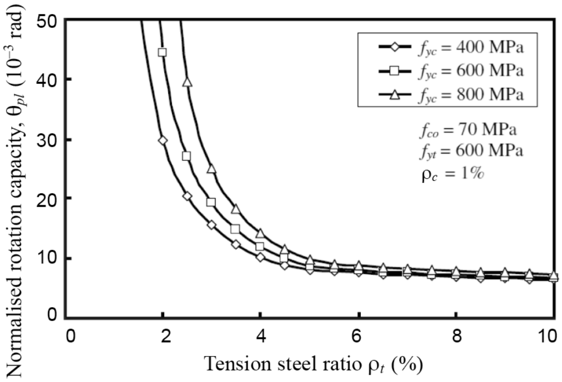


b) θ_{pl} against ρ_t

Fig. 4. Effects of tension steel yield strength on deformability



a) θ_{pl} against λ



b) θ_{pl} against ρ_t

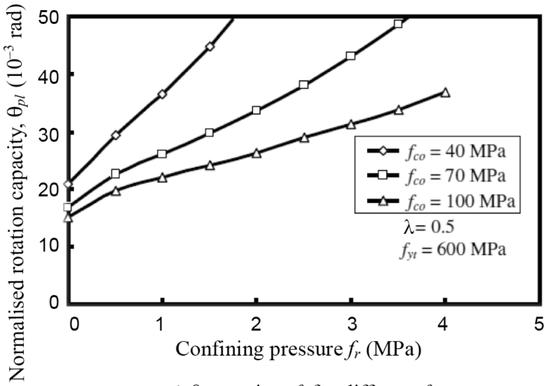
Fig. 5. Effects of compression steel yield strength on deformability

3.4. Effects of confining pressure

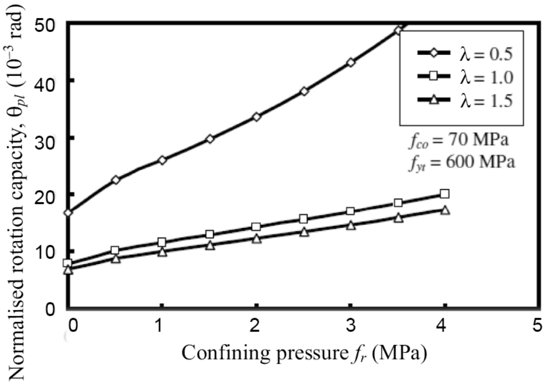
To study the effect of the confining pressure f_r, θ_{pl} is plotted against confining pressures f_r for different concrete strength f_{co}, degree of reinforcement λ and tension steel ratios ρ_t in Fig. 6. It is evident from Fig. 6a that at a given λ, θ_{pl} increases as the f_r increases for all concrete strength. It is also seen from Fig. 6b that at a fixed f_{co}, θ_{pl} increases as f_r increases for all λ. In Fig. 6c, it is seen that at a fixed f_{co}, θ_{pl} increases as the confining pressure f_r increases for all ρ_t.

3.5. Effects of neutral axis depth

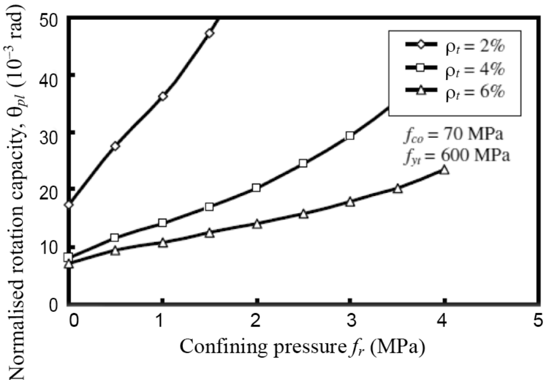
Alternatively, the degree of beam section being under- or over-reinforced may be expressed in terms of x_u/x_{ub}, where x_u and x_{ub} are the neutral axis depths of beam section and the balanced section, respectively. As per the existing RC design codes (Ministry of Construction 2001; ECS 2004; Standards New Zealand 2006; ACI Committee 2008), the value of x_u is measured at the ultimate limit state at maximum moment. To be consistent, the values of x_u and x_{ub} neutral axis depths presented in this study are all taken at the maximum moment point. To study the effect of x_u/x_{ub} and the neutral axis depth itself (expressed in dimensionless form of x_u/d) on the deformability of beams, θ_{pl} is plotted against x_u/x_{ub} and x_u/d in Figs 7 and 8 for different concrete strength and tension steel yield strength respectively. It can be observed from Fig. 7 that the deformability decreases as x_u/x_{ub} or x_u/d increases until x_u/x_{ub} is equal to 1.0, after which the deformability remains relatively constant. At a given ratio of x_u/x_{ub} or x_u/d, the deformability decreases as concrete strength increases. Therefore, adopting HSC would decrease the deformability of concrete beam at a specified ratio of x_u/x_{ub} or x_u/d. It is now evident that the empirical deemed-to-satisfy rules stipulated in the existing RC design codes, which restrict the maximum neutral axis depth for all concrete strength, would provide a smaller deformability to beam when HSC is adopted. For the effects of steel yield strength, it is apparent from Fig. 8a that at a given x_u/x_{ub}, the deformability increases as the tension steel yield strength increases. Nevertheless, at a given x_u/d, it can be seen from Fig. 8b that the deformability is insensitive to the tension steel yield strength.



a) θ_{pl} against f_r for different f_{co}

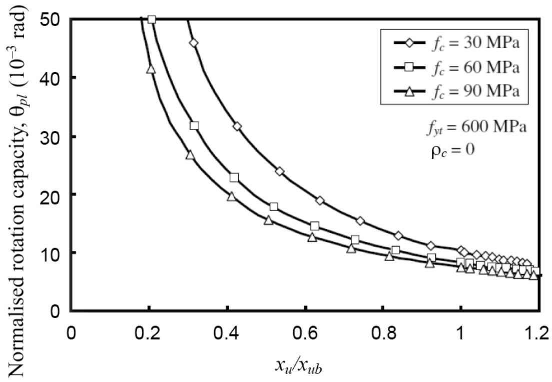


b) θ_{pl} against f_r for different λ

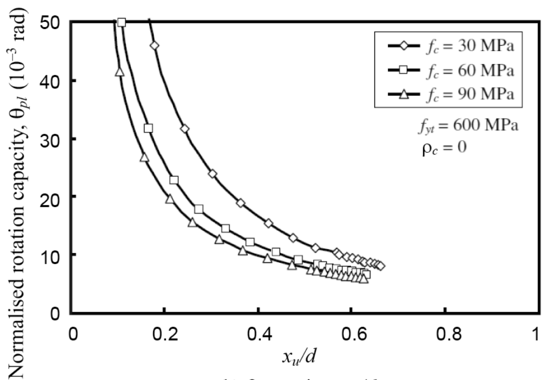


c) θ_{pl} against f_r for different ρ_t

Fig. 6. Effects of confining pressure on deformability

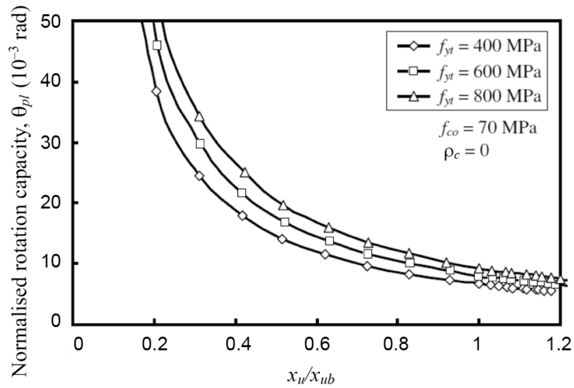


a) θ_{pl} against x_u/x_{ub}

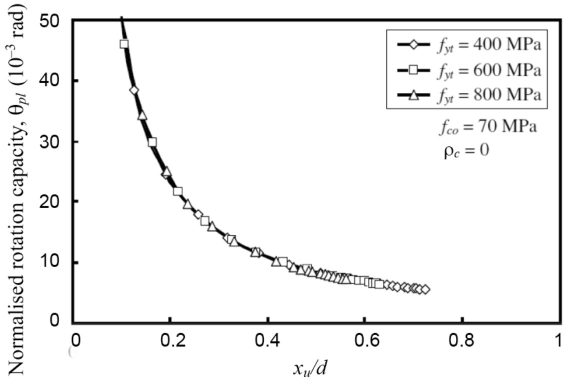


b) θ_{pl} against x_u/d

Fig. 7. Variation of deformability with neutral axis depth at different concrete strength



a) θ_{pl} against x_u/x_{ub}



b) θ_{pl} against x_u/d

Fig. 8. Variation of deformability with neutral axis depth at different steel yield strength

4. Limited deformability design of concrete beams

4.1. Required deformability for RC beams in low-moderate seismicity regions

In the existing RC design codes, the design of deformability for concrete beams subjected to seismic risk is usually governed by some sets of empirical deemed-to-satisfy rules that limit the maximum neutral axis depth or tension steel ratio. This ensures that a large tensile strain would be developed in the tension steel at the ultimate limit state such that adequate rotation capacity is provided to the RC beams for the formation of plastic hinge and moment redistribution to occur. The respective rules of some existing RC design codes are extracted and highlighted as follows:

1. American Code ACI 318 (ACI Committee 2008): In addition to Clause 10.3.5 that limits the maximum tension steel strain to not less than 0.004, confinement of minimum diameter of 10 mm and maximum spacing of not larger than $0.25d$, $8d_b$, $24d_s$ or 300 mm (whichever is the smallest), where d_b and d_s are the diameter of longitudinal and confining steel respectively, should be provided within the critical region as per Clause 7.1.0.5.1.
2. Chinese Code GB50011 (Ministry of Construction 2001): Clause 6.3.3 of the code requires the neutral axis depth to be not larger than 0.35 d. Confinement of minimum diameter of 8 mm and maximum spacing not larger than 0.25 d, 8 d_b or 100 mm (whichever is the smallest) should be provided within the critical region as per Clause 6.3.3.3.
3. European Code EC2 (ECS 2004): Clause 5.6.2.2 of the code limits the neutral axis depth to not more than 0.25 d when $f_c' \leq 50$ MPa or 0.15 d when $f_c' > 50$ MPa, in which f_c' is the concrete cylinder strength.
4. New Zealand Code NZS3101 (Standards New Zealand 2006): In addition to Clause 9.3.8.1 that restricts the neutral axis depth to not more than 0.75 of that in the balanced section, Clause 9.4.3.3 further restricts the maximum tension steel ratio should not be larger than $(f_c'+10)/6f_y \leq 2.5\%$ for reinforcement design within critical region. Compression steel ratio of not less than 0.38 times the tension steel ratio should also be provided as per Clause 9.4.3.4.

From the above, the deformability expressed in terms of normalised rotation capacity θ_{pl} provided by various existing design codes may be evaluated by their respective values of θ_{pl} at different concrete strength and steel yield strength. To reflect the ranges of concrete and steel that are commonly adopted in practical construction, the deformability at f_{co} = 30, 50 and 100 MPa and f_{yt} = 400 and 800 MPa are calculated using nonlinear moment-curvature analysis and summarised in Table 2. Alternatively, the deformability could be calculated using the following formulas previously developed by the authors (Zhou *et al*. 2010):

$$\theta_{pl} = 0.03m\left(f_{co}\right)^{-0.3}(\lambda)^{-1.0n} \left(1+110\left(f_{co}\right)^{-1.1}\left(\frac{f_{yc}\rho_c}{f_{yt}\rho_t}\right)^3\right)\left(\frac{f_{yt}}{460}\right)^{0.3}; \quad (5a)$$

$$m = 1+4f_{co}{}^{0.4}(f_r / f_{co}); \quad (5b)$$

$$n = 1+3f_{co}{}^{0.2}(f_r / f_{co}). \quad (5c)$$

The validity of Eq. (5) has been verified by comparing with the measured deformability of beams tested by other researchers. The comparison is shown in Tables 3 and 4 for NSC and HSC beams respectively.

It can be seen from Table 2 that the actual deformability provided to concrete beams with different materials' strength ranges widely from an average value of 0.0328 rad when f_{co} = 30 MPa and f_{yt} = 400 MPa to 0.0285 rad when f_{co} = 50 MPa and f_{yt} = 400 MPa. Hence, the deformability provided to beam made of HSC is only 86% of that provided to beam made of NSC. Moreover, the deformability reduces more significantly from 0.0296 rad when f_{co} = 30 MPa and f_{yt} = 800 MPa to 0.0181 rad f_{co} = 100 MPa and f_{yt} = 800 MPa. Hence, the deformability provided to beam made of HSS is only 61% of that provided to beam made of NSS. Therefore, the existing empirical rules cannot provide a consistent level of deformability to concrete beams containing HSC and/or HSS. To avoid overestimating the deformability provided, it is proposed herein that a new set of deformability design rules, which depend on the strength of concrete and steels, should be established to replace the existing empirical deemed-to-satisfy rules. The new rules should propose a deformability level for designing RC beams in low to moderate seismicity regions, as well as the respective maximum allowable limits of tension steel and neutral axis depth.

4.2. Derivation of limited deformability

For RC beams located in low to moderate seismicity regions, the beam should be designed to have adequate rotation capacity to enable the formation of plastic hinges, and hence allow moment redistribution to occur. According to Eurocode 2 (ECS 2004), no direct check of rotation capacity is needed and the required deformability is deemed to satisfied if the ratio of $x_u/d \leq 0.25$ for $f_c' \leq$ 50 MPa and $x_u/d \leq 0.15$ if $f_c' > 50$ MPa. As seen from Table 2, the normalised rotation capacity calculated as per the deemed-to-satisfy requirements of Eurocode 2 is about 0.03 rad in all circumstances. The authors have also carried out an independent check of this value against the recommended maximum allowable tension steel ratio stipulated in Eurocode 8 Part 1 (2004). Using Clauses 5.2.3.4.3(3) and 5.4.3.1.2(4), as well as Table 5 of EC8, it can be calculated that for f_{yt} = 500 MPa, the maximum allowable tension steel ratios are 1.15%, 1.73% and 2.3% for beam design as per medium ductility class. These steel ratios can be converted to the normalised rotation capacity using Eq. (5.4), and the respective values are

$\theta_{pl,\lim}$ = 0.033 rad, 0.028 rad and 0.023 rad. The average value of these normalised rotation capacities is 0.028 rad, which is very close to the previous obtained value of $\theta_{pl,\lim}$ = 0.03 rad. Therefore, the authors suggest in this study to adopt this normalised rotation capacity $\theta_{pl,\lim}$ = 0.03 rad as the benchmark for providing limited deformability to RC beams in low to moderate seismicity regions.

Table 2. Deformability provided in design codes for RC beams subjected to seismic risk

Design codes	Normalised rotation capacity θ_{pl} (rad)					
	f_{co} = 30 MPa		f_{co} = 50 MPa		f_{co} = 100 MPa	
	f_{yt} = 400 MPa	f_{yt} = 800 MPa	f_{yt} = 400 MPa	f_{yt} = 800 MPa	f_{yt} = 400 MPa	f_{yt} = 800 MPa
American Code ACI318	0.0157	0.0183	0.0195	0.0147	0.0253	0.0118
Chinese Code GB50011	0.0410	0.0410	0.0363*	0.0271	0.0433*	0.0184*
Eurocode EC2	0.0304	0.0304	0.0218	0.0218	0.0270	0.0270
New Zealand Code NZS3101	0.0442	0.0286	0.0363*	0.0209	0.0427*	0.0153*
Average	0.0328	0.0296	0.0285	0.0211	0.0346	0.0181

*Deformability is calculated for beam section containing the largest allowable tension steel ratio of 2.5% instead of complying with the empirical deemed-to-satisfy rules.

Table 3. Comparison with experimental results on rotation capacities of NSC beams

Code	f_c' (MPa)	f_r (Mpa)	f_{yt} (Mpa)	ρ_t (%)	ρ_c (%)	θ_{pl} by Eq. (5) (rad) [1]	θ_{pl} by others (rad) [2]	θ_{pl} by EC2 (rad) [3]	$\frac{[1]}{[2]}$	$\frac{[3]}{[2]}$
Nawy *et al*. (1968)										
P9G1	33.6	0.00	328	1.73	0.71	0.0870	0.0650	0.0330	1.34	0.51
P11G3	35.1	0.50	328	1.73	0.71	0.1536	0.1110	0.0320	1.38	0.29
P3G4	37.5	1.30	452	1.73	0.71	0.1232	0.1340	0.0260	0.92	0.19
P4G5	39.1	1.30	452	1.73	0.71	0.1217	0.1360	0.0265	0.89	0.19
Pecce and Fabbocino (1999)										
A	41.3	0.98	471	2.60	0.05	0.0255	0.0220	0.0100	1.16	0.45
B	41.3	0.94	454	1.10	0.05	0.0736	0.1220	0.0265	0.60	0.22
Debernardi and Taliano (2002)										
T1A1	27.7	0.46	587	0.67	0.30	0.1433	0.1035	0.0310	1.38	0.30
T3A1	27.7	0.46	587	2.00	0.59	0.0270	0.0290	0.0080	0.93	0.28
T5A1	27.7	0.35	587	0.63	0.22	0.0978	0.1130	0.0300	0.87	0.27
T6A1	27.7	0.35	587	1.28	0.22	0.0311	0.0245	0.0160	1.27	0.65
Haskett *et al*. (2009)										
A1	38.2	0.67	315	1.47	0.0	0.0313	0.0360	0.0269	0.87	0.75
A2	42.3	0.32	318	1.47	0.0	0.0226	0.0205	0.0280	1.10	1.37
A3	41.0	0.31	336	1.47	0.0	0.0209	0.0168	0.0270	1.24	1.61
A4	42.9	1.29	315	2.95	0.0	0.0222	0.0305	0.0172	0.73	0.56
A5	39.6	0.59	314	2.95	0.0	0.0136	0.0207	0.0154	0.66	0.74
A6	41.1	0.31	328	2.95	0.0	0.0103	0.0118	0.0153	0.87	1.30
B1	43.0	0.65	329	1.47	0.0	0.0293	0.0277	0.0278	1.06	1.00
B2	41.8	0.31	322	1.47	0.0	0.0222	0.0152	0.0277	1.46	1.82
B3	42.9	1.29	321	2.95	0.0	0.0217	0.0218	0.0168	1.00	0.77
B4	42.9	0.64	323	2.95	0.0	0.0138	0.0120	0.0166	1.15	1.38
C2	26.0	0.39	329	1.47	0.0	0.0219	0.0258	0.0203	0.85	0.79
C3	25.6	0.32	330	1.47	0.0	0.0201	0.0187	0.0200	1.07	1.07
C4	25.9	1.23	325	2.95	0.0	0.0205	0.0297	0.0080	0.69	0.27
C5	23.4	0.64	328	2.95	0.0	0.0126	0.0130	0.0080	0.97	0.62
C6	27.4	0.34	319	2.95	0.0	0.0102	0.0125	0.0080	0.82	0.64
								Average	1.01	0.72
								Standard deviation	0.24	0.47

Table 4. Comparison with experimental results on rotation capacities of HSC beams

Code	f_c' (MPa)	f_r (Mpa)	f_{yt} (Mpa)	ρ_t (%)	ρ_c (%)	θ_{pl} by Eq. (5) (rad) [1]	θ_{pl} by others (rad) [2]	θ_{pl} by EC2 (rad) [3]	$\frac{[1]}{[2]}$	$\frac{[3]}{[2]}$
Pecce and Fabbocino (1999)										
AH	93.8	0.98	471	2.60	0.05	0.0271	0.0220	0.0170	1.23	0.77
CH	95.4	1.11	534	2.20	0.04	0.0300	0.0380	0.0170	0.79	0.45
Ko *et al*. (2001)										
6-65-1	66.6	2.26	415	3.59	0.79	0.0547	0.0472	0.0150	1.16	0.32
6-75-1	66.6	2.33	427	4.27	0.77	0.0399	0.0412	0.0100	0.97	0.24
8-50-1	82.1	2.42	443	3.35	0.80	0.0580	0.0482	0.0160	1.20	0.33
8-65-1	82.1	2.33	427	4.27	0.77	0.0398	0.0450	0.0100	0.88	0.22
8-75-1	82.1	2.15	394	4.97	0.79	0.0338	0.0484	0.0080	0.70	0.17
$7\text{-}62^{00}\text{-}1$	70.8	1.91	408	3.16	0.00	0.0403	0.0530	0.0135	0.76	0.25
$7\text{-}62^{15}\text{-}1$	70.8	1.91	408	3.16	0.79	0.0587	0.0510	0.0160	1.15	0.31
Lopes and Bernardo (2003)										
A(64.9-2.04)	64.9	0.59	555	2.04	0.20	0.0248	0.0200	0.0210	1.24	1.05
A(63.2-2.86)	63.2	0.62	575	2.86	0.20	0.0161	0.0180	0.0110	0.89	0.61
A(65.1-2.86)	65.1	0.62	575	2.86	0.20	0.0161	0.0150	0.0110	1.07	0.73
B(82.9-2.11)	82.9	0.59	555	2.11	0.20	0.0243	0.0210	0.0180	1.16	0.86
B(83.9-2.16)	83.9	0.59	555	2.16	0.20	0.0237	0.0200	0.0180	1.19	0.90
B(83.6-2.69)	83.6	0.62	575	2.69	0.20	0.0178	0.0210	0.0150	0.85	0.71
B(83.4-2.70)	83.4	0.62	575	2.70	0.20	0.0177	0.0200	0.0150	0.89	0.75
								Average	1.01	0.54
								Standard deviation	0.18	0.28

5. Methods of providing limited deformability

5.1. By controlling the maximum degree of reinforcement

Based on this specified value of normalised rotation capacity $\theta_{pl,\lim}$ = 0.03 rad, it can be seen from Eq. (5a) that a corresponding maximum allowable value of λ, denoted by $\lambda_{\max}$, exists for each chosen f_{co} and f_{yt}. The expression for $\lambda_{\max}$ is shown in Eq. (6a):

$$\lambda_{\max} = \left\{0.03m\left(f_{co}\right)^{-0.3}\left(\frac{f_{yt}}{460}\right)^{0.3} \left(1+110\left(f_{co}\right)^{-1.1}\left(\frac{f_{yc}\rho_c}{f_{yt}\rho_t}\right)^{3}\right)\left(\theta_{pl,\lim}\right)^{-1.0}\right\}^{\frac{1}{n}}, \quad (6a)$$

the values of $\lambda_{\max}$ with respect to the specified deformability $\theta_{pl,\lim}$ = 0.03 rad are evaluated rigorously by nonlinear moment-curvature analysis and are summarised in Tables 5 to 7 for different combination of concrete and steel yield strengths of f_{co} = 30 – 100 MPa and f_{yt} = 400, 600 and 800 MPa. Alternatively, the value of $\lambda_{\max}$ for unconfined singly RC beams can be calculated by substituting $\theta_{pl,\lim}$ = 0.03 rad into Eq. (6a):

$$\lambda_{\max} = \left(f_{co}\right)^{-0.3}\left(\frac{f_{yt}}{460}\right)^{0.3}. \quad (6b)$$

It can be seen from the above tables that the value of $\lambda_{\max}$ decreases significantly as the concrete strength increases from 30 to 100 MPa for a given steel yield strength. On the contrary, the value of $\lambda_{\max}$ increases slightly as the steel yield strength increases from 400 to 800 MPa for a given concrete strength. It is apparent that a lower value of $\lambda_{\max}$ should be set for designing RC beams when HSC is adopted in order to achieve the provision of limited deformability of $\theta_{pl,\lim}$ = 0.03 rad. And a slightly higher value of $\lambda_{\max}$ should be set for the design of RC beams when HSS is adopted. However, since the effects of concrete strength is not the same as that of steel yield strength, the value of $\lambda_{\max}$ may or may not decrease when both HSC and HSS are adopted.

The corresponding maximum allowable tension steel ratio $\rho_{t,\max}$ for singly-reinforced concrete beam section (i.e. ρ_c = 0%) having different concrete and steel yield strength can be determined by multiplying the value of $\lambda_{\max}$ with the respective balanced steel ratio ρ_{bo}. The evaluated values of $\rho_{t,\max}$ have been listed in Tables 5 to 7. It can be observed from these tables that although the value of $\lambda_{\max}$ decreases substantially as the concrete strength increases, the value of $\rho_{t,\max}$ increases as the concrete strength increases because the value of ρ_{bo} increases considerably with the concrete strength. Therefore, the use of HSC has the major advantage of increasing the maximum design flexural strength and providing limited deformability. On the other hand, it is seen that the value of $\rho_{t,\max}$ decreases significantly as the steel yield strength increases because the balanced steel ratio ρ_{bo} decreases as the steel yield strength increases. Nevertheless, since higher strength steel is adopted, the provision of a lower tension steel ratio may or may not lead to a reduction in the maximum design limit of flexural strength.

In order to investigate numerically the flexural strength that can be achieved by the beam sections designed for the proposed deformability, the maximum moment capacity expressed in terms of $M_p/(bd^2)$ for singly RC beam section (ρ_c = 0%) having different concrete and steel yield strength are calculated. The results are tabulated in Tables 8 to 10. It can be observed from these tables that the maximum flexural strength achieved by the beam section designed with $\theta_{pl,\lim}$ = 0.03 rad increases significantly as the concrete strength increases. However, the flexural strength remains relatively constant as tension steel yield strength increases. The advantages of using higher strength materials are now obvious. The use of HSC with/without HSS in RC beams would allow a higher flexural strength to be achieved at limited deformability level, despite that HSC and HSS are less deformable *per se*. And the use of HSS solely in RC beams would not increase the flexural strength at the limited deformability level, but nevertheless reduce significantly the amount of tension steel and hence avoid steel congestion in the critical regions and beam-column joints.

Table 5. Values of $\lambda_{\max}$ and $\rho_{t,\max}$ for $\theta_{pl,\lim}$ = 0.03 rad when f_{yt} = 400 MPa

f_{co} (MPa)	ρ_{bo} (%)	$\lambda_{\max}$	$\rho_{t,\max}$ (%)
30	3.815	0.366	1.396
40	4.735	0.320	1.513
50	5.625	0.289	1.627
60	6.455	0.269	1.734
70	7.285	0.252	1.838
80	8.055	0.237	1.907
90	8.765	0.227	1.994
100	9.415	0.221	2.083

Table 6. Values of $\lambda_{\max}$ and $\rho_{t,\max}$ for $\theta_{pl,\lim}$ = 0.03 rad when f_{yt} = 600 MPa

f_{co} (MPa)	ρ_{bo} (%)	$\lambda_{\max}$	$\rho_{t,\max}$ (%)
30	2.225	0.418	0.929
40	2.735	0.369	1.008
50	3.225	0.336	1.083
60	3.685	0.314	1.156
70	4.125	0.297	1.225
80	4.555	0.279	1.270
90	4.935	0.269	1.328
100	5.285	0.262	1.387

Table 7. Values of $\lambda_{\max}$ and $\rho_{t,\max}$ for $\theta_{pl,\lim}$ = 0.03 rad when f_{yt} = 800 MPa

f_{co} (MPa)	ρ_{bo} (%)	$\lambda_{\max}$	$\rho_{t,\max}$ (%)
30	1.495	0.466	0.697
40	1.815	0.417	0.757
50	2.125	0.383	0.813
60	2.425	0.358	0.869
70	2.695	0.341	0.919
80	2.965	0.321	0.953
90	3.215	0.309	0.995
100	3.435	0.302	1.036

(Note f_r = 0 MPa and ρ_c = 0% in Tables 5 to 7)

5.2. By controlling the maximum neutral axis depth

As an alternative method, the provision of limited deformability to RC beams can be achieved by restricting the maximum neutral axis depth at ultimate state. This method of limiting the neutral axis depth for the provision of flexural deformability has been adopted by some of the existing RC design codes. The maximum limits of neutral axis depth are normally expressed in a dimensionless form in the ratio of x_u/d, e.g. EC 2 and GB50011, where x_u and d are the neutral axis and effective beam depths; or x_u/x_{ub}, e.g. New Zealand Code, where x_{ub} is the neutral axis depth of balanced section at ultimate state. However, since not all of these limits are dependent on the concrete and steel yield strength, the deformability provided to HSC beam will be lower than that of beam section made of NSC and/or NSS.

To ensure that a consistent level of deformability is provided to RC beams with different concrete and steel yield strength, the maximum limits of neutral axis depth should depend on the strengths of concrete and steel reinforcement. The maximum limit of neutral axis depth is expressed in this study by the ratio x_u/d, which is more commonly adopted by the existing RC design codes. For designing beam section having the proposed limited deformability of $\theta_{pl,\lim}$ = 0.03 rad, the respective maximum limits of x_u/d for different concrete and steel yield strength are derived using moment-curvature analysis, which are summarised in Table 11. It is noted that the maximum value of x_u/d decreases significantly as the concrete strength increases from 30 to 100 MPa. Thus, a lower maximum limit should be set to the value of x_u/d when HSC is used. On the other hand, that the maximum value of x_u/d is constant when steel yield strength is varied.

5.3. Improving flexural strength and deformability by adding compression steel

From Tables 8 to 10, it is easily observed that at a prescribed concrete strength, there is a maximum design limit of flexural strength that can be achieved in association with the proposed limited deformability $\theta_{pl,\lim}$ = 0.03 rad. To increase the flexural strength of the beam section at the same concrete strength, the beam dimensions should be enlarged appropriately. However, in real construction practice especially in the design of building structures, it is often not practicable to increase the beam size because of maximising the usable floor space to the users. Under such a circumstance, some compression steel could be added to the beam to increase the maximum design limit of flexural strength while maintaining the provision of limited deformability. The enhanced maximum allowable tension steel ratios and design limit of flexural strength are calculated and presented in Tables 8 to 10 for ρ_c = 1% and 2%. It is evident from the table that adding compression steel can increase both the maximum allowable limits of tension steel ratio and flexural strength to achieve limited deformability. It is also observed that the rate of increase in both maximum limits decreases as concrete strength increases.

Table 8. Values of $\rho_{t,\max}$ and $M_p/(bd^2)$for $\theta_{pl,\lim}$ = 0.03 rad when $f_{yt} = f_{yc}$ = 400 MPa (at different compression steel ratios ρ_c)

f_{co} (MPa)	$\rho_{t,\max}$ (%)			$M_p/(bd^2)$ (MPa)		
	ρ_c = 0%	ρ_c = 1.0%	ρ_c = 2.0%	ρ_c = 0%	ρ_c = 1.0%	ρ_c = 2.0%
30	1.396	2.601	3.785	5.030	9.301	13.501
40	1.513	2.702	3.869	5.557	9.783	13.969
50	1.627	2.798	3.970	6.043	10.214	14.417
60	1.734	2.903	4.075	6.494	10.665	14.860
70	1.838	2.976	4.148	6.918	10.996	15.196
80	1.907	3.066	4.198	7.217	11.380	15.442
90	1.994	3.160	4.269	7.573	11.769	15.757
100	2.083	3.193	4.333	7.932	11.940	16.039

Table 9. Values of $\rho_{t,\max}$ and $M_p/(bd^2)$for $\theta_{pl,\lim}$ = 0.03 rad when $f_{yt} = f_{yc}$ = 600 MPa (at different compression steel ratios ρ_c)

f_{co} (MPa)	Maximum value of ρ_t (%)			$M_p/(bd^2)$ (MPa)		
	ρ_c = 0%	ρ_c = 1.0%	ρ_c = 2.0%	ρ_c = 0%	ρ_c = 1.0%	ρ_c = 2.0%
30	0.929	1.733	2.527	5.022	9.294	13.517
40	1.008	1.799	2.582	5.552	9.772	13.982
50	1.083	1.868	2.646	6.037	10.227	14.416
60	1.156	1.933	2.711	6.494	10.652	14.833
70	1.225	1.983	2.767	6.917	10.991	15.203
80	1.270	2.042	2.099	7.211	11.364	15.442
90	1.328	2.107	2.838	7.568	11.769	15.716
100	1.387	2.140	2.888	7.923	12.000	16.036

Table 10. Values of $\rho_{t,\max}$ and $M_p/(bd^2)$for $\theta_{pl,\lim}$ = 0.03 rad when $f_{yt} = f_{yc}$ = 800 MPa (at different compression steel ratios ρ_c)

f_{co} (MPa)	Maximum value of ρ_t (%)			$M_p/(bd^2)$ (MPa)		
	ρ_c = 0%	ρ_c = 1.0%	ρ_c = 2.0%	ρ_c = 0%	ρ_c = 1.0%	ρ_c = 2.0%
30	0.697	1.300	1.894	5.020	9.293	13.512
40	0.757	1.349	1.936	5.556	9.772	13.979
50	0.813	1.403	1.985	6.042	10.240	14.417
60	0.869	1.448	2.033	6.500	10.644	14.834
70	0.919	1.489	2.075	6.918	11.002	15.203
80	0.953	1.531	2.799	7.212	11.364	15.444
90	0.995	1.578	2.128	7.557	11.759	15.712
100	1.036	1.605	2.165	7.894	12.000	16.030

Table 11. Maximum value of x_u/d for $\theta_{pl,\lim}$ = 0.03 rad

f_{co} (MPa)	Maximum value of x_u/d		
	f_y = 400 MPa	f_y = 600 MPa	f_y = 800 MPa
30	0.251	0.253	0.251
40	0.214	0.214	0.214
50	0.190	0.190	0.190
60	0.172	0.172	0.172
70	0.159	0.159	0.159
80	0.151	0.151	0.151
90	0.142	0.142	0.143
100	0.136	0.136	0.136

(Note $f_y = f_{yt} = f_{yc}$ in Table 11)

5.4. Improving flexural strength and deformability by adding confinement

As an alternative method of extending the limits of maximum allowable tension steel ratios and flexural strength for limited deformability design of RC beams, additional confinement could be added. The enhanced limits are calculated and presented in Tables 12 to 14 for f_r = 1, 2 and 3 MPa at different steel yield strengths. Similar to Tables 5 to 7, it is seen that adding confinement can increase both the maximum allowable limits of tension steel ratio and flexural strength to achieved limited deformability. It is also noted that the rate of increase in both maximum limits decreases as concrete strength increases. At a fixed concrete strength and steel yield strength, it is found that the increase in the maximum limit of design flexural strength decreases as the confinement increases. Therefore, the effectiveness of adding confinement decreases as confining pressure increases. It should also note that for beam section with f_{co} = 30 MPa and f_{yt} = 600 or 800 MPa, the maximum allowable tension steel ratio is larger than the respective balanced steel ratio. In these cases, the maximum tension steel ratio equal to the balanced steel ratio is proposed instead of the actual tension steel ratio in order to avoid the design of over-reinforced beam.

6. Simplified design guidelines

The maximum limits of the degree of reinforcement and neutral axis to effective depths ratio have been derived and presented in Tables 5 to 14. For incorporation into RC design codes, more simplified guidelines are preferred to the above tables. Referring to the maximum allowable values of the degree of reinforcement λ_{max} summarised in Tables 5 to 7, it can be observed that the variation of λ_{max} with steel yield strength f_{yt} can be represented fairly accurately by the following equation:

$$\frac{(\lambda_{max})_1}{(\lambda_{max})_2} = \left[\frac{(f_{yt})_1}{(f_{yt})_2}\right]^{0.35}, \tag{7}$$

where $(\lambda_{max})_i$ is the maximum allowable value of degree of reinforcement at limited deformability level with respect to tension steel yield strength $(f_{yt})_i$. Therefore, it is recommended herein only to propose the design values of λ_{max} for f_{yt} = 400 MPa, and use Eq. (7) to evaluate the values of λ_{max} for other steel yield strength. The guidelines for f_{yt} = 400 MPa are shown below:

In case of f_{yt} (and f_{yc}) = 400 MPa: λ_{max} should not exceed 0.35 when $f_{co} \leq$ 30 MPa, should not exceed 0.25 when 30 MPa $\leq f_{co} <$ 60 MPa, and should not exceed 0.2 when 60 MPa $\leq f_{co} <$ 100 MPa.

Table 12. Values of $\rho_{t,\max}$ and $M_p/(bd^2)$ for $\theta_{pl,\lim}$ = 0.03 rad when $f_{yt} = f_{yc}$ = 400 MPa (at different confining pressure f_r)

f_{co} (MPa)	$\rho_{t,\max}$ (%)			$M_p/(bd^2)$ (MPa)		
	f_r = 1.0	f_r = 2.0	f_r = 3.0	f_r = 1.0	f_r = 2.0	f_r = 3.0
30	2.843	4.233	6.017	9.176	12.325	14.925
40	3.049	4.379	5.930	10.237	13.620	16.905
50	3.205	4.486	5.908	11.048	14.554	17.863
60	3.332	4.568	5.889	11.702	15.271	18.656
70	3.440	4.637	5.893	12.251	15.844	19.263
80	3.520	4.702	5.896	12.682	16.335	19.732
90	3.588	4.754	5.907	13.042	16.730	20.133
100	3.620	4.772	5.925	13.265	16.990	20.493

Table 13. Values of $\rho_{t,\max}$ and $M_p/(bd^2)$ for $\theta_{pl,\lim}$ = 0.03 rad when $f_{yt} = f_{yc}$ = 600 MPa (at different confining pressure f_r)

f_{co} (MPa)	Maximum value of ρ_t (%)			$M_p/(bd^2)$ (MPa)		
	f_r = 1.0	f_r = 2.0	f_r = 3.0	f_r = 1.0	f_r = 2.0	f_r = 3.0
30	1.896	2.899	4.145*	9.178	12.336	14.774
40	2.033	2.961	4.110	10.238	13.656	16.692
50	2.137	2.994	4.089	11.048	14.558	17.954
60	2.222	3.045	4.042	11.705	15.271	18.782
70	2.293	3.091	3.934	12.252	15.845	19.270
80	2.347	3.135	3.931	12.684	16.335	19.733
90	2.392	3.170	3.938	13.042	16.731	20.133
100	2.417	3.181	3.950	13.290	16.990	20.493

Note: The mark * indicates that evaluated value is larger than the respective balanced steel ratio and consequently the latter is used.

Table 14. Values of $\rho_{t,\max}$ and $M_p/(bd^2)$ for $\theta_{pl,\lim}$ = 0.03 rad when $f_{yt} = f_{yc}$ = 800 MPa (at different confining pressure f_r) (Confining pressure f_r is expressed in MPa in Tables 12 to 14)

f_{co} (MPa)	Maximum value of ρ_t (%)			$M_p/(bd^2)$ (MPa)		
	f_r = 1.0	f_r = 2.0	f_r = 3.0	f_r = 1.0	f_r = 2.0	f_r = 3.0
30	1.432	2.293	2.965*	9.184	12.306	14.792
40	1.525	2.295	3.399	10.235	13.717	16.530
50	1.602	2.311	3.190	11.047	14.656	17.996
60	1.666	2.297	3.156	11.702	15.294	18.908
70	1.720	2.320	3.128	12.251	15.852	19.527
80	1.760	2.351	3.023	12.682	16.335	19.863
90	1.794	2.377	2.953	13.042	16.730	20.131
100	1.816	2.386	2.962	13.309	16.990	20.487

Note: The mark * indicates that evaluated value is larger than the respective balanced steel ratio and consequently the latter is used.

Furthermore, by substituting Eqs. (3) and (4) into Eq. (7), it can be seen from Eq. (8) that the maximum allowable tension steel ratio for singly-reinforced beam (i.e. $\rho_c = 0$) is inversely proportional to the yield strength of tension steel at a given concrete strength for providing limited deformability:

$$\frac{(\rho_{t,\max})_1}{(\rho_{t,\max})_2} = \frac{(f_{yt})_2}{(f_{yt})_1}, \tag{8}$$

where $(\rho_{t,\max})_i$ is the maximum allowable value of tension steel ratio at limited deformability level with respect to its yield strength $(f_{yt})_i$. For beam sections with or without compression steel, Eq. (9) is derived that correlates the maximum allowable tension steel ratio to concrete and steel yield strength:

$$\rho_{t,\max} = 4(f_{co} + 100 + 100\rho_c)/ f_{yt}, \tag{9}$$

where $\rho_{t,\max}$ and ρ_c are in %, f_{co} and f_y are in MPa.

As seen in Table 11, the effects of steel yield strength on the maximum allowable neutral axis to effective depth ratio are considered insignificant. Hence, it is proposed to ignore the effect of steel yield strength in the simplified design guidelines. Accordingly, the following guidelines are developed:

In the case of $400 \leq f_{yc} = f_{yt} \leq 800$ MPa, x_u/d should not exceed 0.25 when $f_{co} \leq 30$ MPa, should not exceed 0.17 when 30 MPa $\leq f_{co} <$ 60 MPa, and should not exceed 0.13 when 60 MPa $\leq f_{co} <$ 100 MPa.

7. Conclusions

The flexural deformability of RC beams in terms of normalised rotation capacity was studied by nonlinear moment-curvature analysis. From the study, it was found that the variation of deformability with degree of reinforcement λ, confining pressure f_r and the neutral axis depth at maximum moment (expressed in dimensionless ratio of x_u/x_{ub} or x_u/d) are not unique and dependent on the concrete and steel yield strength. Because of such dependence, the current empirical deemed-to-satisfy rules stipulated in most of the RC design codes, which are concrete and/or steel yield strength independent, are not able to provide a consistent level of deformability to RC beams. Most importantly, the deformability provided by the existing empirical rules to HSC beams with HSS is much lower than that provided to NSC beams containing NSS (~55%).

In order to provide a consistent level of deformability to RC beams to cater for the seismic demand in low to moderate seismicity regions, it is proposed to set a consistent level of deformability in the design of RC beams. The design of RC beams possessing this required level of deformability $\theta_{pl,\lim}$ is named the limited deformability design. This proposed deformability is set at the normalised rotation capacity provided to NSC beams in accordance with the plastic design method stipulated in Eurocode 2. To achieve the provision of limited deformability to RC beams, the maximum degree of reinforcement $\lambda_{\max}$ or tension steel ratio $\rho_{t,\max}$ or neutral axis to effective depth ratio x_u/d should be limited. In this study, these maximum allowable values were derived for different combination of concrete strength (30–100 MPa) and steel yield strength (400–800 MPa). From the results, it is evident that maximum allowable values of $\lambda_{\max}$, $\rho_{t,\max}$ and x_u/d decrease significantly as the concrete strength increases. Moreover, it was also found that there exists a maximum flexural strength of a singly-reinforced beam section for the provision of limited deformability at a given concrete strength.

To improve the maximum flexural strength limit of RC beams designed for limited deformability, compression steel and/or confinement can be added without the need of enlarging beam size. The maximum tension steel ratio and the design limit of flexural strength for different compression steel ratio from 0 to 2% and confining pressure from 0 to 3 MPa have also been derived in this study. From the results, it can be concluded that the use of HSC with or without HSS would improve the maximum design limit of flexural strength of RC beams at limited deformability. And the use of HSS solely would not increase the flexural strength of RC beam section at limited deformability level, but nevertheless reduce significantly the amount of tension steel and hence avoid steel congestion. Lastly, simplified guidelines for incorporation into RC design codes that limit the value of $\lambda_{\max}$, $\rho_{t,\max}$ and x_u/d to ensure provision of the proposed limited deformability have been developed.

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STIPRIOJO BETONO SIJŲ RIBINIO DEFORMATYVUMO SKAIČIAVIMAS ŽEMO IR VIDUTINIO SEISMINGUMO REGIONUOSE

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S a n t r a u k a

Mažo ir vidutinio seismingumo regionuose projektuojant seisminių apkrovų veikiamas gelžbetonines sijas turi būti užtikrinta ne tik jų laikomoji galia, bet ir tinkamas deformatyvumas. Pagal šiuo metu galiojančias gelžbetoninių konstrukcijų projektavimo normas tai pasiekiama ribojant didžiausiąjį skerspjūvio aukštį, tempiamosios zonos armavimo procentą arba ribojant mažiausią armatūros kiekį. Tačiau šios konstravimo taisyklės buvo sukurtos prieš daugelį metų ir skirtos elementams iš normalaus stiprio betono ir armatūros. Jas tiesiogiai taikant gelžbetoninėms sijoms, pagamintoms iš stipriųjų medžiagų, gaunamas sumažintos deformacijos. Šiais problemai spręsti siūlomas naujas projektavimo metodas, paremtas nustatytu deformatyvumu. Autoriai siūlo mažo ir vidutinio seismingumo regionuose visas gelžbetonines sijas projektuoti pasirenkant deformacijas pagal „Eurokode 2“ pateiktus deformatyvumo reikalavimus. Pagal anksčiau autorių gautas teorines formules skirtingiems betono ir plieno stipriams pateiktos dvi atskiros skaičiuojamųjų rodiklių grupės išreikštos maksimaliu tempiamosios zonos armavimo koeficientu ir efektyviuoju sijos skerspjūvio aukščiu. Praktiniam taikymui sukurtos supaprastintos gelžbetoninių sijų projektavimo rekomendacijos, atitinkančios deformatyvumo reikalavimus.

Reikšminiai žodžiai: sijos, rišamoji armatūra, kreivis, deformacijos, projektavimo formulės, stiprusis betonas, stiprusis plienas, mažas ir vidutinis teismingumas, gelžbetonis, sukamoji galia.

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