SENSITIVITY ANALYSIS OF TANGENTIAL DIGGING FORCES OF THE BUCKET WHEEL EXCAVATOR SCHRS 1320 FOR DIFFERENT TERRACES

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Abstract. Researches in the field of economy of control of mining processes of minerals are continually met with a number of unsolved and newly arising problems. New deposits of brown coal are located at greater depths than before and are more difficult to mine. Problems in the field of rock mining stem from the greater diversity of rocks in mining, and it results in more difficult mining conditions. On the other hand, even greater demands are made to decrease the economic demand of mining. Current research is aimed at the most optimal utilization of energy provided on the bucket wheel of excavators, such that the energy demand of the mining process decreases, whilst preserving maximum performance. These requirements place greater demands on the precision of planning of mining and maximum economic efficiency. The presented article illustrates the possibilities of the utilization of Sobol sensitivity analysis during the investigation of the influence of the parameters of mining processes on tangential digging forces. Analyses are carried out for the mining process of bucket wheel excavator SchRs 1320. For purposes of the study, detailed measurements of operational parameters were performed on the excavator during a whole work cycle.

Keywords: bucket wheel excavator, sensitivity analysis, tangential digging force, SchRs 1320, measurement, probability, efficiency.

1. Introduction

Maximum performance with minimum energy consumption, which presents contradictory demands that cannot be satisfied simultaneously, is required for each working machine. It is however possible to optimize the working activity of the machinery and to find the most optimal working conditions which are a compromise of both contradictory requirements.

Current researches in the field of mining technology are aimed at the optimization of the energy demand of mining, and at the optimization of maintenance and operation of mining technology. Operators of mining technology focus mainly on the effective utilization of energy in the mining of coal. Proper choice of mining approach can significantly increase the efficiency of bucket wheel excavators. Savings are also made on the energy necessary for mining of a given block, and mining time is also reduced significantly. This has a positive influence on the resulting price of the mined mineral.

The issue is topical in the Czech Republic but also elsewhere in the world due to the fact that easily accessible deposits of brown coal have mostly been mined and at present coal is mined in locations with very hard overburden and more frequently occurring sandstone bands. In the Czech Republic at present, mining is being performed in the location of Severočeské doly (North-

Bohemian pits) in places that were considered, in the 50's of the last century, as not-mineable by continuous mining. This all leads to significantly higher dynamic loading of mining equipment. A secondary effect stemming from worsened geological conditions is increased strain of utilized mining machinery. The cutting teeth wear out faster and must be exchanged more frequently. Shutdowns of machinery necessary for replacements increase costs of the mined coal once again. However, non-replacement of cutting teeth manifests itself as increased vibrations of the machinery during mining. This increased strain may lead to fatigue cracks on the bearing steel parts of the machinery which may further lead to serious damage of machinery (Bošnjak *et al.* 2010, 2011; Rusiński *et al.* 2010a).

In practice, the influence of digging forces on the vibration of bucket wheel excavator (BWE) and bearing steel structures is frequently analysed. Critical points the damage of which due to aging could lead to fatal consequences are determined numerically and experimentally (Bošnjak *et al.* 2009a; Rusiński *et al.* 2010b). The degree of vibration is determined and the possibility of resonance of frequency of mining forces with the natural frequencies of the machinery is examined (Gottvald 2010). Analyses of material properties of steel are performed (Melcher *et al.* 2004; Kala *et al.* 2009). Parallel attention is paid to better understanding of

structural safety and reliability (Kala 2007, 2008; Kowal 2011; Radoń 2011; Karmazínová *et al.* 2009; Kala *et al.* 2010). Current research is aimed mainly at nonlinear analysis of the coupling systems (e.g., Melcher *et al.* 2009; J. Králik, J. Králik 2009; Juozapaitis *et al.* 2010; Sandovič *et al.* 2011) and advanced models for solving coupled safety problems (e.g., Liaudanskiene *et al.* 2012; Jiang *et al.* 2012; Turskis *et al.* 2012). Let us note that relatively sufficient statistical information is provided for the material characteristics of steel members in comparison to other related branches (e.g., Amšiejus *et al.* 2009, 2010; Lashgari *et al.* 2011).

Despite all maintenance efforts of machinery, breakdown of mining technology is not rare (Bošnjak *et al.* 2009b). It is usually the case of accidents occurring as a result of production errors or the design of unsuitable structural modifications. We often encounter breakdown occurring as a result of wear of rotational parts of the machinery (Savković *et al.* 2011). Another example is breakdown due to extreme loading, that was not taken into consideration during design (Gottvald 2011). All these undesirable effects are reflected in the final price of the mined coal. As a result, great emphasis is put on the correct planning of mining, such that operational costs are within a tolerable level.

Through optimal design of the mining parts of the BWE, we can minimize the energy required for mining and decrease vibrations of machinery evoked by the mining process. Problems of optimization of BWEs are specific, however we can use a number of advanced optimization approaches (Venskus *et al.* 2010; Sušinskas *et al.* 2011; Torno *et al.* 2011; Zavadskas *et al.* 2011). The optimal design of the bucket wheel and its motors requires the correct knowledge of the size of digging forces acting on the buckets. It is apparent that the size of digging forces is dependent on the properties of the mined material, and on the construction of the bucket wheel. These are parameters that we cannot influence, or rather can only in the course of design. During mining we can however choose the size of the mining block, which determines the volume of the actual mined overburden and also the amount of energy required for its cutting. We are thus able to satisfy both requirements through optimal setting.

Extensive studies examining the most appropriate ratios of the cutting depth and cutting width on the terrace have been carried out (Vladimirov *et al.* 1968; Durst, Vogt 1986; Fries *et al.* 2010). Conditions of when and how to increase or decrease the horizontal speed of bucket wheel boom, such that the ratio of cutting depth and width is optimal and constant if possible, were set.

Results of sensitivity analyses of parameters having an influence on the resulting size of the digging force are presented in the article. Obtained conclusions may be used for subsequent optimization of the mining process. Sobol decomposition was applied as a method for global sensitivity analysis (Saltelli *et al.* 2004). An analytical approach of the evaluation of the total tangential digging force based on the power of the motor, parameters of the mining block and properties of overburden is presented in the article. Probabilistic characteristics of considered random variables were evaluated from measurements during mining with BWE SchRs 1320, in the opencast mine Doly Nástup Tušimice, Czech Republic.

2. Introduction of BWE SchRS 1320

BWE SchRs 1320 (in accurate description SchRs 1320/4x30, where 4x30 is the depth and height reach of the bucket wheel boom) is operated at the opencast mine Doly Nástup Tušimice, Czech Republic. The main supplier of the BWE was the Prodeco Inc. in cooperation with the German company TryssenKrupp. The BWE was put into operation in 2006.

BWE SchRs 1320 is conceived as an excavator with non-telescopic bucket wheel boom. Movement of the excavator is facilitated by crawlers. The theoretical capacity of the excavator is 5500 m^3 of overburden per hour. The excavator is approx. 100 m long and approx. 60 m high. Together with the loading unit and connecting bridge, it weighs 4193 tonnes. The diameter of the bucket wheel including cutting teeth, which are 26 in number, is 12.5 m.

An overall view of the BWE SchRs 1320 is presented in Fig. 1.

Fig. 1. Overall view of bucket wheel excavator SchRs 1320

3. Calculation of tangential digging force

As it already was mentioned, the analytic calculation of the total tangential digging force is evaluated from the input power of bucket wheel motors, from the parameters of the mining block and properties of overburden. The tangential digging force F_{grges} is expressed as the ratio of the power P_{gr} required in overcoming the tangential component of digging resistance to the angular velocity of the bucket wheel ω and bucket wheel radius R_k. The whole ratio is multiplied by the efficiency of the digging process η_{grges}:

$$F_{grges} = \frac{P_{gr} \cdot \eta_{grges}}{\omega \cdot R_k} . \qquad (1)$$

Power P_{gr} is calculated from the total power of driving motors of the bucket wheel:

$$P_{gr} = P - \left(P_h + P_z + P_L \right), \qquad (2)$$

where P_h is the power required to transport mined material to place of discharge; P_z is the power needed to rotate the bucket wheel when empty; P_L is the power needed to overcome friction between soil and bucket wheel ring (for cell-less and combined bucket wheels only).

The drive power of BWE SchRs 1320 consists of two Siemens ARNR 630Y-6 motors. Input power of both motors is measured. Motor power P is then obtained from the equation:

$$P = P_p \cdot \eta_m \cdot \cos \varphi_m \cdot \eta_p, \qquad (3)$$

where P_p is the sum of measured input power of both motors of bucket wheel; η_m is the efficiency of electro-motors; $\cos \varphi_m$ is the power factor of electromotors; η_p is the efficiency of the gearbox of the bucket wheel.

Power P_h for transport according to Durst and Vogt (1986) can be evaluated from Eq. (4). The significance of the equation is graphically illustrated in Fig. 2.

$$P_h = \frac{F_{gF} \cdot h_h}{\eta_h \cdot t_\upsilon}, \qquad (4)$$

where: F_{gF} is the lifting force (see Fig. 2); h_h is the median height of material lift; η_h is the efficiency; t_υ is the rise time.
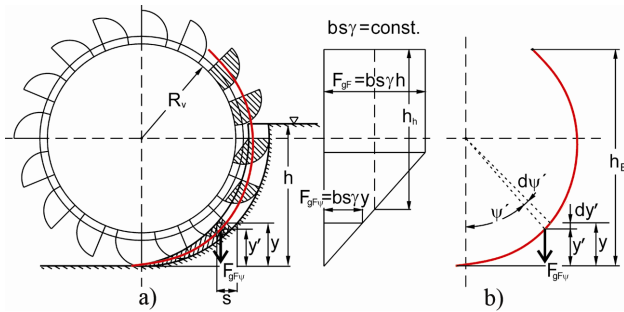

Fig. 2. Scheme for evaluation of power P_h (Durst, Vogt 1986)

The lifting force F_{gF} is calculated according to Fig. 2 from the equation:

$$F_{gF} = b \cdot s \cdot \gamma \cdot h, \qquad (5)$$

where s is the cutting depth; b the cutting width; h the height of the terrace; γ is the specific weight of the mined soil.

The rise time t_υ is calculated from parameters of the gearbox and measured revolutions of the motors of the bucket wheel. In this approach, little inaccuracies occur for the current structural arrangement of the motor of BWE SchRs 1320 because the rotations of the bucket wheel may be influenced by the inserted hydrodynamic fluid coupling Voith. Hydrodynamic fluid coupling is placed on the cardan shafts between the motor and gear box. The potential influence of this member in the system was neglected during analyses. The error obtained in such a manner is neglible for our purposes. The rise time t_υ is evaluated as follows:

$$t_\upsilon = \frac{60 \cdot R}{n_m \cdot n_b}, \qquad (6)$$

where n_m is the average revolution of motors of the bucket wheel; R is the gear ratio of gearbox of bucket wheel; n_b is the number of buckets. In case of BWE SchRs 1320, $R = 172.05$ and $n_b = 26$. The value of 60 in Eq. (6) means conversion to seconds because revolutions are measured per minutes.

Median lift height h_h is calculated according to Fig. 2 using the equation:

$$h_h \approx h_E - \frac{y_{\max}}{2}, \qquad (7)$$

where height y_{max} is the value obtained from Eq. (9) for maximum angle ψ_{max} which is given by the equations:

For $\qquad h < R_k ; \ \psi_{\max} = \arccos \dfrac{R_k - h}{R_k} ; \qquad (8a)$

For $\qquad h = R_k ; \ \psi_{\max} = 90° ; \qquad (8b)$

For $\qquad h > R_k ; \ \psi_{\max} = 90° + \arcsin \dfrac{h - R_k}{R_k} . \qquad (8c)$

Height y_{max} is calculated from the equation:

$$y_{\max} = R_k - \left(R_k - \frac{s}{2} \right) \cdot \cos \psi_{\max} . \qquad (9)$$

The resulting total height of lift h_E is determined in accordance with Fig. 2 from the equation:

$$h_E \approx R_k + R_v \cdot \cos \left(180 - \varphi'_{v,1} \right), \qquad (10)$$

where R_v is the internal radius of the bucket wheel; $\varphi'_{v,1}$ is the angle at the start of soil emptying of the bucket wheel, which is obtained from the equation:

$$\varphi'_{v,1} = \varphi_{v,1} + \Delta \varphi, \qquad (11)$$

where angle $\varphi_{v,1}$ is the start of bucket emptying for ideal soil (see Fig. 3). The angle $\Delta \varphi$ expresses the delay in emptying of bucket according to soil type (13). The angle $\varphi_{v,1}$ is calculated according to Vladimirov *et al.* (1968):

$$\varphi_{v,1} = 180° - \arccos \frac{R_v \cdot \omega^2}{g} \pm \beta, \qquad (12)$$

where β is the angle of tilt of the bucket wheel boom, and g is acceleration due to gravity 9.81 ms^{-2}.

Calculation of angle $\Delta \varphi$ is performed according to the work of Vladimirov *et al.* (1968). The overburden in Doly Nástup Tušimice consists mainly of clay which exhibits considerable block size during mining. In the course of evaluation, an approach based on the size of the block of mined overburden a_z that is formed during mining is thus assumed. The following equation is considered:

$$\Delta \varphi = 2 \cdot \arcsin \frac{a_z}{2 \cdot R_v \cdot \cos \alpha_v}, \qquad (13)$$

where a_z is the so-called size of blocks of mined material, and angle α_v is calculated according to Vladimirov *et al.* (1968):
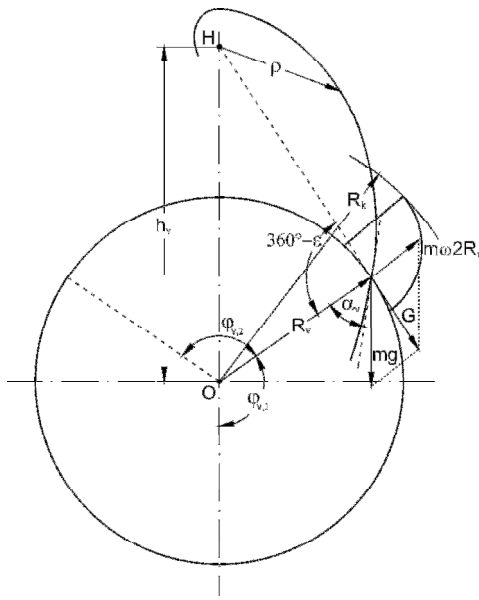
Fig. 3. Scheme of emptying of bucket (Vladimirov *et al.* 1968)

$$\alpha_v = 90° - \arctan\left(\frac{R_v \cdot \omega^2}{g \cdot \sin\varphi_{v,1}} + \tan(\varphi_{v,1})^{-1}\right)^{-1} + \varphi_z, \quad (14)$$

where φ_z is the angle of repose of soil. The angular velocity of the bucket wheel is evaluated from measured revolutions of the bucket wheel motors. During evaluation, we commit the same error as it was in the case of the evaluation of rise time t_v. The resulting inaccuracy is negligible for our purposes. Calculation of angular velocity of the bucket wheel upon substitution of conversion constants for BWE SchRs 1320 can be expressed by the equation:

$$\omega = \frac{2 \cdot \pi \cdot n_m}{n_s \cdot R}. \quad (15)$$

The mutual relationship between angular velocity of the bucket wheel ω and tangential speed on the bucket cutting edge υ is given as:

$$\upsilon = \omega \cdot R_k. \quad (16)$$

The power P_z of the motor for operation of the bucket wheel whilst empty was determined experimentally from measurements of the power of bucket wheel motors during its free rotation. The same conversion from equation (3) holds for the measured values. We can write:

$$P_Z = P_{Zp} \cdot \eta_m \cdot \cos\varphi_m \cdot \eta_p, \quad (17)$$

where P_{Zp} is the power of the motors of the bucket wheel measured during its free rotation.

Frictional power P_L can be calculated according to the equation listed in (Durst *et al.* 1986). For measured cutting width b and cutting depth s, we obtain the equation:

$$P_L = s \cdot b \cdot h \cdot \upsilon \cdot \gamma \cdot f_t, \quad (18)$$

where f_t is the coefficient of friction between the cutting wheel and soil.

4. Measurements on the BWE

During mining, a significant amount of operational data is measured on the BWE. The measured data are processed by the control computer in the BWE operator's cabin. Operational data are visually displayed to the operator of the excavator on monitors on the control panel. With the aid of these data, he is able to control the manner of mining, and check proper functionality of all parts of the excavator.

If necessary, the control computer can be set to save measured data to hard disk and utilize the measured data for offline processing. This option was used in the presented study. Saving of selected data was set for a period of approx. one month on the computer. Frequency of data logging was 1 Hz which was determined by the data bus line on BWE SchRs 1320. Measured data necessary for the study are listed in Table 1.

Table 1. Measured values

Value	Unit
Input power of bucket wheel motor 2	[kW]
Input power of bucket wheel motor 2	[kW]
Revolutions of bucket wheel motor 1	[rpm]
Revolutions of bucket wheel motor 2	[rpm]
Horizontal position of bucket wheel boom	[°]
Vertical position of bucket wheel boom	[°]
Distance moved by BWE	[m]

The cutting width b is evaluated from the measured horizontal position of the bucket wheel boom. The vertical position of the bucket wheel boom is used for the calculation of the height of the terrace h. The total block height is obtained from geodetic measurement. Records on the distance moved by the BWE are used to calculate the cutting depth s.

A mining record of a mining block of height of approx. 20.5 m was selected for the presented analysis. The working face was mined in four terraces. The first terrace had a height of 7.185 m, second terrace a height of 5.084 m, third terrace a height of 5.278 m and the fourth terrace was 3.172 m high. From the description, it is apparent that the terrace cutting method was used. This mining method is predominantly used in the Czech Republic. An illustration of the working face is shown in Fig. 4.
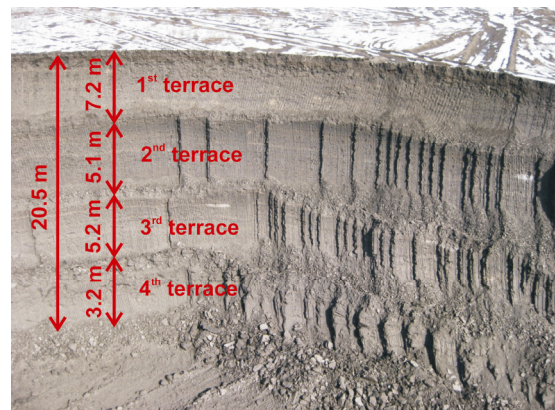

Fig. 4. Mining block during measurement

4.1. Statistical evaluation of measured variables

Measured variables were divided into individual terraces during evaluation. Events not dealing directly with mining were filtered from the time course of data. They include, above all, excavator movement during change of terrace, and movements made during the cleaning of buckets. Thus, only data pertinent to standard mining using the terrace cutting method were evaluated. The statistical evaluation of measured variables for mining on the 1st terrace is described in the following text and figures of the present chapter.

The time course of averaged revolutions of bucket wheel motors is depicted in Fig. 5. Statistical evaluation yielded a histogram which we approximated using the Gauss probability density function, see Fig. 6.

Fig. 7. Total input power of bucket wheel motors vs. mining time

Fig. 5. Averaged revolutions of bucket wheel motors vs. mining time
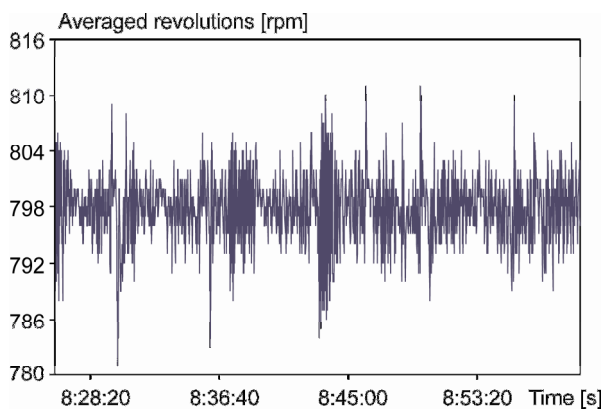
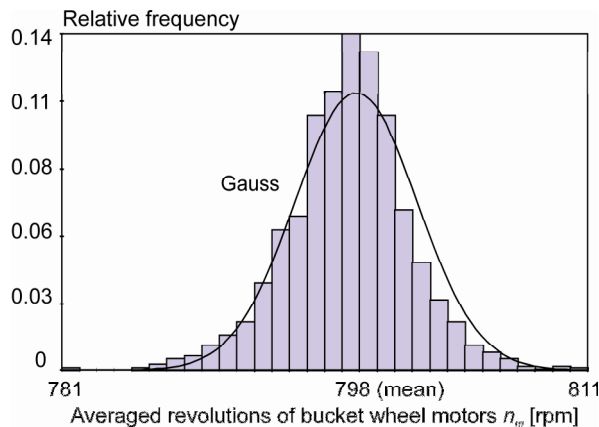**Fig. 8.** Histogram of total input power of bucket wheel motors

Fig. 6. Histogram of revolutions of bucket wheel motors

Fig. 7 illustrates the time course of the sum of input powers of bucket wheel motors. Regular decrease of input power of bucket wheel motors, which is given by retraction from the bench during change in the direction of rotation of the bucket wheel boom, is apparent in the Fig. 7. This phenomen was filtered off during evaluation, in order to avoid distortion of the histogram of input power of the bucket wheel motors in the mining process during the calculation of tangential digging force. The filtered time course of input power of bucket wheel
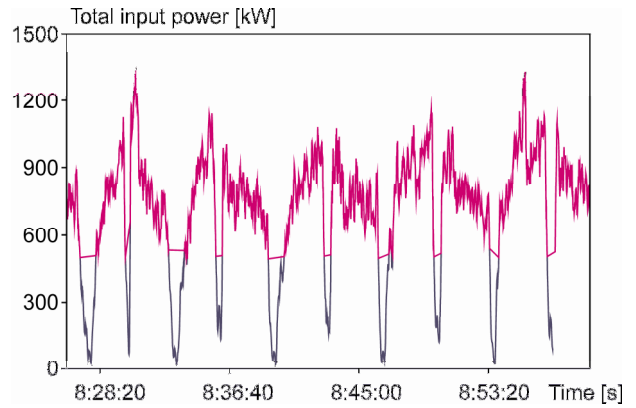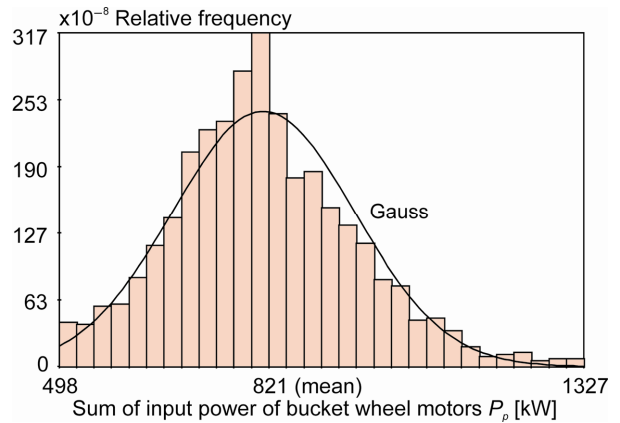
motors is depicted in purple in Fig. 7. The Gauss probability density function was used once again for approximation; see Fig. 8.

The cutting width was calculated from the measured horizontal position of the bucket wheel boom. The time course of the cutting width is depicted in Fig. 9. The mined terrace has the shape of a sickle, i.e., cutting depth decreases at its borders. In order to maintain maximum efficiency of mining, the speed of rotation is regulated, thus increasing or decreasing the cutting width. Ideally, the aim is for the ratio of s/b to be constant during mining (Durst, Vogt 1986; Fries *et al.* 2010). The cutting width is thus a variable, see Fig. 9. Decrease of the value of the cutting width to zero is due to change of direction of rotation of the bucket wheel boom. Statistical evaluation of the cutting width is difficult due to its physical significance. The obtained histogram is depicted in Fig. 10, approximation was performed using the Gauss probability distribution after taking into account the geometry of the mining block.

The basic cutting depth is calculated from the record of advanced meters of the crawler tracks of the BWE. The BWE moves in steps during mining using the terrace cutting method where each step characterizes the digging of the excavator into the working face which corresponds to the cutting depth. The cutting depth is dependent on the horizontal position of the bucket wheel boom. The cutting depth decreases and increases according to the

character of the sickle-form terrace. The recalculated time record of the cutting depth is depicted in Fig. 11. Rotation of the bucket wheel boom is apparent from the Fig. 11 where one arc corresponds to one slice on terrace. Statistical evaluation of such an event is very challenging. Hermite probability density function, which is the Gaussian probability density function multiplied by the Hermite polynom with respected skewness and kurtosis (programme Statrel 3.10), was used to approximate the histogram depicted in Fig. 12.
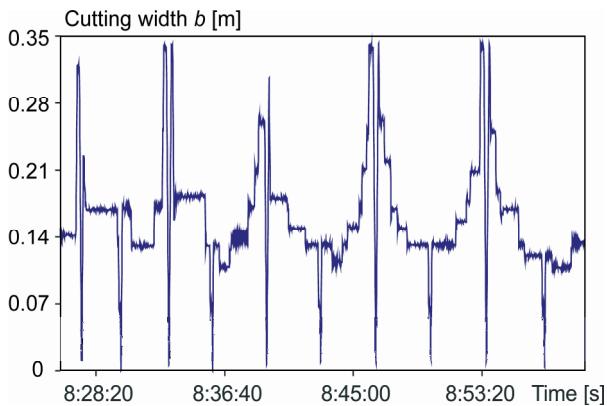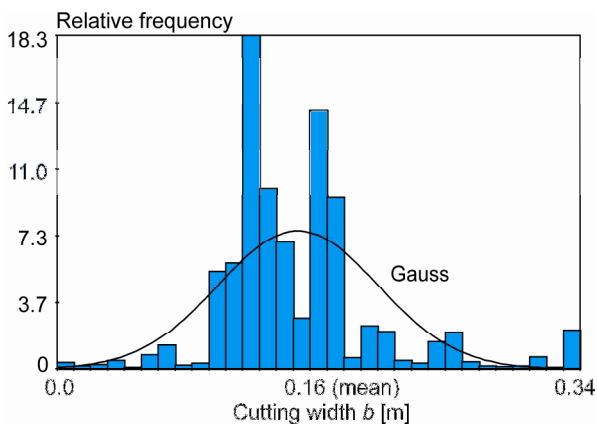

Fig. 9. Cutting width *b* vs. mining time

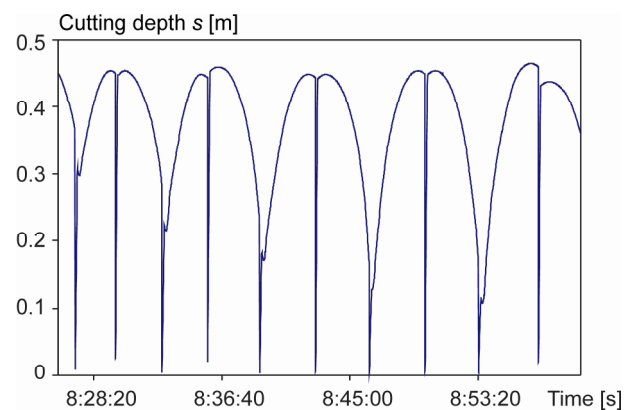**Fig. 10.** Histogram of cutting width *b*

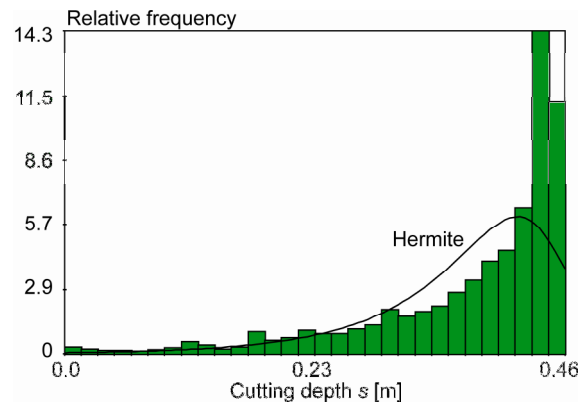**Fig. 11.** Cutting depth *s* vs. mining time

Fig. 12. Histogram of cutting depth *s*

The input power of bucket wheel motors during free rotation was measured for the determination of the power of the motor for operation of the bucket wheel whilst empty. Regulation of the revolution of the bucket wheel within 80% to 100% of the nominal revolution can be performed for the current configuration of the drive power of the bucket wheel with hydrodynamic fluid coupling Voith. This option is used during mining of clayey soil when the revolutions are decreased to the bottom limit of 80%. The power P_{Zp} was determined for two cases: 100% and 80% of nominal revolutions of bucket wheel motors. These measurements were performed during the whole series of measurements of individual movements of BWE SchRs 1320, see (Gottvald 2011). Mining had to be interrupted during measurements. Lengths of individual measurements were therefore minimized according to permissions from operational control. Due to this, the time record of measurements is approx. 1.5 minutes long in both cases. Approximately 90 values were measured within this time interval during sampling frequency of 1 Hz which is used for BWE SchRs 1320. It thus follows that the statistical evaluation is not entirely optimal. A uniform distribution was therefore chosen for approximation; see its statistical characteristics in Tables 2 to 5. Sensitivity analysis results generally depend more on the selected ranges than on the assigned distribution (Saltelli *et al.* 2004; Arwade *et al.* 2010).

Fig. 5 shows that the mean value of revolutions of bucket wheel motors during mining of the first terrace is approx. equal to 800 rpm. This value corresponds to the level of 80% of nominal revolutions of bucket wheel motors. For the remaining terraces, the motors were at 100% of nominal revolutions. Descriptive characteristics of selected distributions are listed in Tables 2 to 5, where Table 2 lists results for the first terrace, Table 3 lists the results for the second terrace, Table 4 lists the results for the third terrace, and Table 5 lists the results for the fourth terrace. The statistical characteristics of probability density functions that characterize variables φ_z, a_z and γ are also listed in the Tables 2 to 5. These variables were evaluated from additional measurements performed during power testing of BWE SchRs 1320, (see Gottvald 2011).

Table 2. Random variables for the first terrace

Symbol	Unit	Distribution	Mean value	Std. deviation
n_m	[rpm]	Gauss	797.799	3.498
P_p	[W]	Gauss	820641	143621
P_{Zp}	[W]	Uniform	49500	6639.53
b	[m]	Gauss	0.158	0.053
φ_z	[°]	Uniform	30	5.77
γ	[N.m^{-3}]	Gauss	19559	875.77
a_z	[m]	Lognormal	0.4	0.15
s	[m]	Hermite	0.377	0.094
			Skewness	Kurtosis
			−1.66	5.44

Table 3. Random variables for the second terrace

Symbol	Unit	Distribution	Mean value	Std. deviation
n_m	[rpm]	Gauss	978.733	4.536
P_p	[W]	Gauss	1227430	151649
P_{Zp}	[W]	Uniform	75500	18186.5
b	[m]	Gauss	0.155	0.047
φ_z	[°]	Uniform	30	5.77
γ	[N.m^{-3}]	Gauss	19559	875.77
a_z	[m]	Lognormal	0.4	0.15
s	[m]	Hermite	0.506	0.138
			Skewness	Kurtosis
			−1.224	3.445

Table 4. Random variables for the third terrace

Symbol	Unit	Distribution	Mean value	Std. deviation
n_m	[rpm]	Gauss	997.694	10.890
P_p	[W]	Gauss	1244370	219840
P_{Zp}	[W]	Uniform	75500	18186.5
b	[m]	Gauss	0.185	0.052
φ_z	[°]	Uniform	30	5.774
γ	[N.m^{-3}]	Gauss	19559	875.77
a_z	[m]	Lognormal	0.4	0.15
s	[m]	Hermite	0.367	0.104
			Skewness	Kurtosis
			−1.860	6.929

Table 5. Random variables for the fourth terrace

Symbol	Unit	Distribution	Mean value	Std. deviation
n_m	[rpm]	Gauss	997.46	10.74
P_p	[W]	Gauss	1313200	223770
P_{Zp}	[W]	Uniform	75500	18186.5
b	[m]	Gauss	0.148	0.041
φ_z	[°]	Uniform	30	5.773
γ	[N.m^{-3}]	Gauss	19559	875.77
a_z	[m]	Lognormal	0.4	0.15
s	[m]	Hermite	0.587	0.174
			Skewness	Kurtosis
			−1.826	6.098

4.2. Other input variables

Deterministic (non-random) variables are present in the above described calculation of the tangential digging forces F_{grges}. Their constant values are listed in Table 6.

Table 6. Constant variables for SchRs 1320

Symbol	Value	Unit
η_{grges}	90	[%]
η_m	96.5	[%]
η_p	95	[%]
η_h	90	[%]
$\cos\varphi_m$	0.85	[–]
f	0.5	[–]
R_k	6.25	[m]
R_v	5.45	[m]
g	9.81	[m.s^{-2}]

The terrace height h and angle of tilt of the bucket wheel boom β are variables that are fixed for each terrace always at the beginning of minig. Values of these variables are thus always chosen at the beginning, and remain unchanged during mining of the terrace. The variables are therefore non-random. Variables which correspond to the evaluated mining are listed in Table 7.

Table 7. Constant variables for SchRs 1320

Terrace	h [m]	β [°]
First	7.185	3.972
Second	5.084	−2.866
Third	5.278	−9.707
Fourth	3.172	−14.900

5. Sensitivity analysis

The sensitivity analysis studies the relationships between information flowing in and out of the model (Saltelli *et al.* 2004). Sensitivity analysis, within the scope of modelling, means different things to different people (see, e.g., Ferreira 2010; Mang *et al.* 2011; Keitel *et al.* 2011; Vaitkus 2011; Zhou *et al.* 2011). The basic measure of sensitivity is given by Spearman rank-order correlation (Kala 2005; Idoro 2012; Rafiei *et al.* 2012). The Sobol decomposition is an advanced method for the so-called global sensitivity analysis (Sobol 1993). One of the advantages of Sobol sensitivity analysis is that it enables the identification of interaction effects among input quantities (e.g. imperfections) on the monitored output (Kala 2009, 2011a, b, 2012).

5.1. Sobol sensitivity analysis

The coherent concept of sensitivity analysis enabling an analysis of the influence of arbitrary subgroups of input factors (doubles, triples, etc.) on the monitored output was elaborated by (Sobol 1993). Sobol sensitivity analysis was applied to analyze the influence of input variables X_i (input quantities) on the tangential digging force F_{grges} (output quantity Y) in the presented article.

The sensitivity coefficient of the first order for Sobol analysis may be written as:

$$S_i = \frac{V\big(E\big(Y|X_i\big)\big)}{V(Y)}. \tag{19}$$

Sobol proposed an alternate definition $S_i = corr\big(Y, E\big(Y\big|X_i\big)\big)$ which is based on the calculation of the correlation between output random variable Y and conditional random arithmetical mean $E\big(Y\big|X_i\big)$. Analogously as in (19), we can write the second order coefficient as:

$$S_{ij} = \frac{V\big(E\big(Y\big|X_i, X_j\big)\big)}{V(Y)} - S_i - S_j. \qquad (20)$$

Sensitivity coefficient S_{ij} expresses the influence of doubles on the monitored output. In a similar manner, we can write other Sobol sensitivity coefficients enabling the quantification of higher order interactions.

The advanced Monte Carlo method was applied (Vořechovský, Novák 2009; Vořechovský 2012). The case with statistically independent input random variables X_i was studied. The variance $V(Y)$ of tangential digging force is calculated under the assumption that all input variables are considered as random; one million simulation runs were applied. The conditional random arithmetical mean $E\big(Y\big|X_i\big)$ was evaluated for $N = 30\,000$ simulation runs; the variance $V\big(E\big(Y\big|X_i\big)\big)$ was calculated for $N = 30\,000$ simulation runs as well, i.e., the numerical demanding difficulty of the calculation is N^2. The procedure was similar for the calculation of the second order indices (20).

Decomposition of Sobol sensitivity coefficients can be written as:

$$\sum_i S_i + \sum_i \sum_{j>i} S_{ij} + \sum_i \sum_{j>i} \sum_{k>j} S_{ijk} + \ldots + S_{123\ldots M} = 1. \qquad (21)$$

The number of members in (21) is $2^M - 1$, i.e., for $M = 3$, we obtain 7 sensitivity coefficients S_1, S_2, S_3, S_{12}, S_{23}, S_{13}, S_{123}; for $M = 8$, we obtain 255 sensitivity coefficients, which is too much for practical application. The main obstacle in the determination of all members (21) is the high numerical demand of their calculation.

5.2. Sensitivity analysis results

Results of sensitivity analyses of the tangential digging force for the first terrace are depicted in Fig. 13. Results for the second terrace are shown in Fig. 14 and obtained results for the third and fourth terraces are shown in Fig. 15 and Fig. 16 respectively. The presented approach in the calculation of the total tangential digging force is based on the knowledge of the input powers of the bucket wheel motors, which can be measured easily. One could say that it is the inverse method of the calculation of the total tangential digging force F_{grges}, because the measured input power of bucket wheel motors P_p is in reality the reaction to the tangential digging force, i.e., the size of the input power of bucket wheel motors is dependent on the size of the total tangential digging force. It is apparent from the graphs in Figs 13 to 16 that the input power P_p of the bucket wheel motors has the greatest influence on the size of the total tangential digging force F_{grges} for the selected approach in the calculation. According to the physical essence of the mining process, the input power of the bucket wheel motors is, in reality, a reaction to the total tangential digging force.
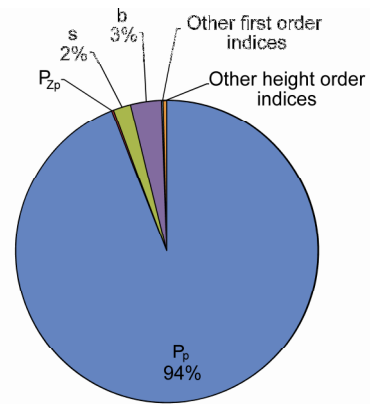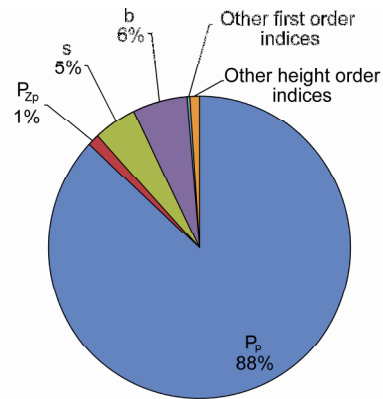

Fig. 13. Results for the first terrace

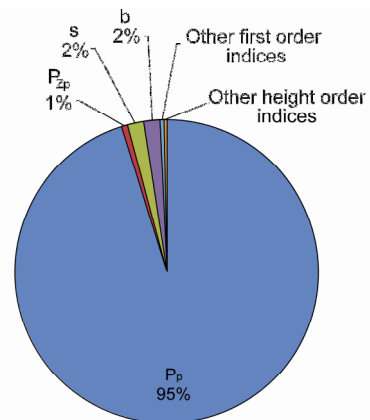**Fig. 14.** Results for the second terrace

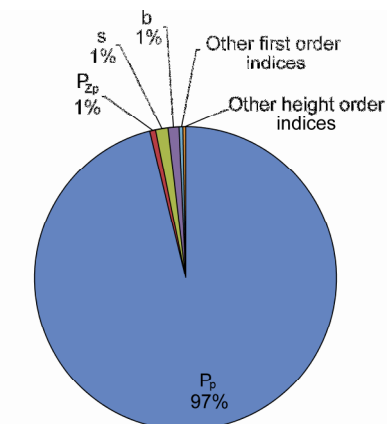**Fig. 15.** Results for the third terrace

Fig. 16. Results for the fourth terrace

The percentual representation of the input power of bucket wheel motors P_p is significant in all cases, and that leads to a less accurate description (overlap) of the influence of other random variables. Therefore, other sensitivity analyses were performed with the input power of the bucket wheel motors considered as a non-random value which was determined as the mean value of a priori considered random variables. The influences of the other quantities on the total tangential digging force is more evident from the rendered graphs. Results for the first terrace are shown in Fig. 17, for the second terrace in Fig. 18, for the third terrace in Fig. 19 and for the fourth terrace in Fig. 20.

The significantly changing effect of the cutting width b on the size of the total tangential digging force F_{grges} is apparent from the results. The effect is 57% in the first terrace, and it decreases up to 32% in the fourth terrace. On the contrary, the effect of the revolutions of bucket wheel motors n_m increases (from 1% in the first terrace to 10% in the fourth terrace) as does the input power of the bucket wheel motors during free rotation P_{Zp} (from 4% in the first terrace till 17% in the fourth terrace). The effect of the cutting depth s is 33% in the first terrace and from the second terrace, it has a constant value of 37%. It is important to note that the first terrace was mined with less revolutions of the bucket wheel, respectively bucket wheel motors than the other terraces. Analyses further showed that the effects of high order interactions of input random variables on the total tangential digging force were practically negligible. Let us note that each calculation was performed for the height of the terrace corresponding to conditions during measurement, see Table 7.

Analyses considering same terrace heights for all cases were performed in order to enable better comparison of results. The following terrace heights were gradually considered: 3, 4, 5, 6 and 7 m. During the determination of the trend of sensitivity, we also focused only on the following variables:

− Average revolution of bucket wheel motors n_m, see Fig. 21;
− Input power of bucket wheel motor during free rotation P_{Zp}, see Fig. 22;
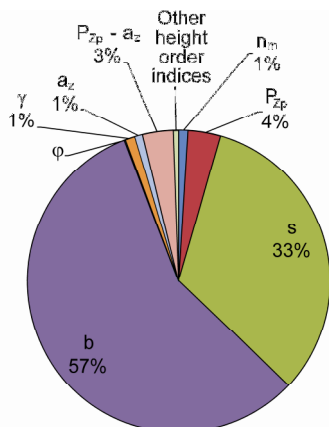− Cutting depth s, see Fig. 23;
− Cutting width b, see Fig. 24.

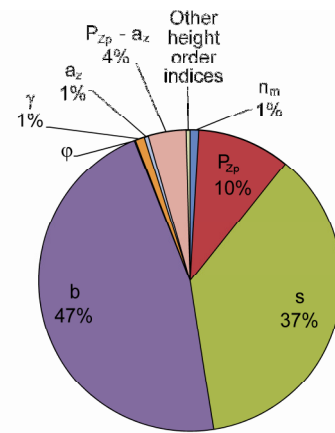**Fig. 17.** Results for the first terrace (P_p is a non-random variable)

Fig. 18. Results for the second terrace (P_p is a non-random variable)
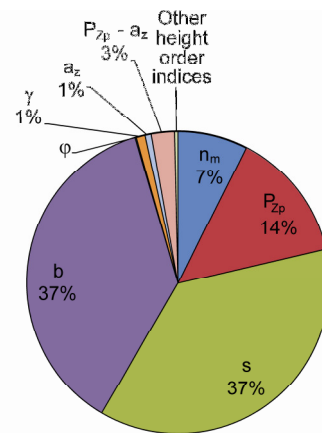

Fig. 19. Results for the third terrace (P_p is a non-random variable)
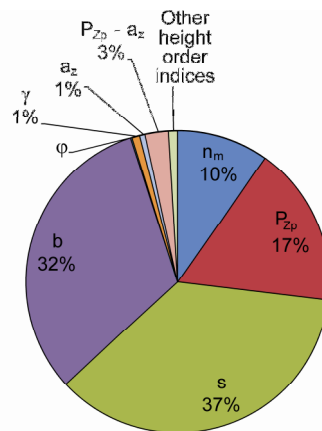

Fig. 20. Results for the fourth terrace (P_p is a non-random variable)

In the following graphs, the terrace heights are plotted on the horizontal axes, and the effects in percentages of experimentally obtained random variables on the total tangential digging force are plotted on the vertical axes. Results where the input power of bucket wheel motors was considered as a non-random value are plotted in the graphs.
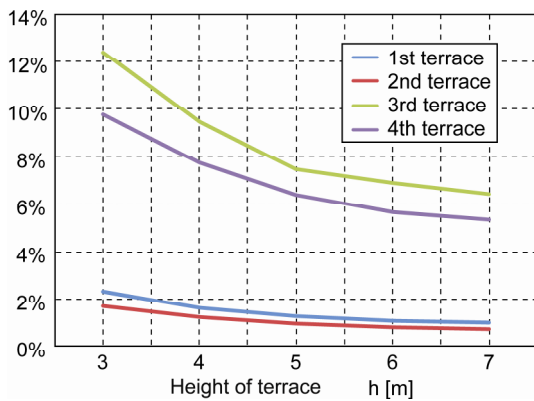
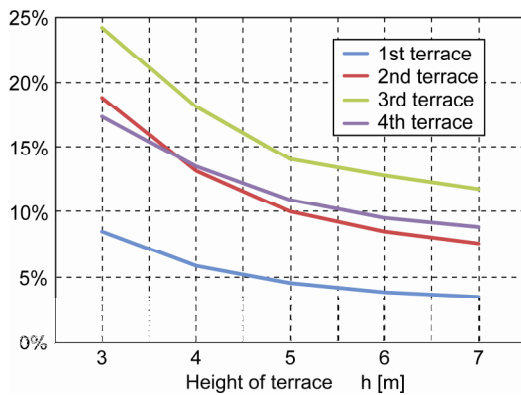**Fig. 21.** Trend of change of effect of revolutions of motor n_m

Fig. 22. Trend of change of effect of input power of motors during free rotation of the bucket wheel P_{Zp}
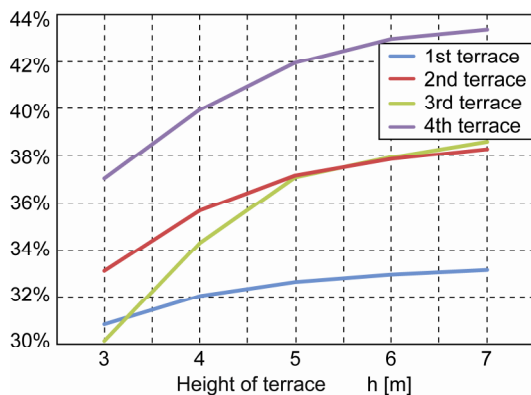

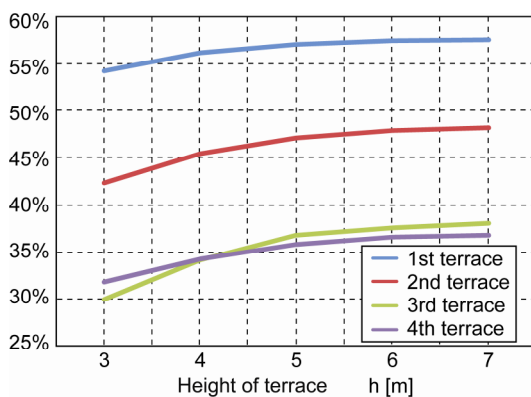**Fig. 23.** Trend of change of effect of cutting depth s

Fig. 24. Trend of change of effect of cutting width b

The effect of individual variables on the size of the total tangential digging force in respective terraces is apparent from the resulting graphs. Sobol sensitivity coefficients of cutting width and cutting depth increase with terrace height. This trend is identical for the analysis of inputs from all four terraces. On the contrary, Sobol sensitivity coefficients of revolutions of motors and input power of the motors during free rotation of the bucket wheel decrease with terrace height.

6. Conclusions

The aim of the sensitivity analysis was to determine the effect of experimentally determined random variables on the total tangential digging force. This effect was quantified using Sobol sensitivity coefficients.

For the adopted calculation method of the tangential digging force, the percentage of the effect of the input power of the motors of the bucket wheel is substantial, it is 94% for the first terrace, 88% for the second terrace, 95% for the third terrace, and 97% for the fourth terrace.

The variability of the input power of the bucket wheel motors overlaps the effects of other random variables. It was therefore considered in further studies as a non-random value which was obtained as the mean value of a priori considered random variables. The varying effect of the cutting width and the relatively unvarying effect of the cutting depth, where the first variable is within the interval of 32–57% and the second one, in the interval of 33–37%, can be observed in the obtained results. This fact points to considerable inefficiency of mining process. It is known from comprehensive studies (Durst, Vogt 1986) that the optional dimensions of the mining block are influenced by the intensity of digging force, but, in particular, by the efficiency of mining. For the maximum efficiency of mining, it is necessary to maintain the constant horizontal area of the mining block. Thus, it is necessary to maintain the fixed ratio of two variable parameters, namely of the cutting depth and cutting width, i.e., s/b = constant. It follows from the results that, in particular in case of the first terrace which was mined, rotations of the bucket wheel being different from those in the other terraces, the percentual proportion of the variance effect is considerably different for both quantities.

In subsequential steps of the sensitivity analysis, the change of the effect of random variables on the tangential digging force on all four terraces for different terrace heights 3 m, 4 m, 5 m, 6 m and 7 m were examined. Based on the results obtained from analysis, we can state that the effect of the revolution of the motors of the bucket wheel on the tangential digging force significantly increases for the third and fourth terraces and their effect is enhanced by decrease in terrace height. The effect of the input powers of the bucket wheel motors during free rotation of the bucket wheel has a similar character. Neither position of the terrace nor height of the terrace has an effect on a significant change in the effect of cutting depth. Change in the effect of the cutting width uniformly decreases from the first to the fourth terrace. Change in terrace height has not significant effect on its influence. This knowledge can be used to determine the

optimum of the terrace height, the optimum mining effectiveness, and loading of mining equipment.

Sensitivity analyses further show that the higher order interaction effects of input random variables on the tangential digging force are practically negligible.

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