

LITHUANIAN CASE STUDY OF MASONRY BUILDINGS FROM THE SOVIET PERIOD

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Abstract. This paper presents the process of effective selection of building elements for renovation important for energy saving in buildings. A large part of energy is lost in non-effective buildings. Therefore, in renovation of buildings, it is important to select effective structural improvements. Building insulation could not only save energy but also time, money and materials, which means that different objectives expressed in different units have to be fulfilled. Although different methods exist for the application of Multi Objective Optimisation, MULTIMOORA, which is composed of three sub-methods – Ratio System, Reference Point Method that uses ratios from the ratio system, and the Full Multiplicative Form – was preferred. Consequently, different solutions for building envelope renovation were ranked by MULTIMOORA as applied for masonry buildings from the Soviet period.

Keywords: thermal renovation, decision making, building envelope, insulation, Ratio System, Reference Point Method, Full Multiplicative Form, MULTIMOORA.

1. Introduction

This paper discusses the traditional way of construction specific to the Soviet period. Soviet period means the period before the independence of Lithuania in 1990. The average lifetime of a building is one hundred year (STR. 1.12.06:2002). Consequently the buildings of the Soviet period have still some lifetime.

Over time, the quality requirements and characteristics pertaining to building structures changed. We know the increasing importance of dry construction and of stricter requirements for thermal insulation appeared during the past decades.

In 2007, the European Council adopted ambitious energy and climate change objectives for 2020 – to reduce greenhouse gas emissions by 20% if the conditions are right, to increase the share of renewable energy to 20% and to make a 20% improvement in energy effectiveness. By 2020, the EU aims to reach that all new constructions and renovations would be passive (less than 15 kWh/m² per year). The European Parliament has continuously supported these goals. The European Council has also given a long term commitment to the decarbonisation path with a target for the EU and other industrialised countries of 80 to 95% cuts in emissions by 2050 (European Commission 2011).

Many countries have their own directives. For instance: BREEAM (Building Research Establishment Environmental Assessment Method) in Great Britain; LEED (Leader Energy Environment Design) in the USA; HQE (High Environmental Quality) in France; MINERGIE in

Switzerland; PASSIVHAUS in Germany; HB BEAM in Hong Kong; CASBEE (Comprehensive Assessment System for Building Environmental Efficiency) in Japan and TOTAL QUALITY in Austria. All directives attempt to reach the sustainability in construction, however in different ways.

Cost of heating is one of the most important factors that influence the real estate market (Huang, Hsueh 2010). Thermal renovation of a building can significantly reduce heat losses and costs, thereby increasing the market value of such building. Although renovation does not necessarily increase energy effectiveness, it is a useful instrument that helps upgrading thermal performance.

Reduction of building energy consumption can be carried out in several stages namely: architectural optimisation of the building, and optimisation of building engineering systems. For architectural optimisation, properly selected building structures, orientation of partitions in respect to compass points, size of windows and thermal storage capabilities of devices are of the utmost importance (Motuziene, Juodis 2010; Kaklauskas *et al.* 2008; Kalibatas *et al.* 2011; Flores-Colen, de Brito 2010). Improvements under the form of optimized heat inflows and of heat losses allow reduced consumption of power for heating.

In order to select a rational alternative for renovation of building, is it necessary to investigate and to assess the conditions of the building, as well as to analyse investments.

The degree of deterioration of a building can be determined by the following studies:

- Prior to the inspection stage – an opinion poll of residents regarding the physical, economic and functional deterioration of the building;
- Investigation of the technical condition of elements of the building (i.e. walls, windows, roof and floors);
- Study of energy supply and more effective fuel utilisation;
- Investigation of the condition of engineering equipment;
- During the investigation – an inventory of reasons for renovation, expected activities and tools that can improve characteristics of the building.

Every building is different in its construction, architecture, purpose, and needs of residents. Therefore, there cannot be one solution that would fit all buildings. Limiting the research only to residential buildings without considering commercial or public buildings forms a first step is to diminish these differences.

Evaluation of renovation possibilities is a difficult task as a building and its environment form complex systems (for instance in terms of technical, technological, ecological, social, comfort and aesthetical conditions). All subsystems influence the total effective performance and the interdependence between subsystems plays a significant role.

Successful choice of a renovation measures depends on the following three stages:

- During the first stage, the elements of building envelope for renovation are determined;
- During the second stage, top-ranked alternatives for building envelope renovation are chosen;
- During the third stage, effective alternatives are selected by multiple objectives analysis in order to obtain alternative priorities and an alternative level of performance.

Building insulation is a safe choice that helps preventing damage to a building and improves energy saving undertaken in different parts of the building.

Insulation of the building will be studied as a function of the different partitions of the building: walls, windows, roofs and ceilings. Prior to renovation works, it is important to identify the existing state of walls, roof, floors, windows, doors and other building elements, as well as technical characteristics, i.e. materials, thickness of structures and orientation in respect of compass points (Tupenaite *et al.* 2010; Pasanisi, Ojalvo 2008). Such assessment of existing structures and complex exploration of building characteristics helps deciding on partitions that require further improvement and identifying changes that would be effective.

Parts of building envelope are assessed with respect to their degree of insulation and the impact of such insulation on reduction of heating costs. Sometimes, calculations reveal that modification of building structures is not effective due to a lengthy payback period that considerably exceeds heat losses for the partition. A multiple objective decision support system has been designed to properly select and implement thermal effectiveness tools for a building.

2. The scope of the existing problem

Sixty years ago, masonry buildings were the most popular in the Eastern Europe. Typically, the envelope of such buildings had no insulation (Figs 1–3). The built-up area was 10 m width to 50 m length. A five-storey building was selected for the analysis. From ceiling to ceiling, the height of a storey was 2.85 m. The building had 90 windows in total: 45 windows facing north and the remaining 45 facing south. The building was ventilated in the natural way. Heating networks from a central heating unit of the city supplied heating to the building. Now obsolete heat network pipes are changed to new ones.

The thermal insulation performance of such a building is rather poor and does not comply with the current hygienic requirements. The main important areas of reduced masonry thicknesses are situated along the wall and slab juncture as demonstrated in Fig. 1; along the edge of the roof as shown in Fig. 2; and along the basement and slab (Fig. 3); and windows (not represented in Figs.). The building was designed with a large part of windows facing north. Because of thin masonry thickness in the partition juncture, thermal bridges cause low surface temperature, which could present a risk for mould growth. Condensation can cause dampness together with favorable temperature, mostly cause mould grow up.

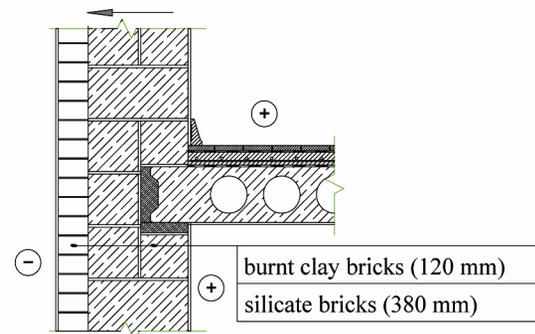


Fig. 1. Vertical section of the wall before renovation. The wall consist of two masonry layers, the first is 380 mm in width and the second is 120 mm in width

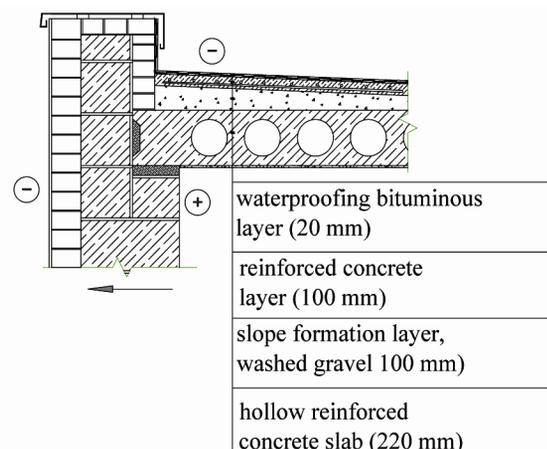


Fig. 2. Cross section of the roof before renovation. The roof detail consists of a hollow concrete slab, sand, and a waterproof bituminous layer

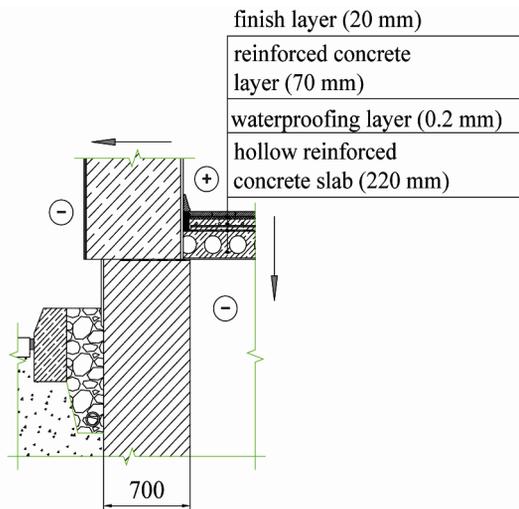


Fig. 3. Vertical section of the cellar ceiling before renovation. The ceiling consist of a hollow concrete slab of 220 mm, waterproofing layer, a reinforced concrete layer of 70 mm and a finish layer of 20 mm

The main purpose of this study is to assess elements of the building envelope using multiple objective models. Heating energy losses and inflows were calculated using equations from the technical construction regulation (STR.2.05.01:2005). A residential building has been chosen for this reason, with partition characteristics listed in Table 1.

Table 1. Heat losses through existing partitions

No.	Heat losses	Calculated values (kWh/m ² per year)
1	Heat losses through the walls	90.21
2	Heat losses through the roof	73.69
3	Heat losses through the insulated cellar ceilings	14.84
4	Heat losses through the windows	68.29
5	Heat losses through the external doors	0.36
6	Heat losses through the bearer thermal bridges	32.40
7	Heat losses through opening external doors	1.23
8	Energy consumption for natural ventilation of the building	24.04
9	Heat losses through the external air infiltration	75.64
10	Heat inflows in the building	-28.76
11	Internal heat divergence in the building	-6.34
12	Total energy consumption in the building (without the assessment of the efficiency of the heating system)	345.6

3. Complex renovation of building partitions

Often, thermal insulation layers were selected for each partition individually, but the final result was often neither accurate nor effective. Authors of the paper offer a

methodology that helps selecting complex heat insulation layers for the entire building. Complex thermal insulation selection helps to estimate better payback time of the insulation, lost heat amounts and durability.

The method for insulation of building envelope is selected depending on the structure, building materials and the purpose of the partition as well as other objectives. Exterior insulation (Figs 5, 6) is the best way for elimination of thermal bridges in wall and ceiling junctures. With the help of the exterior wall insulation, the existing wall structure falls within the positive temperature range. In addition, the wall gets protected from atmospheric and other external effects (Fig. 4). Thermal renovation of the facade doesn't require a temporary removal of residents and the work can be done quickly. Renovated facade is more resistant to atmospheric impacts and has a greater aesthetic value.

Wall insulation from inside can be used for various buildings, especially for heritage buildings. Heritage building means a building possessing historical, architectural or cultural values. A heritage building is declared by Competent Authority in whose jurisdiction such building is situated.

Internal wall insulation, in terms of thermal and humidity effectiveness, is less effective. A wall supporting structure, which is insulated from the inside, has significant temperature fluctuations. This is due to external climate impact. Consequently operational conditions deteriorate. To avoid the risk of condensation, it is not recommended to install interior thermal insulation that would be wider than 5 cm. Internal wall insulation reduces the living space, requires recoating of walls and reinstallation of engineering equipment (Golic *et al.* 2011).

Insulation improvements should ensure that thermal insulation layers of different partitions connect and overlap. New insulation layer placed onto the wall has to be connected or overlap the insulation layer placed onto the roof.

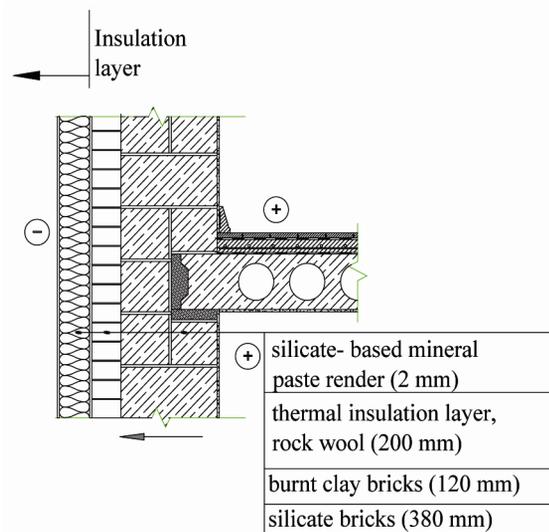


Fig. 4. Improvement of the thermal insulation layer (200 mm in width) on walls of the building. Thermal bridge is eliminated along the wall and slab juncture

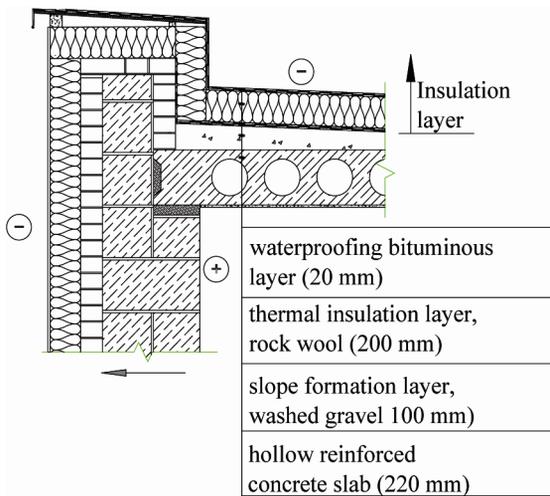


Fig. 5. Improvement of the thermal insulation layer (200 mm in width) on the roof of the building. The external insulation layer on walls is connected to the insulation layer on the roof

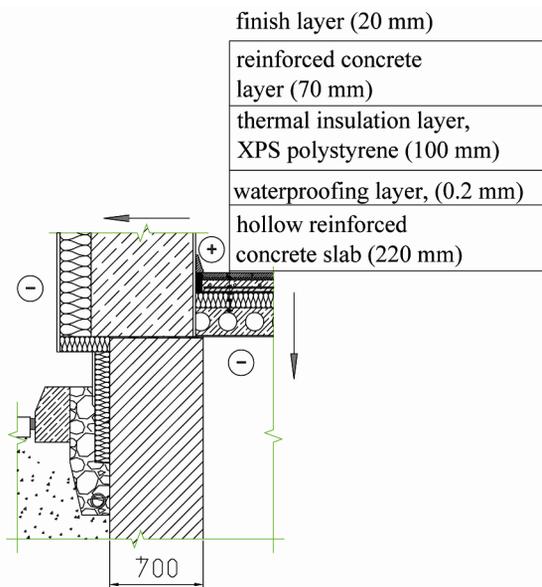


Fig. 6. Improvement of the thermal insulation layer (100 mm in width) on the ceiling of the building cellar

With the help of multi-objective decision-making methods, the research aims to create a technique for an effective selection of insulation for building envelope.

4. Multi-objective optimisation on the basis of the ratio analysis method

Multi-objective Decision Making MODM – a simplified and less time-consuming method for mathematical investigation of comparative performance was selected (Chakraborty 2011). Multiple Objective decision methods MOORA and MULTIMOORA were selected for further calculation.

For the first time, multi-objective optimization on the basis of the ratio analysis was introduced by Brauers and Zavadskas (2006). The method starts with a matrix of responses of different alternatives to different objectives:

$$X = \begin{bmatrix} x_{11} & \dots & x_{1i} & \dots & x_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{j1} & \dots & x_{ji} & \dots & x_{jn} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{m1} & \dots & x_{mi} & \dots & x_{mn} \end{bmatrix}, \quad (1)$$

where: x_{ij} – the response of alternative j on objective i ;
 $i = 1, 2, \dots, n$ – the number of objectives;
 $j = 1, 2, \dots, m$ – the number of alternatives.

The MOORA method consists of two components: (4.1) the ratio system and (4.2) the reference point approach.

4.1. The ratio system as a part of MOORA

In the ratio system of MOORA, each response of an alternative to an objective is compared to a denominator, which represents all alternatives related to that objective. The square root of the sum of squares of each alternative to the objective was used as denominator in the MOORA formula (Van Delft, Nijkamp 1977):

$$\bar{x}_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^m x_{ij}^2}}, \quad (2)$$

with: x_{ij} – response of alternative j to objective i ;
 $j = 1, 2, \dots, m$; m – the number of alternatives;
 $i = 1, 2, \dots, n$; n – the number of objectives;
 \bar{x}_{ij} – a dimensionless number representing the response of alternative j to objective i .

Brauers and Zavadskas (2006) proved that this formula is the best choice between many other possible ones.

Dimensionless numbers have no specific unit of measurement. They are obtained, for instance, by deduction, multiplication or division. The normalised response of the alternatives to the objectives belongs to the interval [0; 1]. However, sometimes the interval could be [-1; 1]. Indeed, in the case of the productivity growth, some sector, region or country may show a decrease instead of an increase in productivity, i.e. a negative dimensionless number.

For optimisation, in case of maximisation these responses are added and in case of minimisation – subtracted:

$$\bar{y}_j = \sum_{i=1}^{i=g} \bar{x}_{ij} - \sum_{i=g+1}^{i=n} \bar{x}_{ij}, \quad (3)$$

where: $i = 1, 2, \dots, g$ – the objectives to be maximized;
 $i = g + 1, g + 2, \dots, n$ – the objectives to be minimised.

\bar{y}_j – the normalised assessment of alternative j with respect to all objectives.

An ordinal ranking of \bar{y}_j shows the final preference. Nevertheless, it may turn out to be necessary to stress that some objectives are more important than the others. In order to give more importance to an objective, it could be multiplied by a significance coefficient.

4.2. The reference point approach as a part of MOORA

The reference point theory is based on the ratios found in equation (2), whereby a maximal objective reference point is also deduced. The maximal objective reference point approach is called realistic and non-subjective when the coordinates (r_i) selected for the reference point are realised in one of the candidate alternatives. For example, we have three alternatives described as follows: A (10; 100), B (100; 20) and C (50; 50). In this case, the maximal objective reference point R_m results in (100; 100). The maximal objective vector is self-evident if the alternatives are well defined as for the projects in the area of project analysis and planning.

Given the dimensionless number representing the response of alternative j to objective i , i.e. \bar{x}_{ij} in equation (2), we come to:

$$(r_i - \bar{x}_{ij}), \tag{4}$$

- where: $i = 1, 2, \dots, n$ as the objective;
- $j = 1, 2, \dots, m$ as the alternatives;
- r_i – the i^{th} coordinate of the reference point;
- \bar{x}_{ij} – the ratio found in the ratio system (formula 2).

This matrix is subject to the Min–Max metric of Tchebycheff (Karlin, Studden 1966):

$$Min_{(j)} \left\{ \max_{(i)} (r_i - \bar{x}_{ij}) \right\}. \tag{5}$$

The Min–Max metric is the best choice between all the possible metrics of the reference point theory (Brauers 2004: 159–161).

4.3. The Full Multiplicative Form

From now, in order to distinguish it from the mixed forms, the following form of n -power for multi-objectives is called a full-multiplicative form:

$$U_j = \prod_{i=1}^n x_{ij}, \tag{6}$$

- with: $j = 1, 2, \dots, m$; m the number of alternatives;
- $i = 1, 2, \dots, n$; n being the number of objectives;
- x_{ij} response of alternative j to objective i ;
- U_j overall utility of alternative j .

The overall utilities (U_j), obtained by multiplication of different units of measurement, become dimensionless.

The objectives to be minimised are denominators in the formula:

$$U_j = \frac{A_j}{B_j}, \tag{6'}$$

with: $A_j = \prod_{g=1}^i x_{gi}$,

i the number of objectives to be maximized,

with: $B_j = \prod_{k=i+1}^n x_{ki}$,

$n-i$ – the number of objectives to be minimized,
with: U_j – the utility of alternative j with objectives to be maximised and objectives to be minimised.

5. MULTIMOORA

MULTIMOORA is the combination of the MOORA method and of the Full Multiplicative Form of Multiple Objectives. For the first time, MULTIMOORA was introduced by Brauers and Zavadskas in the beginning of 2010 (Brauers, Zavadskas 2010a). MULTIMOORA became the most robust system of multiple optimisations under condition of support from the Ameliorated Nominal Group Technique and Delphi method (Brauers, Zavadskas 2010b, 2011a; Brauers, Ginevicius 2010).

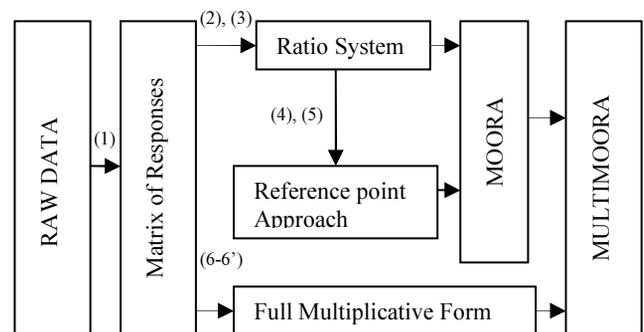


Fig. 7. Diagram of MULTIMOORA

Diagram Fig. 7 represents the combination of the three different methods of MULTIMOORA (Brauers, Zavadskas 2011b). Nominal Group Technique: 1) the Ratio System; 2) the Reference Point Approach; 3) the Full Multiplicative Form. The Nominal Group Technique and the Delphi method can also be used to reduce the remaining subjectivity. Numbers and arrows in the picture are referring by formulas rank and the priority in the multi-objective method.

6. Effective selection of building envelope insulation alternatives using multi-objective decision-making methods

In order to find an optimal alternative the multi-objective optimization method MOORA (Multi-Objective Optimisation by Ratio analysis) and MULTIMOORA (MOORA plus Full Multiplicative Form) were used.

When using multi-objective optimisation methods, all objectives must be measurable. Multi-objective techniques seem to be an appropriate tool for ranking or selecting one or more alternatives from a set of available options based on the multiple, sometimes conflicting objectives. A large number of methods have been developed for solving multi-objective problems (Balezentis et al. 2011; Zavadskas et al. 2010; Medineckiene et al. 2011). Multi-objective optimisation frameworks vary from simple approaches, requiring very little information,

to methods based on mathematical programming techniques, requiring extensive information on each objective and on the preferences of the stakeholders.

Various objectives of effectiveness are proposed with different dimensions, different significances as well as different directions of optimization.

The main steps of multiple attribute decision-making are as follow (Kaklauskas *et al.* 2008; Nativida-de-Jesus *et al.* 2007):

- the system of evaluation attributes that relates system capabilities to goals;
- the alternative systems for attaining the goals (generating alternatives);
- the initial data;
- application of a multiple-objectives analysis method;
- calculation of results and selection of the “optimal” (preferred) alternative.

A case study considers 20 possible alternatives of external walls, roofs, ceilings and windows. Some alternatives given in Table 2 (wall 1, roof 1, roof 2, ceiling 1, windows 1) don't comply the requirements of Lithuanian standards for thermal insulation.

In alternatives windows 2 and windows 4 it was decided to change windows only on the northern side, in total around 258.30 m².

Each alternative provided in Table 2, is described by seven objectives:

- x_1 – Thermal resistance [W/K·m²];
- x_2 – Partition price per square meter [€/m²];
- x_3 – Total partition price [EUR]; EUR 1= LTL 3.458;
- x_4 – Counted partition heat losses [kWh/m² per year];
- x_5 – Savings in the amount of heat loss [kWh/ m²·year];
- x_6 – Total amount saved per year [EUR/year];
- x_7 – Simple payback period [year].

The main purpose of a residential building is to create good living conditions with appropriate air ventilation, level of humidity and heating (Sobotka, Rolak 2009; Thomas 2010). The basic design of a building must comply with certain requirements. External walls, roof, ground storey, doors and windows have a particular role in the construction of a building. These elements can influence the choice of heating and cooling systems used depending on the season. Only comprehensive assessment of elements that are responsible for heat losses can greatly reduce related costs, especially in renovated buildings (Juodis *et al.* 2009; Ginevicius *et al.* 2008).

The effective selection of walls with the help of calculations applied in the multiple objective methods are described in Tables 3 and 4. Calculations pertaining to other partitions are provided in annexes A, B and C.

Objectives that define heat losses, payback period and partition renovation prices are minimised; whereas heat resistance and heat savings subsequent to renovation of the building are maximized.

Table 2. Heating energy consumptions, costs and payback periods of the main partitions

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
		max	min	min	min	max	max	min
1	wall 1	2.10	145.00	182 700.00	32.56	57.65	17 294.42	10.56
	wall 2	3.45	150.00	189 000.00	19.67	70.54	21 161.29	8.93
	wall 3	4.76	160.00	201 600.00	14.24	75.97	22 790.24	8.85
	wall 4	5.00	180.00	226 800.00	13.57	76.64	22 991.23	9.86
	wall 5	5.99	190.00	239 400.00	11.53	78.68	23 603.21	10.14
2	roof 1	1.73	144.51	74 646.64	16.13	57.56	17 267.42	4.32
	roof 2	3.08	150.29	77 632.30	9.18	64.51	19 352.35	4.01
	roof 3	5.78	158.96	82 110.79	4.73	68.96	20 687.31	3.97
	roof 4	6.25	170.52	88 082.11	4.45	69.24	20 771.31	4.24
	roof 5	7.14	179.19	92 560.59	3.89	69.80	20 939.30	4.42
3	ceiling 1	1.95	57.80	29 856.59	8.13	6.710	2 012.93	14.83
	ceiling 2	3.34	66.47	34 335.08	5.49	9.35	2 804.91	12.24
	ceiling 3	4.00	72.25	37 320.74	4.73	10.11	3 032.90	12.31
	ceiling 4	4.70	89.6	46 282.88	4.08	10.76	3 227.89	14.34
	ceiling 5	6.06	104.05	53 747.03	3.39	11.45	3 434.89	15.65
4	windows 1	0.59	202.31	102 656.14	46.44	69.14	20 741.31	4.95
	windows 2	0.63	216.76	55 989.11	55.77	59.81	17 942.40	3.12
	windows 3	0.63	216.76	109 988.36	43.71	71.87	21 560.28	5.10
	windows 4	0.83	245.66	63 453.98	50.21	65.37	19 610.35	3.24
	windows 5	0.83	245.66	124 652.80	32.78	82.80	24 839.17	5.02

Table 3. MOORA applied on 5 wall alternatives with 7 conditions3a. Matrix of responses of alternatives on objectives: (x_{ij})

	x_1 max	x_2 min	x_3 min	x_4 min	x_5 max	x_6 max	x_7 min
wall 1	2.10	145	182 700.00	32.56	57.65	17 294.42	10.56
wall 2	3.45	150	189 000.00	19.67	70.54	21 161.29	8.93
wall 3	4.76	160	201 600.00	14.24	75.97	22 790.24	8.85
wall 4	5.00	180	226 800.00	13.57	76.64	22 991.23	9.86
wall 5	5.99	190	239 400.00	11.53	78.68	23 603.21	10.14

3b. Sum of squares and their square roots

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
wall 1	4.41	21025	$33\,379.29 \cdot 10^6$	1 060.15	3 323.52	$2\,990.97 \cdot 10^5$	111.60
wall 2	11.89	22500	$35\,721 \cdot 10^6$	386.91	4 975.89	$4\,478.00 \cdot 10^5$	79.77
wall 3	22.68	25600	$40\,642.56 \cdot 10^6$	202.78	5 771.44	$5\,193.950 \cdot 10^5$	78.25
wall 4	25.00	32400	$51\,438.24 \cdot 10^6$	184.15	5 873.69	$5\,285.96 \cdot 10^5$	97.31
wall 5	35.86	36100	$57\,312.36 \cdot 10^6$	132.94	6 190.54	$5\,571.11 \cdot 10^5$	102.87
Sum of squares.	99.84	137 625.00	$218\,493450 \cdot 10^6$	1 966.93	26 135.09	$23\,520.01 \cdot 10^5$	469.81
Square roots	9.99	370.98	467 432.82	44.35	161.66	48 497.43	21.68

3c. Objectives divided by their square roots and MOORA

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	total	Rank max
wall 1	0.210	0.391	0.391	0.734	0.357	0.357	0.487	-1.080	5
wall 2	0.345	0.404	0.404	0.444	0.436	0.436	0.412	-0.446	4
wall 3	0.477	0.431	0.431	0.321	0.470	0.470	0.408	-0.175	1
wall 4	0.500	0.485	0.485	0.306	0.474	0.474	0.455	-0.283	3
wall 5	0.599	0.512	0.512	0.260	0.487	0.487	0.468	-0.180	2

3d. Reference point theory with ratios: coordinates of the reference point equal to the maximal objective values

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
r_i	0.599	0.391	0.391	0.260	0.487	0.487	0.408

3e. Reference point theory: deviations from the reference point

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	max	Rank min
wall 1	0.389	0.000	0.000	0.474	0.130	0.130	0.079	0.474	5
wall 2	0.254	0.013	0.013	0.184	0.050	0.050	0.004	0.254	4
wall 3	0.123	0.040	0.040	0.061	0.017	0.017	0.000	0.123	3
wall 4	0.099	0.094	0.094	0.046	0.013	0.013	0.047	0.099	1
wall 5	0.000	0.121	0.121	0.000	0.000	0.000	0.060	0.121	2

Table 4. The Full Multiplicative Form

	A	B	C	D	E	F	G	H	I
	max	min	C=A:B	min	E=C:D	min	G=E:F	max	I=G:H
wall 1	2.10	145	0.01	182 700.00	$7.93 \cdot 10^{-8}$	32.56	$2.44 \cdot 10^{-9}$	57.65	$1.4 \cdot 10^{-7}$
wall 2	3.45	150	0.02	189 000.00	$1.21 \cdot 10^{-7}$	19.67	$6.18 \cdot 10^{-9}$	70.54	$4.36 \cdot 10^{-7}$
wall 3	4.76	160	0.03	201 600.00	$1.47 \cdot 10^{-7}$	14.24	$1.04 \cdot 10^{-8}$	75.97	$7.88 \cdot 10^{-7}$
wall 4	5.00	180	0.03	226 800.00	$1.22 \cdot 10^{-7}$	13.57	$9.02 \cdot 10^{-9}$	76.64	$6.92 \cdot 10^{-7}$
wall 5	5.99	190	0.03	239 400.00	$1.31 \cdot 10^{-7}$	11.53	$1.14 \cdot 10^{-8}$	78.68	$8.98 \cdot 10^{-7}$

J	K	L	M	Result	Project
max	K=I:J	min	M=K:L		
17 294.42	$0.24 \cdot 10^{-2}$	10.56	$0.23 \cdot 10^{-3}$	5	wall 1
21 161.29	$0.92 \cdot 10^{-2}$	8.93	$1.03 \cdot 10^{-3}$	4	wall 2
22 790.24	$1.79 \cdot 10^{-2}$	8.85	$2.03 \cdot 10^{-3}$	2	wall 3
22 991.23	$1.59 \cdot 10^{-2}$	9.86	$1.61 \cdot 10^{-3}$	3	wall 4
23 603.21	$2.12 \cdot 10^{-2}$	10.14	$2.09 \cdot 10^{-3}$	1	wall 5

Table 5. MULTIMOORA as a consequence of the MOORA method and of the Full Multiplicative Form

		MOORA		Full Multiplicative Form	MULTIMOORA
		Ratio system	Reference point		Rank
1	wall 1	5	5	5	5
	wall 2	4	4	4	4
	wall 3	1	3	2	2
	wall 4	3	1	3	3
	wall 5	2	2	1	1
2	roof 1	5	5	5	5
	roof 2	4	4	4	4
	roof 3	2	3	2	2
	roof 4	3	1	3	3
	roof 5	1	2	1	1
3	ceiling 1	5	5	5	5
	ceiling 2	4	4	4	4
	ceiling 3	2	2	2	2
	ceiling 4	3	1	3	3
	ceiling 5	1	3	1	1
4	windows 1	5	2	5	5
	windows 2	3	3	3	3
	windows 3	4	4	4	4
	windows 4	1	1	1	1
	windows 5	2	5	2	2

7. Calculation results

MULTIMOORA optimisation techniques with discrete dimensionless measures were used for ranking alternatives in this study. Three results were obtained. Indeed MOORA consists of two components: the ratio system and the reference point, whereas MULTIMOORA adds the Full Multiplicative Form: Table 5 represents the final outcomes.

MULTIMOORA optimised the best thermal insulation variant for renovation of a building structure: wall number 5, roof number 5, ceiling number 5 and window number 4.

Instead of the insulation problem for buildings from the Soviet period in Lithuania, a much heavier problem of Multi-Objective Decision Making will arrive when the economic depreciation of the Soviet time buildings will come to its end. At that moment, with an exception for the historical buildings, the following alternative choice will arrive. On the one hand, the keeping in operation of the Soviet time buildings with huge maintenance costs or on the other hand new constructions with lower maintenance costs, better and cheaper material, such as the replacement of wet construction by dry construction, lower labour costs and more modern comfort but with a new starting economic depreciation.

8. Conclusions

One of the most difficult tasks, during the building audit is ensuring a proper design of thermal insulation for building envelope. The method presented in this article, called MULTIMOORA, optimises building envelope improvements concerning insulation. This method can facilitate and accelerate selection of insulation for building envelope on the basis of local technical specifications.

Inside MULTIMOORA three methods are assumed to have the same importance. These three methods represent all existing methods with dimensionless measures in multi-objective optimisation. With the help of these Multi-Objective Optimisation methods, the case study focuses on an effective selection between 20 alternatives of building partitions. Five renovation scenarios based on small (alternatives 1), medium (alternative 2 to alternatives 3) and basic (alternative 4 to alternatives 5) investment packages were proposed for residential multi-apartment buildings in Lithuania and the priority order for their application was determined.

MULTIMOORA optimised the best thermal insulation variant for renovation of a building structure concerning walls, roof, ceiling and windows.

With the help of MULTIMOORA, this study proved that the proposed theoretical model was effective in real life and could be successfully applied for a solution of similar utility problems in construction as well as in other fields

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Annex A

Table 6. MOORA applied on 5 roof alternatives with 7 conditions

6a. Matrix of responses of alternatives on objectives: (x_{ij})

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
	max	min	min	min	max	max	min
roof 1	1.73	144.51	74 646.64	16.13	57.56	17 267.42	4.32
roof 2	3.08	150.29	77 632.30	9.18	64.51	19 352.35	4.01
roof 3	5.78	158.96	82 110.79	4.73	68.96	20 687.31	3.97
roof 4	6.25	170.52	88 082.11	4.45	69.24	20 771.31	4.24
roof 5	7.14	179.19	92 560.59	3.89	69.80	20 939.30	4.42

6b. Sum of squares and their square roots

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
roof 1	2.98	20 883.14	5 572.12·10 ⁶	260.18	3 313.15	298.16·10 ⁶	18.69
roof 2	9.48	22 587.08	6 026.77·10 ⁶	84.27	4 161.54	374.51·10 ⁶	16.09
roof 3	33.41	25 268.28	6 742.18·10 ⁶	22.37	4 755.48	427.96·10 ⁶	15.75
roof 4	39.06	29 077.07	7 758.45·10 ⁶	19.80	4 794.18	431.45·10 ⁶	17.98
roof 5	51.02	32 109.06	8 567.46·10 ⁶	15.13	4 872.04	438.45·10 ⁶	19.54
Sum of squares.	135.95	129 924.63	34 666.99·10 ⁶	401.76	21 896.39	1970.54·10 ⁶	88.06
Square roots	11.66	360.45	186190.755	20.044	147.974	44390.81	9.38

6c. Objectives divided by their square roots and MOORA

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	total	rank
roof 1	0.148	0.401	0.401	0.805	0.389	0.389	0.461	-1.141	5
roof 2	0.264	0.417	0.417	0.458	0.436	0.436	0.427	-0.584	4
roof 3	0.496	0.441	0.441	0.236	0.466	0.466	0.423	-0.113	2
roof 4	0.536	0.473	0.473	0.222	0.468	0.468	0.452	-0.148	3
roof 5	0.613	0.497	0.497	0.194	0.472	0.472	0.471	-0.103	1

6d. Reference point theory with ratios: coordinates of the reference point equal to the maximal objective values

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
r_i	0.613	0.401	0.401	0.194	0.472	0.472	0.423

6e. Reference point theory: deviations from the reference point

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	max	Rank min
roof 1	0.464	0.000	0.000	0.611	0.083	0.083	0.038	0.611	5
roof 2	0.349	0.016	0.016	0.264	0.036	0.036	0.005	0.349	4
roof 3	0.117	0.040	0.040	0.042	0.006	0.006	0.000	0.117	3
roof 4	0.077	0.072	0.072	0.028	0.004	0.004	0.029	0.0766	1
roof 5	0.000	0.096	0.096	0.000	0.000	0.000	0.048	0.096	2

Table 6'. The full Multiplicative Form

	A	B	C	D	E	F	G	H	I
	max	min	C=A:B	min	E=C:D	min	G=E:F	max	I=G:H
roof 1	1.73	144.51	0.012	74'646.64	1.6011·10 ⁻⁷	16.13	9.926·10 ⁻⁹	57.56	5.71·10 ⁻⁷
roof 2	3.08	150.29	0.021	77'632.30	2.6372·10 ⁻⁷	9.18	2.872·10 ⁻⁸	64.51	1.85·10 ⁻⁶
roof 3	5.78	158.96	0.036	82'110.79	4.4286·10 ⁻⁷	4.73	9.362·10 ⁻⁸	68.96	6.46·10 ⁻⁶
roof 4	6.25	170.52	0.037	88'082.11	4.1612·10 ⁻⁷	4.45	9.351·10 ⁻⁸	69.24	6.47·10 ⁻⁶
roof 5	7.14	179.19	0.040	92'560.59	4.3066·10 ⁻⁷	3.89	1.107·10 ⁻⁷	69.80	7.73·10 ⁻⁶

J	K	L	M	Result	Project
max	K=I·J	min	M=K:L		
17 267.42	0.99·10 ⁻²	4.32	2.30·10 ⁻³	5	roof 1
19 352.35	3.59·10 ⁻²	4.01	8.90 ·10 ⁻³	4	roof 2
20 687.31	13.36·10 ⁻²	3.97	33.70·10 ⁻³	2	roof 3
20 771.31	13.45·10 ⁻²	4.24	31.70·10 ⁻³	3	roof 4
20 939.30	16.18·10 ⁻²	4.42	36.60·10 ⁻³	1	roof 5

Annex B

Table 7. MOORA applied on 5 ceiling alternatives with 7 conditions

7a. Matrix of responses of alternatives on objectives: (x_{ij})

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
	max	min	min	min	max	max	min
ceiling 1	1.95	57.80	29 856.59	8.13	6.71	2 012.93	14.83
ceiling 2	3.34	65.00	33 575.75	5.49	9.35	2 804.91	11.97
ceiling 3	4.00	75.00	38 741.25	4.73	10.11	3 032.90	12.77
ceiling 4	4.700	90.00	46 489.50	4.08	10.76	3 227.89	14.40
ceiling 5	6.061	104.05	53 747.03	3.39	11.45	3 434.89	15.65

7b. Sum of squares and their square roots

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
ceiling 1	3.82	3 340.84	891.42·10 ⁶	66.097	45.02	4 051898.86	220.00
ceiling 2	11.19	4225.00	1 127.33·10 ⁶	30.140	87.42	7 867500.47	143.29
ceiling 3	16.00	5625.00	1 500.88·10 ⁶	22.373	102.21	9 198475.74	163.17
ceiling 4	22.04	8100.00	2 161.27·10 ⁶	16.646	115.78	10 419289.35	207.43
ceiling 5	36.73	10 826.40	2 888.74·10 ⁶	11.492	131.10	11 798438.40	244.84
Sum of squares.	89.77	32 117.243	8 569.65·10 ⁶	146.748	481.54	43 335602.82	978.73
Square roots	9.475	179.213	92 572.393	12.114	21.94	6 582.98	31.29

7c. Objectives divided by their square roots and MOORA

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	total	rank
ceiling 1	0.206	0.323	0.323	0.671	0.306	0.306	0.474	-0.973	5
ceiling 2	0.353	0.363	0.363	0.453	0.426	0.426	0.383	-0.356	4
ceiling 3	0.422	0.418	0.418	0.390	0.461	0.461	0.408	-0.292	3
ceiling 4	0.496	0.502	0.502	0.337	0.490	0.490	0.460	-0.325	2
ceiling 5	0.640	0.581	0.581	0.280	0.522	0.522	0.500	-0.258	1

7d. Reference point theory with ratios: coordinates of the reference point equal to the maximal objective values

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
r_i	0.640	0.323	0.323	0.280	0.522	0.522	0.383

7e. Reference point theory: deviations from the reference point

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	max	Rank min
ceiling 1	0.434	0.000	0.000	0.391	0.216	0.216	0.091	0.434	5
ceiling 2	0.287	0.040	0.040	0.173	0.096	0.096	0.000	0.287	4
ceiling 3	0.217	0.096	0.096	0.111	0.061	0.061	0.026	0.218	2
ceiling 4	0.144	0.180	0.180	0.057	0.031	0.031	0.078	0.180	1
ceiling 5	0.000	0.258	0.258	0.000	0.000	0.000	0.118	0.258	3

Table 7'. The full Multiplicative Form

	A	B	C	D	E	F	G	H	I
	max	min	C=A:B	min	E=C:D	min	G=E:F	max	I=G:H
ceiling 1	1.95	57.80	0.034	29 856.59	1.132·10 ⁻⁶	8.13	1.392·10 ⁻⁷	6.71	9.34·10 ⁻⁷
ceiling 2	3.34	65.00	0.052	33 575.75	1.533·10 ⁻⁶	5.49	2.791·10 ⁻⁷	9.35	2.61·10 ⁻⁶
ceiling 3	4.00	75.00	0.053	38 741.25	1.377·10 ⁻⁶	4.73	2.911·10 ⁻⁷	10.11	2.94·10 ⁻⁶
ceiling 4	4.70	90.00	0.052	46 489.50	1.122·10 ⁻⁶	4.08	2.750·10 ⁻⁷	10.76	2.96·10 ⁻⁶
ceiling 5	6.06	104.05	0.058	53 747.03	1.084·10 ⁻⁶	3.39	3.197·10 ⁻⁷	11.45	3.66·10 ⁻⁶

J	K	L	M	Result	Project
max	K=I:J	min	M=K:L		
2 012.93	0.19·10 ⁻²	14.83	0.127·10 ⁻³	5	ceiling 1
2 804.91	0.73·10 ⁻²	11.97	0.612·10 ⁻³	4	ceiling 2
3 032.90	0.89·10 ⁻²	12.77	0.699·10 ⁻³	2	ceiling 3
3 227.89	0.96·10 ⁻²	14.40	0.663·10 ⁻³	3	ceiling 4
3 434.89	0.13·10 ⁻²	15.65	0.804·10 ⁻³	1	ceiling 5

Annex C

Table 8. MOORA applied on 5 windows alternatives with 7 conditions

8a. Matrix of responses of alternatives on objectives: (x_{ij})

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
	max	min	min	min	max	max	min
windows 1	0.588	202.31	102 656.14	46.44	69.14	20 741.31	4.95
windows 2	0.625	216.76	55 989.11	55.77	59.81	17 942.40	3.12
windows 3	0.625	216.76	109 988.36	43.71	71.87	21 560.28	5.10
windows 4	0.833	245.66	63 453.98	50.21	65.37	19 610.35	3.24
windows 5	0.833	245.66	124 652.80	32.78	82.80	24 839.17	5.02

8b. Sum of squares and their square roots

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
windows 1	0.35	40 929.34	10 538.28·10 ⁶	2 156.67	4 780.34	430.20·10 ⁶	24.50
windows 2	0.39	46 984.90	3 134.78·10 ⁶	3 110.29	3 577.24	321.93·10 ⁶	9.74
windows 3	0.39	46 984.90	12 097.44·10 ⁶	1 910.56	5 165.30	464.85·10 ⁶	26.02
windows 4	0.69	60 348.84	4 026.41·10 ⁶	2 521.04	4 273.24	384.57·10 ⁶	10.47
windows 5	0.69	60348.84	15 538.32·10 ⁶	1 074.53	6 855.84	616.99·10 ⁶	25.18
Sum of squares.	2.52	255 596.80	45 335.23·10 ⁶	10 773.10	24 651.95	2 218.52·10 ⁶	95.91
Square roots	1.59	505.57	212 920.71	103.79	157.01	47 101.25	9.79

8c. Objectives divided by their square roots and MOORA

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	total	rank
windows 1	0.371	0.400	0.482	0.447	0.440	0.440	0.505	-0.584	5
windows 2	0.394	0.429	0.263	0.537	0.381	0.381	0.319	-0.392	3
windows 3	0.394	0.429	0.517	0.421	0.458	0.458	0.521	-0.578	4
windows 4	0.525	0.486	0.298	0.484	0.416	0.416	0.330	-0.240	1
windows 5	0.525	0.486	0.585	0.316	0.527	0.527	0.512	-0.320	2

8d. Reference point theory with ratios: coordinates of the reference point equal to the maximal objective values

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
r_i	0.525	0.400	0.263	0.316	0.527	0.527	0.319

8e. Reference point theory: deviations from the reference point

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	max	Rank min
windows 1	0.155	0.000	0.219	0.132	0.087	0.087	0.187	0.219	2
windows 2	0.131	0.029	0.000	0.221	0.146	0.146	0.000	0.221	3
windows 3	0.131	0.029	0.254	0.105	0.070	0.070	0.202	0.254	4
windows 4	0.000	0.086	0.035	0.168	0.111	0.111	0.012	0.168	1
windows 5	0.000	0.086	0.322	0.000	0.000	0.000	0.194	0.322	5

Table 8'. The full Multiplicative Form

	A	B	C	D	E	F	G	H	I
	max	min	C=A:B	min	E=C:D	min	G=E:F	max	I=G·H
windows 1	0.588	202.31	0.0029	102 656.14	2.83·10 ⁻⁸	46.44	6.1·10 ⁻¹⁰	69.14	4.217·10 ⁻⁸
windows 2	0.625	216.76	0.0029	55 989.11	5.15·10 ⁻⁸	55.77	9.23·10 ⁻¹⁰	59.81	5.523·10 ⁻⁸
windows 3	0.625	216.76	0.0029	109 988.36	2.62·10 ⁻⁸	43.71	6·10 ⁻¹⁰	71.87	4.31·10 ⁻⁸
windows 4	0.833	245.66	0.0034	63 453.98	5.35·10 ⁻⁸	50.21	1.06·10 ⁻⁹	65.37	6.96·10 ⁻⁸
windows 5	0.833	245.66	0.0034	124 652.80	2.72·10 ⁻⁸	32.78	8.3·10 ⁻¹⁰	82.80	6.87·10 ⁻⁸

J	K	L	M	Result	Project
max	K=I·J	min	M=K:L		
20 741.31	0.09·10 ⁻²	4.95	0.177·10 ⁻³	5	windows 1
17 942.40	0.10·10 ⁻²	3.12	0.318·10 ⁻³	3	windows 2
21 560.28	0.09·10 ⁻²	5.10	0.182·10 ⁻³	4	windows 3
19 610.35	0.14·10 ⁻²	3.24	0.422·10 ⁻³	1	windows 4
24 839.17	0.17·10 ⁻²	5.02	0.340·10 ⁻³	2	windows 5

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