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VIBRATION OF A CRACKED CANTILEVER BEAM UNDER MOVING MASS LOAD

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Abstract. This study is devoted to the investigation of the vibration of a cracked cantilever beam under moving mass load. The present formulation contains inertial, centripetal and Coriolis forces that depend on mass and the velocity of the moving load. The existence of crack induces a local flexibility which is a function of the crack depth, thereby changing its vibration behavior and the eigen-values of the system. The response of the system is obtained in terms of Duhamel integral. The differential equation which involves complicated terms on the right side is solved via an iterative procedure. It has been shown that the centripetal and Coriolis forces make an effect to decrease the deformations on the beam since the deformed beam remains concave during the passage of the moving load. It has also been detected that the previous solutions for the case of moving constant force had several mistakes. The results are exemplified for various values of the variables.

Keywords: beam, centripetal, Coriolis, crack, moving mass, vibration.

1. Introduction

There has been much work on the cracked structures since the crack influence the static and dynamic response of the mechanical system. The occurrence of crack at the beam induces a local flexibility which is a function of the crack depth, thereby changing its dynamic behavior and the eigen-values of the system. Several techniques were proposed to determine the eigen-functions of the cracked structures. While some of these researchers were dealt with the detection of crack (Rizos, Aspragathos 1990; Liang *et al.* 1991; Chondros, Dimarogonas 1980), some others were based on investigating the effects of cracks on the frequencies of the beam (Dimarogonas 1996; Lin, Chang 2006; Shifrin, Ruotolo 1999).

Reis *et al.* (2008) has been investigated the dynamic response of the supported bridges under moving load. Khalfallah (2008) analyzed cracked flexural reinforced concrete structures with special highlighting of modeling the interaction between concrete and reinforcement. Parhi and Behera (1997) used the Runge-Kutta method to find the deflection of a cracked circular shaft subjected to a moving load. Mahmoud and Abou Zaid (2002) used an iterative modal analysis approach to determine the cracked beam's response. Most of these works have analyzed the problem numerically or hybrid numerically. Lin *et al*. (2002) presented an extended method for the beam vibrations with an arbitrary number of cracks for obtaining the modes and frequencies of the system. Lin and Chang (2006) analyzed the forced response of a cracked cantilever beam under a concentrated moving load. In this paper, the crack divides the beam into two parts. Each part obeys Euler–Bernoulli beam theory. Forced response was obtained by the modal expansion theory using the determined eigen-functions. However, as proved in the following paragraphs, the method presented is deficient in many respects and must be reformulated.

The results obtained so far in the literature can be only a rough approximation to determine the dynamic behavior and the resonant response of the system for heavy moving masses. Hence, a more complicated, but sensitive method considering the effects of centripetal forces, Coriolis forces and the inertia of the moving mass loads in a cracked beam is inevitable for predicting the more realistic behavior of the system. Therefore, this study is devoted to investigating these effects on the dynamic behavior. And an analytical approach is presented for investigating the dynamic response of cracked beams under moving mass load. The analysis results in a complicated differential equation whose solution requires using iterative techniques. Lin and Chang (2006) have solved a similar problem for a constant moving force. It should be expected that a mass rather than a force in the analysis makes the formulation much more complicated. The reason for this complexity is the existence of coupled terms on the right hand side of the differential equation in the case of mass loads. The mass rotations and the shear effects of the beam are neglecting in the study. The present analysis is performed for a single-side crack. Forced response is obtained in terms of Duhamel integrals.

2. Eigen-value analysis

Let us consider a cracked cantilever beam which has a length of L (Fig. 1). It contains an open crack located at $x = L_0$. The dimensions of the uniform cross-section of the beam are: width B, thickness H, crack depth D. The crack divides the beam into two parts. According to Euler-Bernoulli beam theory, the equation of motion for each part in the case of free vibration can be written as:

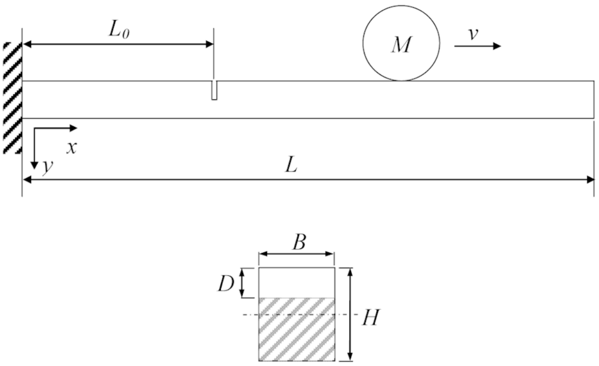


Fig. 1. Cracked cantilever beam under moving mass load

$$EI\frac{\partial^4 y_1}{\partial x^4} + m\frac{\partial^2 y_1}{\partial t^2} = 0, \quad 0 < x < L_0, \tag{1}$$

$$EI\frac{\partial^4 y_2}{\partial x^4} + m\frac{\partial^2 y_2}{\partial t^2} = 0, \quad L_0 \le x \le L,$$

where y_1 and y_2 are the vertical displacements, E is the elastic modulus, I is the moment of inertia and m is the mass per unit length. The boundary conditions for the beam are given by:

$$y_1(0,t) = y_1'(0,t) = 0, \qquad y_2''(L,t) = y_2'''(L,t) = 0\,. \tag{2}$$

The compatibility requirements enforce continuities of the displacement, bending moment and shear force, respectively, across the crack and can be expressed as:

$$\begin{aligned} y_1(L_0^-,t) &= y_2(L_0^+,t), \\ y_1''(L_0^-,t) &= y_2''(L_0^+,t), \\ y_1'''(L_0^-,t) &= y_2'''(L_0^+,t), \end{aligned} \tag{3}$$

here L_0^+, L_0^- denote the locations immediately before and after the crack position, respectively. Discontinuity condition at the crack can be written as:

$$y_2'(L_0^+,t) - y_1'(L_0^-,t) = \theta_1 L y_2''(L_0^+,t), \tag{4}$$

where θ_1 is the non-dimensional crack sectional flexibility, which is a function of the crack extent (Haisty, Springer 1998). For a single sided open crack (Ostachowitz, Krawczuk 1991):

$$\theta_1 = 6\pi\gamma^2 f(\gamma)(H/L), \tag{5}$$

here $\gamma = D/H$ is the non-dimensional crack-depth ratio, and

$$f(\gamma) = 0.6384 - 1.035\gamma + 3.7201\gamma^2 - 5.177\gamma^3 + \ldots\,. \tag{6}$$

Using the separable solution $y_{(i)}(x,t) = \phi_{ni}(x)e^{i\omega_n t}$, $i = 1, 2$ and $n = 1, 2, \ldots$ in Eq. (1) leads to:

$$\phi_{n1}^{IV}(x) - \lambda_n^4\phi_{n1}(x) = 0, \qquad 0 < x < L_0, \tag{7}$$

$$\phi_{n2}^{IV}(x) - \lambda_n^4\phi_{n2}(x) = 0, \qquad L_0 \le x < L, \tag{8}$$

where:

$$\lambda_n^4 = \frac{m\omega_n^2}{EI}. \tag{9}$$

Using Eqs (3) and (4), the conditions on ϕ_{n1} and ϕ_{n2} are readily obtained as:

$$\begin{aligned} \phi_{n1}(L_0^-) &= \phi_{n2}(L_0^+), \\ \phi_{n1}''(L_0^-) &= \phi_{n2}''(L_0^+), \\ \phi_{n1}'''(L_0^-) &= \phi_{n2}'''(L_0^+), \end{aligned} \tag{10}$$

$$\phi_{n2}'(L_0^-) - \phi_{n1}'(L_0^+) = \theta_1 L\phi_{n2}''(L_0^+). \tag{11}$$

On the other hand, using the boundary conditions in Eq. (2), we obtain:

$$\begin{aligned} \phi_{n1}(0) &= 0, \quad \phi_{n1}'(0) = 0, \\ \phi_{n2}''(L) &= 0, \quad \phi_{n2}'''(L) = 0\,. \end{aligned} \tag{12}$$

The solutions of Eq. (7) and Eq. (8) can be shown to be:

$$\begin{aligned} \phi_{n1}(x) = {} & A_{n1}\sin(\lambda_n x) + B_{n1}\cos(\lambda_n x) + \\ & C_{n1}\sinh(\lambda_n x) + D_{n1}\cosh(\lambda_n x), \qquad 0 < x < L_0, \end{aligned} \tag{13}$$

$$\begin{aligned} \phi_{n2}(x) = {} & A_{n2}\sin\lambda_n(x - L_0) + B_{n2}\cos\lambda_n(x - L_0) + \\ & C_{n2}\sinh\lambda_n(x - L_0) + D_{n2}\cosh\lambda_n(x - L_0), \\ & L_0 < x < L, \end{aligned} \tag{14}$$

where A_{ni}, B_{ni}, C_{ni} and D_{ni} are constants to be determined. Using the conditions given by Eqs (10), (11) and (12), and eliminating all the coefficients, the frequency equation is obtained as:

$$\begin{aligned} & u_6\left[-u_2.m_2 + n_1m_1 + u_4m_4 - n_3m_3\right] + \\ & \left[-u_3.m_2 + n_2m_1 + u_5m_4 - n_4m_3\right] = 0\,. \end{aligned} \tag{15}$$

Here the following abbreviations are made:

$$\begin{aligned} n_1 &= \sin\lambda_n L_0, & n_2 &= \cos\lambda_n L_0, \\ n_3 &= \sinh\lambda_n L_0, & n_4 &= \cosh\lambda_n L_0, \\ m_1 &= \sin\lambda_n(L - L_0), & m_2 &= \cos\lambda_n(L - L_0), \\ m_3 &= \sinh\lambda_n(L - L_0), & m_4 &= \cosh\lambda_n(L - L_0), \end{aligned} \tag{16}$$

$$\begin{aligned} u_1 &= (\theta_1 L\lambda_n)/2, \\ u_2 &= n_2 - u_1n_1 - u_1n_3, \\ u_3 &= -n_1 - u_1n_2 - u_1n_4, \\ u_4 &= -n_4 - u_1n_1 - u_1n_3, \\ u_5 &= -n_3 - u_1n_2 - u_1n_4, \\ u_6 &= \frac{u_3m_1 + n_2m_2 - u_5m_3 + n_4m_4}{-u_2m_1 - n_1m_2 + u_4m_3 - n_3m_4}. \end{aligned} \tag{17}$$

After finding the eigen-values of the cracked cantilever beam, these values are written in the Eigen functions of the beam. It is clearly seen that there is only one unknown term (A_{n1}) in the eigen-functions:

$$\phi_{n1}(x) = A_{n1}\left[\sin(\lambda_n x) + u_5\cos(\lambda_n x) - \sinh(\lambda_n x) - u_5\cosh(\lambda_n x)\right], \quad 0 < x < L_1, \tag{18}$$

$$\phi_{n2}(x) = A_{n1}\left[(u_1 + u_2 u_5)\sin\lambda_n(x - L_1) + (n_1 + n_2 u_5)\cos\lambda_n(x - L_1) + (u_3 + u_4 u_5)\sinh\lambda_n (x - L_1) + (-n_3 + n_4 u_5)\cosh\lambda_n(x - L_1)\right], \quad L_1 < x < L. \tag{19}$$

Using the ortonormality condition

$$\left\|\phi_n(x)\right\| = \left(\int_0^L \phi_n^2(x)\,dx\right)^{1/2} = \left(\int_0^{L_1} \phi_{n1}{}^2(x)\,dx + \int_{L_1}^{L} \phi_{n2}{}^2(x)\,dx\right)^{1/2} = 1, \tag{20}$$

A_{n1} can be obtained as:

$$A_{n1} = \sqrt{\frac{1}{\int_0^{L_1}\left[\frac{\phi_{n1}(x)}{A_{n1}}\right]^2 dx + \int_{L_1}^{L}\left[\frac{\phi_{n2}(x)}{A_{n1}}\right]^2 dx}}. \tag{21}$$

3. Dynamic response analysis

The equation of motion of the beam under a moving mass M can be written as (Michaltsos, Kounadis 2001):

$$EI\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^2 y}{\partial t^2} = M\left[g - a_M\right]\delta(x - vt), \tag{22}$$

here $\delta(x - vt)$ is Dirac's delta function. Since the transverse displacement y is a function of x and time t, we obtain the transverse acceleration a_M as:

$$a_M = \ddot{y} + v^2 y'' + 2v\,\dot{y}'. \tag{23}$$

The second and third terms on the right side of Eq. (23) correspond to the centrifugal and Coriolis accelerations. Inserting Eq. (23) into Eq. (22) yields:

$$EI\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^2 y}{\partial t^2} = Mg\delta(x - vt) - M\left[\ddot{y} + v^2 y'' + 2v\,\dot{y}'\right]\delta(x - vt). \tag{24}$$

A series solution of Eq. (24) can be sought in the form:

$$y(x,t) = \sum_{n=1}^{N}\phi_n(x)\,q_n(t), \tag{25}$$

where eigen-functions $\phi_n(x)$ of the cracked system are given by Eqs (13) and (14), $q_n(t)$ are the generalized coordinates and N is the number of eigen-functions used to approximate the solution. Substituting Eq. (25) into Eq. (24), multiplying by $\phi_m(x)$ and integrating from 0 to L lead to:

$$\begin{aligned}\sum_{n=1}^{N}\left[\ddot{q}_n + \omega_n^2 q_n\right]\int_0^L \phi_m(x)\phi_n(x)\,dx = {} & \frac{Mg}{m}\int_0^L \delta(x - vt)\phi_m(x)\,dx - \\ & \frac{M}{m}\left[\sum_{m=1}^{N}\ddot{q}_m\int_0^L \phi_n(x)\phi_m(x)\delta(x - vt)\,dx\right] - \\ & \frac{M}{m}\left[\sum_{m=1}^{N}v^2 q_m\int_0^L \phi_n''(x)\phi_m(x)\delta(x - vt)\,dx\right] - \\ & \frac{M}{m}\left[\sum_{m=1}^{N}2v\dot{q}_m\int_0^L \phi_n'(x)\phi_m(x)\delta(x - vt)\,dx\right].\end{aligned} \tag{26}$$

Using the ortogonality condition of eigen-functions, Eq. (26) can be written as:

$$\ddot{q}_n + \omega_n^2 q_n = \frac{M}{m}\phi_n(vt)\left\{g - \left[\sum_{m=1}^{N}\ddot{q}_m\phi_m(vt)\right] - \left[v^2\sum_{m=1}^{N}q_m\phi_m''(vt)\right] - \left[2v\sum_{m=1}^{N}\dot{q}_m\phi_m'(vt)\right]\right\}. \tag{27}$$

A very important point that must be recalled here is that, although the eigen-functions are orthogonal, they are not orthonormal in general. In other words, they result in:

$$\int_0^L \phi_m^2 dx = \int_0^L \phi_n^2 dx = \int_0^{L_0}\phi_{n1}^2 dx + \int_{L_0}^{L}\phi_{n2}^2 dx = k. \tag{28}$$

Due to the crack, integration is divided into two parts. In order to make this term normalized, the constant A's in the expressions (Eq. (13) and Eq. (14)) for eigen-functions must be chosen as $A = 1/\sqrt{k}$. However, this value must also take place on the right hand side of Eq. (27).

A closed form solution to Eq. (27) is not possible. However, we can seek an approximate solution of Eq. (27). In order to solve Eq. (27), a technique developed by Michaltsos and Kounadis (2001) will be used. This method has also been used by Kounadis (1985). In fact, this method is a different version of Picard's method applied to this differential equation. According to this method, a first approximate solution of the differential equation is obtained by keeping only the first term on the right side of Eq. (27). This leads to:

$$\ddot{q}_n + \omega_n^2 q_n = \begin{cases}\dfrac{Mg}{m}\phi_{n1}(vt), & t \le \dfrac{L_0}{v},\\[2mm] \dfrac{Mg}{m}\phi_{n2}(vt), & t > \dfrac{L_0}{v},\end{cases} \tag{29}$$

where:

$$\phi_{n1}(vt) = A_{n1}\sin\lambda_n(vt) + B_{n1}\cos\lambda_n(vt) + C_{n1}\sinh\lambda_n(vt) + D_{n1}\cosh\lambda_n(vt), \tag{30}$$

$$\phi_{n2}(vt) = A_{n2}\sin\lambda_n(L-vt) + B_{n2}\cos\lambda_n(L-vt) + C_{n2}\sinh\lambda_n(L-vt) + D_{n2}\cosh\lambda_n(L-vt). \tag{31}$$

The solution of the homogenous part of Eq. (27) for $t \le L_0/v$ is:

$$\left(q_{n1}\right)_h = d_1\sin\omega_n t + d_2\cos\omega_n t\ , \tag{32}$$

where d_1, d_2 are constants to be determined. Let us assume that the proper solution to Eq. (27) has the form:

$$\left(q_{n1}\right)_p = \bar{A}_{n1}\sin\Omega_n t + \bar{B}_{n1}\cos\Omega_n t + \bar{C}_{n1}\sinh\Omega_n t + \bar{D}_{n1}\cosh\Omega_n t\ , \tag{33}$$

where $\Omega_n = \lambda_n v$. Substituting Eq. (33) into Eq. (29) yields:

$$\bar{A}_{n1} = \frac{Mg}{m}\frac{A_{n1}}{\left(\omega_n^2-\Omega_n^2\right)}, \qquad \bar{B}_{n1} = \frac{Mg}{m}\frac{B_{n1}}{\left(\omega_n^2-\Omega_n^2\right)},$$
$$\bar{C}_{n1} = \frac{Mg}{m}\frac{C_{n1}}{\left(\omega_n^2+\Omega_n^2\right)}, \qquad \bar{D}_{n1} = \frac{Mg}{m}\frac{D_{n1}}{\left(\omega_n^2+\Omega_n^2\right)}. \tag{34}$$

Thus, the general solution to Eq. (27) for $t < L_0/v$ takes the form:

$$q_{n1}(t) = d_1\sin\omega_n t + d_2\cos\omega_n t + \left(q_{n1}\right)_p . \tag{35}$$

In order to determine d_1 and d_2 in Eq. (35), we use the initial conditions $q_{n1}(0)=0$ and $\dot{q}_{n1}(0)=0$. After some operations, one readily finds:

$$d_1 = -\frac{\Omega_n}{\omega_n}\left(\bar{A}_{n1}+\bar{C}_{n1}\right), \qquad d_2 = -\bar{B}_{n1}-\bar{D}_{n1}. \tag{36}$$

The last form of $q_{n1}(t)$ in the first region can thus be written as:

$$q_{n1}(t) = q_{m1}(t) = (q_{n1})_h + (q_{n1})_p = d_1\sin\omega_n t + d_2\cos\omega_n t + \bar{A}_{n1}\sin\Omega_n t + \bar{B}_{n1}\cos\Omega_n t + \bar{C}_{n1}\sinh\Omega_n t + \bar{D}_{n1}\cosh\Omega_n t. \tag{37}$$

Now, we insert Eq. (37) into Eq. (27) for the first part $t \le L_0/v$:

$$\ddot{q}_{n1} + \omega_n^2 q_{n1} = \frac{2M}{mL}\ \phi_{n1}(vt)\left\{g - \left[\sum_{m=1}^{N}\ddot{q}_{m1}\phi_{m1}\right] - v^2\left[\sum_{m=1}^{N}q_{m1}\phi''_{m1}\right] - 2v\left[\sum_{m=1}^{N}\dot{q}_{m1}\phi'_{m1}\right]\right\} = Q_{n1}, \tag{38}$$

here q_{m1}, $\dot{q}_{m1}$, $\ddot{q}_{m1}$ are to be found from Eq. (37). Substituting Eq. (37) into right hand side of Eq. (38), we have:

$$\ddot{q}_n + \omega_n^2 q_n = Q_{n1}, \qquad t \le L_0/v . \tag{39}$$

Under zero initial conditions, the solution of Eq. (39) has the form:

$$q_n(t) = \frac{1}{\omega_n}\int_0^t Q_{n1}(\tau)\sin\omega_n(t-\tau)\,d\tau, \qquad t \le L_0/v . \tag{40}$$

In the same manner, for $t > L_0/v$ the solution of homogeneous part and the proper solution can be written as:

$$\left(q_{n2}\right)_h = \bar{d}_1\sin\omega_n t + \bar{d}_2\cos\omega_n t\ , \tag{41}$$

$$\left(q_{n2}\right)_p = \bar{A}_{n2}\sin\Omega_n t + \bar{B}_{n2}\cos\Omega_n t + \bar{C}_{n2}\sinh\Omega_n t + \bar{D}_{n2}\cosh\Omega_n t. \tag{42}$$

One can readily show that the coefficients in Eq. (42) are in the forms:

$$\bar{A}_{n2} = \frac{Mg}{m}\frac{A_{n2}}{\left(\omega_n^2-\Omega_n^2\right)}, \quad \bar{B}_{n2} = \frac{Mg}{m}\frac{B_{n2}}{\left(\omega_n^2-\Omega_n^2\right)},$$
$$\bar{C}_{n2} = \frac{Mg}{m}\frac{C_{n2}}{\left(\omega_n^2+\Omega_n^2\right)}, \quad \bar{D}_{n2} = \frac{Mg}{m}\frac{D_{n2}}{\left(\omega_n^2+\Omega_n^2\right)}. \tag{43}$$

Hence, the general solution of $q_{n2}(t)$ in the second region can be written as:

$$q_{n2}(t)\ \Big|_{t>\frac{L_0}{v}} = \bar{d}_1\sin\omega_n t + \bar{d}_2\cos\omega_n t + (q_{n2})_p . \tag{44}$$

In order to determine $\bar{d}_1$ and $\bar{d}_2$ in Eq. (44), we use the initial conditions:

$$q_{n2}\left(\frac{L_0}{v}\right) = q_{n1}\left(\frac{L_0}{v}\right) = H_1 = \frac{1}{\omega_n}\int_0^{L_0/v} Q_{n1}(\tau)\sin\omega_n\left(\frac{L_0}{v}-\tau\right)d\tau \tag{45}$$

and

$$\dot{q}_{n2}\left(\frac{L_0}{v}\right) = \dot{q}_{n1}\left(\frac{L_0}{v}\right) = H_2 = \int_0^{L_0/v} Q_{n1}(\tau)\cos\omega_n\left(\frac{L_0}{v}-\tau\right)d\tau . \tag{46}$$

After some operations, one readily finds:

$$\bar{d}_1 = \sin\frac{\omega_n L_0}{v}\left(H_1 - \bar{B}_{n2} - \bar{D}_{n2}\right) + \frac{\cos\frac{\omega_n L_0}{v}}{\omega_n}\left[H_2 - \Omega_n\left(\bar{A}_{n2}+\bar{C}_{n2}\right)\right], \tag{47}$$

$$\bar{d}_2 = \cos\frac{\omega_n L_0}{v}\left(H_1 - \bar{B}_{n2} - \bar{D}_{n2}\right) - \frac{\sin\frac{\omega_n L_0}{v}}{\omega_n}\left[H_2 - \Omega_n\left(\bar{A}_{n2} + \bar{C}_{n2}\right)\right]. \tag{48}$$

The last form of $q_{n2}(t)$ in the second region can thus be written as:

$$q_{n2}(t) = q_{m2}(t) = (q_{n2})_h + (q_{n2})_p = \bar{d}_1 \sin\omega_n t + \bar{d}_2 \cos\omega_n t + \bar{A}_{n2}\sin\Omega_n t + \bar{B}_{n2}\cos\Omega_n t + \bar{C}_{n2}\sinh\Omega_n t + \bar{D}_{n2}\cosh\Omega_n t. \tag{49}$$

For the second part $t > L_0 / v$, we have

$$\ddot{q}_{n2} + \omega_n^2 q_{n2} = \frac{2M}{mL}\,\phi_{n2}(vt)\left\{g - \left[\sum_{m=1}^{N}\ddot{q}_{m2}\phi_{m2}\right] - v^2\left[\sum_{m=1}^{N} q_{m2}\phi''_{m2}\right] - 2v\left[\sum_{m=1}^{N}\dot{q}_{m2}\phi'_{m2}\right]\right\} = Q_{n2}. \tag{50}$$

here, q_{m2}, $\dot{q}_{m2}$, $\ddot{q}_{m2}$ are to be found from Eq. (49). Writing Eq. (49) in Eq. (50) together, we have:

$$\ddot{q}_n + \omega_n^2 q_n = Q_{n2}, \qquad t > L_0 / v. \tag{51}$$

The solution of Eq. (51) has the form:

$$q_n(t) = \frac{1}{\omega_n}\int_0^{L_0/v} Q_{n1}(\tau)\sin\omega_n(t-\tau)\,d\tau + \frac{1}{\omega_n}\int_{L_0/v}^{t} Q_{n2}(\tau)\sin\omega_n(t-\tau)\,d\tau, \qquad t > \frac{L_0}{v}. \tag{52}$$

A very important point here is that if the time at which we wish to plot the curve is smaller than the time required for the load to arrive at the crack, Eq. (40) is the answer of Eq. (27). When the load passes across the crack, then Eq. (52) is the answer of Eq. (27).

4. Results and discussion

The maximum deflection that the beam would have is the deflection of the free end of the beam. Therefore, dynamic deflection of the free end is divided by these static deflections of the free end in the graphics below. This static deflection (depending upon the position of the mass) is given by $y(L) = \frac{Mg(x)^2}{6EI}\left[(x) - 3L\right]$, where x is the location of the mass. In the following calculations, as an example, we assume $L = 8$ m, $H = 0.2$ m, $B = 0.1$ m, $A = B \times H$ $E = 2.06 \times 10^{11}\,\text{N/m}^2$, $\rho = 7800\ \text{kg/m}^3$, $m = \rho \times A$. The number of terms in the series has been taken as $n = 4$. It has been observed that the series rapidly converges (Lin, Chang 2006). Since the right hand side of the differential equation involves the unknown functions, as the number of terms in the series is increased, the solution time highly increases.

In the first place, the effects of the crack depth and its location on the dynamic deflections must be determined. To do this, in Fig. 2, the normalized deflection at the free end $\left(\bar{y} = y_{\text{dynamic}} / y_{\text{static}}\right)$ versus normalized position of the moving load $\left(\bar{t} = vt / L\right)$ has been plotted for various values of crack-depth ratios $\left(\gamma = 0.25 - 0.5 - 0.75\right)$. The crack is located at the midpoint of the beam in this case ($L_0 = 4$ m). As expected, the normalized deflections at the free end increase as the crack-depth ratio's (γ) increase. In Fig. 3, the normalized deflections at the free end versus normalized position of the moving load has been plotted for different values of the crack position L_0 = (2 m, 4 m, 6 m) and $\gamma = 0.5$. As seen in the figure, as the crack location goes away from the fixed end, the normalized deflection at the free end decreases and approaches to that of the un-cracked case.

One of the main purposes of the present work is to see the effects of inertia, centripetal and Coriolis forces on the dynamic response. The velocity and the mass of the moving load are two parameters which affect these force terms directly. Therefore, in Figs. 4 and Figs. 5 the normalized deflections at the free end versus normalized position of the moving load have been plotted for various values of the velocity and the mass of the moving load. The crack is assumed to be at the middle of the beam $\left(L_0 = 4\ \text{m},\ \gamma = 0.5\right)$ in Figs. 4 and 5.

In Fig. 4, the effect of the constant force corresponding to $F = Mg$, the mass load involving inertia force and the mass load with inertia force plus centripetal and Coriolis forces have been presented in separate curves in each figure. Dotted lines correspond to the case of constant force; dashed lines correspond to the case of mass load with inertia effect while solid lines correspond to the case of the case of mass load involving the totality of the effects.

In Fig. 5, the effect of the mass has been presented in separate curves in each figure for various values of the velocity of the moving load. Dotted lines correspond to the case of M = 100 kg; dashed lines correspond to the case of M = 500 kg while solid lines correspond to the case of the M = 1000 kg.

When the graphics are observed, it can be said that the velocity of the moving load has a great effect on the dynamic response of the beam. At low velocities, it is seen that the effects of inertia, centripetal and Coriolis forces remain so small that they can be neglected compared to the constant force ($F = Mg$) (Fig. 4, a). As the velocity increases, these effects become comparable with the constant force (Fig. 4, b). Neglecting these terms cause a considerable difference at the dynamic response of the system. In addition to this, in a vast interval of velocity, it is seen that the effect of centripetal and Coriolis forces on the normalized deflection of the free end is bigger than the effects of inertial forces (Fig. 4, c).

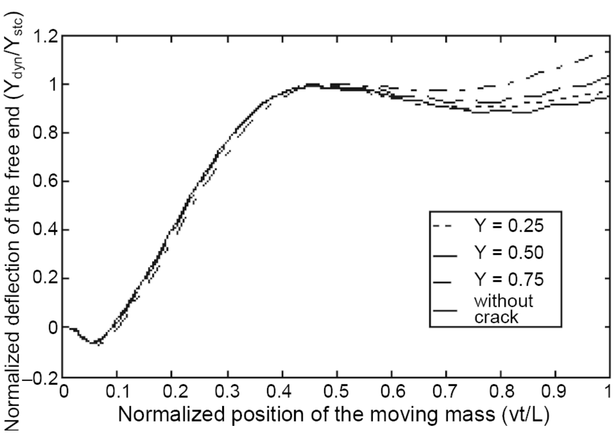


Fig. 2. Variation of the normalized deflection at the free end with respect to normalized position of moving mass for various values of crack depth ratio (γ). $L_0 = 4$ m, $v = 10$ m/s, $M = 1000$ kg

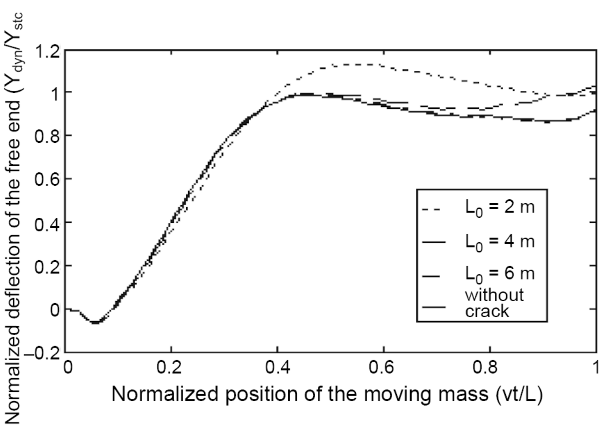


Fig. 3. Variation of the normalized deflection at the free end with respect to normalized position of moving mass for various values of crack position (L_0). $\gamma = 0.5$, $v = 10$ m/s, $M = 1000$ kg

The mass of the moving load has no important effect of the dynamic response of the beam at low velocities of the moving load (Fig. 5, a). As the velocity increases, the effect of the mass starts to becomes apparent (Fig. 5, b, c). It is clearly seen that the normalized deflections at the free end decrease with increasing values of the mass at high velocities of the moving load (Fig. 5, c). Since deformed cantilever beam is concave, the effects of centripetal and Coriolis forces are negative and these forces are functions of the moving mass. That is the reason for the decrease in the deflection at the free end when the mass increases.

It is seen in all figures that the velocity has a considerable effect on the dynamic response of the beam. In Fig. 6, the normalized deflections at the free end versus normalized position of the moving load have been plotted for various values of the velocity of the moving load. The crack is assumed to be at the middle of the beam ($L_0 = 4$ m, $\gamma = 0.5$) in Fig. 6. It is evident that when the velocity decreases the ratio between the dynamic and the static response of the beam approaches to 1. We have called this ratio as 'normalized deflection'. When the velocity increases, the normalized deflection at the free end decreases. The reason for this behavior is that the

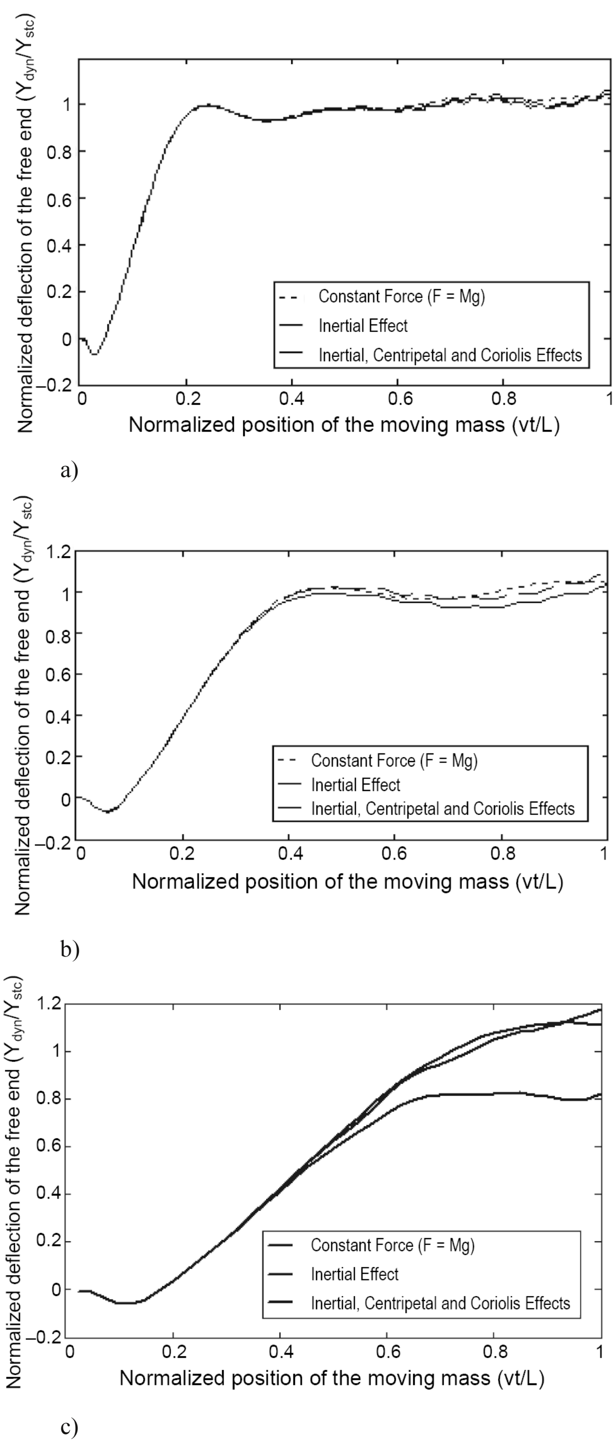
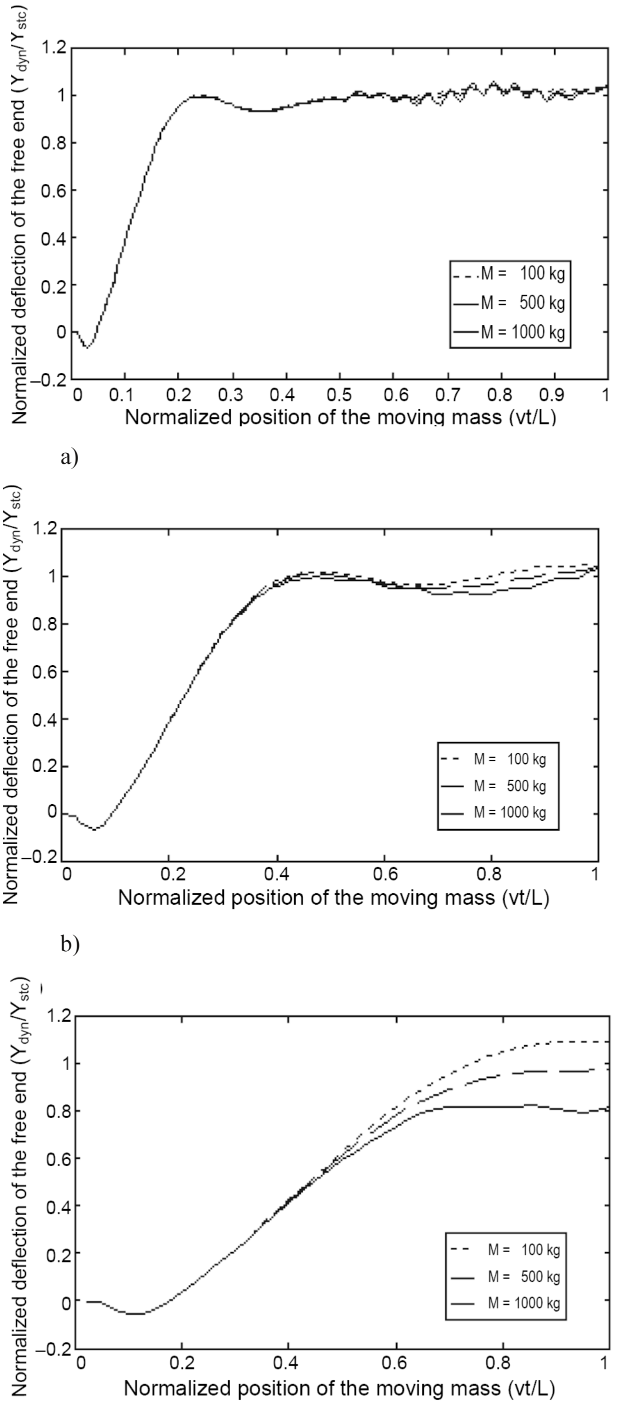


Fig. 4. Variation of the normalized deflection at the free end with respect to normalized position of moving mass for different values of the velocity of the moving load. $L_0 = 4$ m, $\gamma = 0.5$, $M = 1000$ kg: a) $v = 5$ m/s; b) $v = 10$ m/s; c) $v = 20$ m/s

centripetal and Coriolis effects are upwards in a concave beam. In addition, at high velocities, the mass leaves the beam before the beam can not undergo remarkable deflections.

While formulating the present study, we have also corrected several mistakes in the literature. For example, we have pointed out that the eigen-functions are orthogonal, but not orthonormal, in general. We have proposed a normalization procedure to normalize them. The present problem has been fully described and solved by giving all the details. Since some of the details were not given in some of the previous papers, it has been detected that the previous solution strategy has deficiency in many ways. It has been pointed out that the coefficients of the eigen-functions cannot be found explicitly, but they must be stated in terms of A's, as in Eq. (34). The coefficients A's are obtained by mean of fact that eigen-functions must also become orthonormal.



a)

b)

c)

Fig. 5. Variation of the normalized deflection at the free end with respect to normalized position of moving mass for different values of the moving mass. $L_0 = 4$ m, $\gamma = 0.5$: a) $v = 5$ m/s; b) $v = 10$ m/s; c) $v = 20$ m/s

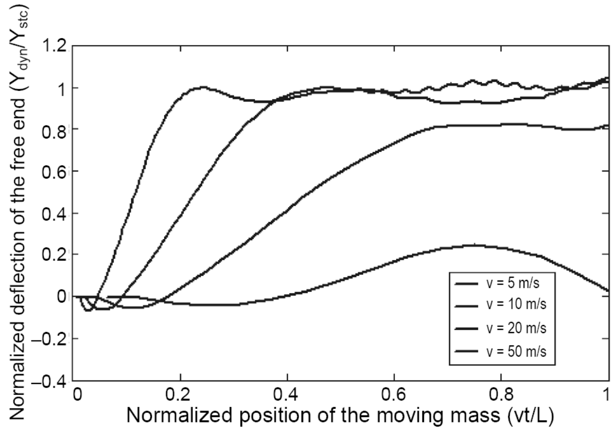


Fig. 6. Variation of the normalized deflections at the free end with respect to normalized position of moving mass for different values of the velocity of the moving mass. $L_0 = 4$ m, $\gamma = 0.5$, $M = 1000$ m/s

5. Conclusions

Vibration analysis of a cracked cantilever beam under moving mass loads has been investigated in the present paper. Euler-Bernoulli beam theory has been used. Centripetal and Coriolis forces which become important in the case of long beams and mass load with high velocity have been inserted into the theory. The insertion of these terms into the theory makes the differential equation quite complicated. This complicated differential equation has been solved by iterative approach. The response of the system has been obtained in the form of Duhamel integral, divided into two parts depending upon the position of the crack.

In the present work, some erroneous results given in some previous papers have been corrected and reformulated. For example, eigen-functions are orthogonal, but not orthonormal in general. But, they have been assumed to be orthonormal, a result which is wrong. The present study involves the correct solution of this problem and adds novelty.

In order to see the effects of centripetal force, Coriolis force and the crack existence, the results have been exemplified for various values of the parameters. It has been concluded that the response in the case of mass load appreciably differs from that of constant force F. In addition, it has been observed that the response of the system is appreciably affected by the inertial, centripetal and Coriolis forces. The velocity of the moving load affect these terms strongly.

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ĮTRŪKUSIOS GEMBINĖS SIJOS VIRPESIAI VEIKIANT JUDANČIA MASĖS APKROVA

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Santrauka

Straipsnyje nagrinėjami įtrūkusios gembinės sijos, kurią veikia judančios masės apkrova, virpesiai. Šiuo atveju veikia inercinės, įcentrinės ir Koriolio jėgos, kurios priklauso nuo judančios apkrovos masės ir judėjimo greičio. Įtrūkis sukelia vietinį sijos lankstumą, kuris priklauso nuo įtrūkio gylio, tokiu būdu keičia virpesių pobūdį ir sistemos reikšmes. Sistemos atsakas nustatomas taikant Duhamelo integralą. Diferencialinė lygtis, kurios dešinėje pusėje yra sudėtingos sąlygos, sprendžiama taikant iteracijas. Nustatyta, kad įcentrinė ir Koriolio jėgos sumažina sijos deformacijas, nes deformuota sija išlieka išsigaubusi, kai per ją juda masės apkrova. Taip pat nustatyta, kad ankstesniuose sprendimuose judančios nekintamos masės apkrovų atvejais buvo keletas klaidų. Kaip pavyzdžiai pateikiami rezultatai esant skirtingoms kintamųjų reikšmėms.

Reikšminiai žodžiai: sija, įcentrinė jėga, Koriolio jėga, įtrūkis, judanti masė, virpesiai.

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