2014 Volume 20(1): 59–69

doi:10.3846/13923730.2013.801906

A COMPARISON OF FUZZY MULTICRITERIA DECISION MAKING METHODS FOR INTELLIGENT BUILDING ASSESSMENT

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Received 20 Jan 2012; accepted 27 Mar 2012

Abstract. The methodology, Multicriteria Decision Making (MCDM), refers to finding the best alternative from all of the feasible alternatives in the presence of multiple, usually conflicting, decision criteria. Nowadays, intelligent buildings' performance that is increasingly evidenced in building design and construction has been analyzed by using MCDM techniques. Intelligent buildings (IBs) are also under assessment according to their IB related characteristics and actual circumstances as a MCDM problem. In this paper, two most known MCDM methodologies, Analytic Hierarchy Process (AHP) and TOPSIS (Technique for Order Preference by Similarity to Ideal Solution), are used for intelligent building assessment under fuzzy environment for dealing with the evaluations' uncertainty and imprecision in which the expert's comparisons that are represented as fuzzy numbers. For this aim, three intelligent building alternatives for a business centre in Istanbul are evaluated by using these two fuzzy MCDM methods and the obtained ranking results are compared.

Keywords: the fuzzy set theory, decision making, intelligent building, assessment, TOPSIS, AHP.

Reference to this paper should be made as follows: Kaya, İ.; Kahraman, C. 2014. A comparison of fuzzy multicriteria decision making methods for intelligent building assessment, *Journal of Civil Engineering and Management* 20(1): 59–69. http://dx.doi.org/10.3846/13923730.2013.801906

Introduction

For many years, buildings that offer comfortable, flexible and energy efficient living environment at a minimal cost has been the expectation of building owners and occupiers. To attain this aspiration, a variety of advanced building technologies have been developed in the past two decades, aiming to improve the building performance to satisfy a variety of human needs and environmental sustainability. While a plethora of advanced building products have been accessible, it has become increasingly evident that developers are confronted with the quandary of choosing components or products to suit the needs and to accomplish the unique configuration of a particular intelligent building (IB) project (Wong, Li 2008).

There has been a myriad of academic and technical literature discussing the definition of intelligent buildings. Early intelligent building definitions focused almost entirely on technology aspect and did not suggest user interaction at all. An intelligent building can be defined as 'one which has fully automated building service control systems'. The Intelligent Building Institution in Washington defined an intelligent building as 'one which integrates various systems to effectively manage resources in

a coordinated mode to maximize: technical performance, investment and operating cost savings, flexibility'. Most recently, a number of authors have extended the definition of an intelligent building and have added 'learning ability' and 'performance adjustment from its occupancy and the environment' in the definition. They proposed that an intelligent building is not only able to react and change accordingly to individual, organizational and environmental requirement, but is also capable of learning and adjusting performance from its occupancy and the environment (Wong *et al.* 2005).

An intelligent building is defined as "a building that integrates technology and process to create a facility that is safer, more comfortable and productive for its occupants, and more operationally efficient for its owners. Advanced technology combined with improved processes for design, construction and operations provide a superior indoor environment that improves occupant comfort and productivity while reducing energy consumption and operations staffing" (Intelligent Building Dictionary 2012).

An intelligent building is a single building or a complex of buildings that offers a coherent set of facilities for

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the building managers and the occupants. For the building managers, it provides an integrated set of management, control maintenance, and inter-building communications facilities that allow efficient and cost-effective environmental control, security surveillance, alarm monitoring and communications, both inside the building and out to municipal authorities (police, fire stations, and hospitals). For the building occupants (tenants) in the workplace, it provides an environment ergonomically designed to increase productivity and encourage creativity and in residences and hotels, environments that will foster comfort and a "humanizing" atmosphere as well as provide sophisticated computer and telecommunications services. An essential part of both these cases is an integrated set of computer and communications facilities that respond cost-effectively to office automation and internal and external communications needs and a holistic, ergonomic approach to the design of the building and the work, living, and recreation spaces. The design extends from a macro level encompassing the building and its internal and external spaces to a micro level involving furniture, workplace, and residential equipment, local atmospheric control, and lighting (Finley et al. 1991). IBs should be sustainable, healthy, and technologically aware, meeting the needs of the occupants and business, and should be flexible and adaptable to deal with change (ALwaer et al. 2010; Kahraman, Kaya 2012).

In this regard, building assessment is becoming popular in order to have a standard method to evaluate new and existing building design. For example, the U.S. Green Building Council developed the Leadership in Energy and Environmental Design Green Building Rating System as a voluntary, consensus based national standard for developing high-performance, sustainable buildings. The Japan Sustainable Building Consortium developed the comprehensive assessment system for building environmental efficiency system as a new environmental assessment system to meet both the political requirements and market needs for achieving a sustainable society. The Building Research Establishment Ltd. (BRE) from UK developed the Building Research Establishment Environmental Assessment Method to assess the environmental performance of both new and existing buildings. Meanwhile, intelligent buildings (IBs) are also under assessment according to their IB related characteristics and actual circumstances. For example, the Asian Institute of Intelligent Buildings developed an IB Index system to specifically assess the performance of IBs; and the BRE developed a matrix tool called MATOOL for assessing the performance of IBs (Chen et al. 2006; Kahraman, Kaya 2012).

In recent years, some papers have concentrated on the assessment of IBs. Kahraman and Kaya (2012) proposed a fuzzy multiple attribute utility (MAUT) model for an intelligent building assessment and three alternative intelligent buildings for a business centre in Istanbul were evaluated. ALwaer and Clements-Croome (2010) used a consensus-based analytical hierarchical process (AHP) model for multi-criteria decision-making to identify key issues related to sustainable intelligent buildings. They developed a conceptual model for the selection of the appropriate key performance indicators (KPIs). Wong and Li (2008) proposed a MCDM model using the analytic hierarchy process (AHP) approach to evaluate the selection of IB systems. Kolokotsa et al. (2007) proposed a methodology for the buildings' intelligence assessment through the development of a matrix tool. Chen et al. (2006) developed an analytic network process (ANP) based MCDM model which was called IBAssessor for lifespan energy efficiency assessment of IBs. Hong et al. (2006) presented a Knowledge-oriented Information Visualization (KIV) approach to facilitate the implementation of building rating systems for the postassessment of IBs. Asian Institute of Intelligent Buildings constructed a quantitative assessment method, called the intelligent building index (IBI) that originated from the nine 'Quality Environment Modules' (M1–M9), each index possesses a score which is a real number (within the range of 1-100). A building can be ranked from A to E to indicate the overall intelligent performance (So, Wong 2002). Preiser and Schramm (2002) developed the post-occupancy evaluation process model that consists of three stages in order to determine the intelligence level of intelligent buildings. Arkin and Paciuk (1997) developed a "Magnitude of Systems' Integration" Index (MSIR) to examine the level of systems' integration of intelligent buildings. This assessment methodology was used for evaluation and comparison of single aspect of building's intelligence. Yang and Peng (2001) adapted the MSIR model for using in the IB performance evaluation.

Differently from the above papers, this paper proposes two multicriteria decision-making model that are the fuzzy AHP and fuzzy TOPSIS to evaluate IB assessment under fuzzy environment. For this aim three building alternatives from Istanbul, Turkey are evaluated. The fuzzy set theory that first introduced by Zadeh (1965) in order to deal with vagueness of human thought is used to represent the linguistic evaluations of decision makers (DMs).

The rest of this paper is organized as follows: fuzzy TOPSIS and fuzzy AHP methodologies that are used in this paper are briefly introduced in Sections 1 and 2, respectively. An application for intelligent building assessment in Istanbul is detailed in Section 3. The obtained results are compared and future research directions are indicated in final section.

1. Fuzzy TOPSIS

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is one of the useful MCDM techniques to rank different alternatives through numerical evaluations the decision maker performs with respect to certain criteria. Weights can also be specified for each criterion, in order to introduce a measure of the relative

importance felt by the decision maker (Gamberini et al. 2006; Kahraman et al. 2009). The method is based on the consideration that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. TOPSIS defines an index called similarity to the positive ideal solution and remoteness from the negative ideal solution. Then the method chooses an alternative with the maximum similarity to the ideal solution (Yoon, Hwang 1995). In this study, firstly, fuzzy TOPSIS approach is used to specify the ranking of alternatives according to aggregated decision matrix and weight vector as well as the individual decision matrices and weigh vectors.

The Fuzzy TOPSIS (FTOPSIS) method is first presented in Chen and Hwang (1992), with reference to Hwang and Yoon (1981). The basic principle of the fuzzy TOPSIS is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative-ideal solution in a geometrical (i.e. Euclidean) sense (Hwang, Yoon 1981). The steps of this algorithm can be summarized as follows (Chen 2000; Aydın *et al.* 2012; Baysal *et al.* 2013):

- **Step 1:** form a group of decision-makers and then identify the evaluation criteria.
- **Step 2:** choose the appropriate linguistic variables for the importance weight of the criteria and the linguistic ratings for alternatives with respect to criteria. For this aim, Tables 1 and 2 can be used.
- **Step 3:** pool the decision makers' opinions to get the aggregated fuzzy rating \tilde{x}_{ij} of alternative A_j under criterion C_j and aggregate the weights of criteria to get the aggregated fuzzy weight \tilde{w}_j of criterion C_j by using Eqns (1) and (2), respectively:

$$\tilde{x}_{ij} = \frac{1}{K} \left[\tilde{x}_{ij}^1(+) \tilde{x}_{ij}^2(+) \cdots (+) \tilde{x}_{ij}^K \right]; \tag{1}$$

Table 1. Linguistic variables for the importance weight of each criterion

Very low (VL)	(0,0,1)
Low (L)	(0, 0.1, 0.3)
Medium low (ML)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
Medium high (MH)	(0.5, 0.7, 0.9)
High (H)	(0.7, 0.9, 1)
Very high (VH)	(0.9, 1, 1)

Table 2. Linguistic variables for the ratings

Very poor (VP)	(0,0,1)
Poor (P)	(0, 1, 3)
Medium poor (MP)	(1, 3, 5)
Fair (F)	(3, 5, 7)
Medium good (MG)	(5, 7, 9)
Good (G)	(7, 9, 10)
Very good (VG)	(9, 10, 10)

$$\tilde{w}_j = \frac{1}{K} \left[\tilde{w}_j^1(+) \tilde{w}_j^2(+) \cdots (+) \tilde{w}_j^K \right], \tag{2}$$

where K is the number of decision makers, \tilde{x}_{ij}^{K} and \tilde{w}_{j}^{K} are the rating and the importance weight of the K^{th} decision maker.

Step 4: construct the fuzzy decision matrix and the normalized fuzzy decision matrix as in Eqns (3) and (4):

$$\tilde{R} = \left[\tilde{r}_{ij}\right]_{m^*n};$$

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}\right), j \in B;$$

$$\tilde{r}_{ij} = \left(\frac{a_{-j}^-}{c_{ij}}, \frac{a_{-j}^-}{b_{ij}}, \frac{a_{-j}^-}{a_{ij}}\right), j \in C;$$

$$c_j^* = \max_i c_{ij} \quad \text{if } j \in B;$$

$$a_j^- = \min_i a_{ij} \quad \text{if } j \in C,$$
(3)

where *B* and *C* are the set of benefit criteria and cost criteria, respectively.

Step 5: construct the weighted normalized fuzzy decision matrix by using Eqns (5) and (6):

$$\tilde{V} = \left[\tilde{v}_{ij}\right]_{m \times n};\tag{5}$$

$$\tilde{v}_{ij} = \tilde{r}_{ij} \otimes \tilde{w}_{j}. \tag{6}$$

- **Step 6:** determine fuzzy positive-ideal solution (FPIS) and fuzzy negative-ideal solution (FNIS).
- **Step 7:** calculate the distances of each alternative from the FPIS (A^*) and the FNIS (A^-) as in Eqn (7), respectively:

$$A^* = (\tilde{v}_1^*, \tilde{v}_2^*, ..., \tilde{v}_n^*),$$

$$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, ..., \tilde{v}_n^-).$$
(7)

Step 8: calculate the closeness coefficient of each alternative as in Eqn (8):

$$d_{i}^{*} = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_{ij}^{*}), \quad i = 1, 2, ..., m,$$

$$d_{i}^{-} = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_{ij}^{-}), \quad i = 1, 2, ..., m,$$
(8)

where (d_i^*, d_i^-) is the distance measurement between two fuzzy numbers.

Then a closeness coefficient is defined to determine the ranking order of all alternatives as in Eqn (9):

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-}, \ i = 1, 2, ..., m.$$
 (9)

Step 9: according to the closeness coefficient, the ranking order of all alternatives can be determined.

2. Fuzzy AHP

AHP is developed to solve complex MCDM problems involving multiple qualitative and quantitative criteria. It allows decision-makers to specify their preferences us-

ing the Saaty's 1–9 scale (Saaty 1980). This scale can be very useful in helping a group of experts or an individual to make a decision. The purpose of AHP is to provide weights for each criterion and alternatives. AHP requires three steps: (i) identifying evaluation criteria, (ii) assessing the decision-maker evaluations by pairwise comparisons, and (iii) calculating the weights for criteria and alternatives. In AHP, logical consistency is also considered by evaluating the validity of the pairwise comparison process obtained from decision-makers' preferences. A number of fuzzy AHP methods or their applications have been published in recent years.

In this paper, Buckley's (1985) fuzzy AHP method is used. The steps of this method are given in the following (Hsieh *et al.* 2004; Kahraman *et al.* 2013; Baysal *et al.* 2013):

Step 1: Pairwise comparison matrices are constructed. Each element (\tilde{c}_{ij}) of the pairwise comparison matrix (C) is a linguistic term representing the importance of one criterion over the other. The pairwise comparison matrix is given by:

$$\tilde{C}_{k} = \begin{vmatrix}
1 & \tilde{c}_{12} & \dots & \tilde{c}_{1n} \\
\tilde{c}_{21} & 1 & \dots & \tilde{c}_{2n} \\
\vdots & \vdots & \dots & \vdots \\
\tilde{c}_{n1} & \tilde{c}_{n2} & \dots & 1
\end{vmatrix}, k = 1, 2, 3, \dots, K, (10)$$

where \tilde{C}_k is a pairwise comparison matrix that belongs to the k^{th} expert. For the evaluation procedure, the linguistic terms given in Table 3 are used. By the way, geometric mean is used to aggregate expert opinions.

Table 3. Linguistic scale for weight matrix (Hsieh *et al.* 2004)

Linguistic scales	Scale of fuzzy number	
(1,1,3)	Equally important	(Eq)
(1,3,5)	Weakly important	(Wk)
(3,5,7)	Essentially important	(Es)
(5,7,9)	Very strongly important	(Vs)
(7,9,9)	Absolutely important	(Ab)

Step 2: Weights are calculated. At first, the fuzzy weight matrix is calculated as follows:

$$\tilde{r}_i = (\tilde{c}_{i1} \otimes \tilde{c}_{i2} \otimes ... \otimes \tilde{c}_{in})^{1/n}; \qquad (11)$$

$$\tilde{w}_i = \tilde{r}_i \otimes (\tilde{r}_1 + \tilde{r}_2 + \dots + \tilde{r}_n)^{-1}, \qquad (12)$$

where \tilde{r}_i is the geometric mean of fuzzy comparison value and \tilde{w}_i indicated by triangular fuzzy numbers \tilde{w}_i (L_i, M_i, U_i) is fuzzy weight of i^{th} criterion.

Step 3: After the fuzzy relative weight matrix is obtained, defuzzification process which converts a fuzzy number into a crisp value is utilized. In this paper, the total integral method is used for this aim. Liou and Wang (1992) proposed the total integral value method with an index of op-

timism $\omega \in [0,1]$. Let \tilde{A} be a fuzzy number with left membership function $f_{\tilde{A}}^L$ and right membership function $f_{\tilde{A}}^R$. Then the total integral value is defined as (Kahraman *et al.* 2004):

$$E_{\omega}(\tilde{A}) = \omega E_R(\tilde{A}) + (1 - \omega) E_L(\tilde{A}), \tag{13}$$

where:

$$E_{R}\left(\tilde{A}\right) = \int_{\tilde{A}}^{\beta} x f_{\tilde{A}}^{R}\left(x\right) dx,\tag{14}$$

and

$$E_{L}\left(\tilde{A}\right) = \int_{\gamma}^{\delta} x f_{\tilde{A}}^{L}\left(x\right) dx,\tag{15}$$

where: $-\infty < \alpha \le \beta \le \gamma \le \delta < \infty$. For a triangular fuzzy number, $\tilde{A} = (a, b, c)$, the total integral value is obtained by:

$$E_{\omega}\left(\tilde{A}\right) = \frac{1}{2} \left[\omega(a+b) + (1-\omega)(b+c)\right]. \tag{16}$$

3. Application

Istanbul is simultaneously located on the two continents and it is known as a link between the east and the west. Istanbul has always been the centre of trade and commerce due to its strategic location. The rapid development of the service sector, demand for high-rise office buildings in Istanbul, has led to the completion of many high rises in the recent years. Most of these high rises are IBs and all the future high rises are planned to be IBs thereafter. Since the assessment of an IB requires the consideration of many criteria, two fuzzy MCDM models for intelligent building assessment are proposed for a business centre project in this paper. The criteria which are determined from a literature survey (ALwaer, Clements-Croome 2010; Wong, Li 2006, 2008; Chen et al. 2006; Kahraman, Kaya 2012) and decision makers' evaluations are used for intelligent building assessment. These criteria are shown in Table 4.

Three intelligent building alternatives for a business centre project are evaluated with respect to these criteria and the hierarchical structure which is shown in Figure 1. The building alternatives are coded as IB-A, IB-B, and IB-C, respectively.

All criteria are evaluated by using the linguistic scales which are explained in Section 2. In this real case application, four decision makers evaluate the alternatives for intelligent building assessment. The decision makers are three professors from Departments of Civil Engineering, Mechatronics Engineering, and Computer Engineering, respectively. And the last one is a top manager in the construction sector.

3.1. The application phase by using Fuzzy TOPSIS

In this section, Fuzzy TOPSIS is used to for intelligent building assessment under fuzzy environment for dealing with the evaluations' uncertainty and imprecision in which the expert's comparisons are represented as fuzzy numbers. The criteria weights for TOPSIS are determined as shown in Table 5.

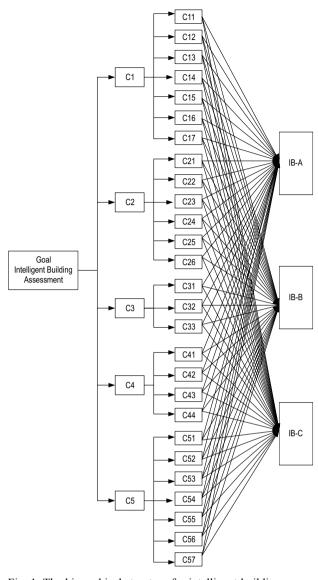


Fig. 1. The hierarchical structure for intelligent building assessment (Kahraman, Kaya 2011)

The decision-makers use the linguistic variables to evaluate the ratings of alternatives with respect to each criterion and these evaluations are shown in Table 6. Then the fuzzy decision matrix is obtained and fuzzy weights of alternatives are shown in Table 7. The normalized fuzzy decision matrix and the weighted normalized fuzzy decision matrix are obtained as shown in Tables 8 and 9, respectively.

The distances of each IB alternative from FPIS and FNIS are calculated and they are shown in Table 10.

In the last step, the closeness coefficient of each alternative is calculated as shown in Table 11.

According to Table 11, the alternative IB-B is determined as the best alternative for an intelligent building and the ranking of alternative is determined as {IB-B; IB-A; IB-C}.

This result is also similar with the result of the fuzzy multi-attribute utility theory (MAUT) proposed by Kahraman and Kaya (2012).

Table 4. Intelligent Building Assessment Criteria (Kahraman, Kaya 2011)

Main Attributes	Sub-Attributes
	Functionality (C11)
	Safety and structure (C12)
	Working efficiency (C13)
Engineering (C1)	Responsiveness (C14)
	Office automation (C15)
	Power supply (C16)
	System integration (C17)
	Energy consumption (C21)
	Water and Water Conservation (C22)
F : (1/C2)	Materials used, Durability and Waste (C23)
Environmental (C2)	Land use and Site selection (C24)
	Greenhouse Gas Emissions (Pollution) (C25)
	Indoor Environmental Quality (C26)
	Economic performance and affordability (C31)
Economical (C3)	Initial costs, operating and maintenance costs (C32)
	Life cycle costing (C33)
	Functionality, Usability and Aesthetic aspects (C41)
	Human comfort (C42)
Socio-Cultural (C4)	Health and sanitation (C43)
,	Architectural considerations – cultural heritage integration and the compatibility with local heritage value (C44)
	Work efficiency (C51)
	Use of high-tech system (C52)
	Use of advanced artificial intelligence (C53)
echnological (C5)	Telecom and data system- Connectibility (C54)
	Security monitoring and access control system (C55)
	Addressable fire detection and alarm system (C56)
	Digital addressable lighting control system (C57)

Table 5. Linguistic evaluations for criteria weights

C		C			
	D1	D2	D3	D4	Aggregation of the weights (\tilde{w}_j)
C11	VH	Н	VH	VH	(0.85; 0.975; 1)
C12	L	L	L	VL	(0; 0.075; 0.25)
C13	Н	Н	Н	VH	(0.75; 0.925; 1)
C14	M	M	ML	M	(0.25; 0.45; 0.65)
C15	VH	VH	VH	VH	(0.9; 1; 1)
C16	VL	L	VL	VL	(0; 0.025; 0.15)
C17	MH	Н	MH	Н	(0.6; 0.8; 0.95)
C21	L	VL	VL	VL	(0; 0.025; 0.15)
C22	L	VL	VL	VL	(0; 0.025; 0.15)
C23	Н	Н	VH	Н	(0.75; 0.925; 1)
C24	Н	Н	Н	Н	(0.7; 0.9; 1)
C25	L	L	L	VL	(0; 0.075; 0.25)
C26	ML	ML	L	L	(0.05; 0.2; 0.4)
C31	VL	VL	VL	VL	(0; 0; 0.1)
C32	L	VL	VL	VL	(0; 0.025; 0.15)
C33	MH	MH	M	M	(0.4; 0.6; 0.8)
C41	L	L	VL	VL	(0; 0.05; 0.2)
C42	Н	Н	Н	Н	(0.7; 0.9; 1)
C43	MH	MH	MH	Н	(0.55; 0.75; 0.925)
C44	МН	MH	M	M	(0.4; 0.6; 0.8)
C51	Н	Н	VH	VH	(0.8; 0.95; 1)
C52	Н	VH	VH	VH	(0.85; 0.975; 1)
C53	Н	Н	VH	VH	(0.8; 0.95; 1)
C54	Н	Н	VH	VH	(0.8; 0.95; 1)
C55	MH	MH	Н	Н	(0.6; 0.8; 0.95)
C56	M	ML	L	L	(0.1; 0.25; 0.45)
C57	ML	ML	M	M	(0.2; 0.4; 0.6)

Table 6. The rating of alternatives

		С	11			С	12			С	13			С	14			С	15			С	16			C	17	
	D1	D2	D3	D4	D1	D2	D3	D4	D1	D2	D3	D4	D1	D2	D3	D4	D1	D2	D3	D4	D1	D2	D3	D4	D1	D2	D3	D4
IB-A	VG	G	VG	G	MG	M	M	G	VG	G	VG	VG	G	MG	G	G	MG	G	G	G	F	F	F	M	MG	F	M	MG
IB-B	VG	G	VG	G	VG	G	VG	G	G	G	VG	G	VG	VG	VG	VG	VG	VG	G	G	M	M	MG	M	G	G	G	MG
IB-C	F	M	MG	F	G	MG	M	F	G	G	G	G	G	G	MG	G	MG	G	G	G	P	F	VP	M	M	M	M	F
		С	21			C.	22			C	23			C:	24			C.	25			C.	26					
	D1	D2	D3	D4	D1	D2	D3	D4	D1	D2	D3	D4	D1	D2	D3	D4	D1	D2	D3	D4	D1	D2	D3	D4				
IB-A	M	F	P	F	M	M	MG	M	VG	VG	G	G	M	MG	M	F	MG	M	M	F	G	VG	G	VG				
IB-B	MG	M	F	F	M	M	MG	M	VG	VG	G	G	M	MG	M	M	G	MG	G	G	VG	VG	VG	VG				
IB-C	G	G	M	G	M	M	MG	M	M	F	G	F	M	M	M	F	G	MG	G	G	G	VG	G	VG				
		С	31			C.	32			C.	33			C	41			C ₄	42			C	43			C ₂	14	
	D1	D2	D3	D4	D1	D2	D3	D4	D1	D2	D3	D4	D1	D2	D3	D4	D1	D2	D3	D4	D1	D2	D3	D4	D1	D2	D3	D4
IB-A	VG	VG	VG	G	VG	VG	G	G	F	F	P	F	G	G	G	G	M	G	G	G	M	M	MG	F	VG	G	G	VG
IB-B	G	G	VG	G	VG	VG	VG	G	MG	G	MG	G	VG	VG	VG	VG	VG	VG	VG	VG	VG	VG	VG	VG	MG	MG	M	M
IB-C	G	G	VG	G	M	M	MG	G	G	G	G	G	G	G	G	G	M	G	MG	G	VG	VG	VG	VG	G	MG	M	M
		С	51			C:	52			C	53			C	54			C:	55			C	56			C:	57	
	D1	D2	D3	D4	D1	D2	D3	D4	D1	D2	D3	D4	D1	D2	D3	D4	D1	D2	D3	D4	D1	D2	D3	D4	D1	D2	D3	D4
IB-A	G	G	G	MG	G	MG	G	G	MG	G	VG	MG	G	G	VG	G	M	M	M	M	F	MG	M	MG	F	P	F	F
IB-B	VG	G	VG	G	G	G	MG	G	VG	VG	VG	VG	VG	VG	VG	VG	F	P	P	F	M	MG	M	MG	M	F	F	G
IB-C	MG	G	G	MG	G	G	G	MG	G	G	VG	MG	М	G	MG	G	G	MG	MG	G	VG	G	VG	VG	M	F	F	G

Table 7. The fuzzy decision matrix and fuzzy weights of alternatives

		C11			C12			C13			C14			C15			C16			C17	
IB-A	8.00	9.50	10.00	4.50	6.50	8.25	8.50	9.75	10.00	6.50	8.50	9.75	6.50	8.50	9.75	1.50	3.50	5.50	3.50	5.50	7.50
IB-B	8.00	9.50	10.00	8.00	9.50	10.00	7.50	9.25	10.00	9.00	10.00	10.00	8.00	9.50	10.00	3.50	5.50	7.50	6.50	8.50	9.75
IB-C	2.50	4.50	6.50	4.00	6.00	7.75	7.00	9.00	10.00	6.50	8.50	9.75	6.50	8.50	9.75	1.00	2.25	4.00	2.50	4.50	6.50
		C21			C22			C23			C24			C25			C26				
IB-A	1.25	3.00	5.00	3.50	5.50	7.50	8.00	9.50	10.00	3.00	5.00	7.00	3.00	5.00	7.00	8.00	9.50	10.00			
IB-B	2.50	4.50	6.50	3.50	5.50	7.50	8.00	9.50	10.00	3.50	5.50	7.50	6.50	8.50	9.75	9.00	10.00	10.00			
IB-C	6.00	8.00	9.25	3.50	5.50	7.50	3.00	5.00	6.75	2.50	4.50	6.50	6.50	8.50	9.75	8.00	9.50	10.00			
		C31			C32			C33			C41			C42			C43			C44	
IB-A	8.50	9.75	10.00	8.00	9.50	10.00	0.75	2.50	4.50	7.00	9.00	10.00	6.00	8.00	9.25	3.00	5.00	7.00	8.00	9.50	10.00
IB-B	7.50	9.25	10.00	8.50	9.75	10.00	6.00	8.00	9.50	9.00	10.00	10.00	9.00	10.00	10.00	9.00	10.00	10.00	4.00	6.00	8.00
IB-C	7.50	9.25	10.00	4.50	6.50	8.25	7.00	9.00	10.00	7.00	9.00	10.00	5.50	7.50	9.00	9.00	10.00	10.00	4.50	6.50	8.25
		C51			C52			C53			C54			C55			C56			C57	
IB-A	6.50	8.50	9.75	6.50	8.50	9.75	6.50	8.25	9.50	7.50	9.25	10.00	3.00	5.00	7.00	3.50	5.50	7.50	0.75	2.50	4.50
IB-B	8.00	9.50	10.00	6.50	8.50	9.75	9.00	10.00	10.00	9.00	10.00	10.00	0.50	2.00	4.00	4.00	6.00	8.00	3.00	5.00	6.75
IB-C	6.00	8.00	9.50	6.50	8.50	9.75	7.00	8.75	9.75	5.50	7.50	9.00	6.00	8.00	9.50	8.50	9.75	10.00	3.00	5.00	6.75
ID-C	0.00	0.00	7.00	0.00	0.00																

Table 8. The fuzzy normalized decision matrix

		C11			C12			C13			C14			C15			C16			C17	
IB-A	0.80	0.95	1.00	0.45	0.65	0.83	0.85	0.98	1.00	0.65	0.85	0.98	0.65	0.85	0.98	0.20	0.47	0.73	0.36	0.56	0.77
IB-B	0.80	0.95	1.00	0.80	0.95	1.00	0.75	0.93	1.00	0.90	1.00	1.00	0.80	0.95	1.00	0.47	0.73	1.00	0.67	0.87	1.00
IB-C	0.25	0.45	0.65	0.40	0.60	0.78	0.70	0.90	1.00	0.65	0.85	0.98	0.65	0.85	0.98	0.13	0.30	0.53	0.26	0.46	0.67
		C21			C22			C23			C24			C25			C26				
IB-A	0.14	0.32	0.54	0.47	0.73	1.00	0.80	0.95	1.00	0.40	0.67	0.93	0.31	0.51	0.72	0.80	0.95	1.00			
IB-B	0.27	0.49	0.70	0.47	0.73	1.00	0.80	0.95	1.00	0.47	0.73	1.00	0.67	0.87	1.00	0.90	1.00	1.00			
IB-C	0.65	0.86	1.00	0.47	0.73	1.00	0.30	0.50	0.68	0.33	0.60	0.87	0.67	0.87	1.00	0.80	0.95	1.00			
		C31			C32			C33			C41			C42			C43			C44	
IB-A	0.85	0.98	1.00	0.80	0.95	1.00	0.08	0.25	0.45	0.70	0.90	1.00	0.60	0.80	0.93	0.30	0.50	0.70	0.80	0.95	1.00
IB-B	0.75	0.93	1.00	0.85	0.98	1.00	0.60	0.80	0.95	0.90	1.00	1.00	0.90	1.00	1.00	0.90	1.00	1.00	0.40	0.60	0.80
IB-C	0.75	0.93	1.00	0.45	0.65	0.83	0.70	0.90	1.00	0.70	0.90	1.00	0.55	0.75	0.90	0.90	1.00	1.00	0.45	0.65	0.83
		C51			C52			C53			C54			C55			C56			C57	
IB-A	0.65	0.85	0.98	0.67	0.87	1.00	0.65	0.83	0.95	0.75	0.93	1.00	0.32	0.53	0.74	0.35	0.55	0.75	0.11	0.37	0.67
IB-B	0.80	0.95	1.00	0.67	0.87	1.00	0.90	1.00	1.00	0.90	1.00	1.00	0.05	0.21	0.42	0.40	0.60	0.80	0.44	0.74	1.00
IB-C	0.60	0.80	0.95	0.67	0.87	1.00	0.70	0.88	0.98	0.55	0.75	0.90	0.63	0.84	1.00	0.85	0.98	1.00	0.44	0.74	1.00

Table 9. The fuzzy weighted normalized decision matrix

	C11			C12			C13			C14			C15			C16			C17		
IB-A	0.68	0.93	1.00	0.00	0.05	0.21	0.64	0.90	1.00	0.16	0.38	0.63	0.59	0.85	0.98	0.00	0.01	0.11	0.22	0.45	0.73
IB-B	0.68	0.93	1.00	0.00	0.07	0.25	0.56	0.86	1.00	0.23	0.45	0.65	0.72	0.95	1.00	0.00	0.02	0.15	0.40	0.70	0.95
IB-C	0.21	0.44	0.65	0.00	0.05	0.19	0.53	0.83	1.00	0.16	0.38	0.63	0.59	0.85	0.98	0.00	0.01	0.08	0.15	0.37	0.63
	C21			C22			C23			C24			C25			C26					
IB-A	0.00	0.01	0.08	0.00	0.02	0.15	0.60	0.88	1.00	0.28	0.60	0.93	0.00	0.04	0.18	0.04	0.19	0.40			
IB-B	0.00	0.01	0.11	0.00	0.02	0.15	0.60	0.88	1.00	0.33	0.66	1.00	0.00	0.07	0.25	0.05	0.20	0.40			
IB-C	0.00	0.02	0.15	0.00	0.02	0.15	0.23	0.46	0.68	0.23	0.54	0.87	0.00	0.07	0.25	0.04	0.19	0.40			
	C31			C32			C33			C41			C42			C43			C44		
IB-A	0.00	0.00	0.10	0.00	0.02	0.15	0.03	0.15	0.36	0.00	0.05	0.20	0.42	0.72	0.93	0.17	0.38	0.65	0.32	0.57	0.80
IB-B	0.00	0.00	0.10	0.00	0.02	0.15	0.24	0.48	0.76	0.00	0.05	0.20	0.63	0.90	1.00	0.50	0.75	0.93	0.16	0.36	0.64
IB-C	0.00	0.00	0.10	0.00	0.02	0.12	0.28	0.54	0.80	0.00	0.05	0.20	0.39	0.68	0.90	0.50	0.75	0.93	0.18	0.39	0.66
	C51			C52			C53			C54			C55			C56			C57		
IB-A	0.52	0.81	0.98	0.57	0.85	1.00	0.52	0.78	0.95	0.60	0.88	1.00	0.19	0.42	0.70	0.04	0.14	0.34	0.02	0.15	0.40
IB-B	0.64	0.90	1.00	0.57	0.85	1.00	0.72	0.95	1.00	0.72	0.95	1.00	0.03	0.17	0.40	0.04	0.15	0.36	0.09	0.30	0.60
IB-C	0.48	0.76	0.95	0.57	0.85	1.00	0.56	0.83	0.98	0.44	0.71	0.90	0.38	0.67	0.95	0.09	0.24	0.45	0.09	0.30	0.60

Table 10. The distance measurements from FPIS and FNIS

IB-A IB-B IB-C IB-A IB-B IB-C C11 0.190 0.190 0.594 0.879 0.879 0.469 C12 0.919 0.899 0.924 0.122 0.150 0.115 C13 0.217 0.266 0.291 0.860 0.826 0.810 C14 0.637 0.585 0.637 0.438 0.475 0.438 C15 0.255 0.164 0.255 0.820 0.898 0.820 C16 0.961 0.946 0.972 0.064 0.087 0.046 C17 0.574 0.389 0.645 0.511 0.719 0.432 C21 0.971 0.962 0.945 0.047 0.061 0.087 C22 0.946 0.946 0.946 0.087 0.087 0.087 C23 0.241 0.241 0.576 0.843 0.843 0.490 C24 0.477 0.435 0.522 0.]	FPIS (A*)		FNIS(A-))
C12 0.919 0.899 0.924 0.122 0.150 0.115 C13 0.217 0.266 0.291 0.860 0.826 0.810 C14 0.637 0.585 0.637 0.438 0.475 0.438 C15 0.255 0.164 0.255 0.820 0.898 0.820 C16 0.961 0.946 0.972 0.064 0.087 0.046 C17 0.574 0.389 0.645 0.511 0.719 0.432 C21 0.971 0.962 0.945 0.047 0.061 0.087 C22 0.946 0.946 0.946 0.087 0.087 0.087 C23 0.241 0.241 0.576 0.843 0.843 0.490 C24 0.477 0.435 0.522 0.661 0.717 0.605 C25 0.931 0.901 0.901 0.106 0.149 0.149 C26 0.804 0.798 <t< td=""><td>-</td><td>IB-A</td><td>IB-B</td><td>IB-C</td><td>IB-A</td><td>IB-B</td><td>IB-C</td></t<>	-	IB-A	IB-B	IB-C	IB-A	IB-B	IB-C
C13 0.217 0.266 0.291 0.860 0.826 0.810 C14 0.637 0.585 0.637 0.438 0.475 0.438 C15 0.255 0.164 0.255 0.820 0.898 0.820 C16 0.961 0.946 0.972 0.064 0.087 0.046 C17 0.574 0.389 0.645 0.511 0.719 0.432 C21 0.971 0.962 0.945 0.047 0.061 0.087 C22 0.946 0.946 0.946 0.087 0.087 0.087 C23 0.241 0.241 0.576 0.843 0.843 0.490 C24 0.477 0.435 0.522 0.661 0.717 0.605 C25 0.931 0.901 0.901 0.106 0.149 0.149 C26 0.804 0.798 0.804 0.257 0.260 0.257 C31 0.968 0.968 <t< td=""><td>C11</td><td>0.190</td><td>0.190</td><td>0.594</td><td>0.879</td><td>0.879</td><td>0.469</td></t<>	C11	0.190	0.190	0.594	0.879	0.879	0.469
C14 0.637 0.585 0.637 0.438 0.475 0.438 C15 0.255 0.164 0.255 0.820 0.898 0.820 C16 0.961 0.946 0.972 0.064 0.087 0.046 C17 0.574 0.389 0.645 0.511 0.719 0.432 C21 0.971 0.962 0.945 0.047 0.061 0.087 C22 0.946 0.946 0.946 0.087 0.087 0.087 C23 0.241 0.241 0.576 0.843 0.843 0.490 C24 0.477 0.435 0.522 0.661 0.717 0.605 C25 0.931 0.901 0.901 0.106 0.149 0.149 C26 0.804 0.798 0.804 0.257 0.260 0.257 C31 0.968 0.968 0.968 0.058 0.058 0.058 C32 0.944 0.944 <t< td=""><td>C12</td><td>0.919</td><td>0.899</td><td>0.924</td><td>0.122</td><td>0.150</td><td>0.115</td></t<>	C12	0.919	0.899	0.924	0.122	0.150	0.115
C15 0.255 0.164 0.255 0.820 0.898 0.820 C16 0.961 0.946 0.972 0.064 0.087 0.046 C17 0.574 0.389 0.645 0.511 0.719 0.432 C21 0.971 0.962 0.945 0.047 0.061 0.087 C22 0.946 0.946 0.946 0.087 0.087 0.087 C23 0.241 0.241 0.576 0.843 0.843 0.490 C24 0.477 0.435 0.522 0.661 0.717 0.605 C25 0.931 0.901 0.901 0.106 0.149 0.149 C26 0.804 0.798 0.804 0.257 0.260 0.257 C31 0.968 0.968 0.968 0.058 0.058 0.058 C32 0.944 0.944 0.955 0.088 0.088 0.072 C33 0.831 0.549 <t< td=""><td>C13</td><td>0.217</td><td>0.266</td><td>0.291</td><td>0.860</td><td>0.826</td><td>0.810</td></t<>	C13	0.217	0.266	0.291	0.860	0.826	0.810
C16 0.961 0.946 0.972 0.064 0.087 0.046 C17 0.574 0.389 0.645 0.511 0.719 0.432 C21 0.971 0.962 0.945 0.047 0.061 0.087 C22 0.946 0.946 0.946 0.087 0.087 0.087 C23 0.241 0.241 0.576 0.843 0.843 0.490 C24 0.477 0.435 0.522 0.661 0.717 0.605 C25 0.931 0.901 0.901 0.106 0.149 0.149 C26 0.804 0.798 0.804 0.257 0.260 0.257 C31 0.968 0.968 0.968 0.058 0.058 0.058 C32 0.944 0.944 0.955 0.088 0.088 0.072 C33 0.831 0.549 0.507 0.226 0.537 0.580 C41 0.922 0.921 <t< td=""><td>C14</td><td>0.637</td><td>0.585</td><td>0.637</td><td>0.438</td><td>0.475</td><td>0.438</td></t<>	C14	0.637	0.585	0.637	0.438	0.475	0.438
C17 0.574 0.389 0.645 0.511 0.719 0.432 C21 0.971 0.962 0.945 0.047 0.061 0.087 C22 0.946 0.946 0.946 0.087 0.087 0.087 C23 0.241 0.241 0.576 0.843 0.843 0.490 C24 0.477 0.435 0.522 0.661 0.717 0.605 C25 0.931 0.901 0.901 0.106 0.149 0.149 C26 0.804 0.798 0.804 0.257 0.260 0.257 C31 0.968 0.968 0.968 0.058 0.058 0.058 C32 0.944 0.944 0.955 0.088 0.088 0.072 C33 0.831 0.549 0.507 0.226 0.537 0.580 C41 0.922 0.921 0.922 0.118 0.119 0.118 C42 0.374 0.221 <t< td=""><td>C15</td><td>0.255</td><td>0.164</td><td>0.255</td><td>0.820</td><td>0.898</td><td>0.820</td></t<>	C15	0.255	0.164	0.255	0.820	0.898	0.820
C21 0.971 0.962 0.945 0.047 0.061 0.087 C22 0.946 0.946 0.946 0.087 0.087 0.087 C23 0.241 0.241 0.576 0.843 0.843 0.490 C24 0.477 0.435 0.522 0.661 0.717 0.605 C25 0.931 0.901 0.901 0.106 0.149 0.149 C26 0.804 0.798 0.804 0.257 0.260 0.257 C31 0.968 0.968 0.968 0.058 0.058 0.058 C32 0.944 0.944 0.955 0.088 0.088 0.072 C33 0.831 0.549 0.507 0.226 0.537 0.580 C41 0.922 0.921 0.922 0.118 0.119 0.118 C42 0.374 0.221 0.406 0.719 0.858 0.687 C43 0.636 0.328 <t< td=""><td>C16</td><td>0.961</td><td>0.946</td><td>0.972</td><td>0.064</td><td>0.087</td><td>0.046</td></t<>	C16	0.961	0.946	0.972	0.064	0.087	0.046
C22 0.946 0.946 0.946 0.087 0.087 0.087 C23 0.241 0.241 0.576 0.843 0.843 0.490 C24 0.477 0.435 0.522 0.661 0.717 0.605 C25 0.931 0.901 0.901 0.106 0.149 0.149 C26 0.804 0.798 0.804 0.257 0.260 0.257 C31 0.968 0.968 0.968 0.058 0.058 0.058 C32 0.944 0.944 0.955 0.088 0.088 0.072 C33 0.831 0.549 0.507 0.226 0.537 0.580 C41 0.922 0.921 0.922 0.118 0.119 0.118 C42 0.374 0.221 0.406 0.719 0.858 0.687 C43 0.636 0.328 0.328 0.442 0.745 0.745 C44 0.479 0.644 <t< td=""><td>C17</td><td>0.574</td><td>0.389</td><td>0.645</td><td>0.511</td><td>0.719</td><td>0.432</td></t<>	C17	0.574	0.389	0.645	0.511	0.719	0.432
C23 0.241 0.241 0.576 0.843 0.843 0.490 C24 0.477 0.435 0.522 0.661 0.717 0.605 C25 0.931 0.901 0.901 0.106 0.149 0.149 C26 0.804 0.798 0.804 0.257 0.260 0.257 C31 0.968 0.968 0.968 0.058 0.058 0.058 C32 0.944 0.944 0.955 0.088 0.088 0.072 C33 0.831 0.549 0.507 0.226 0.537 0.580 C41 0.922 0.921 0.922 0.118 0.119 0.118 C42 0.374 0.221 0.406 0.719 0.858 0.687 C43 0.636 0.328 0.328 0.442 0.745 0.745 C44 0.479 0.644 0.622 0.596 0.434 0.455 C51 0.299 0.215 <t< td=""><td>C21</td><td>0.971</td><td>0.962</td><td>0.945</td><td>0.047</td><td>0.061</td><td>0.087</td></t<>	C21	0.971	0.962	0.945	0.047	0.061	0.087
C24 0.477 0.435 0.522 0.661 0.717 0.605 C25 0.931 0.901 0.901 0.106 0.149 0.149 C26 0.804 0.798 0.804 0.257 0.260 0.257 C31 0.968 0.968 0.968 0.058 0.058 0.058 C32 0.944 0.944 0.955 0.088 0.088 0.072 C33 0.831 0.549 0.507 0.226 0.537 0.580 C41 0.922 0.921 0.922 0.118 0.119 0.118 C42 0.374 0.221 0.406 0.719 0.858 0.687 C43 0.636 0.328 0.328 0.442 0.745 0.745 C44 0.479 0.644 0.622 0.596 0.434 0.455 C51 0.299 0.215 0.332 0.790 0.861 0.755 C52 0.265 0.265 <t< td=""><td>C22</td><td>0.946</td><td>0.946</td><td>0.946</td><td>0.087</td><td>0.087</td><td>0.087</td></t<>	C22	0.946	0.946	0.946	0.087	0.087	0.087
C25 0.931 0.901 0.901 0.106 0.149 0.149 C26 0.804 0.798 0.804 0.257 0.260 0.257 C31 0.968 0.968 0.968 0.058 0.058 0.058 C32 0.944 0.944 0.955 0.088 0.088 0.072 C33 0.831 0.549 0.507 0.226 0.537 0.580 C41 0.922 0.921 0.922 0.118 0.119 0.118 C42 0.374 0.221 0.406 0.719 0.858 0.687 C43 0.636 0.328 0.328 0.442 0.745 0.745 C44 0.479 0.644 0.622 0.596 0.434 0.455 C51 0.299 0.215 0.332 0.790 0.861 0.755 C52 0.265 0.265 0.265 0.825 0.825 0.825 C53 0.305 0.164 <t< td=""><td>C23</td><td>0.241</td><td>0.241</td><td>0.576</td><td>0.843</td><td>0.843</td><td>0.490</td></t<>	C23	0.241	0.241	0.576	0.843	0.843	0.490
C26 0.804 0.798 0.804 0.257 0.260 0.257 C31 0.968 0.968 0.968 0.058 0.058 0.058 C32 0.944 0.944 0.955 0.088 0.088 0.072 C33 0.831 0.549 0.507 0.226 0.537 0.580 C41 0.922 0.921 0.922 0.118 0.119 0.118 C42 0.374 0.221 0.406 0.719 0.858 0.687 C43 0.636 0.328 0.328 0.442 0.745 0.745 C44 0.479 0.644 0.622 0.596 0.434 0.455 C51 0.299 0.215 0.332 0.790 0.861 0.755 C52 0.265 0.265 0.265 0.825 0.825 0.825 C53 0.305 0.164 0.272 0.772 0.898 0.710 C55 0.601 0.814 <t< td=""><td>C24</td><td>0.477</td><td>0.435</td><td>0.522</td><td>0.661</td><td>0.717</td><td>0.605</td></t<>	C24	0.477	0.435	0.522	0.661	0.717	0.605
C31 0.968 0.968 0.968 0.058 0.058 0.058 C32 0.944 0.944 0.955 0.088 0.088 0.072 C33 0.831 0.549 0.507 0.226 0.537 0.580 C41 0.922 0.921 0.922 0.118 0.119 0.118 C42 0.374 0.221 0.406 0.719 0.858 0.687 C43 0.636 0.328 0.328 0.442 0.745 0.745 C44 0.479 0.644 0.622 0.596 0.434 0.455 C51 0.299 0.215 0.332 0.790 0.861 0.755 C52 0.265 0.265 0.265 0.825 0.825 0.825 C53 0.305 0.164 0.272 0.772 0.898 0.710 C54 0.241 0.164 0.368 0.843 0.898 0.710 C55 0.601 0.814 <t< td=""><td>C25</td><td>0.931</td><td>0.901</td><td>0.901</td><td>0.106</td><td>0.149</td><td>0.149</td></t<>	C25	0.931	0.901	0.901	0.106	0.149	0.149
C32 0.944 0.944 0.955 0.088 0.088 0.072 C33 0.831 0.549 0.507 0.226 0.537 0.580 C41 0.922 0.921 0.922 0.118 0.119 0.118 C42 0.374 0.221 0.406 0.719 0.858 0.687 C43 0.636 0.328 0.328 0.442 0.745 0.745 C44 0.479 0.644 0.622 0.596 0.434 0.455 C51 0.299 0.215 0.332 0.790 0.861 0.755 C52 0.265 0.265 0.265 0.825 0.825 0.825 C53 0.305 0.164 0.272 0.772 0.898 0.807 C54 0.241 0.164 0.368 0.843 0.898 0.710 C55 0.601 0.814 0.406 0.484 0.251 0.707 C56 0.839 0.827 <t< td=""><td>C26</td><td>0.804</td><td>0.798</td><td>0.804</td><td>0.257</td><td>0.260</td><td>0.257</td></t<>	C26	0.804	0.798	0.804	0.257	0.260	0.257
C33 0.831 0.549 0.507 0.226 0.537 0.580 C41 0.922 0.921 0.922 0.118 0.119 0.118 C42 0.374 0.221 0.406 0.719 0.858 0.687 C43 0.636 0.328 0.328 0.442 0.745 0.745 C44 0.479 0.644 0.622 0.596 0.434 0.455 C51 0.299 0.215 0.332 0.790 0.861 0.755 C52 0.265 0.265 0.265 0.825 0.825 0.825 C53 0.305 0.164 0.272 0.772 0.898 0.807 C54 0.241 0.164 0.368 0.843 0.898 0.710 C55 0.601 0.814 0.406 0.484 0.251 0.707 C56 0.839 0.827 0.755 0.211 0.226 0.300	C31	0.968	0.968	0.968	0.058	0.058	0.058
C41 0.922 0.921 0.922 0.118 0.119 0.118 C42 0.374 0.221 0.406 0.719 0.858 0.687 C43 0.636 0.328 0.328 0.442 0.745 0.745 C44 0.479 0.644 0.622 0.596 0.434 0.455 C51 0.299 0.215 0.332 0.790 0.861 0.755 C52 0.265 0.265 0.265 0.825 0.825 0.825 C53 0.305 0.164 0.272 0.772 0.898 0.807 C54 0.241 0.164 0.368 0.843 0.898 0.710 C55 0.601 0.814 0.406 0.484 0.251 0.707 C56 0.839 0.827 0.755 0.211 0.226 0.300	C32	0.944	0.944	0.955	0.088	0.088	0.072
C42 0.374 0.221 0.406 0.719 0.858 0.687 C43 0.636 0.328 0.328 0.442 0.745 0.745 C44 0.479 0.644 0.622 0.596 0.434 0.455 C51 0.299 0.215 0.332 0.790 0.861 0.755 C52 0.265 0.265 0.265 0.825 0.825 0.825 C53 0.305 0.164 0.272 0.772 0.898 0.807 C54 0.241 0.164 0.368 0.843 0.898 0.710 C55 0.601 0.814 0.406 0.484 0.251 0.707 C56 0.839 0.827 0.755 0.211 0.226 0.300	C33	0.831	0.549	0.507	0.226	0.537	0.580
C43 0.636 0.328 0.328 0.442 0.745 0.745 C44 0.479 0.644 0.622 0.596 0.434 0.455 C51 0.299 0.215 0.332 0.790 0.861 0.755 C52 0.265 0.265 0.265 0.825 0.825 0.825 C53 0.305 0.164 0.272 0.772 0.898 0.807 C54 0.241 0.164 0.368 0.843 0.898 0.710 C55 0.601 0.814 0.406 0.484 0.251 0.707 C56 0.839 0.827 0.755 0.211 0.226 0.300	C41	0.922	0.921	0.922	0.118	0.119	0.118
C44 0.479 0.644 0.622 0.596 0.434 0.455 C51 0.299 0.215 0.332 0.790 0.861 0.755 C52 0.265 0.265 0.825 0.825 0.825 C53 0.305 0.164 0.272 0.772 0.898 0.807 C54 0.241 0.164 0.368 0.843 0.898 0.710 C55 0.601 0.814 0.406 0.484 0.251 0.707 C56 0.839 0.827 0.755 0.211 0.226 0.300	C42	0.374	0.221	0.406	0.719	0.858	0.687
C51 0.299 0.215 0.332 0.790 0.861 0.755 C52 0.265 0.265 0.825 0.825 0.825 0.825 C53 0.305 0.164 0.272 0.772 0.898 0.807 C54 0.241 0.164 0.368 0.843 0.898 0.710 C55 0.601 0.814 0.406 0.484 0.251 0.707 C56 0.839 0.827 0.755 0.211 0.226 0.300	C43	0.636	0.328	0.328	0.442	0.745	0.745
C52 0.265 0.265 0.265 0.825 0.825 0.825 C53 0.305 0.164 0.272 0.772 0.898 0.807 C54 0.241 0.164 0.368 0.843 0.898 0.710 C55 0.601 0.814 0.406 0.484 0.251 0.707 C56 0.839 0.827 0.755 0.211 0.226 0.300	C44	0.479	0.644	0.622	0.596	0.434	0.455
C53 0.305 0.164 0.272 0.772 0.898 0.807 C54 0.241 0.164 0.368 0.843 0.898 0.710 C55 0.601 0.814 0.406 0.484 0.251 0.707 C56 0.839 0.827 0.755 0.211 0.226 0.300	C51	0.299	0.215	0.332	0.790	0.861	0.755
C54 0.241 0.164 0.368 0.843 0.898 0.710 C55 0.601 0.814 0.406 0.484 0.251 0.707 C56 0.839 0.827 0.755 0.211 0.226 0.300	C52	0.265	0.265	0.265	0.825	0.825	0.825
C55 0.601 0.814 0.406 0.484 0.251 0.707 C56 0.839 0.827 0.755 0.211 0.226 0.300	C53	0.305	0.164	0.272	0.772	0.898	0.807
C56 0.839 0.827 0.755 0.211 0.226 0.300	C54	0.241	0.164	0.368	0.843	0.898	0.710
	C55	0.601	0.814	0.406	0.484	0.251	0.707
C57 0.825 0.704 0.704 0.247 0.390 0.390	C56	0.839	0.827	0.755	0.211	0.226	0.300
	C57	0.825	0.704	0.704	0.247	0.390	0.390

Table 11. the closeness coefficient of each alternative

	A*	A-	CC_i	Ranking
IB-A	16.652	12.115	0.4211	2
IB-B	15.453	13.340	0.4633	1
IB-C	16.821	12.013	0.4166	3

Table 12. Decision maker's evaluations for criteria

3.2. The application phase by using fuzzy AHP

In this paper, intelligent building assessment is also analysed and is re-evaluated by using a fuzzy AHP methodology. For this aim, the similar criteria and hierarchical structure are used.

The linguistic evaluations of DMs for criteria which are shown in Table 12 are converted to fuzzy numbers as shown in Table 13. The pairwise comparison matrices for criteria and sub-criteria are obtained by using the geometric mean method as shown in Table 14.

In the last step, the fuzzy weights can be calculated and defuzzified by using Liou and Wang's total integral method (1992). The fuzzy and crisp weights of criteria and subcriteria with global weights are shown in Table 15.

After the weights of criteria and sub-criteria are determined, the alternatives are evaluated with respect to the evaluation criteria. The obtained results are summarized in Table 16.

According to Table 16, the alternative "IB-B" is determined as the best alternative for intelligent building. The rank of the alternatives is as follows: {IB-B; IB-A; IB-C} and this result is similar with the result of fuzzy TOPSIS and fuzzy MAUT.

The obtained results for fuzzy decision making methodologies are summarized in Table 17.

According to Table 17, three fuzzy decision making methodologies have determined the similar results.

Conclusion

Although the AHP and TOPSIS are two of the most widely used MCDM tools to capture the expert's evaluation, the traditional AHP and TOPSIS still cannot really reflect the expert evaluations since they use exact value to express the expert opinion in a comparison of alternatives. Therefore, they cannot succeed in handling the inherent uncertainty and imprecision in the decision process. To overcome this, the fuzzy set theory can be successfully used.

In this paper, the fuzzy set theory is integrated with AHP and TOPSIS methodologies to increase their flexibility and sensitiveness. In the evaluation process, the fuzzy set theory brings many advantages on decision making process such as a possibility to evaluate immeasurable criteria and to take into consideration evaluation of human judgments. The fuzzy set theory also gives an advantage that is an ease definition of score for alternative and is a flexible scale for expert judgments.

	D1 D2												D3	3		D4						
	C1	C2	C3	C4	C5	C1	C2	C3	C4	C5	C1	C2	C3	C4	C5	C1	C2	C3	C4	C5		
C1		Vs	Ab	Es	Eq		Es	Vs	Es	1/Eq		Es	Es	Es	Eq		Es	Es	Es	1/Eq		
C2			Vs	Eq	1/Vs			Vs	Eq	1/Vs			Eq	Wk	1/Es			Es	Eq	1/Es		
C2 C3				1/Vs	1/As				1/Es	1/Es				1/Es	1/Vs				1/Vs	1/Ab		
C4					1/Es					1/Wk					1/Es					1/Es		
C5																						

Table 13. Converted Fuzzy Numbers of DMs' Evaluations for Criteria

	D1				D2					
	C1	C2	С3	C4	C5	C1	C2	С3	C4	C5
C1		(5, 7, 9)	(7, 9, 9)	(3, 5, 7)	(1, 1, 3)		(3, 5, 7)	(5, 7, 9)	(3, 5, 7)	(0.33, 1, 1)
C2	(0.11, 0.14, 0.2)		(5, 7, 9)	(1, 1, 3)	(0.11, 0.14, 0.2)	(0.14, 0.2, 0.33)		(5, 7, 9)	(1, 1, 3)	(0.11, 0.14, 0.2)
C3	(0.11, 0.11, 0.14)	(0.11, 0.14, 0.2)		(0.11, 0.14, 0.2)	(0.11, 0.11, 0.14)	(0.11, 0.14, 0.2)	(0.11, 0.14, 0.2)		(0.14, 0.2, 0.33)	(0.14, 0.2, 0.33)
C4	(0.14, 0.2, 0.33)	(0.33, 1, 1)	(5, 7.14, 9.09)		(0.14, 0.2, 0.33)	(0.14, 0.2, 0.33)	(0.33, 1, 1)	(3.03, 5, 7.14)		(0.2, 0.33, 1)
C5	(0.33, 1, 1)	(5, 7.14, 9.09)	(7.14, 9.09, 9.09)	(3.03, 5, 7.14)		(1, 1, 3.03)	(5, 7.14, 9.09)	(3.03, 5, 7.14)	(1, 3.03, 5)	
D3							D4			
	C1	C2	C3	C4	C5	C1	C2	C3	C4	C5
C1		(3, 5, 7)	(3, 5, 7)	(3, 5, 7)	(1, 1, 3)		(3, 5, 7)	(3, 5, 7)	(3, 5, 7)	(0.33, 1, 1)
C2	(0.14, 0.2, 0.33)		(1, 1, 3)	(1, 3, 5)	(0.14, 0.2, 0.33)	(0.14, 0.2, 0.33)		(3, 5, 7)	(1, 1, 3)	(0.14, 0.2, 0.33)
C3	(0.14, 0.2, 0.33)	(0.33, 1, 1)		(0.14, 0.2, 0.33)	(0.11, 0.14, 0.2)	(0.14, 0.2, 0.33)	(0.14, 0.2, 0.33)		(0.11, 0.14, 0.2)	(0.11, 0.11, 0.14)
C4	(0.14, 0.2, 0.33)	(0.2, 0.33, 1)	(3.03, 5, 7.14)		(0.14, 0.2, 0.33)	(0.14, 0.2, 0.33)	(0.33, 1, 1)	(5, 7.14, 9.09)		(0.14, 0.2, 0.33)
C5	(0.33, 1, 1)	(3.03, 5, 7.14)	(5, 7.14, 9.09)	(3.03, 5, 7.14)		(1, 1, 3.03)	(3.03, 5, 7.14)	(7.14, 9.09, 9.09)	(3.03, 5, 7.14)	

Table 14. The pairwise comparison matrices of the evaluation criteria

	C1	C2	C3	C4	C5
C1	(1, 1, 1)	(3.409, 5.439, 14.561)	(4.213, 6.3, 7.937)	(3, 5, 7)	(0.574, 1, 1.732)
C2	(0.134, 0.184, 0.293)	(1, 1, 1)	(2.943, 3.956, 6.422)	(1, 1.316, 3.409)	(0.124, 0.167, 0.257)
C3	(0.126, 0.159, 0.237)	(0.156, 0.253, 0.34)	(1, 1, 1)	(0.124, 0.167, 0.257)	(0.117, 0.136, 0.19)
C4	(0.143, 0.2, 0.333)	(0.293, 0.76, 1)	(3.892, 5.976, 8.058)	(1, 1, 1)	(0.153, 0.227, 0.435)
C5	(0.577, 1, 1.741)	(3.892, 5.976, 8.058)	(5.273, 7.371, 8.559)	(2.297, 4.412, 6.534)	(1, 1, 1)

Table 15. The local and global weights of criteria and sub-criteria

Criteria	Fuzzy	Crisp	Sub-Criteria	Fuzzy	Crisp	Global Weight
		0.39	C11	(0.167, 0.358, 0.697)	0.34	0.134
			C12	(0.017, 0.035, 0.078)	0.04	0.014
			C13	(0.046, 0.114, 0.271)	0.12	0.046
C1	(0.181, 0.381, 0.842)		C14	(0.025, 0.048, 0.143)	0.06	0.023
			C15	(0.18, 0.359, 0.752)	0.36	0.140
			C16	(0.013, 0.028, 0.063)	0.03	0.011
			C17	(0.022, 0.057, 0.127)	0.06	0.022
		0.10	C21	(0.016, 0.034, 0.094)	0.04	0.004
			C22	(0.02, 0.05, 0.101)	0.05	0.005
C2	(0.052, 0.095, 0.219)		C23	(0.214, 0.47, 0.976)	0.45	0.046
C2	(0.032, 0.093, 0.219)		C24	(0.121, 0.276, 0.668)	0.28	0.029
			C25	(0.031, 0.069, 0.194)	0.08	0.008
			C26	(0.038, 0.101, 0.236)	0.10	0.010
		0.03	C31	(0.058, 0.107, 0.225)	0.11	0.004
C3	(0.019, 0.034, 0.065)		C32	(0.062, 0.13, 0.224)	0.12	0.004
			C33	(0.395, 0.762, 1.493)	0.77	0.025
		0.10	C41	(0.041, 0.101, 0.201)	0.09	0.009
C4	(0.045, 0.099, 0.204)		C42	(0.194, 0.495, 1.17)	0.49	0.048
C4			C43	(0.098, 0.219, 0.59)	0.23	0.023
			C44	(0.079, 0.185, 0.469)	0.19	0.019
		0.38	C51	(0.069, 0.155, 0.404)	0.16	0.059
			C52	(0.152, 0.387, 0.918)	0.37	0.139
			C53	(0.041, 0.151, 0.364)	0.14	0.053
C5	(0.184, 0.391, 0.75)		C54	(0.06, 0.142, 0.47)	0.16	0.061
			C55	(0.026, 0.073, 0.183)	0.07	0.027
			C56	(0.02, 0.053, 0.148)	0.06	0.021
			C57	(0.016, 0.04, 0.115)	0.04	0.016

Table 16. The evaluation results for intelligent building assessment

	IB-A	IB-B	IB-C
C11	0.063	0.062	0.010
C12	0.008	0.001	0.005
C13	0.036	0.005	0.006
C14	0.005	0.012	0.005
C15	0.031	0.085	0.025
C16	0.002	0.006	0.003
C17	0.008	0.010	0.005
C21	0.000	0.001	0.002
C22	0.002	0.002	0.002
C23	0.022	0.021	0.003
C24	0.010	0.010	0.009
C25	0.001	0.004	0.003
C26	0.002	0.007	0.001
C31	0.003	0.001	0.001
C32	0.002	0.002	0.000
C33	0.008	0.002	0.016
C41	0.002	0.004	0.003
C42	0.005	0.037	0.005
C43	0.001	0.011	0.011
C44	0.011	0.002	0.005
C51	0.003	0.044	0.012
C52	0.048	0.051	0.040
C53	0.009	0.040	0.004
C54	0.016	0.040	0.005
C55	0.006	0.001	0.020
C56	0.001	0.004	0.016
C57	0.001	0.007	0.007
TOTAL	0.306	0.471	0.222

Table 17. The comparison results of fuzzy DM methodologies for intelligent building alternatives

	IB-A	IB-B	IB-C
Fuzzy AHP	2	1	3
Fuzzy TOPSIS	2	1	3
Fuzzy MAUT (Kahraman, Kaya 2011)	2	1	3

By the way, in this paper, the fuzzy AHP and the fuzzy TOPSIS models for intelligent building assessment have been proposed and they have been successfully applied for the assessment of intelligent building alternatives for a business centre in İstanbul. Then, the obtained results of these techniques are compared.

As a result of evaluation process, these two MCDM methodologies, fuzzy AHP and fuzzy TOPSIS, have determined the similar results. The alternative IB-B is selected as the most suitable building with respect to intelligent level by two methods. The ranking of the alternative

is similar and also determined as follows: {IB-B; IB-A; IB-C}. Also the obtained results are compared with the results of fuzzy MAUT. These three fuzzy decision-making methods have determined the similar results.

In future studies, other fuzzy multi-criteria decision making methods such as VIKOR, ANP, DEMATEL and ELECTRE, etc. can be used for intelligent building assessment.

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