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OPTIMAL TIME-BASED AND COST-BASED CONTRACTS IN CONSTRUCTION PROJECTS UNDER ASYMMETRIC INFORMATION

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Abstract. A project owner (principal) delegates a project to a contractor (agent). Because the contractor has construction experience, he has private information about the project's expected completion time. Besides, the contractor can exert an unobservable effort to shorten the completion time. Under an asymmetric information setting, we provide the optimal time-based contract and the optimal cost-based contract, both of which consists of one payment scheme. Additionally, we consider a menu of time-based contracts that consists of a series of contracts. By comparing three contracts, we demonstrate that the owner has a preference for the menu of time-based contracts over the other two. The pooling time-based contract is superior to the pooling cost-based contract. We also find that the social welfare under the pooling time-based contract is lower than under the menu of time-based contracts if the daily operating cost is low but it may be higher than under the menu of time-based contracts if the daily operating cost is high. For the pooling cost-based contract, whether it is better than the previous two contracts depend greatly on the proportion of cost borne by the contractor and the daily operating cost.

Keywords: construction project management, time-based contract, cost-based contract, principal-agent model.

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1. Introduction

The construction project has many distinctive characteristics. In terms of time, a construction project can take decades (for example, the A20 motorway in Germany takes 4,302 days). From the perspective of space, a construction project can span thousands of territories (for example, the West-East Gas Transmission project is 4,200 kilometers long). From the perspective of investment scale, the cost of the Three Gorges project can reach hundreds of billions of yuan. From an organizational point of view, construction projects usually involve multiple enterprises and departments, and even different industries. It is because of these complex characteristics that the time of the construction project has great uncertainty.

One of the most common uncertainties in construction project management is delay. The Sydney Light Rail project, for example, is two years late and cost overruns of $500 million compared to the original budget. High-speed Rail Service between Guangzhou, Shenzhen and Hong Kong was delayed by three years and cost overruns of HK $17.5 billion. The Royal Adelaide Hospital project is believed to be one of the most expensive buildings in Australia, with a 17-month delay from an estimated 1.7 billion to the final 2.3 billion. In addition, there is a large amount of literature on the problems of overtime and overcost in construction projects. Aibinu and Jagboro (2002) discover that the two most common consequences of project construction delays in Nigerian projects are cost overruns and time overruns. According to Assaf and Al-Hejji (2006), 70% of construction projects are considered to be time overrun. A study conducted by Sweis (2013) shows that 81.5% of construction projects in Jordan experienced delays during the period 1990–1997. The main effects of construction project delays include time overruns and cost overruns (Sambasivan & Soon, 2007). Manavazhi and Adhikari (2002) study a number of road projects in Nepal and found that delays resulted in cost overruns of up to

5% of the total budget. Therefore, time is an important consideration in contract project management.

When the contract is signed, it is still in the early stage of the project, and the project owner cannot judge the final completion time of the project. In practice, few projects are completed on time. In the process of the project, there will be various uncertainties, such as design changes, material delays, etc., these uncertainties eventually lead to the delay of the construction period. Majid and McCaffer (1998) identify five major causes of delay, namely late delivery, material damage, poor planning, equipment failure, and unsuitable equipment. Odeh and Battaineh (2002) attribute delays to inexperienced contractors, financing and payment of completed works, owner intervention, labour supply and subcontractors. Rao and Gul (2017) examine the fundamental risk factors that lead to delays in construction projects and offer five risks: design risks (such as not completing the design in time), financial risks (such as contractor financial difficulties), technical risks (such as inefficient or conventional technology), labor risks (such as labor productivity, field labor accidents, labor availability), and external risks (such as changes in laws and regulations). Some of the reasons for the above delay are controllable and some are not. But it all boils down to three things: talent (one's own ability), hard work (an effort to improve one's ability), and external factors beyond our control.

In practice, there is an asymmetry of time information between owners and contractors (Zeng et al., 2019; Shi & Zhang, 2022; Shi et al., 2021; Zhao et al., 2022). Specifically, owners know roughly when the construction project will be completed through social experience, but lack a more accurate prediction of the specific completion time of the project. This means that the project owner knows the approximate time of expected completion time, but lacks more accurate information about the specific completion times. Because the contractor has been in contact with the construction project for a long time, the contractor can have more information about the specific time of the completion of the project. Contractors then have an incentive to lie about their expected completion times according to the contract form in order to gain more profits. In addition, contractors may have the ability to adopt new technologies, improve their management skills, or spend more labor on projects to shorten completion times. These behaviors are not easily observed by the project owner. In short, there are two aspects of information asymmetry in this process: one is that the contractor has private information about the expected completion time of the project, and the other is that the contractor can make some efforts to reduce the total completion time. The owner has no way to directly observe either of these two pieces of information. This phenomenon is called the adverse selection problem and the moral hazard problem.

The problem of adverse selection and moral hazard is usually solved through contract design. There is many literature that study time-based contracts and cost-based contracts. In time-based contracts, payments are based on actual time to completion, and the longer it takes to complete the project, the lower the pay. In a cost-based contract, the owner pays the contractor based on actual expenses. If the project takes longer to complete and the total cost is higher, the project owner will be paid less. Kwon et al. (2010) examine what kind of time-based and cost-based contracts would make two separate decisions as effective as one. Boarnet (1998) notes that the City of Los Angeles estimated that each day the highway was out of service would cause more than $1 million in damage, and in order to quickly repair the Santa Monica Highway, the project owner (LA government) offered a time-based contract to the contractor (Clint Meyers). Clint Meyers would pay the city $200,000 for every day the highway was closed after the six-month target. But for every day the highway opens early, the city pays Meyers an additional $200,000. As a result, Highway 5 reopened 33 days early and the contractor received a $4.95 million bonus, while Highway 10 reopened 66 days early and the contractor received a $14.8 million bonus. Berends (2000) points out that in large construction projects, especially the very large construction projects with complicated construction process, cost-bases contract is favored by owners because of its incentive effect. Bower et al. (2002) study that in the cost-based contract, the owner can determine the contractor's remuneration according to the difference between the actual and agreed target costs through the contract. However, there is little literature on the application of time information asymmetry to contract design. To enrich the literature, we consider the following research questions:

1. Under an asymmetric information setting, what are the optimal time-based contract and the optimal cost-based contract?
2. From the perspectives of the profit of the owner and the social welfare, is the time-based contract consistently better than the cost-based contract?

To solve the above problems, we consider a model that a project owner delegates a contractor to work on one project. The project's total completion time is determined by the project's expected completion time, the contractor's effort, and some unexpected random noise. We consider two types of expected completion time. If the contractor's expected completion time is long, we call it the low-type contractor while we call the contractor with a short expected completion time the high-type contractor. In the example of Santa Monica Highway above, if the expected completion time of the contractor is 7 months, which is longer than 6 months, then we call this type of contractor an inefficient type of contractor, referred to as a low type contractor. If the expected completion time is 5 months, then we call this type of contractor high efficiency type contractor, referred to as high type contractor.

The contractor has private information about the type of expected completion time while the owner cannot see the contractor's effort that shortens the expected completion time. First, we consider the owner knows the contractor's expected completion time and obtain the optimal time-based contract that the pay is based on the time it takes to complete the project. Second, we consider a pooling time-based and a pooling cost-based contract with unobservable expected completion time. A pooling contract means that the owner only provides one payment scheme no matter what type of contractor. The contractor will choose it if the utility exceeds the outside opportunity. If the owner provides a series of payment scheme in a contract, we called it the menu of contracts. Then, we provide a series of time-based payment scheme to the contractor (i.e., the menu of time-based contracts). And then the contractor will choose the contract that best aligns with their type. We prove that the project owner would like to provide the menu of time-based contracts than the other two contracts. Besides, the pooling time-based contract is more efficient than the cost-based contract for the owner. For social welfare, if the daily operating cost is low, the menu of time-based contracts is better than the pooling time-based contracts, and the pooling time-based contract may be better if the daily operating cost is high. And that whether the cost-based is best greatly decided by the proportion of cost borne by the contractor and the daily operating cost.

The remainder of the paper is structured as follows. A review of the relevant literature in Section 2. In Section 3, we show our base model. And we first consider symmetric information setting and then asymmetric information setting in Sections 4. We consider a menu of time-based contracts in Section 5. Then we examine the numerical examples in Section 6. Finally, we provide the conclusion in Section 7. To complete the presentation, the Appendix consists of all proofs.

2. Literature review

This paper involves research on contract design in project management. Jørgensen et al. (2017) investigate how the different type of contracts affects the software project's outcome. And they find that fixed-price contract is related to higher failure than time-type and material-type contracts. In a supply chain for bioenergy that includes a power plant and farmers who possess information regarding product quality, Jiang et al. (2021) investigate the government's punishment policy. According to Bayiz and Corbett (2005), a time-based contract is one where the payment is based on how quickly the job is really completed. In their paper, there are two concurrent tasks involved in the project, and they are delegated to two separate contractors. The contractor decides how quickly they work but it is unknown to the owner. Compared with Bayiz and Corbett (2005), we classify contractors into two types: inefficient (long expected completion time of projects) and efficient (short expected completion time of projects). We also looked at the menu of contracts that can distinguish agents. In addition, Bayiz and Corbett (2005) are only dealing with hidden action, i.e., the speed of work, whereas we are dealing with not only hidden action (the contractor reduces the completion time through exerting effort) but hidden information (the project's expected completion time). Kwon et al. (2010) study whether the time-based or cost-based contracts can achieve supply chain coordination under invisible work efficiency. They find that time-based contract and cost-based contract can effectively facilitate supply chain coordination. Among them, supply chain coordination contracts are contracts where they engage project owners and contractors as a team to make decisions that are in the best interest of everyone. Chen et al. (2015) examine an "incentive payment" contract in a large project with *n* consecutive stages that are all outsourced to different contractors. They propose that their "incentive payment" contract is widely applied in practice. Hou et al. (2021) consider a scenario where the principal obtains a fixed compensation and consider a declining payment scheme that varies depending on the project's completion time with the exponential form. They focus on how the contract form affects the system coordination. Wang et al. (2022) examine the effect of government subsidies and regulations on the design of optimal BOT transportation project contracts. However, in contrast to the above literature, we study the contract design in project management with asymmetric information. We consider the expected completion time is private information owned by the contractor.

Also, our research is connected to the body of literature that focuses on analyzing optimal decisions in supply chain and marketing. On the one hand, our research has a connection to the literature on supply chain under conditions of asymmetric information. Cachon and Lariviere (2001) consider supply chains where manufacturers with private demand information provide their demand forecasts along with salary contracts to suppliers. Ha (2001) proves that if the buyer has private information about their marginal cost, the single firm solution is impossible to achieve. Chen et al. (2022) study how to induce the supply chain's partner who has inherent quality information to improve product quality. Zhang et al. (2021) create a producer-retailer supply chain structure in which retailers collect and hold customer rate of return information, resulting in asymmetric information between retailers and manufacturers. On the other hand, the literature on salesforce compensation in marketing is related. Gonik (1978) reports a useful scheme. That anticipates the demand in the salesperson's interest and incentives the salesperson to invest in hard work. The salesperson should present their estimate of the expected sales under this plan. And the compensation they received is determined by the actual sales and their submitted forecast. Later, Mantrala and Raman (1990) analyze Gonik's scheme and derive the salesperson's reaction to this scheme in a random environment. They show how management adjusts the coefficients of the scheme to make it work. Chen (2005) considers a company sells a product through the sales agent and the actual sales are determined by the market condition, i.e., the private information, the agent's effort, and random noise. Recently, Rubel and Prasad (2016) investigate the dynamic incentive in sales force compensation and find that the project owner should incentive the high (low) efficient sales agent who is averse to taking risks with a concave (convex) compensation scheme. Dai and Jerath (2019) investigate how the company motivates the sales agent to increase demand through effort. They prove that if the sales agent's effort is invisible to the company, the ideal contract has an extremely convex form, i.e., the reward is offered only when the sales agent achieves the highest sales. Sun et al. (2019) consider the interference of manufacturer with the cost-cutting under symmetric demand information setting or asymmetric demand information setting through a signaling game. Different from the above literature, we regard the expected completion time in project management as asymmetric information and design the contract with time as our main concern. At the same time, we study how asymmetric information affects project owners' returns.

3. The model

A project owner hires a contractor to work on her behalf and pays him once the project is finished. The actual completion time is an additive form consisting of the expected completion time τ that is privately known by the contractor, the effort of shortening completion time e that is also unobservable to the owner, and an exogenously random noise ϵ, and we denote it by T, i.e., $T = \tau - e + \epsilon$. This additive form occurs in many real-life cases. Take one example: if we entrust a professional translation agency to translate an article, the time taken by the translation agency to complete the translation depends on the translation ability of the staff, the effort put into translating the article (e.g., overtime work), and some special circumstances (e.g., computer crash). We assume that the owner knows that the project's expected completion time τ follows a two-point distribution, i.e., $\Pr(\tau = \tau_H) = \mu$ and $\Pr(\tau = \tau_L) = 1 - \mu$ for $0 < \mu < 1$. Without loss of generality, $\tau_H < \tau_L$ because τ_H and τ_L can represent the expected completion time of high-efficient and low-efficient contractors, respectively. The random variable ϵ follows a normal distribution that $\epsilon \sim N(0, \sigma^2)$. If the mean of random variable is not zero, our main conclusion does not change. For ease of calculation, we set it to zero. We assume that τ and ϵ are independent of each other. Furthermore, we assume that τ is enough large and the possibility of T being negative is negligible.

The timeline is shown in Figure 1. First, the project owner offers the contract $S(T)$ to the contractor. Second, the contractor observes his expected completion time τ ($= \tau_H$ or τ_L) and determines whether to sign the contract, and if so, the contractor decides effort e. Finally, the owner pays the contractor the compensation $S(T)$ upon the project is completed. Because the contract depends on the final completion time T, the actual compensation keeps uncertain until T is realized. The flowchart is shown in Figure 2.

For the contractor, the net income of completing the project is the payment received minus the cost of effort. For simplify the calculation, we assume that the contractor's cost of effort is $v(e) = \frac{1}{2}ke^2$ that is increasing with k in the form of convex function. k is the effort cost factor. It generally assumes that the principal is risk neutral and the agent is risk averse. The contractor is less able to handle risk than the project owner, so we assume that the contractor is risk averse and his utility is $-e^{-rx}$. x represents the contractor's net income $\left(x = S(T) - v(e)\right)$; r indicates the contractor's risk aversion. The larger the r, the more risk averse the contractor. Given the compensation contract $S(T)$, the contractor decides her best effort by solving:

$$\max_{e} E\left\{-e^{-r\left[S(T)-v(e)\right]}\right\}. \tag{1}$$

We know that the expected completion time is the private information to the contractor and so the contractor has known his type already. Therefore, the above expectation is about the random variable ϵ. Thus, the optimal effort depends on the payment contract $S(T)$ and the contractor's type τ. Let $e(S, \tau)$ be the best effort and $u(S, \tau)$ be the corresponding expected utility for the given payment contract $S(T)$ and his type τ. If the expected utility is not less than the contractor's best outside opportunity, and then the payment contract $S(T)$ is acceptable to the contractor. We denote the best outside opportunity as $-U_0$. Therefore, when the contractor faces a series of contracts, he will choose one of them to maximize his expected utility. And then he will compare the optimization utility with $-U_0$ and participates only if the optimization utility exceeds $-U_0$.

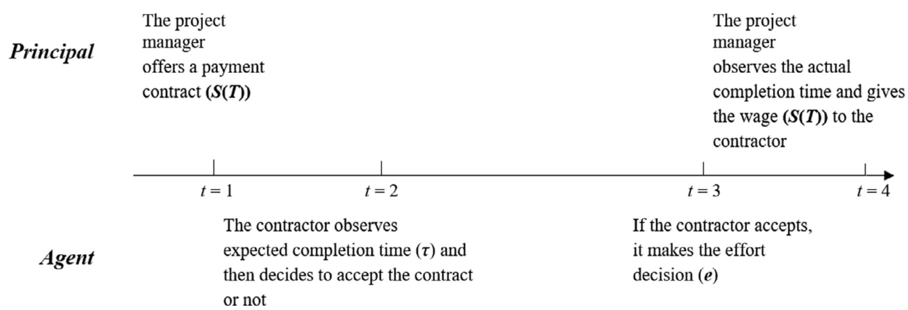


Figure 1. The timeline of events

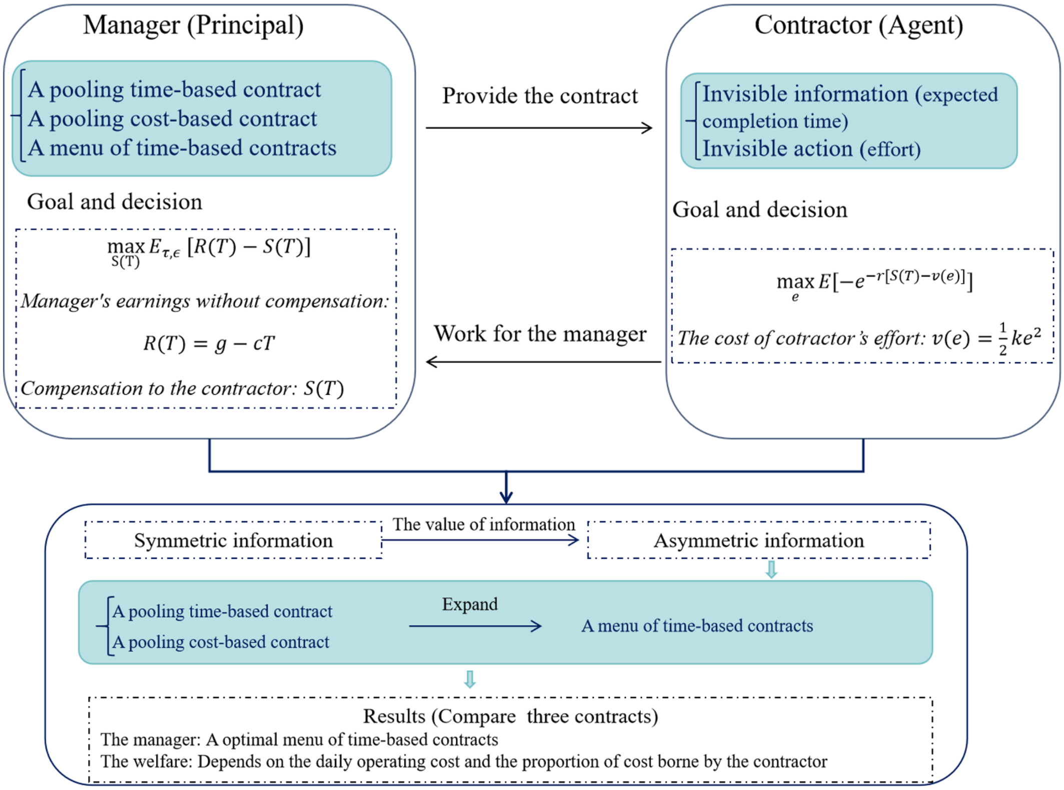


Figure 2. The framework of optimal contract in principle-agent model

For the project owner, the project's revenue is characterized by $R(T)=g-cT$. This means that the revenue is decreasing linearly concerning T. We assume that c indicates the average daily operating cost. We assume that the total cost of the project is the average cost per day multiplied by the number of days. g represents the project owner's fixed income at the completion of the project. Because the project owners have more resources, they are better able to deal with risks. So, we assume that the project owner is indifference to risk. The owner's profit can be described as $R(T)-S(T)$. Let $S_H(T)$ and $S_L(T)$ be the payment contract chosen by the high and low efficient type, respectively. Let e_{ij} represent the type-i contractor's optimal effort if he chooses the payment contract created for the type-j: $\{i,j\}\in\{H,L\}$. We consider the incentive compatibility conditions, i.e.:

$$u(S_H,\tau_H)\ge u(S_H,\tau_L), \qquad \text{(IC-LH) (2)}$$

$$u(S_L,\tau_L)\ge u(S_H,\tau_L). \qquad \text{(IC-HL) (3)}$$

And as discussed before, the contract must be acceptable to both types, i.e., $u(S_H,\tau_H)\ge -U_0$ and $u(S_L,\tau_L)\ge -U_0$. Hence, the following equation characterizes the owner's problem:

$$\max_{S_L,S_H} \ \mu E\left[R(T)-S(T)\,|\,e=e_{HH},\tau=\tau_H\right]+ (1-\mu)E\left[R(T)-S(T)\,|\,e=e_{LL},\tau=\tau_L\right]$$

$$\text{s.t.}\quad e_{LL}=e(S_L,\tau_L), \qquad \text{(IC-L)}$$

$$e_{HH}=e(S_H,\tau_H), \qquad \text{(IC-H)}$$

$$u(S_L,\tau_L)\ge u(S_H,\tau_L), \qquad \text{(IC-LH)}$$

$$u(S_H,\tau_H)\ge u(S_L,\tau_H), \qquad \text{(IC-HL)}$$

$$u(S_L,\tau_L)\ge -U_0, \qquad \text{(IR-L)}$$

$$u(S_H,\tau_H)\ge -U_0. \qquad \text{(IR-H) (4)}$$

The constraints (IC-L) and (IC-H) are the incentive compatibility conditions that means the contractor will make rational decision to maximize his expected utility. The constraint (IC-LH) ensures that the low-type contractor selects the payment contract which is created for him. Similarly, the constraint (IC-HL) ensures the high-type contractor selects the corresponding contract. The last two constraints make the contractor's utility exceeds the best outside opportunity.

Before we analyze different situations, we illustrate a transformation to simplify our states. We assume that the net income of the agent is $Q\sim N(\mu_Q,\sigma_Q)$ and the agent's expected utility is negative-exponential form, i.e., $E\left[-e^{-rQ}\right]$. We can verify that $E\left[-e^{-rQ}\right]=-e^{-rCE[Q]}$, where $CE[Q]=\mu_Q-\frac{1}{2}r\sigma_Q^2$. And μ_Q is the mean of Q and σ_Q^2 is the variance. Hence, when we need maximize the expected utility, it is same to maximize $-e^{-rCE[Q]}$.

The meaning of the symbols is provided in Table 1.

Table 1. The meaning of the symbols

Symbol	Explanation
$\tau_i(\tau_j)$	$i, j \in \{H, L\}$, the expected completion time of the type-*i*(*j*) contractor
e	The contractor's effort
ϵ	The random variable, $\epsilon \sim N(0, \sigma^2)$
μ	Proportion of high-type contractors
k	Cost coefficient
$v(e)$	The cost of effort
$S(T)$	Salary contract
$R(T)$	Project owner's income at project completion
$u(S, \tau)$	Contractor's utility under salary contract S and type τ
α	Fixed salary
β	Excitation factor
g	Project owner's fixed income
c	Average daily cost
m	Fixed payment in the cost-based contract
θ	The proportion of costs borne by the contractor

4. Analysis

In this section, we first study the symmetric information setting that the owner knows the expected completion time and the effort of the contractor. We then consider the asymmetric information with unobservable expected completion time under a pooling time-based contract and a pooling cost-based contract.

4.1. Symmetric information

Suppose that the owner can know the private information of the contractor, i.e., the expected completion time τ_i (i = L or H). For obtaining the optimal contract, we first consider the problem of the contractor according to the sequence of events.

We consider that the owner provides a time-based scheme (α, β) that the payment $S(T) = \alpha - \beta(T - d)$ based on the realized completion time T. Here, d is the project's schedule. Assume that the schedule is determined externally based on historical experience. If the actual project's completion time exceeds the deadline the contractor will receive a penalty, i.e., $\beta(T - d)$ and if the completion time is shorter than the deadline the contractor will receive a reward, i.e., $\beta(d - T)$. Note that α is a fixed compensation and β is the excitation factor.

Consider a contractor with the expected completion τ_i (i = L or H). The contractor needs to maximize his expected utility, i.e., $\max E\left\{-e^{-r\left[S(T)-v(e)\right]}\right\}$. From the simple transformations, this is the same thing as maximizing $CE\left[S(T) - v(e)\right] = \alpha_i - \beta_i\left(\tau_i - e_i - d\right) - \frac{1}{2}r\beta_i^2\sigma^2 - \frac{1}{2}ke_i^2$. Note that it is concave in e_i. Then, we can obtain the optimal effort level $e_i^* = \frac{\beta_i}{k}$ by F.O.C. We know that the effort of the contractor is positively correlated with the penalty factor β_i. Now we turn to the owner's problem. The owner aims to maximize his expected profit, i.e., $E\left[R(T) - S(T)\right] = E\left[g - c\left(\tau_i - e_i + \epsilon\right) - \alpha_i + \beta_i\left(\tau_i - e_i + \epsilon - d\right)\right]$ and must ensure that the contractor's expected utility is greater than the outside opportunity, i.e., $-e^{-r\left[\alpha_i - \beta_i\left(\tau_i - e_i - d\right) - \frac{1}{2}r\beta_i^2\sigma^2 - \frac{1}{2}ke_i^2\right]} \geq -U_0$, where $e_i = \frac{\beta_i}{k}$.

Proposition 1. *Suppose the project owner knows the contractor's expected completion time* τ_i (i = L or H). *The owner will provide the optimal contract* {α_i^*, β_i^*} *for type-i contractor* (i = L or H), *where*

$$\beta_i^* = \frac{c}{r\sigma^2 k + 1}; \tag{5}$$

$$\alpha_i^* = -\frac{\ln(U_0)}{r} + \left(\frac{1}{2}r\sigma^2 - \frac{1}{2k}\right)\left(\beta_i^*\right)^2 + \left(\tau_i - d\right)\beta_i^*. \tag{6}$$

The contractor with τ_i (i = L or H) *will take the optimal effort* $e_i^* = \frac{\beta_i}{k}$.

From Proposition 1, we conclude that if the owner faces the low-type (high-type) contractor he will offer the contract $\{\alpha_L^*, \beta_L^*\}$ ($\{\alpha_H^*, \beta_H^*\}$). The project owner makes the contractor's incentive coefficient as $\beta_i = \frac{c}{r\sigma^2 k + 1} < c$, for $r\sigma^2 k > 0$ and $0 < \frac{1}{r\sigma^2 k + 1} < 1$. β_i is a linear function of c, the greater the daily operating cost for the owner, the greater the incentive coefficient for the contractor received from the owner. Also, we know that the incentive coefficient is independent of the contractor's type, i.e., under symmetric information, no matter what type of contract, the penalty coefficient is the same. However, the fixed payment is different. The fixed payment α_i is a quadratic function of β_i and is an increasing function with the contractor's type τ_i, so lower type of contractors have higher fixed payment. If the owner faces the high-type contractor, his profit π_H^* is $\frac{\ln(U_0)}{r} + g - c\tau_H + \frac{c^2}{2k\left(1 + rk\sigma^2\right)}$. If the owner faces the low-type contractor, his profit π_L^* is $\frac{\ln(U_0)}{r} + g - c\tau_L + \frac{c^2}{2k\left(1 + rk\sigma^2\right)}$. So, if the owner can know the expected completion time and choose one of two type contractors, he will choose the high-type contractor.

4.2. Asymmetric information

Under an asymmetric information setting that the project's expected completion time and the contractor's effort are not directly observable by the project owner, we provide two common pooling contracts, i.e., a pooling time-based contract and a pooling cost-based contract.

4.2.1. A pooling time-based contract

We suppose that the owner provides a time-based contract that treats all contractors equally, i.e., no matter what type the contractor is S = (α, β),which is called a pooling time-based contract. We index the solution in this scenario with a superscript P, meaning a pooling time-based contract.

We can know from the symmetric information setting discussion that the contractor's best effort is $e_i^P(S,\tau_i)=\frac{\beta}{k}, i=L$ or H. Different contractors exert the same effort. Then, we address the owner's problem in the following manner:

$$\max_{\alpha,\beta} \quad (1-\mu)E\left[g-c\left(\tau_L-e_L+\epsilon\right)-\alpha+\beta\left(\tau_L-e_L+\epsilon-d\right)\right]+ \\ (1-\mu)E\left[g-c\left(\tau_L-e_L+\epsilon\right)-\alpha+\beta\left(\tau_L-e_L+\epsilon-d\right)\right]$$

$$\text{s.t.} \quad e_L=\frac{\beta}{k}, \qquad \text{(IC-L)}$$

$$e_H=\frac{\beta}{k}, \qquad \text{(IC-H)}$$

$$\alpha-\beta\left(\tau_H-e_H-d\right)-\frac{1}{2}ke_H^2-\frac{1}{2}r\beta^2\sigma^2\geq-\frac{\ln\left(U_0\right)}{r}, \qquad \text{(IR-L)}$$

$$\alpha-\beta\left(\tau_H-e_H-d\right)-\frac{1}{2}ke_H^2-\frac{1}{2}r\beta^2\sigma^2\geq-\frac{\ln\left(U_0\right)}{r}. \qquad \text{(IR-H)} \ (7)$$

Proposition 2. *Under the asymmetric information, we suppose the owner just provides a pooling time-based contract. We find that the optimal pooling time-based contract design is as follows*:

$$\beta^P=\max\left\{\frac{c}{r\sigma^2k+1}-\frac{\mu k\left(\tau_L-\tau_H\right)}{r\sigma^2k+1},0\right\}, \qquad (8)$$

$$\alpha^P=-\frac{\ln\left(U_0\right)}{r}+\left(\frac{1}{2}r\sigma^2-\frac{1}{2k}\right)\left(\beta^P\right)^2+\left(\tau_L-d\right)\beta^P. \qquad (9)$$

Let π^P represent the optimal expected profit of the project owner with a pooling time-based contract.

$$\pi^P=\frac{\ln\left(U_0\right)}{r}+g-c\left(\left(1-\mu\right)\tau_L+\mu\tau_H\right)+ \\ \frac{c-\mu\left(\tau_L-\tau_H\right)k}{2k}\max\left\{\frac{c}{r\sigma^2k+1}-\frac{\mu k\left(\tau_L-\tau_H\right)}{r\sigma^2k+1},0\right\}. \qquad (10)$$

Because

$$\beta^P=\max\left\{\frac{c}{r\sigma^2k+1}-\frac{\mu k\left(\tau_L-\tau_H\right)}{r\sigma^2k+1},0\right\}\leq\beta^*=\frac{c}{r\sigma^2k+1}, e^*\geq e^P.$$

Under symmetric information, the contractor's effort is lower than under asymmetric information. We know the value of information is:

$$\Delta=\pi^*-\pi^P=\begin{cases}\dfrac{c^2}{2k\left(kr\sigma^2+1\right)}, & c<\mu k\left(\tau_L-\tau_H\right)\\ \dfrac{c^2-\left[c-\mu k\left(\tau_L-\tau_H\right)\right]^2}{2k\left(kr\sigma^2+1\right)}, & c\geq\mu k\left(\tau_L-\tau_H\right).\end{cases} \qquad (11)$$

Corollary 1. *The value of information is increasing with the proportion of high-type contractors. Also, it is increasing with the daily operating cost c.*

We discuss the impact of a proportion of high-type contractors (μ) and daily operating costs (c) on the value of information showed in Figures 3 and 4 (k = 0.8, σ^2 = 0.5, r = 0.5, τ_H = 0.3 and τ_L = 1. In Figure 3, we assume that c = 0.5 and we assume that μ = 0.3 in Figure 4).

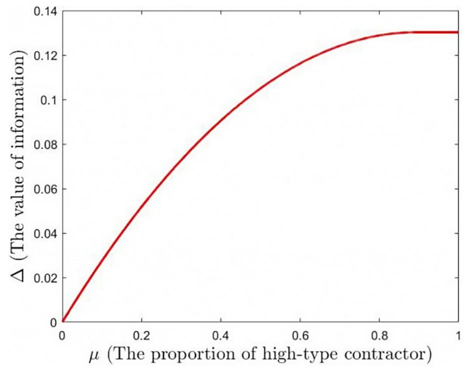


Figure 3. The impact of μ on Δ

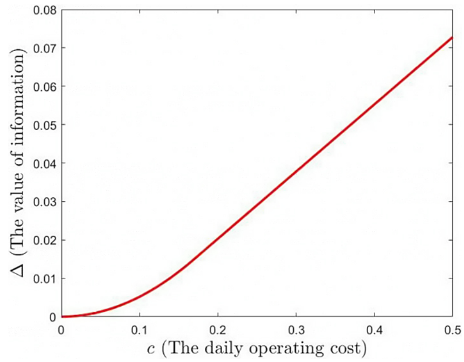


Figure 4. The impact of c on Δ

Figure 3 shows that the value of information is increasing with the proportion of high-type contractor (μ) and achieves the maximum if $\mu>\frac{c}{k\left(\tau_L-\tau_H\right)}$. If the proportion of high-type contractor is high, the incentive given by the owner is very low that induce the contractor exert little effort. Hence, With the increasing of the proportion of high-type contractor, the value of information is increasing. From the Figure 4, we know that the value of information is always increasing with the daily operating cost. We know that the owner's revenue is more sensitive if the daily operating cost is high.

In addition, if we do not consider the contractor's efforts, then this amounts to a fixed salary provided by the project owner. We can calculate that the project owner's optimal return is $\Pi_M^F=\frac{\ln\left(U_0\right)}{r}+g-c\left(\left(1-\mu\right)\tau_L+\mu\tau_H\right)$, and $\Pi_M^F\leq\Pi_M^P$. This can show that we consider the problem of contractor moral hazard in the design of project compensation contract is better than no consideration.

4.2.2. A pooling cost-based contract

We consider another common contract, i.e., a pooling cost-based contract which contains one payment scheme. The payment of a cost-based contract depends on the contractor's daily operating cost c and the period time T. We index the solution in this scenario with a superscript

C, meaning a pooling cost-based contract. We consider one general type of cost-based contracts that the owner is responsible for a friction of the overall project cost. Let the payment be $S(T) = -\theta cT + m$, where $m > 0$ depicts the fix payment. We assume that $\theta \in [0,1]$ represents the percentage of the daily operating cost borne by the contractor. If θ = 0, the project owner bears all costs. If θ = 1, the contractor bears all costs and receives a fixed payment contract. Under this contract, the project owner pays the contractor based on the time to completion of the work as well as the daily cost. The owner will obtain a reward g after the project is finished, i.e., $R(T) = g - cT$ as before. The contractor's net income is $S(T) - v(e)$. Hence the contractor needs to solve:

$$\max_{e} \; -e^{-r\left[S(T)-v(e)\right]}. \tag{12}$$

From the simple transformation, we can solve the problem through

$$\max_{e} \; m - \theta c\left(\tau_i - e\right) - \frac{1}{2}ke^2 - \frac{1}{2}r\theta^2c^2\sigma^2. \tag{13}$$

The objective function is a concave function in e. So by F.O.C, $e_i = \frac{\theta c}{k}$. e_i represents the i-type contractor's optimal effort.

The owner's expected utility can be written as $(1-\mu)\left[g-(1-\theta)c(\tau_L - e_L) - m\right] + \mu\left[g-(1-\theta)c(\tau_H - e_H) - m\right]$. Also, the related constraints must be held. Hence, the owner's problem is:

$$\begin{aligned}
\max_{m} \quad & (1-\mu)\left[g-(1-\theta)c(\tau_L - e_L) - m\right] + \\
& \mu\left[g-(1-\theta)c(\tau_H - e_H) - m\right] \\
\text{s.t.} \quad & m - \theta c\left(\tau_L - e_L\right) - \frac{1}{2}ke_L^2 - \frac{1}{2}r\theta^2c^2\sigma^2 \ge -\frac{ln(U_0)}{r}, && \text{(IR-L)} \\
& m - \theta c\left(\tau_H - e_H\right) - \frac{1}{2}ke_H^2 - \frac{1}{2}r\theta^2c^2\sigma^2 \ge -\frac{ln(U_0)}{r}, && \text{(IR-H)} \; (14) \\
& \text{where } e_i = \frac{\theta c}{k} \text{ for } i \in \{H, L\}.
\end{aligned}$$

Proposition 3. *Under the asymmetric information, we suppose that the owner provides a pooling cost-based contract. Given the exogenous proportion born by the contractor θ, the optimal cost-based solution and the optimal profit of the project owner are*:

$$m^C = \left(\frac{1}{2}rc^2\sigma^2 - \frac{c^2}{2k}\right)\theta^2 + c\tau_L\theta - \frac{ln(U_0)}{r}, \tag{15}$$

$$\pi^C = \frac{ln(U_0)}{r} + g - c\left[\mu\tau_H + (1-\mu)\tau_L\right] - \left(\frac{1}{2}rc^2\sigma^2 + \frac{c^2}{2k}\right)\theta^2 + \left[\frac{c^2}{k} - c\mu\left(\tau_L - \tau_H\right)\right]\theta. \tag{16}$$

Note that the margin and the owner's profit are both concave functions in θ.

Corollary 2. *Compared to the pooling time-based contract, adopting the cost-based contract always results in an inefficient expected profit for the owner. And if the proportion of cost born by the contractor can be determined by the project owner, there is an optimal proportion*

$$\theta = \frac{1 - \frac{\mu k\left(\tau_L - \tau_H\right)}{c}}{1 + kr\sigma^2} \quad \textit{if} \quad 0 \le \frac{1 - \frac{\mu k\left(\tau_L - \tau_H\right)}{c}}{1 + kr\sigma^2} \le 1.$$

We can know that the payment under a cost-based contract can be translated to the payment under a time-based contract by $m = \alpha$ and $\theta c = \beta$. Hence, when we maximize the owner's profit with the fixed payment m, it is the same to the situation, where given a pooling contract. However, under a pooling time-based contract, we also optimize the β, so the profit of the owner under the pooling time-based contract is always more efficient than under the pooling cost-based contract.

5. The menu of contracts

From the Corollary 1, we know that the information on the expected completion time plays a great role in the profit of the owner. To reduce asymmetric information, the project owner provides a series of time-based contracts, i.e., the menu of time-based contracts, to the contractor. And the contractors choose one contract in menu according to the maximization of their utility. We index the solution in this scenario with a superscript S. Let $S_L(\alpha_L, \beta_L)$ be the scheme created for the low-type contractor, and $S_H(\alpha_H, \beta_H)$ be the scheme created for the high-type contractor, with $\alpha_L, \alpha_H, \beta_L, \beta_H > 0$. Let e_{LH} (or e_{HL}) represent the low-type (or high-type) contractor's optimal effort who chooses the high-type (low-type) contractor's contract. The contractor will maximize his expected utility and derives the optimal effort as follows:

$$e_{LL}^S = \frac{\beta_L}{k}, \; e_{HH}^S = \frac{\beta_H}{k}; \tag{17}$$

$$e_{LH}^S = \frac{\beta_H}{k}, \; e_{HL}^S = \frac{\beta_L}{k}. \tag{18}$$

The owner's problem can be rewritten as

$$\begin{aligned}
\max_{\alpha_L, \beta_L, \alpha_H, \beta_H} \quad & (1-\mu)E\left[g - c\left(\tau_L - e_{LL} + \epsilon\right) - \alpha_L + \beta_L\left(\tau_L - e_{LL} + \epsilon - d\right)\right] + \mu E\left[g - c\left(\tau_H - e_{HH} + \epsilon\right) - \alpha_H + \beta_H\left(\tau_H - e_{HH} + \epsilon - d\right)\right] \\
\text{s.t.} \quad & \alpha_L - \beta_L\left(\tau_L - e_{LL} - d\right) - \frac{1}{2}ke_{LL}^2 - \frac{1}{2}r\beta_L^2\sigma^2 \ge && \text{(IC-LH)} \\
& \alpha_H - \beta_H\left(\tau_L - e_{LH} - d\right) - \frac{1}{2}ke_{LH}^2 - \frac{1}{2}r\beta_H^2\sigma^2, \\
& \alpha_H - \beta_H\left(\tau_H - e_{HH} - d\right) - \frac{1}{2}ke_{HH}^2 - \frac{1}{2}r\beta_H^2\sigma^2 \ge \\
& \alpha_L - \beta_L\left(\tau_H - e_{HL} - d\right) - \frac{1}{2}ke_{HL}^2 - \frac{1}{2}r\beta_L^2\sigma^2, && \text{(IC-LH)} \\
& \alpha_L - \beta_L\left(\tau_L - e_{LL} - d\right) - \frac{1}{2}ke_{LL}^2 - \frac{1}{2}r\beta_L^2\sigma^2 \ge -\frac{ln(U_0)}{r}, && \text{(IR-L)}
\end{aligned}$$

$$\alpha_H - \beta_H\left(\tau_H - e_{HH} - d\right) - \frac{1}{2}ke_{HH}^2 - \frac{1}{2}r\beta_H^2\sigma^2 \geq -\frac{ln\left(U_0\right)}{r}. \quad \text{(IR-H) (19)}$$

We can solve the above problem and obtain $\alpha_L^S, \alpha_H^S, \beta_L^S$ and β_H^S.

Proposition 4. *Under the asymmetric information, we assume that the project owner provides a menu of contracts with a series of time-based contracts. We found that the optimal menu of time-based contracts is as follows*:

$$\beta_H^S = \frac{c}{r\sigma^2 k + 1}; \quad (20)$$

$$\beta_L^S = \max\left\{\frac{c}{r\sigma^2 k + 1} - \frac{\mu k\left(\tau_L - \tau_H\right)}{\left(1-\mu\right)\left(r\sigma^2 k + 1\right)}, 0\right\}; \quad (21)$$

$$\alpha_H^S = -\frac{\ln\left(U_0\right)}{r} + \left(\frac{1}{2}r\sigma^2 - \frac{1}{2k}\right)\left(\beta_H^S\right)^2 + \left(\tau_H - d\right)\beta_H^S + \left(\tau_L - \tau_H\right)\beta_L^S; \quad (22)$$

$$\alpha_L^S = -\frac{\ln\left(U_0\right)}{r} + \left(\frac{1}{2}r\sigma^2 - \frac{1}{2k}\right)\left(\beta_L^S\right)^2 + \left(\tau_L - d\right)\beta_L^S. \quad (23)$$

Because

$$\beta_L^S = \max\left\{\frac{c}{r\sigma^2 k + 1} - \frac{\mu k\left(\tau_L - \tau_H\right)}{\left(1-\mu\right)\left(r\sigma^2 k + 1\right)}, 0\right\} \leq \frac{c}{r\sigma^2 k + 1} = \beta_L^*,$$

$e_H^* = e_H^S$ and $e_L^* \geq e_L^S$. Under the menu of time-based contracts, the high-type contractor exerts the same effort as under the symmetric information. However, the low-type contractor will exert lower effort than under the symmetric information.

Corollary 3. We also derive the following corollas under three situations.

(1) *The effort of the contractor.* $e_H^* = e_H^S \geq e_H^P$ *and* $e_L^* \geq e_L^P \geq e_L^S$.

(2) *The profit of the project owner.* $\pi^* \geq \pi^S \geq \pi^P$.

(3) *The utility of the contractor.* $u_L^* = u_L^P = u_L^S$ *and* $u_H^* \leq u_H^S \leq u_H^P$.

(4) *The social welfare. If* $c \leq \mu k\left(\tau_L - \tau_H\right)$, $W^* \geq W^S \geq W^P$.

If $\mu k\left(\tau_L - \tau_H\right) < c < \dfrac{\mu k\left(\tau_L - \tau_H\right)}{1-\mu}$ *and* $U_0 \leq U_1$, $W^S \geq W^P$. *If* $c \geq \dfrac{\mu k\left(\tau_L - \tau_H\right)}{1-\mu}$ *and* $U_0 \leq U_2$, $W^S \geq W^P$.

$$U_1 = \frac{\mu^2 c^2 - \left[c - \mu k\left(\tau_L - \tau_H\right)\right]^2}{2k\mu\left(r\sigma^2 k + 1\right)\left[1 - e^{-rB\left(\tau_L - \tau_H\right)}\right]},$$

$$U_2 = \frac{\mu^3 k\left(\tau_L - \tau_H\right)^2}{2k\mu\left(r\sigma^2 k + 1\right)\left[e^{-rA\left(\tau_L - \tau_H\right)} - e^{-rB\left(\tau_L - \tau_H\right)}\right]},$$

$$A = \frac{c - \dfrac{\mu k\left(\tau_L - \tau_H\right)}{1-\mu}}{kr\sigma^2 + 1} \text{ and } B = \frac{c - \mu k\left(\tau_L - \tau_H\right)}{kr\sigma^2 + 1}.$$

Therefore, the project owner's lack of expected completion time information result in reduced efforts. For the low-type contractor, he exerts more effort if the pooling time-based contract is provided rather than the menu of time-based contracts. However, the high-type contractor will exert more effort if the menu of time-based contracts is provided. For the project owner, the profit of providing a menu of time-based contracts is higher than that of providing a pooling time-based contract, so the owner is more ready to offer a menu of time-based contracts to distinguish between different types of contractors. And the profit in a scenario where information is symmetric is greater than when information is asymmetric. For contractors, different types of contractors have different utility situations under different situations. For the low-type contractors, the utility is $-U_0$, whether it offers a menu of time-based contracts or a pooling time-based contract. For the high-type contractors, he prefers the owner to provide only one contract. For the social welfare, if the daily operating cost is low, the social welfare under the menu of contracts is greater than under the pooling contract. We know that when the daily operating cost is low, the profit of the owner is insensitive to the time so the incentive coefficient is zero. Hence, the utility under the pooling contract is the same as under the menu of contracts and the social welfare is not related to the outside opportunity. If the daily operating cost is high, the social welfare under the menu of time-based contracts is greater than under the pooling time-based contract only if the outside opportunity is high. When we choose the menu of time-based contracts instead of the pooling time-based contract, the decline in the contractor's utility is less than the increase in the owner's utility only if the outside opportunity is relatively high.

6. Numerical examples

To further explore the nature of the results, we furnish the numerical experiments in this section. Taking into account the proportion of high type contractors (μ) has a great impact on the owner's contract, i.e., α, β, contractor's effort (e) and owner's profit (π). We compare the magnitude and trend of these values with the proportion of the high-type contractors (ρ) under the three situations, i.e., symmetric information (the first-best contract), asymmetric information (the menu of time-based contracts and the time-based pooling contract). We also consider the cost-based contract compared to the time-based contract. Suppose that the risk evasion coefficient value of the contractor is moderate (r = 0.5); the daily operating cost is neither large nor small (c = 0.5); the project completion time specified by the project owner, i.e., the deadline is 0.5 year; the cost coefficient of effort is high (k = 0.8); the expected completion time of high-type contractor (τ_H) is 0.3 year and the expected completion time of a low-type contractor (τ_L) is 1 year. Assume that fixed income received by the project owner upon completion of the project is g = 1 and external opportunity utility is $\ln\left(U_0\right) = -0.1$. To simplify the calculations, assume $\sigma^2 = 0.5$. We display the results for a numerical simulation after assigning values in Figures 5 to 8.

As depicted in Figures 5 to 6, the contractor's effort (e) and the fixed payment (α) decrease with the increasing proportion of the high-type contractor (μ). We analyze that the impact of μ on β is similar to the e because they are different in a constant. On the one hand, the effort of the contractor is larger under symmetric information than under a pooling time-based contract, and the menu of time-based contracts increases the high-type contractor's effort and decreases the low-type contractor's effort. On the other hand, under symmetric information, the high-type contractor receives the highest fixed payment and the low-type contractor receives the lowest fixed payment. Given the pooling time-based contract, the contractor receives a fixed payment which is higher than under the menu of time-based contracts. Under the menu of time-based contracts, the low-type contractor always receives a greater fixed payment than the high-type contractor. Although this is not true for other assignments, in general, the fixed payment (α) decreases with the increasing proportion of the high-type contractor (μ).

Figure 7a indicates that the profit of the owner (π) is increasing with the proportion of the high-type contractor (ρ). It is obvious that under symmetric information setting, the owner's profit is the most. In offering the time-based pooling contract, the owner's profit is minimal. It means that the project owner gets more profit if he has more information. In Figure 7b, we consider the influence of θ (the portion of cost borne by the contractor) on the profit and we know that the profit under the cost-based contract is more inefficient than the time-based contract for any θ. Also, it identifies the corollary 3 that if θ can be determined by the owner, there is an optimal sharing cost proportion θ.

In Figure 8a, we know that the social welfare of the cost-based contract is greater than that of the menu of time-based contracts in the crescent region. Outside this area, the opposite is true. This is equivalent to, the side of the graph is when the social welfares of the two form contracts are equal. From the Figures 8b to 8d, we check if the pattern will occur in different situations. We find that the graph still exists in many cases, indicating that it is not because of the assignment problem that the cost-based contract is likely to be superior to the menu of time-based contracts. Figure 8a indicates that the menu of time-based contracts is always better than the cost-based contract if the cost is too high. However, if the cost is moderate, there is a proportion of cost born by the contractor that makes the cost-based contract better than the time-based contract.

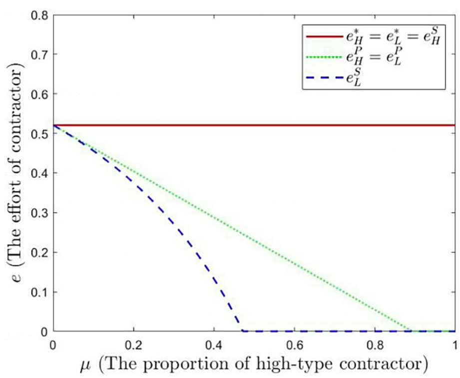


Figure 5. The impact of μ on *e*

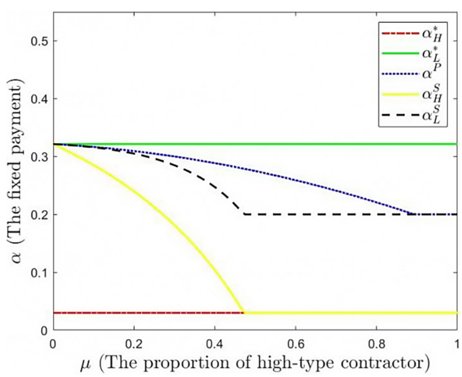
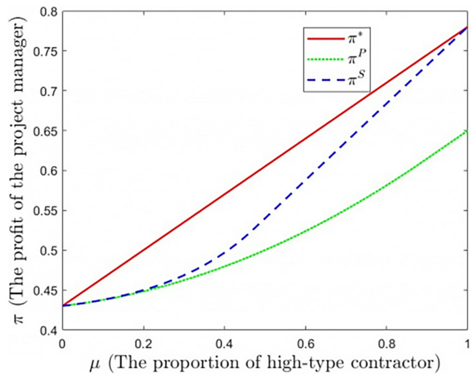


Figure 6. The impact of μ on α

a) for μ



b) for θ

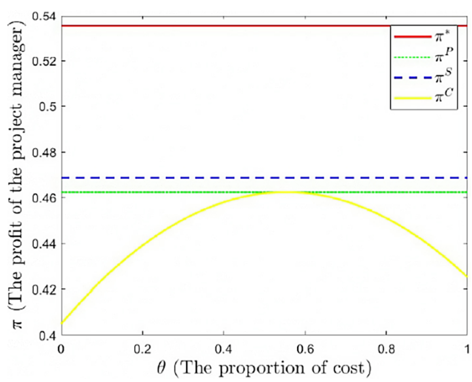


Figure 7. The profit of the project owner

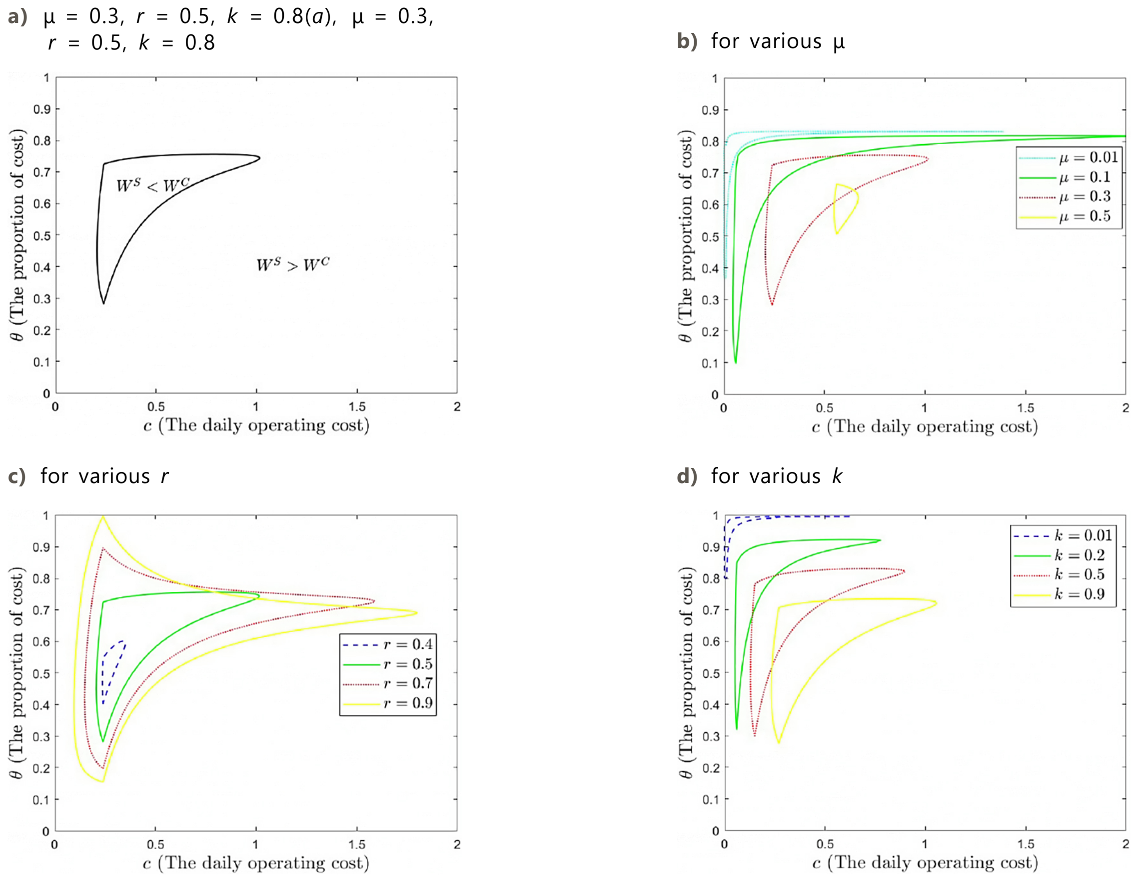


Figure 8. The impact of θ and *c* on the social welfare

7. Conclusions

This paper analyzes the problem that how a project owner designs a time-based contract and a cost-based contract if he delegates a project to a contractor endowed with private information and hidden effort action. As mentioned above, when we entrust a professional translation agency to translate an article, the translation agency is more experienced with translation skills and knows the approximate time of complete the translation task (hidden information), and it can shorten the time through overtime work (hidden action). And then we construct the principal-agent models where both adverse selection problem and moral hazard problem are present. We suppose that the owner provides a time-based contract or a cost-based contract to obtain the maximum profit. We derive the solution under symmetric or asymmetric information setting. We calculate the owner's optimal incentive coefficient (β), the fixed compensation (α), the margin of contractor (*m*) and also the optimal effort decision (*e*) of the contractor.

We derive some findings. The optimal effort (*e*) of the contractor is related to the incentive coefficient (β). The higher incentive coefficient means that there is a higher payoff for finishing early and a higher loss if the contractor delays. So a higher incentive coefficient provided by the owner leads to an increase effort exerted by th contractor. The owner's lack of expected completion time information result in reduced efforts. For the low-type contractor, he exerts more effort if the pooling time-based contract is provided rather than the menu of time-based contracts. However, the high-type contractor will exert more effort if the menu of time-based contracts is provided.

For the contractor, the more information, the more profit. And the cost-based contract is inefficient than the time-based contract no matter how much the contractor shares the cost (θ). On the contrary, the more information revealed, the less utility the high-type contractor has. For the low-type contractor, whatever which situation, his utility is the same which is equal to the outside opportunity. On the whole, the high-type contractor's utility is higher than the low-type contractor's utility. Hence, the project owner must provide the menu of the time-based contracts.

For the social welfare, if the daily operating cost is low, the welfare under the menu of time-based contracts is greater than under the pooling time-based contract. If the daily operating cost is high, the social welfare under the menu of time-based contracts is greater than under the pooling contract only if the outside opportunity is high. From the numerical examples, we know that the social welfare under the pooling cost-based contract may exceed under the time-based contract if the daily operating cost is moderate. It means that when we choose the cost-based contract over the menu of time-based contracts, in some proportion of cost born by the contractor, the increase in the contractor's utility is greater than the decrease in the owner's utility.

In general, we design an incentive mechanism in the contracting process of construction projects. We pay particular attention to completion times in construction projects, taking into account both private information about the expected completion time of the contractor and the incentive pay design for invisible effort actions. Compared with the previous literature, we paid more attention to the asymmetric information of expected completion time, which enriched the literature of incentive contract design. Our research helps project owners to effectively control the completion time of the project and maximize the utility of the project. At the same time, we extend this incentive system to different entities. Enterprises or governments can get an optimal incentive design by inputting some known information.

These findings effectively provide some new insights for enterprises to be entrusted with the agency in compensation contract design. Also, we have some limitations. We make too many assumptions in our models and the setting of the expected completion time of the project only considers the two-point distribution, and does not consider the continuous distribution. There are several potential directions for future work. For example, the extensions of this framework could include an audit setting in which the owner can examine the progress and set a penalty for the contractors. The model can also be extended to multiple parallel or serial projects and see the differences under different project structures. Also, the literature can extend our model to set the cost-based contract's share fraction.

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Author contributions

Zhiyuan Chen and Guanqun Shi conceived and designed the study. Zhiyuan Chen contributed to the theoretical framework and validation of the model. Guanqun Shi was responsible for the development of the model and oversaw the data analysis. Jingru Tong wrote the first draft of the article. Yichen Wang was responsible for the interpretation of the results. All authors contributed to the revision of the manuscript and approved the final version.

Disclosure statement

All the author(s) declare no competing financial or any other kind of interest.

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APPENDIX

Proof of Proposition 1

Under the symmetric information situation, the contractor's problem is as follows:

$$\max_{e_i} \ E\left\{-e^{-r\left[\alpha_i-\beta_i\left(\tau_i-e_i+\epsilon-d\right)-\frac{1}{2}ke_i^2\right]}\right\}.$$

We can easily know that $E\left(-e^{-rY}\right)=-e^{-rCE\left[Y\right]}$ and $CE\left[Y\right]=\mu_Y-\frac{1}{2}r\sigma_Y^2$. Hence,

$$E\left\{-e^{-r\left[\alpha_i-\beta_i\left(\tau_i-e_i+\epsilon-d\right)-\frac{1}{2}ke_i^2\right]}\right\}=-e^{-r\left[\alpha_i-\beta_i\left(\tau_i-e_i-d\right)-\frac{1}{2}ke_i^2-\frac{1}{2}r\beta_i^2\sigma^2\right]}.$$

We know $-e^{-rx}$ is an increasing function of x because $\frac{\partial\left(-e^{-rx}\right)}{\partial x}=re^{-rx}>0$. Hence the contractor's problem can be reduced to

$$\max_{e_i} \ \alpha_i-\beta_i\left(\tau_i-e_i-d\right)-\frac{1}{2}ke_i^2-\frac{1}{2}r\beta_i^2\sigma^2.$$

Let $I\left(e_i\right)=\alpha_i-\beta_i\left(\tau_i-e_i-d\right)-\frac{1}{2}ke_i^2-\frac{1}{2}r\beta_i^2\sigma^2$. Because $\frac{\partial^2 I}{\partial e_i^2}=-k<0$, $I\left(e_i\right)$ is a concave function. By F.O.C, we can obtain the optimal solution, i.e., $e_i^*=\frac{\beta_i}{k}$. Turn to the owner's problem as follows:

$$\max_{\alpha_i,\beta_i} \quad E\left[g-c\left(\tau_i-e_i+\epsilon\right)-\alpha_i+\beta_i\left(\tau_i-e_i+\epsilon-d\right)\right]$$

$$\text{s.t.} \quad e_i^*=\frac{\beta_i}{k},$$

$$-e^{-r\left[\alpha_i-\beta_i\left(\tau_i-e_i-d\right)-\frac{1}{2}ke_i^2-\frac{1}{2}r\beta_i^2\sigma^2\right]}\geq -U_0.$$

The objective function has only one random variable ϵ and $\epsilon\sim N\left(0,\sigma^2\right)$. Let Π_M be the owner's profit and then $\pi_M=g-c\left(\tau_i-\frac{\beta_i}{k}\right)-\alpha_i+\beta_i\left(\tau_i-\frac{\beta_i}{k}-d\right)$ using $\left(IC-i\right)$. Now we consider the constraint (IR-i). By simplifying, we know that the condition (IR-i) is equal to $\alpha_i-\beta_i\left(\tau_i-\frac{\beta_i}{k}-d\right)-\frac{1}{2}ke_i^2-\frac{1}{2}r\beta_i^2\sigma^2\geq-\frac{\ln\left(U_0\right)}{r}$. If $\alpha_i-\beta_i\left(\tau_i-\frac{\beta_i}{k}-d\right)-\frac{1}{2}k\left(\frac{\beta_i}{k}\right)^2-\frac{1}{2}r\beta_i^2\sigma^2>-\frac{\ln\left(U_0\right)}{r}$, we can reduce α_i without violating any constraint and increase the objective function Π_M. Therefore, the constraint (IR-i) must hold as an equality. Then $\alpha_i=\beta_i\left(\tau_i-\frac{\beta_i}{k}-d\right)+\frac{1}{2}k\left(\frac{\beta_i}{k}\right)^2+\frac{1}{2}r\beta_i^2\sigma^2-\frac{\ln\left(U_0\right)}{r}=-\frac{\ln\left(U_0\right)}{r}+\left(\frac{1}{2}r\sigma^2-\frac{1}{2k}\right)\left(\beta_i\right)^2+\left(\tau_i-d\right)\beta_i$. We substitute the expression of α_i in the owner's objective function and obtain the objective function as

$$g-c\left(\tau_i-\frac{\beta_i}{k}\right)-\left[-\frac{\ln\left(U_0\right)}{r}+\left(\frac{1}{2}r\sigma^2-\frac{1}{2k}\right)\left(\beta_i\right)^2+\left(\tau_i-d\right)\beta_i\right]+\beta_i\left(\tau_i-\frac{\beta_i}{k}-d\right)=-\left(\frac{1}{2k}+\frac{1}{2}\sigma^2 r\right)\beta_i^2+\frac{c}{k}\beta_i+g-c\tau_i+\frac{\ln\left(U_0\right)}{r}.$$

Because

$$\frac{\partial^2\pi_M}{\partial\beta_i^2}=-\left(\sigma^2 r+\frac{1}{k}\right)<0,$$

we obtain the optimal solution of β_i by F.O.C, i.e.,

$$\beta_i^*=\frac{c}{r\sigma^2 k+1},$$

$$\alpha_i^*=-\frac{\ln\left(U_0\right)}{r}+\left(\frac{1}{2}r\sigma^2-\frac{1}{2k}\right)\left(\beta_i^*\right)^2+\left(\tau_i-d\right)\beta_i^*.$$

Proof of Proposition 2

Under a pooling time-based contract, the contractor maximizes his expected utility to get the optimal effort, i.e., $e_L = e_H = \frac{\beta}{k}$. The owner needs to solve the problem as following:

$$\max_{\alpha,\beta} \quad (1-\mu)E\left[g - c\left(\tau_L - e_L + \epsilon\right) - \alpha + \beta\left(\tau_L - e_L + \epsilon - d\right)\right] + \mu E\left[g - c\left(\tau_H - e_H + \epsilon\right) - \alpha + \beta\left(\tau_H - e_H + \epsilon - d\right)\right]$$

$$\text{s.t.} \quad e_L = \frac{\beta}{k},$$

$$e_H = \frac{\beta}{k},$$

$$-e^{-r\left[\alpha - \beta\left(\tau_L - e_L - d\right) - \frac{1}{2}ke_L^2 - \frac{1}{2}r\beta^2\sigma^2\right]} \ge -U_0,$$

$$-e^{-r\left[\alpha - \beta\left(\tau_H - e_H - d\right) - \frac{1}{2}ke_H^2 - \frac{1}{2}r\beta^2\sigma^2\right]} \ge -U_0.$$

The constraints (IR-L) and (IR-H) can be simplified to

$$\alpha - \beta\left(\tau_L - e_L - d\right) - \frac{1}{2}ke_L^2 - \frac{1}{2}r\beta^2\sigma^2 \ge -\frac{\ln\left(U_0\right)}{r},$$

$$\alpha - \beta\left(\tau_H - e_H - d\right) - \frac{1}{2}ke_H^2 - \frac{1}{2}r\beta^2\sigma^2 \ge -\frac{\ln\left(U_0\right)}{r}.$$

Let $U_L = \alpha - \beta\left(\tau_L - e_L - d\right) - \frac{1}{2}ke_L^2 - \frac{1}{2}r\beta^2\sigma^2$ and $U_H = \alpha - \beta\left(\tau_H - e_H - d\right) - \frac{1}{2}ke_H^2 - \frac{1}{2}r\beta^2\sigma^2$. Then, using $e_L = e_H = \frac{\beta}{k}$, we can know that $U_H - U_L = \beta\left(\tau_L - \tau_H\right) > 0$ for $\tau_L > \tau_H$ and $\beta > 0$. Therefore, if the constraint (IR-L) is satisfied, the constraint (IR-H) must be satisfied. We can eliminate the constraint (IR-H). Let π_M be the principal's profit and then $\pi_M = (1-\mu)\left[g - c\left(\tau_L - \frac{\beta}{k}\right) - \alpha + \beta\left(\tau_L - \frac{\beta}{k} - d\right)\right] + \mu\left[g - c\left(\tau_H - \frac{\beta}{k}\right) - \alpha + \beta\left(\tau_H - \frac{\beta}{k} - d\right)\right]$. The problem can be reduced to

$$\max_{\alpha,\beta} \quad (1-\mu)\left[g - c\left(\tau_L - \frac{\beta}{k}\right) - \alpha + \beta\left(\tau_L - \frac{\beta}{k} - d\right)\right] + \mu\left[g - c\left(\tau_H - \frac{\beta}{k}\right) - \alpha + \beta\left(\tau_H - \frac{\beta}{k} - d\right)\right]$$

$$\text{s.t.} \quad \alpha - \beta\left(\tau_L - \frac{\beta}{k} - d\right) - \frac{1}{2}k\left(\frac{\beta}{k}\right)^2 - \frac{1}{2}r\beta^2\sigma^2 \ge -\frac{\ln\left(U_0\right)}{r}.$$

Similarly, If the constraint (IR-L) is not an equality, we can reduce α without violating any constraint and increase the objective function. So $\alpha = \beta\left(\tau_L - \frac{\beta}{k} - d\right) + \frac{1}{2}k\left(\frac{\beta}{k}\right)^2 + \frac{1}{2}r\beta^2\sigma^2 - \frac{\ln\left(U_0\right)}{r} = -\frac{\ln\left(U_0\right)}{r} + \left(\frac{1}{2}r\sigma^2 - \frac{1}{2k}\right)\beta^2 + \left(\tau_L - d\right)\beta$. We substitute the expression of α in the objective function and obtain

$$\pi_M = \mu\left[g - c\left(\tau_H - \frac{\beta}{k}\right) - \alpha + \beta\left(\tau_H - \frac{\beta}{k} - d\right)\right] + (1-\mu)\left[g - c\left(\tau_L - \frac{\beta}{k}\right) - \alpha + \beta\left(\tau_L - \frac{\beta}{k} - d\right)\right] =$$
$$-\left(\frac{1}{2k} + \frac{1}{2}r\sigma^2\right)\beta^2 + \left[\frac{c}{k} - \mu\left(\tau_L - \tau_H\right)\right]\beta + g - c\left[\mu\tau_H + (1-\mu)\tau_L\right] + \frac{\ln\left(U_0\right)}{r}.$$

Because

$$\frac{\partial^2\pi_M}{\partial\beta^2} = -\left(\sigma^2 r + \frac{1}{k}\right) < 0,$$

by F.O.C,

$$\beta^P = \max\left\{\frac{c}{r\sigma^2 k + 1} - \frac{\mu k\left(\tau_H - \tau_L\right)}{r\sigma^2 k + 1}, 0\right\}.$$

So

$$\alpha^P = -\frac{\ln\left(U_0\right)}{r} + \left(\frac{1}{2}r\sigma^2 - \frac{1}{2k}\right)\left(\beta^P\right)^2 + \left(\tau_L - d\right)\beta^P.$$

Proof of Π_M^F. The owner's problem is as follows:

$$\max_{\alpha_L,\beta_L,\alpha_H,\beta_H} (1-\mu)E\left[g-c\left(\tau_L+\epsilon\right)-\alpha_L+\beta_L\left(\tau_L+\epsilon-d\right)\right]+\mu E\left[g-c\left(\tau_H+\epsilon\right)-\alpha_H+\beta_H\left(\tau_H+\epsilon-d\right)\right]$$

s.t.

$$\alpha_L-\beta_L\left(\tau_L-d\right)-\frac{1}{2}r\beta_L^2\sigma^2\ge\alpha_H-\beta_H\left(\tau_L-d\right)-\frac{1}{2}r\beta_H^2\sigma^2,$$

$$\alpha_H-\beta_H\left(\tau_H-d\right)-\frac{1}{2}r\beta_H^2\sigma^2\ge\alpha_L-\beta_L\left(\tau_H-d\right)-\frac{1}{2}r\beta_L^2\sigma^2,$$

$$\alpha_L-\beta_L\left(\tau_L-d\right)-\frac{1}{2}r\beta_L^2\sigma^2\ge-\frac{\ln\left(U_0\right)}{r},$$

$$\alpha_H-\beta_H\left(\tau_H-d\right)-\frac{1}{2}r\beta_H^2\sigma^2\ge-\frac{\ln\left(U_0\right)}{r},$$

We obtain that $\alpha_L=\alpha_H=-\frac{\ln\left(U_0\right)}{r}$, $\beta_H=\beta_L=0$ by calculated.

Proof of Proposition 3

The contractor's optimal effort decision is $e_i=\frac{\theta c}{k}$, $i\in\{L,H\}$. The owner's problem is:

$$\max_m (1-\mu)\left[g-(1-\theta)c\left(\tau_L-e_L\right)-m\right]+\mu\left[g-(1-\theta)c\left(\tau_H-e_H\right)-m\right],$$

$$m-\theta c\left(\tau_L-e_L\right)-\frac{1}{2}ke_L^2-\frac{1}{2}r\theta^2c^2\sigma^2\ge-\frac{\ln\left(U_0\right)}{r}, \tag{IR-L}$$

$$m-\theta c\left(\tau_H-e_H\right)-\frac{1}{2}ke_H^2-\frac{1}{2}r\theta^2c^2\sigma^2\ge-\frac{\ln\left(U_0\right)}{r}, \tag{IR-H}$$

where $e_i=\frac{\theta c}{k}$ for $i\in\{H,L\}$. Let $U_L=m-\theta c\left(\tau_L-e_L\right)-\frac{1}{2}ke_L^2-\frac{1}{2}r\theta^2c^2\sigma^2$ and $U_H=m-\theta c\left(\tau_H-e_H\right)-\frac{1}{2}ke_H^2-\frac{1}{2}r\theta^2c^2\sigma^2$. Because $U_H-U_L=\theta c\left(\tau_L-\tau_H\right)>0$, if the constraint (IR-L) holds, the constraint (IR-H) must be satisfied. So we can omit the constraint (IR-H). From the objective function, we know that if *m* is smaller, the value is greater. So the optimal $m^C=-\frac{\ln\left(U_0\right)}{r}+\theta c\left(\tau_L-e_L\right)+\frac{1}{2}ke_L^2+\frac{1}{2}r\theta^2c^2\sigma^2=\left(\frac{1}{2}rc^2\sigma^2-\frac{c^2}{2k}\right)\theta^2+c\tau_L\theta-\frac{\ln\left(U_0\right)}{r}$.

Proof of Proposition 4

Under the asymmetric information, if the owner provides a menu of contracts, the contractor maximizes his expected utility to get the optimal effort, i.e., $e_{LL}=e_{HL}=\frac{\beta_L}{k}$ and $e_{HH}=e_{LH}=\frac{\beta_H}{k}$. e_{ij} represents the optimal effort if the type-*i* contractor pretends to be the type-*j* contractor. The owner's problem is as follows:

$$\max_{\alpha_L,\beta_L,\alpha_H,\beta_H} (1-\mu)E\left[g-c\left(\tau_L-e_{LL}+\epsilon\right)-\alpha_L+\beta_L\left(\tau_L-e_{LL}+\epsilon-d\right)\right]+\mu E\left[g-c\left(\tau_H-e_{HH}+\epsilon\right)-\alpha_H+\beta_H\left(\tau_H-e_{HH}+\epsilon-d\right)\right]$$

s.t.

$$a_L-\beta_L\left(\tau_L-e_{LL}-d\right)-\frac{1}{2}ke_{LL}^2-\frac{1}{2}r\beta_L^2\sigma^2\ge\alpha_H-\beta_H\left(\tau_L-e_{LH}-d\right)-\frac{1}{2}ke_{LH}^2-\frac{1}{2}r\beta_H^2\sigma^2,$$

$$a_H-\beta_H\left(\tau_H-e_{HH}-d\right)-\frac{1}{2}ke_{HH}^2-\frac{1}{2}r\beta_H^2\sigma^2\ge\alpha_L-\beta_L\left(\tau_H-e_{HL}-d\right)-\frac{1}{2}ke_{HL}^2-\frac{1}{2}r\beta_L^2\sigma^2,$$

$$a_L-\beta_L\left(\tau_L-e_{LL}-d\right)-\frac{1}{2}ke_{LL}^2-\frac{1}{2}r\beta_L^2\sigma^2\ge-\frac{\ln\left(U_0\right)}{r},$$

$$a_H-\beta_H\left(\tau_H-e_{HH}-d\right)-\frac{1}{2}ke_{HH}^2-\frac{1}{2}r\beta_H^2\sigma^2\ge-\frac{\ln\left(U_0\right)}{r},$$

where $e_{LL}=\frac{\beta_L}{k}$, $e_{HH}=\frac{\beta_H}{k}$, $e_{LH}=\frac{\beta_H}{k}$, and $e_{HL}=\frac{\beta_L}{k}$. Let

$$U_{LL}=\alpha_L-\beta_L\left(\tau_L-e_{LL}-d\right)-\frac{1}{2}ke_{LL}^2-\frac{1}{2}r\beta_L^2\sigma^2,$$

$$U_{LH}=\alpha_H-\beta_H\left(\tau_L-e_{LH}-d\right)-\frac{1}{2}ke_{LH}^2-\frac{1}{2}r\beta_H^2\sigma^2,$$

$$U_{HH}=\alpha_H-\beta_H\left(\tau_H-e_{HH}-d\right)-\frac{1}{2}ke_{HH}^2-\frac{1}{2}r\beta_H^2\sigma^2,$$

$$U_{HL}=\alpha_L-\beta_L\left(\tau_H-e_{HL}-d\right)-\frac{1}{2}ke_{HL}^2-\frac{1}{2}r\beta_L^2\sigma^2.$$

So, the constraints are equal to

$U_{LL}\ge U_{LH}$,

$U_{HH}\ge U_{HL}$,

$$U_{LL} \geq -\frac{\ln(U_0)}{r},$$
$$U_{HH} \geq -\frac{\ln(U_0)}{r}.$$

We observe the constraints and simplify the above optimization problem. First, the constraints (IC-HL) and (IR-L) together imply (IR-H). Because if $U_{LL} \geq -\frac{\ln(U_0)}{r}$, then $U_{HH} \geq U_{HL} = U_{LL} + \beta_L(\tau_L - \tau_H) \geq -\frac{\ln(U_0)}{r}$, i.e., $U_{HH} \geq -\frac{\ln(U_0)}{r}$. Thus, we eliminates the constraint (IR-H). Second, we guess that the constraint (IC-LH) always holds and check it ex post. Let Π_M be the owner's profit. Using $e_{LL} = \frac{\beta_L}{k}$ and $e_{HH} = \frac{\beta_H}{k}$, we know that

$$\pi_M = (1-\mu)\left[g - c\left(\tau_L - \frac{\beta_L}{k}\right) - \alpha_L + \beta_L\left(\tau_L - \frac{\beta_L}{k} - d\right)\right] + \mu\left[g - c\left(\tau_H - \frac{\beta_H}{k}\right) - \alpha_H + \beta_H\left(\tau_H - \frac{\beta_H}{k} - d\right)\right].$$

Thus, the problem can be simplified to

$$\max_{\alpha_L,\beta_L,\alpha_H,\beta_H} \quad (1-\mu)\left[g - c\left(\tau_L - \frac{\beta_L}{k}\right) - \alpha_L + \beta_L\left(\tau_L - \frac{\beta_L}{k} - d\right)\right] + \mu\left[g - c\left(\tau_H - \frac{\beta_H}{k}\right) - \alpha_H + \beta_H\left(\tau_H - \frac{\beta_H}{k} - d\right)\right]$$

$$\text{s.t.} \quad \alpha_H - \beta_H\left(\tau_H - \frac{\beta_H}{k} - d\right) - \frac{1}{2}k\left(\frac{\beta_H}{k}\right)^2 - \frac{1}{2}r\beta_H^2\sigma^2 \geq \alpha_L - \beta_L\left(\tau_H - \frac{\beta_L}{k} - d\right) - \frac{1}{2}k\left(\frac{\beta_L}{k}\right)^2 - \frac{1}{2}r\beta_L^2\sigma^2, \quad \text{(IC-HL)}$$

$$\alpha_L - \beta_L\left(\tau_L - \frac{\beta_L}{k} - d\right) - \frac{1}{2}k\left(\frac{\beta_L}{k}\right)^2 - \frac{1}{2}r\beta_L^2\sigma^2 \geq -\frac{\ln(U_0)}{r}. \quad \text{(IR-L)}$$

The constraint (IR-L) must hold as an equality because otherwise the owner can decrease α_L without violating any constraint and increase the objective function. Similarly, the constraint (IC-HL) must hold as an equality. Then

$$\alpha_L = \beta_L\left(\tau_L - \frac{\beta_L}{k} - d\right) + \frac{1}{2}k\left(\frac{\beta_L}{k}\right)^2 + \frac{1}{2}r\beta_L^2\sigma^2 - \frac{\ln(U_0)}{r} = \left(\frac{1}{2}\sigma^2 r - \frac{1}{2k}\right)\beta_L^2 + (\tau_L - d)\beta_L - \frac{\ln(U_0)}{r},$$
$$\alpha_H = \left(\frac{1}{2}\sigma^2 r - \frac{1}{2k}\right)\beta_H^2 + (\tau_H - d)\beta_H - \frac{\ln(U_0)}{r} + \beta_L(\tau_L - \tau_H).$$

We substitute the expressions of α_H, α_L in the owner's objective function and

$$\pi_M = g + \frac{\ln(U_0)}{r} + \mu\left\{-\left(\frac{1}{2k} + \frac{1}{2}r\sigma^2\right)\beta_H^2 + \frac{c}{k}\beta_H - c\tau_H\right\} + (1-\mu)\left\{-\left(\frac{1}{2k} + \frac{1}{2}r\sigma^2\right)\beta_L^2 + \left[\frac{c}{k} - \frac{\mu(\tau_L - \tau_H)}{1-\mu}\right]\beta_L - c\tau_L\right\}.$$

Because

$$\frac{\partial^2 \pi_M}{\partial \beta_H^2} = -\mu\left(r\sigma^2 + \frac{1}{k}\right) < 0 \text{ and } \frac{\partial^2 \pi_M}{\partial \beta_L^2} = -(1-\mu)\left(r\sigma^2 + \frac{1}{k}\right) < 0,$$

we obtain the optimal solution of β_H and β_L by F.O.C, i.e.,

$$\beta_H^S = \frac{c}{r\sigma^2 k + 1},$$
$$\beta_L^S = \max\left\{\frac{c}{r\sigma^2 k + 1} - \frac{\mu k(\tau_L - \tau_H)}{(1-\mu)(r\sigma^2 k + 1)}, 0\right\},$$
$$\alpha_H^S = -\frac{\ln(U_0)}{r} + \left(\frac{1}{2}r\sigma^2 - \frac{1}{2k}\right)\left(\beta_H^S\right)^2 + (\tau_H - d)\beta_H^S + (\tau_L - \tau_H)\beta_L^S,$$
$$\alpha_L^S = -\frac{\ln(U_0)}{r} + \left(\frac{1}{2}r\sigma^2 - \frac{1}{2k}\right)\left(\beta_L^S\right)^2 + (\tau_L - d)\beta_L^S.$$

Final, we check the constraint (IC-LH). If $c \geq \frac{\mu k(\tau_L - \tau_H)}{(1-\mu)}$, i.e., $\beta_L^S = \frac{c}{r\sigma^2 k + 1} - \frac{\mu k(\tau_L - \tau_H)}{(1-\mu)(r\sigma^2 k + 1)}$, $U_{LL} - U_{LH} = \frac{k\mu(\tau_L - \tau_H)^2}{(1-\mu)(r\sigma^2 k + 1)} \geq 0$. If $c < \frac{\mu k(\tau_L - \tau_H)}{(1-\mu)}$, i.e., $\beta_L^S = 0$, $U_{LL} - U_{LH} = \frac{c(\tau_L - \tau_H)}{r\sigma^2 k + 1} > 0$. Thus, the constraint (IC-LH) holds.

Proof of Corollary 3

Proof of the statement (1): From before propositions, we know that

$$e_L^* = \frac{\beta_L^*}{k} \text{ and } e_H^* = \frac{\beta_H^*}{k},$$

$$e_L^P = e_H^P = \frac{\beta^P}{k},$$

$$e_L^S = \frac{\beta_L^S}{k} \text{ and } e_H^S = \frac{\beta_H^S}{k},$$

where $\beta_L^* = \beta_H^* = \frac{c}{r\sigma^2 k+1}$, $\beta_L^P = \beta_H^P = \max\left\{\frac{c}{r\sigma^2 k+1} - \frac{\mu k(\tau_L-\tau_H)}{r\sigma^2 k+1}, 0\right\}$, $\beta_H^S = \frac{c}{r\sigma^2 k+1}$ and $\beta_L^S = \max\left\{\frac{c}{r\sigma^2 k+1} - \frac{\mu k(\tau_L-\tau_H)}{(1-\mu)(r\sigma^2 k+1)}, 0\right\}$.

Because $\frac{1}{1-\mu} \geq 1$, $\frac{c}{r\sigma^2 k+1} - \frac{\mu k(\tau_L-\tau_H)}{r\sigma^2 k+1} \geq \frac{c}{r\sigma^2 k+1} - \frac{\mu k(\tau_L-\tau_H)}{(1-\mu)(r\sigma^2 k+1)}$, i.e., $\beta_L^P \geq \beta_L^S$. Because $\frac{\mu k(\tau_L-\tau_H)}{r\sigma^2 k+1} \geq 0$ and $\frac{\mu k(\tau_L-\tau_H)}{(1-\mu)(r\sigma^2 k+1)} \geq 0$, $\beta_H^* = \beta_H^S \geq \beta_H^P$ and $\beta_L^* \geq \beta_L^P \geq \beta_L^S$. And then easily to know that $e_H^* = e_H^S \geq e_H^P$ and $e_L^* \geq e_L^P \geq e_L^S$.

Proof of the statement (2):

$$\beta_L^P = \beta_H^P = \max\left\{\frac{c}{r\sigma^2 k+1} - \frac{\mu k(\tau_L-\tau_H)}{r\sigma^2 k+1}, 0\right\}, \ \beta_L^S = \max\left\{\frac{c}{r\sigma^2 k+1} - \frac{\mu k(\tau_L-\tau_H)}{(1-\mu)(r\sigma^2 k+1)}, 0\right\}.$$

$$\pi_M^* - \pi_M^S = (1-\mu)\left\{\frac{c^2}{2k(r\sigma^2 k+1)} - \frac{c - \frac{\mu}{1-\mu}k(\tau_L-\tau_H)}{2k}\max\left\{\frac{c - \frac{\mu}{1-\mu}k(\tau_L-\tau_H)}{r\sigma^2 k+1}, 0\right\}\right\} =$$

$$\begin{cases} (1-\mu)\dfrac{c^2}{2k(r\sigma^2 k+1)} \geq 0, & c < \dfrac{\mu}{1-\mu}k(\tau_L-\tau_H) \\ (1-\mu)\dfrac{c^2 - \left[c - \frac{\mu}{1-\mu}k(\tau_L-\tau_H)\right]^2}{2k(r\sigma^2 k+1)} \geq 0, & c \geq \dfrac{\mu}{1-\mu}k(\tau_L-\tau_H). \end{cases}$$

Then, we consider $\pi_M^S - \pi_M^P$ according to the range of c.

(1) $c \geq \frac{\mu k(\tau_L-\tau_H)}{1-\mu}$.

$$\pi_M^S - \pi_M^P = -\frac{\rho^3 k(\tau_L-\tau_H)^2}{2(1-\mu)(r\sigma^2 k+1)} \geq 0, \text{ i.e., } \pi_M^P \leq \pi_M^S.$$

(2) $\frac{\mu k(\tau_L-\tau_H)}{1-\mu} > c \geq \mu k(\tau_L-\tau_H)$.

$$\pi_M^S - \pi_M^P = -\frac{1}{2k(r\sigma^2 k+1)}\left\{\left[c-\mu(\tau_L-\tau_H)k - \sqrt{\mu c}\right]\left[c-\mu(\tau_L-\tau_H)k + \sqrt{\mu c}\right]\right\}.$$

Because $c \geq \mu k(\tau_H-\tau_L)$, $c-\mu(\tau_H-\tau_L)k+\sqrt{\mu c} \geq 0$. Because $\frac{\mu(\tau_H-\tau_L)k}{1-\sqrt{\mu}} \geq \frac{\mu(\tau_H-\tau_L)k}{1-\mu} \geq c$, $\frac{\mu(\tau_H-\tau_L)k}{1-\sqrt{\mu}} \geq c$, i.e., $c - c\sqrt{\mu} - \mu(\tau_H-\tau_L)k \leq 0$. Hence, $\pi_M^P \leq \pi_M^S$.

(3) $c < \mu k(\tau_L-\tau_H)$.

$$\pi_M^S - \pi_M^P = \frac{\mu c^2}{2k(r\sigma^2 k+1)} \geq 0, \text{ i.e., } \pi_M^P \leq \pi_M^S.$$

Hence, $\pi_M^* \geq \pi_M^S \geq \pi_M^P$.

Proof of the statement (3): We know that

$$U_H^* = U_L^* = -\frac{\ln(U_0)}{r},$$

$$U_L^P = -\frac{\ln(U_0)}{r} \text{ and } U_H^P = -\frac{\ln(U_0)}{r} + \beta^P(\tau_L-\tau_H),$$

$$U_{LL}^S = -\frac{\ln(U_0)}{r} \text{ and } U_{HH}^S = -\frac{\ln(U_0)}{r} + \beta_L^S(\tau_L-\tau_H).$$

From the above discussion, we know that

$$\beta^P = \max\left\{\frac{c}{r\sigma^2k+1} - \frac{\mu k\left(\tau_L-\tau_H\right)}{r\sigma^2k+1}, 0\right\} \ge \beta_L^S = \max\left\{\frac{c}{r\sigma^2k+1} - \frac{\mu k\left(\tau_L-\tau_H\right)}{\left(1-\mu\right)\left(r\sigma^2k+1\right)}, 0\right\}.$$

The utility of the contractor are:

$u_H^* = u_L^* = -U_0$,

$u_L^P = -U_0$ and $u_H^P = -U_0 e^{-r\beta^P\left(\tau_L-\tau_H\right)}$,

$u_{LL} = -U_0$ and $u_H^P = -U_0 e^{-r\beta_L^S\left(\tau_L-\tau_H\right)}$.

Hence, $u_L^* = u_L^P = u_{LL}^S$ and $u_H^* \le u_{HH}^S \le u_H^P$.

Proof of the statement (4):

$$W^S - W^P = \begin{cases} \dfrac{\mu c^2}{2k\left(kr\sigma^2+1\right)}, \ c < \mu k\left(\tau_L-\tau_H\right), \\ \dfrac{\mu c^2 - \left[c-\mu k\left(\tau_L-\tau_H\right)\right]^2}{2k\left(kr\sigma^2+1\right)} - \mu\left[U_0 - U_0 e^{-rA\left(\tau_L-\tau_H\right)}\right], \ \mu k\left(\tau_L-\tau_H\right) \le c < \dfrac{\mu}{1-\mu}k\left(\tau_L-\tau_H\right), \\ \dfrac{\mu^3 k\left(\tau_L-\tau_H\right)^3}{2\left(1-\mu\right)\left(rk\sigma^2+1\right)} - \mu\left[U_0 e^{-rB\left(\tau_L-\tau_H\right)} - U_0 e^{-rA\left(\tau_L-\tau_H\right)}\right], \ c \ge \dfrac{\mu}{1-\mu}k\left(\tau_L-\tau_H\right), \end{cases}$$

where $A = \dfrac{c}{r\sigma^2k+1} - \dfrac{\mu k\left(\tau_L-\tau_H\right)}{r\sigma^2k+1}$ and $B = \dfrac{c}{r\sigma^2k+1} - \dfrac{\rho k\left(\tau_L-\tau_H\right)}{\left(1-\rho\right)\left(r\sigma^2k+1\right)}$.

Similarly, we consider three cases according to the range of c.

(1) $c < \mu k\left(\tau_L-\tau_H\right)$.

$W^S - W^P \ge 0$, i.e., $W^S \ge W^P$.

(2) $\mu k\left(\tau_H-\tau_L\right) \le c < \dfrac{\mu k\left(\tau_L-\tau_H\right)}{1-\mu}$.

If $W^S - W^P \ge 0$, $U_0 \le \dfrac{\mu^2c^2 - \left[c-\mu k\left(\tau_L-\tau_H\right)\right]^2}{2k\mu\left(r\sigma^2k+1\right)\left[1-e^{-rB\left(\tau_L-\tau_H\right)}\right]}$. Let $U_1 = \dfrac{\mu^2c^2 - \left[c-\mu k\left(\tau_L-\tau_H\right)\right]^2}{2k\mu\left(r\sigma^2k+1\right)\left[1-e^{-rB\left(\tau_L-\tau_H\right)}\right]}$.

Hence, if $\dfrac{\mu k\left(\tau_L-\tau_H\right)}{1-\mu} \ge c \ge \mu k\left(\tau_H-\tau_L\right)$ and $U_0 \le U_1$, $W^S \ge W^P$.

(3) $c \ge \dfrac{\mu}{1-\mu}k\left(\tau_L-\tau_H\right)$.

If $W^S \ge W^P$, $U_0 \le \dfrac{\mu^3 k\left(\tau_L-\tau_H\right)^2}{2k\mu\left(r\sigma^2k+1\right)\left[e^{-rA\left(\tau_L-\tau_H\right)} - e^{-rB\left(\tau_L-\tau_H\right)}\right]}$.

Let $U_2 = \dfrac{\mu^3 k\left(\tau_L-\tau_H\right)^2}{2k\mu\left(r\sigma^2k+1\right)\left[e^{-rA\left(\tau_L-\tau_H\right)} - e^{-rB\left(\tau_L-\tau_H\right)}\right]}$.

Hence, if $c \ge \dfrac{\mu}{1-\mu}k\left(\tau_L-\tau_H\right)$ and $U_0 \le U_2$, $W^S \ge W^P$.