ANALYTICAL MODELS FOR ESTIMATION OF THE MAXIMUM STRAIN OF BEAM STRUCTURES BASED ON OPTICAL FIBER BRAGG GRATING SENSORS

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Abstract. The structural safety of a beam structure is assessed by a comparison between the maximum stress measured during monitoring and the allowable stress of the beam. However, the strain directly measured from a fiber Bragg grating (FBG) strain sensor may not be identical with the actual maximum strain induced in the structural member. Unless a FBG strain sensor is installed exactly on where maximum strain occurs, the reliability of the evaluated safety based on the measured strain depends on the number and location of sensors. Therefore, in this paper, analytical models are presented for estimation of the maximum values of strains in a linear elastic beam using the local strains measured from FBG sensors. The model is tested in an experiment by comparing estimated maximum strain from FBG sensors and directly measured strain from electrical gages. For the assessment of safety of typical beam structures in buildings and infrastructures, analytical models for various loading and boundary conditions are provided.

Keywords: fiber optic sensor, strain sensing, health monitoring, FBG strain sensor, steel beams.

Introduction

Maximum stress in a structural member for buildings and infrastructures can exceed an allowable stress when the structure is subjected to various loads such as earthquake, wind, or unexpected loads. Additionally, structural members in complex-shaped buildings can experience unexpected overloads or stresses due to the complexity of architectural configurations (Keras *et al.* 2005; Scott *et al.* 2007). If a member is subjected to a higher stress than the allowable stress, the safety and serviceability of the member should be checked to prevent unexpected damages. For this reason, strain-based structural health monitoring (SHM), the maximum stress in a structural member has been widely used as clear index to assess the state of the member (AISC 1989; Farrar, Worden 2007).

Since the actual stress distribution induced in a beam structure by varying amounts and types of loads is non-uniform, the strain directly measured from a strain sensor is not identical with the actual maximum strain induced in the structural member. Unless a strain sensor is installed exactly on where maximum strain occurs, the reliability of the evaluated safety based on the measured strain depends on the number and location of sensors. Therefore, to reduce the number of sensors, the average strains from long gauge fiber optic sensors (LGFOS) or vibrating wire strain sensors (VWSGs) have been used in

estimating the maximum strain, since LGFOS or VWSGs measure integrated strain over a relatively long gauge length (Park *et al.* 2005, 2007; Lee, Park 2011).

Although point sensors measure local strain, fiber Bragg grating (FBG) strain sensors can successfully overcome the limitation on the gage length by a multiplexing technology (Grattan, Sun 2000; Li *et al.* 2004; Majumder *et al.* 2008). The multiplexing is a method by which multiple signals are combined into one signal, and hence it makes only single cable suffice for the multiple sensors. In addition, FBG sensors are easy to install and increase the number of sensors compared to long gauge sensors including LGFOSs and VWSGs. For these reasons, FBG strain sensors have been widely applied in various fields even though a FBG sensor measures strain at a local point.

Therefore, in this paper, analytical models are presented for the determination of the maximum values of strains or stresses in a beam structure based on the local strain measured from FBG sensors. The models define the relation between measured local strains and maximum strains for various types of loading and boundary conditions for linear elastic beam structures in buildings or infrastructures. The model is tested in an experiment by comparing estimated maximum strain from FBG sensors and directly measured strain from electrical gages.

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1. FBG strain sensors

The FBG sensor utilizes Bragg grating technology. A Bragg grating can be understood as an optical fiber with a periodic refractive index perturbation pattern inscribed in the core such that it diffracts the optical signal in the guided mode at specific wavelengths into other core-bounded modes provided the index period is of the order of the interrogation wavelength (López-Higuera *et al.* 2011; Delepine-Lesoille *et al.* 2008). If the broad-band light source collide against the Bragg grating, a light with particular wavelength, called as Bragg wavelength (λ_B), satisfying the relationship shown in Eqn (1) is reflected. This principle is illustrated in Figure 1 and the relation of Bragg grating and Bragg wavelength is given by:

$$\lambda_B = 2n_{eff}\,\Lambda, \tag{1}$$

where: Λ is the period of Bragg grating; n_{eff} is effective refractive index. Based on the fact that Bragg grating varies according to changes of temperature, ΔT, pressure, ΔP, and elongation, ΔL, strain can be calculated by detecting the change of Bragg wavelength:

$$\frac{\Delta\lambda_B}{\lambda_B} = a\Delta T + b\frac{\Delta L}{L} + c\Delta P, \tag{2}$$

where: a, b and c are constant coefficients with $a \approx 8 \times 10^{-6}\,°C$, $b \approx 0.8 \times 10^{-6}\,\mu m^{-1}$ and $c \approx 3 \times 10^{-6}\,MPa^{-1}$, respectively.

As the beam structure deforms, either by expansion or contraction, FBG sensor bonded or embedded into the structure elongates or contracts accordingly. Then, strain due to the elongation or contraction induced in an FBG can be measured by the relative change of Bragg wave length in Eqn (2). A wide range of applications of FBG strain sensors can be found in the field of structural health monitoring of buildings, bridges, and civil structures (Méndez 2007).

2. Analytical model

Analytical model is presented to estimate the maximum strain of a linear elastic beam structure by defining the relation between the local strains measured from FBG and the maximum strain. Seven types of typical beam structures subjected to various loading conditions are considered in this study.
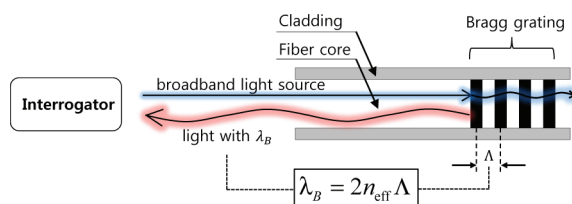

Fig. 1. Principle of fiber Bragg grating

2.1. Simply supported beams subjected to uniformly distributed load

For a simply supported beam subjected to the uniformly distributed load with magnitude of ω, as in Figure 2, the longitudinal strain of the beam, $\varepsilon(x)$, is expressed as a function of the distance x from the left-hand support:

$$\varepsilon(x) = \frac{1}{EZ}\left(\frac{\omega L}{2}x - \frac{\omega}{2}x^2\right), \tag{3}$$

where: E is the modulus of elasticity; Z is the elastic section modulus; L is the length of the beam and x is distance from left-hand support. If the FBG sensor is installed at the distance of x_{FBG} from the left-hand support for the measurement of local strain, as in Figure 2, the measured strain from the FBG sensor ε_{FBG} is given by:

$$\varepsilon_{FBG} = \frac{1}{EZ}\left(\frac{\omega L}{2}x_{FBG} - \frac{\omega}{2}x_{FBG}^2\right). \tag{4}$$

From Eqn (4), the distributed load ω can be expressed by:

$$\omega = \frac{2EZ\varepsilon_{FBG}}{x_{FBG}(L - x_{FBG})}. \tag{5}$$

By substituting ω in Eqn (5) into Eqn (3), longitudinal strain at arbitrary point can be expressed by:

$$\varepsilon(x) = \frac{x^2 - Lx}{x_{FBG}(x_{FBG} - L)}\omega_{FBG}. \tag{6}$$

Then, the location of the maximum strain in the beam structure can be found by taking derivative of Eqn (6) with respect to and setting to zero:

$$\frac{\partial\varepsilon(x)}{\partial x} = \frac{2x - L}{x_{FBG}(x_{FBG} - L)}\varepsilon_{FBG} = 0. \tag{7}$$

Therefore, using the location for the maximum strain from Eqn (7), the analytical model that defines the relationship between the maximum strain ε_{max} and the measured local strain from FBG ε_{FBG} is given by:

$$\varepsilon_{max} = \frac{L^2}{4x_{FBG}(L - x_{FBG})}\varepsilon_{FBG}. \tag{8}$$

Regardless of the location of the FBG sensor, the maximum stress in the simply supported beam structure
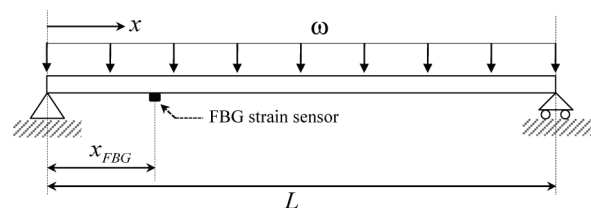

Fig. 2. Simply supported beam subjected to uniformly distributed load

subjected to the distributed load of ω can be estimated by Eqn (8).

2.2. Simply supported beams subjected to a concentrated load

For a simple-supported beam, as in Figure 3, the concentrated load P acts at the arbitrary distance a from the left-hand support and distance b from the right-hand support. Based on general concepts in engineering mechanics, the longitudinal strain of the beam, $\varepsilon(x)$, can be expressed as a function of the distance x from the left-hand support:

$$\varepsilon(x) = \begin{cases} \dfrac{Pb}{EZ}x & (0 \leq x \leq a) \\ \dfrac{Pa}{EZ}(L-x) & (a \leq x \leq L) \end{cases}. \tag{9}$$

If the FBG sensor is installed at the distance of x_{FBG} from the left-hand support for the measurement of local strain, as Figure 3, the measured strain from the FBG sensor ε_{FBG} is given by:

$$\varepsilon_{FBG} = \begin{cases} \dfrac{Pb}{EZ}x_{FBG} & 0 \leq x_{FBG} \leq a \\ \dfrac{Pa}{EZ}(L-x_{FBG}) & a \leq x_{FBG} \leq L \end{cases}. \tag{10}$$

From Eqn (10), the concentrated load P can be expressed by:

$$P = \begin{cases} \dfrac{EZ}{bx_{FBG}}\varepsilon_{FBG} & 0 < x_{FBG} \leq a \\ \dfrac{EZ}{a(L-x_{FBG})}\varepsilon_{FBG} & a \leq x_{FBG} < L \end{cases}. \tag{11}$$

By substituting P in Eqn (11) into Eqn (9), longitudinal strain at arbitrary point can be expressed by:

$$\varepsilon(x) = \begin{cases} \begin{cases} \dfrac{x}{x_{FBG}}\varepsilon_{FBG} & 0 < x_{FBG} \leq a \\ \dfrac{bx}{a(L-x_{FBG})}\varepsilon_{FBG} & a \leq x_{FBG} < L \end{cases} & (0 \leq x \leq a) \\ \begin{cases} \dfrac{a(L-x)}{bx_{FBG}}\varepsilon_{FBG} & 0 < x_{FBG} \leq a \\ \dfrac{L-x}{L-x_{FBG}}\varepsilon_{FBG} & a \leq x_{FBG} < L \end{cases} & (a \leq x \leq L) \end{cases}. \tag{12}$$
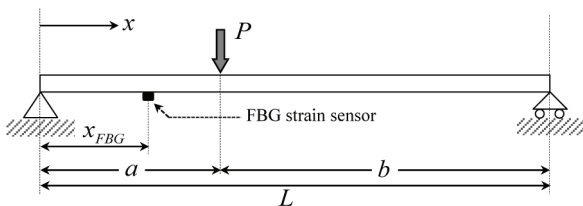

Fig. 3. Simply supported beam subjected to concentrated load

In the case of the beam subjected to a concentrated load, the maximum strain occurs at the point where the load is applied. Then, the analytical model that defines the relationship between the maximum strain ε_{max} and the measured local strain from FBG ε_{FBG} is given by:

$$\varepsilon_{max} = \begin{cases} \dfrac{a}{x_{FBG}}\varepsilon_{FBG} & 0 < x_{FBG} \leq a \\ \dfrac{b}{L-x_{FBG}}\varepsilon_{FBG} & a \leq x_{FBG} < L \end{cases}. \tag{13}$$

Therefore, regardless of the location of the FBG sensor, the maximum stress in the simply supported beam structure subjected to the point load P can be estimated by Eqn (13).

2.3. Analytical models for beams

Determining the maximum values of stresses in beam structures is essential in assessing the safety of beams. To estimate the maximum stresses with a minimized number of FBG sensors, the relationships between the maximum and local strains are established in the previous sections. Based on the relationships, the analytical model is derived to estimate the maximum strain using the local strains measured from FBG sensors. The analytical models can be given in the general form:

$$\varepsilon_{max} = \beta \times \varepsilon_{FBG}. \tag{14}$$

To provide convenient access to the relationships, the values of β for typical beam structures subjected to various loading conditions are given in Tables 1 and 2.

3. Test of the model

To verify the performance of the analytical model, a bending test of the simply supported steel beam subjected to a concentrated load was conducted. The schematic diagram of the test model is shown in Figure 4. A concentrate load was applied at the midpoint of the steel beam of H-100×100×6×8 section with the length of 4000 mm. The concentrated load was increased in three steps: 4.8 kN, 8 kN, and 11.2 kN. As shown in Figure 4, measurements during static testing were performed with both 7 FBG sensors and 7 electrical strain gauges (ESGs) attached at the surface of the bottom flange of the steel beam with equal intervals of 500 mm. The gage lengths of FBG strain sensors and ESGs used in this study were 10 mm and 5 mm, respectively.

When the concentrated load is applied by means of a hydraulic jack as shown in Figure 5, the beam deflects downward and the maximum tensile strain occurs at the midpoint. Using the measured strains from both 7 FBGs and 7 ESGs, the distributions of strains along the length of the beam are shown in Figure 6. In Table 3, the maximum strain directly measured from the ESG at the midpoint is compared with the estimated maximum

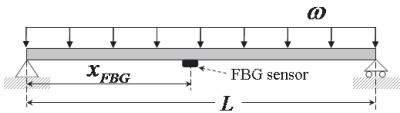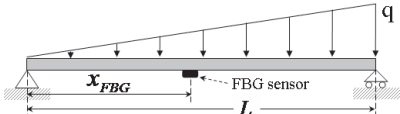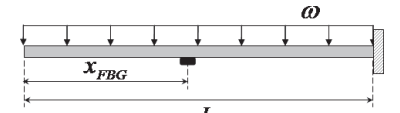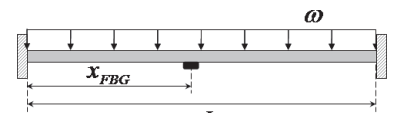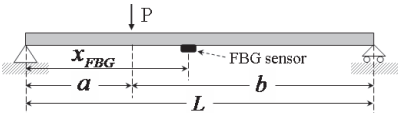Table 1. Values of β for the beams subjected to distributed load
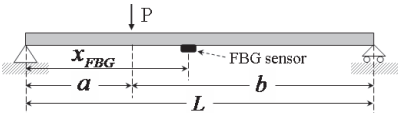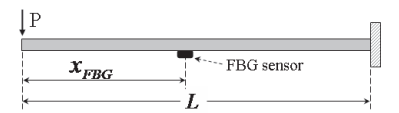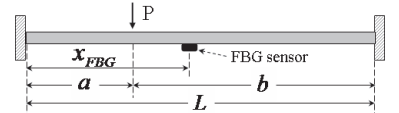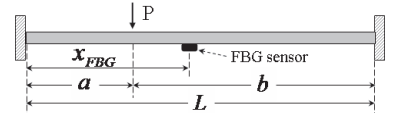
Loading condition	Condition of FBG sensor location	β value $\left(\beta = \varepsilon_{max} / \varepsilon_{FBG}\right)$
	$0 < x_{FBG} < L$	$\beta = \dfrac{L^2}{4x_{FBG}\left(L - x_{FBG}\right)}$
	$0 < x_{FBG} < L$	$\beta = \dfrac{2\sqrt{3}L^3}{9x_{FBG}\left(L^2 - x_{FBG}{}^2\right)}$
	$0 < x_{FBG} \leq L$	$\beta = \dfrac{L^2}{x_{FBG}{}^2}$
	$\begin{cases} 0 \leq x_{FBG} \leq L \\ x_{FBG} \neq 0.211L \\ x_{FBG} \neq 0.789L \end{cases}$	$\beta = \dfrac{-L^2}{2\left(L^2 - 6Lx_{FBG} + 6x_{FBG}{}^2\right)}$

Table 2. Values of β for the beams subjected to concentrated load

Loading condition	Condition of FBG sensor location	β value $\left(\beta = \varepsilon_{max} / \varepsilon_{FBG}\right)$
	$0 < x_{FBG} \leq a$	$\beta = \dfrac{a}{x_{FBG}}$
	$a \leq x_{FBG} < 0$	$\beta = \dfrac{b}{L - x_{FBG}}$
	$0 < x_{FBG} \leq L$	$\beta = \dfrac{L}{x_{FBG}}$
	$\begin{cases} 0 \leq x_{FBG} \leq a \\ x_{FBG} \neq \dfrac{La}{2a + L} \end{cases}$	$\beta = \dfrac{2a^2}{2ax_{FBG} - L\left(L - b - x_{FBG}\right)}$
	$\begin{cases} a \leq x_{FBG} \leq L \\ x_{FBG} \neq \dfrac{L(L+b)}{L+2b} \end{cases}$	$\beta = \dfrac{2b^2}{L\left(L + b - x_{FBG}\right) - 2bx_{FBG}}$

strains based on the analytical model in Eqn (14). For each loading step, the maximum strains can be estimated by 7 measured strains from 7 FBGs located at different locations as shown in Figure 4. The maximum differences are found to be less than 2.6% for each load step. The

results indicate a good agreement between the measured and estimated strains. Figure 7 shows a comparison between the estimated maximum strains obtained through the analytical model and the maximum strain measured directly by electrical strain gauge.
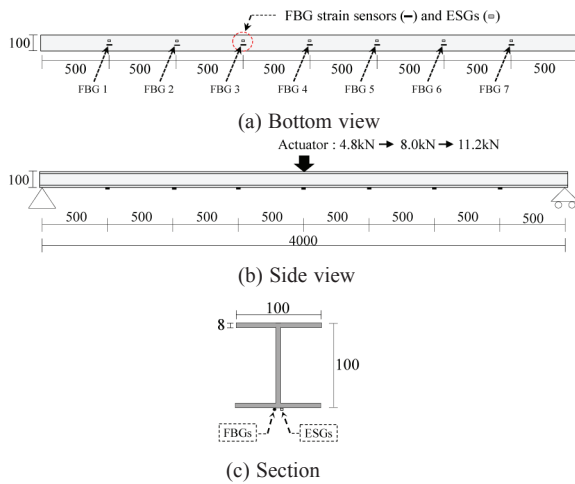

Fig. 4. Experimental setup for test model (unit: mm)
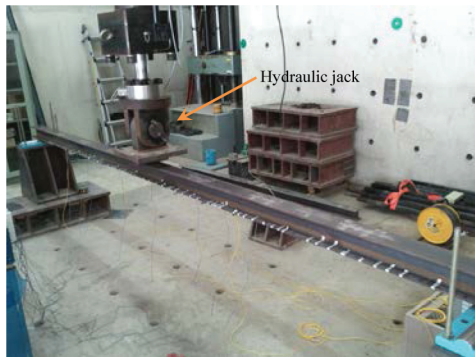

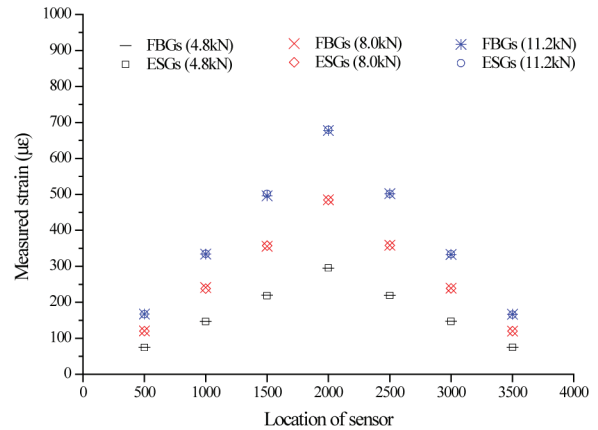Fig. 5. Simply supported beam and hydraulic jack

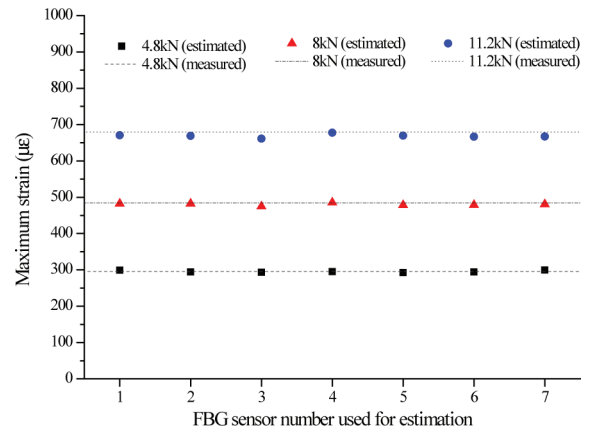Fig. 6. Measured strains by both of 7 FBG sensors and 7 ESGs

Fig. 7. Comparison between maximum strains estimated by analytical model using FBG sensor and direct measured

Table 3. Comparison of the maximum strains between estimation and measurement

Loading step	Maximum strain directly measured by ESG (με)	Estimated maximum strain by analytical model using a FBG sensor (με) (Relative error of estimation to the measurement (%))						
		FBG sensor located on:						
		500 mm	1000 mm	1500 mm	2000 mm	2500 mm	3000 mm	3500 mm
step 1 (4.8 kN)	295.7	299.3 (1.2)	294.3 (0.5)	293.4 (0.8)	294.7 (0.3)	292.8 (1.0)	294.4 (0.5)	299.7 (1.4)
step 2 (8.0 kN)	484.5	482.1 (0.5)	482.0 (0.5)	474.7 (2.0)	485.2 (0.2)	478.1 (1.3)	478.5 (1.2)	480.5 (0.8)
step 3 (11.2 kN)	679.2	670.5 (1.3)	668.7 (1.5)	661.5 (2.6)	677.5 (0.2)	669.2 (1.5)	666.3 (1.9)	667.2 (1.8)

Conclusions

In this paper, analytical models to estimate the maximum strains of linear elastic beam structures subjected to various loading conditions are presented. The models are derived by defining the relationship between the maximum strain induced in a beam structure and the measured local strain from a FBG attached at an arbitrary location. The estimation model was tested in an experiment by comparing the maximum strains measured directly from ESGs and the estimated maximum strains based on local strains from FBG strain sensors. The maximum values of strains estimated from the analytical model agreed quite well with the directly measured values from electrical strain gauges. For the assessment of safety of a beam in buildings and infrastructures, analytical models for beam structures with typical boundary conditions subjected to various loadings are provided. For more practical applications, the values of β in Eqn (14) need to be derived according to boundary and loading conditions.

Acknowledgements

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