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Q-INDETERMINATE CORRELATION COEFFICIENT BETWEEN SIMPLIFIED NEUTROSOPHIC INDETERMINATE SETS AND ITS MULTICRITERIA DECISION-MAKING METHOD

Shigui DU[1], Jun YE[1,2*], Rui YONG[2], Fangwei ZHANG[1]

[1]*Institute of Rock Mechanics, Ningbo University, 315211 Ningbo, P. R. China*
[2]*Department of Civil Engineering, Shaoxing University, 312000 Shaoxing, P. R. China*

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Abstract. Owing to the indeterminacy, incompleteness, and inconsistency of decision makers' arguments/cognitions regarding complicated decision-making problems, the truth, falsity, and indeterminacy degrees given by decision makers may imply the partial certainty and partial uncertainty information. In this case, a simplified neutrosophic set (SNS) cannot express the uncertainty degrees of the truth, falsity, indeterminacy arguments. To depict the hybrid information of SNS and neutrosophic (indeterminate) numbers (NNs) together, this study presents a simplified neutrosophic indeterminate set (SNIS) to describe the uncertainty degrees of the truth, falsity, indeterminacy, and then based on the de-neutrosophication technology using the parameterized SNSs of SNISs we introduce the q-indeterminate correlation coefficients of SNISs with a parameter $q \in [0, 1]$. Next, a simplified neutrosophic indeterminate multicriteria decision-making method using the q-indeterminate correlation coefficients of SNISs is established along with decision makers' risk attitudes, such as the small risk for $q = 0$, the moderate risk for $q = 0.5$, and the large risk for $q = 1$, to carry out multicriteria decision-making problems in SNIS setting. Eventually, the proposed decision-making approach is applied in an example of selecting a satisfactory slope design scheme for an open pit mine to indicate the practicality and flexibility in SNIS setting.

Keywords: q-indeterminate correlation coefficient, simplified neutrosophic indeterminate set, multicriteria decision making, neutrosophic number, slope design scheme.

Introduction

Smarandache (1998) firstly presented a neutrosophic set from philosophical viewpoint as a powerful general formal framework to depict the inconsistent and indeterminate information in the real life. Owing to an important mathematical tool of correlation coefficients in decision-making and pattern recognition problems, Hanafy et al. (2012) presented the centroid-based correlation coefficient of neutrosophic sets. In single-valued neutrosophic situations Wang et al. (2010) and Ye (2013a, 2013b) proposed correlation coefficients between single-valued neutrosophic sets (SvNSs) and utilized them in multicriteria decision-making (MDM) problems with SvNS information. In interval-valued neutrosophic setting Wang et al. (2005), Broumi and Smarandache (2013) proposed the correlation coefficient of interval-valued neutrosophic sets (IvNSs). Then, Ye (2014a) further presented the improved correlation coefficients of SvNSs and IvNSs for MDM problems. Salama et al. (2014) introduced a correlation coefficient of neutrosophic data from probability spaces. Zhang et al. (2015) presented an improved weighted correlation coefficient of IvNSs for MDM applications. Regarding interval-valued neutrosophic hesitant fuzzy sets, Ye (2016) also put forward their correlation coefficients for MDM problems. Based on simplified neutrosophic sets (SNSs) (Ye, 2014b) implying SvNSs and IvNSs, Shi (2016) proposed the correlation coefficient of SNSs and applied it to the vibration fault diagnosis of rolling bearing with SNS information. Şahin and Liu (2017) presented a single-valued neutrosophic correlation coefficient for MDM problems. Next, Ye (2017a) introduced the correlation coefficient of dynamic single-valued neutrosophic multisets for MDM applications. Recently, Hu et al. (2018) proposed element-weighted neutrosophic correlation coefficient for improving CAMShift tracker in RGBD video. Xue et al. (2019) introduced a neutrosophic cubic correlation coefficient for pattern recognition problems with neutrosophic cubic information.

*Corresponding author. E-mails: *yejun1@nbu.edu.cn, yehjun@aliyun.com*

However, the truth, falsity, indeterminacy degrees yielded by decision makers in complicated MDM problems may imply the partial certainty and partial uncertainty information owing to inconsistency, incompleteness, and indeterminacy of decision makers' cognitions. Then, SNSs (Ye, 2014b) contain the single-valued and interval-valued neutrosophic sets described by the truth, falsity, and indeterminacy degrees as the generalization of fuzzy sets (Zadeh, 1965), (interval-valued) intuitionistic fuzzy sets (Atanassov, 1986; Atanassov & Gargov, 1989), while a neutrosophic number (NN) (i.e., an indeterminate number) (Smarandache, 1998, 2013, 2014), denoted by $p = \alpha + \delta I$ for $\alpha, \delta \in \Re$ (all real numbers) and $I \in [I^L, I^U]$, consists of its certain term α and its uncertain term δI with indeterminacy $I \in [I^L, I^U]$ to flexibly depict the partial determinacy and partial indeterminacy information in actual applications. Thus, NN easily depicts a changeable interval number ($p = [\alpha + \delta I^L, \alpha + \delta I^U]$) depending on a specified indeterminate range of $I \in [I^L, I^U]$, which shows its main highlight in the expression of indeterminate information. Hence, NNs have been wildly applied in many areas, such as decision making (Ye, 2017b) and slope stability assessment (Li et al., 2019).

Although SNSs and NNs were applied in MDM problems (Peng et al., 2014, 2016; Wu et al., 2016; Ye, 2017b; Zhou et al., 2019), either SNS or NN cannot express the uncertainty degrees of the truth, falsity, indeterminacy arguments. Furthermore, existing neutrosophic correlation coefficients cannot deal with such a MDM problem with the uncertainty degrees of the truth, falsity, indeterminacy arguments and also lack decision makers' risk attitudes in indeterminate decision-making applications. Motivated by both the hybrid information of SNS and NN and the decision makers' risk attitudes in indeterminate decision-making problems, this study proposes a simplified neutrosophic indeterminate set (SNIS) for the first time to describe the uncertainty degrees of the truth, indeterminacy, falsity, and then based on the de-neutrosophication technology using the parameterized SvNSs of SNISs we introduce two q-indeterminate correlation coefficients of SNISs with a parameter $q \in [0, 1]$ and their MDM approach with decision makers' risk attitudes in SNIS setting to solve MDM problems with SNIS information under inconsistent and indeterminate environment.

This work is constructed as the following framework. Section 1 introduces preliminaries of SvNSs for further study. In Section 2, SNIS is proposed to describe the uncertainty degrees of the truth, indeterminacy, falsity, and then q-indeterminate correlation coefficients between SNISs are introduced based on the de-neutrosophication technology using the parameterized SvNSs of SNISs with a parameter $q \in [0, 1]$. Section 3 presents a MDM method using the proposed q-indeterminate correlation coefficients is established along with decision makers' risk attitudes, such as the small risk for $q = 0$, the moderate risk for $q = 0.5$, and the large risk for $q = 1$, in SNIS setting. In Section 4, the proposed MDM method is applied to a MDM example of selecting a satisfactory slope design scheme for an open pit mine regarding decision makers' risk attitudes to indicate the applicability and flexibility of the proposed MDM method in SNIS setting. Lastly, conclusions and further study are presented.

1. Preliminaries of SvNSs

Regarding a subset of a neutrosophic set Smarandache (1998) and Wang et al. (2010) defined a SvNS $E = \{\langle u_k, a_E(u_k), b_E(u_k), c_E(u_E)\rangle \mid u_k \in U\}$ in the universe set $U = \{u_1, u_2, \ldots, u_n\}$, where $a_E(u_k)$: $U \to [0, 1]$, $b_E(u_k)$: $U \to [0, 1]$, and $c_E(u_k)$: $U \to [0, 1]$ ($k = 1, 2, \ldots, n$) are the truth, indeterminacy, and falsity membership functions of the element u_k to the set E, such that $0 \le a_E(u_k) + b_E(u_k) + c_E(u_k) \le 3$ for $u_k \in U$.

Then, an element $\langle u_k, a_E(u_k), b_E(u_k), c_S(u_E)\rangle$ in E is denoted as the single-valued neutrosophic number (SvNN) $e_k = <a_k, b_k, c_k>$ for the simplified representation.

Set two SvNNs as $e_1 = <a_1, b_1, c_1>$ and $e_2 = <a_2, b_2, c_2>$ and $v > 0$. Then, there exist the following relations (Smarandache, 1998; Wang et al., 2010):

(1) $e_1 \subseteq e_2 \Leftrightarrow a_1 \le a_2, b_1 \ge b_2, c_1 \ge c_2$;

(2) $e_1 = e_2 \Leftrightarrow e_1 \subseteq e_2$ and $e_2 \subseteq e_1$;

(3) $(e_1)^c = <c_1, 1 - b_1, a_1>$ (Complement of e_1).

Suppose that two SvNSs are $E_1 = \{e_{11}, e_{12}, \ldots, e_{1n}\}$ and $E_2 = \{e_{21}, e_{22}, \ldots, e_{2n}\}$, where $e_{1k} = <a_{1k}, b_{1k}, c_{1k}>$ and $e_{1k} = <a_{1k}, b_{1k}, c_{1k}>$ ($k = 1, 2, \ldots, n$) are SvNNs. Ye (2013a, 2013b) proposed two weighted correlation coefficients of SvNSs:

$$W_1\,(E_1, E_2) = \frac{\sum_{k=1}^{n} v_k (a_{1k}a_{2k} + b_{1k}b_{2k} + c_{1k}c_{2k})}{\sqrt{\sum_{k=1}^{n} v_k \left(a_{1k}^2 + b_{1k}^2 + c_{1k}^2\right)}\sqrt{\sum_{k=1}^{n} v_k \left(a_{2k}^2 + b_{2k}^2 + c_{2k}^2\right)}}, \quad (1)$$

$$W_2\,(E_1, E_2) = \frac{\sum_{k=1}^{n} v_k (a_{1k}a_{2k} + b_{1k}b_{2k} + c_{1k}c_{2k})}{\max\left(\sum_{k=1}^{n} v_k \left(a_{1k}^2 + b_{1k}^2 + c_{1k}^2\right), \sum_{k=1}^{n} v_k \left(a_{2k}^2 + b_{2k}^2 + c_{2k}^2\right)\right)}. \quad (2)$$

Then, the correlation coefficients of Eqns (1) and (2) contain the following properties (Ye, 2013a, 2013b):

(1) $W_1(E_1, E_2) = W_1(E_2, E_1)$ and $W_2(E_1, E_2) = W_2(E_2, E_1)$;

(2) $W_1(E_1, E_2) = W_2(E_1, E_2) = 1$ if $E_1 = E_2$;

(3) $W_1(E_1, E_2), W_2(E_1, E_2) \in [0, 1]$.

2. Simplified neutrosophic indeterminate sets and their q-indeterminate correlation coefficients

By combining SNS with NNs (indeterminate numbers), a SNIS concept is introduced as the generalization of a SNS concept in indeterminate and inconsistent setting.

Definition 1. Set $U = \{u_1, u_2, \ldots, u_n\}$ as a universe set. A SNIS P in U is defined as the following form:

$$P=\left\{\left\langle u_k, A_P(u_k,I), B_P(u_k,I), C_P(u_k,I)\right\rangle | u_k \in U\right\},$$

where $A_P(u_k,I)=\alpha_k+\delta_k I \subseteq [0,1]$, $B_P(u_k,I)=\beta_k+\rho_k I \subseteq [0,1]$, and $C_P(u_k,I)=\gamma_k+\eta_k I \subseteq [0,1]$ with indeterminacy $I \in [I^L, I^U]$ for $u_k \in U$ (k = 1, 2, .., n) are the truth, indeterminacy, and falsity NNs that consist of their certain terms α_k, β_k, γ_k for α_k, β_k, $\gamma_k \in \Re$ and their uncertain terms $\delta_k I$, $\rho_k I$, $\eta_k I$ for δ_k, ρ_k, $\eta_k \in \Re$ and $I \in [I^L, I^U]$, such that the condition $0 \le \sup A_P(u_k,I)+\sup B_P(u_k,I)+\sup C_P(u_k,I) \le 3$.

Thus, it is obvious that SNIS implies the SvNS family and IvNS family depending on the indeterminate values and ranges of $I \in [I^L, I^U]$.

In a SNIS P, the element $\left\langle u_k, A_P(u_k,I), B_P(u_k,I), C_P(u_k,I)\right\rangle$ for $u_k \in U$ (k = 1, 2, …, n) and $I \in [I^L, I^U]$ is denoted by its simplified form $\left\langle A_k(I), B_k(I), C_k(I)\right\rangle=\left\langle \alpha_k+\delta_k I, \beta_k+\rho_k I, \gamma_k+\eta_k I\right\rangle$, which is called a simplified neutrosophic indeterminate number (SNIN).

Considering the de-neutrosophication of SNIS, we can introduce a parameter $q \in [0, 1]$ to transform SNIS into the parameterized SvNS, which is defined as a q-indeterminate SvNS with $q \in [0, 1]$.

Definition 2. Let $P = \{p_1, p_2, \ldots, p_n\}$ be a SNIS, where $p_k=\left\langle A_k(I), B_k(I), C_k(I)\right\rangle$ (k = 1, 2, …, n) for $I \in [I^L, I^U]$ are SNINs, and let $q \in [0, 1]$ be a parameter. Then, the parameterized SvNS of P is defined as a q-indeterminate SvNS $P(q) = \{p_1(q), p_2(q), \ldots, p_n(q)\}$ for $q \in [0, 1]$, where $p_k(q)=\left\langle A_k(q), B_k(q), C_k(q)\right\rangle=\left\langle \alpha_k+\delta_k I^L+\delta_k q(I^U-I^L), \beta_k+\rho_k I^L+\rho_k q(I^U-I^L), \gamma_k+\eta_k I^L+\eta_k q(I^U-I^L)\right\rangle$ (k = 1, 2, …, n) are q-indeterminate SvNNs with $q \in [0, 1]$.

Clearly, each $p_k(q)$ (k = 1, 2, …, n) is changed when the value of $q \in [0, 1]$ is changed for a specified indeterminate range of $I \in [I^L, I^U]$. Obviously, the q-indeterminate SvNN implies a family of SvNNs depending on different values of $q \in [0, 1]$, while SvNN is only a special case of q-indeterminate SvNN if q is equal to a specified value.

As the extension of existing correlation coefficients of SvNSs (Ye, 2013a, 2013b), we define the q-indeterminate correlation and information energy of SNISs with $q \in [0, 1]$ and propose two q-indeterminate correlation coefficients between two SNISs.

Definition 3. Let two SNISs be $P_1 = \{p_{11}, p_{12}, \ldots, p_{1n}\}$ and $P_2 = \{p_{21}, p_{22}, \ldots, p_{2n}\}$, where $p_{1k}=\left\langle A_{1k}(I), B_{1k}(I), C_{1k}(I)\right\rangle$ and $p_{2k}=\left\langle A_{2k}(I), B_{2k}(I), C_{2k}(I)\right\rangle$ (k = 1, 2, …, n) for $I \in [I^L, I^U]$ are two groups of SNINs. Then the q-indeterminate correlation of SNISs P_1 and P_2 with $q \in [0, 1]$ is presented below:

$$N_q\left(P_1,P_2\right)=\sum_{k=1}^{n}\left\{\begin{array}{l}[\alpha_{1k}+\delta_{1k}I^L+\delta_{1k}q(I^U-I^L)][\alpha_{2k}+\delta_{2k}I^L+\delta_{2k}q(I^U-I^L)]\\ +[\beta_{1k}+\rho_{1k}I^L+\rho_{1k}q(I^U-I^L)][\beta_{2k}+\rho_{2k}I^L+\rho_{2k}q(I^U-I^L)]\\ +[\gamma_{1k}+\eta_{1k}I^L+\eta_{1k}q(I^U-I^L)][\gamma_{2k}+\eta_{2k}I^L+\eta_{2k}q(I^U-I^L)]\end{array}\right\}. \tag{3}$$

By Eqn (3), the q-indeterminate correlations between P_1 and P_1 and between P_2 and P_2 are obtained by the following forms:

$$N_q\left(P_1,P_1\right)=\sum_{k=1}^{n}\left\{[\alpha_{1k}+\delta_{1k}I^L+\delta_{1k}q(I^U-I^L)]^2+[\beta_{1k}+\rho_{1k}I^L+\rho_{1k}q(I^U-I^L)]^2+[\gamma_{1k}+\eta_{1k}I^L+\eta_{1k}q(I^U-I^L)]^2\right\}; \tag{4}$$

$$N_q\left(P_2,P_2\right)=\sum_{k=1}^{n}\left\{[\alpha_{2k}+\delta_{2k}I^L+\delta_{2k}q(I^U-I^L)]^2+[\beta_{2k}+\rho_{2k}I^L+\rho_{2k}q(I^U-I^L)]^2+[\gamma_{2k}+\eta_{2k}I^L+\eta_{2k}q(I^U-I^L)]^2\right\}, \tag{5}$$

which are also called the q-indeterminate informational energy of SNISs P_1 and P_2.

Thus, the two q-indeterminate correlation coefficients of SNISs P_1 and P_2 are given by the following formulae:

$$R_1^q(P_1,P_2)=\frac{N_q(P_1,P_2)}{\sqrt{N_q(P_1,P_1)}\sqrt{N_q(P_2,P_2)}}=$$

$$\frac{\sum_{k=1}^{n}\left\{\begin{array}{l}[\alpha_{1k}+\delta_{1k}I^L+\delta_{1k}q(I^U-I^L)][\alpha_{2k}+\delta_{2k}I^L+\delta_{2k}q(I^U-I^L)]\\+[\beta_{1k}+\rho_{1k}I^L+\rho_{1k}q(I^U-I^L)][\beta_{2k}+\rho_{2k}I^L+\rho_{2k}q(I^U-I^L)]\\+[\gamma_{1k}+\eta_{1k}I^L+\eta_{1k}q(I^U-I^L)][\gamma_{2k}+\eta_{2k}I^L+\eta_{2k}q(I^U-I^L)]\end{array}\right\}}{\left\{\begin{array}{l}\sqrt{\sum_{k=1}^{n}\left([\alpha_{1k}+\delta_{1k}I^L+\delta_{1k}q(I^U-I^L)]^2+[\beta_{1k}+\rho_{1k}I^L+\rho_{1k}q(I^U-I^L)]^2+[\gamma_{1k}+\eta_{1k}I^L+\eta_{1k}q(I^U-I^L)]^2\right)}\\\times\sqrt{\sum_{k=1}^{n}\left([\alpha_{2k}+\delta_{2k}I^L+\delta_{2k}q(I^U-I^L)]^2+[\beta_{2k}+\rho_{2k}I^L+\rho_{2k}q(I^U-I^L)]^2+[\gamma_{2k}+\eta_{2k}I^L+\eta_{2k}q(I^U-I^L)]^2\right)}\end{array}\right\}}; \tag{6}$$

$$R_2^q(P_1,P_2)=\frac{N_q(P_1,P_2)}{\max\{N_q(P_1,P_1),N_q(P_2,P_2)\}}=$$

$$\frac{\sum_{k=1}^{n}\left\{\begin{array}{l}[\alpha_{1k}+\delta_{1k}I^L+\delta_{1k}q(I^U-I^L)][\alpha_{2k}+\delta_{2k}I^L+\delta_{2k}q(I^U-I^L)]\\+[\beta_{1k}+\rho_{1k}I^L+\rho_{1k}q(I^U-I^L)][\beta_{2k}+\rho_{2k}I^L+\rho_{2k}q(I^U-I^L)]\\+[\gamma_{1k}+\eta_{1k}I^L+\eta_{1k}q(I^U-I^L)][\gamma_{2k}+\eta_{2k}I^L+\eta_{2k}q(I^U-I^L)]\end{array}\right\}}{\max\left\{\begin{array}{l}\sum_{k=1}^{n}\left([\alpha_{1k}+\delta_{1k}I^L+\delta_{1k}q(I^U-I^L)]^2+[\beta_{1k}+\rho_{1k}I^L+\rho_{1k}q(I^U-I^L)]^2+[\gamma_{1k}+\eta_{1k}I^L+\eta_{1k}q(I^U-I^L)]^2\right),\\\sum_{k=1}^{n}\left([\alpha_{2k}+\delta_{2k}I^L+\delta_{2k}q(I^U-I^L)]^2+[\beta_{2k}+\rho_{2k}I^L+\rho_{2k}q(I^U-I^L)]^2+[\gamma_{2k}+\eta_{2k}I^L+\eta_{2k}q(I^U-I^L)]^2\right)\end{array}\right\}}. \tag{7}$$

Based on the properties of single-valued neutrosophic correlation coefficients (Ye, 2013a, 2013b), it is obvious that the q-indeterminate correlation coefficients of Eqns (6) and (7) for $q \in [0, 1]$ also contain the following properties:

(P1) $R_1^q(P_1,P_2)=R_1^q(P_2,P_1)$ and $R_2^q(P_1,P_2)=R_2^q(P_2,P_1)$;

(P2) $R_1^q(P_1,P_2)=R_2^q(P_1,P_2)=1$ if $P_1 = P_2$;

(P3) $R_1^q(P_1,P_2), R_2^q(P_1,P_2)\in[0,1]$.

Proof. It is obvious that the properties (P1) and (P2) are straightforward. Thus, one only verifies the property (P3).

Based on the Cauchy-Schwarz inequality $\left(\sum_{k=1}^{n}(x_k y_k)\right)^2 \le \sum_{k=1}^{n}x_k^2\sum_{k=1}^{n}y_k^2$, there is $\sum_{k=1}^{n}(x_k y_k)\le\sqrt{\sum_{k=1}^{n}x_k^2}\sqrt{\sum_{k=1}^{n}y_k^2}$. Thus, there exist the following inequalities for $q \in [0, 1]$:

$$\sum_{k=1}^{n}\left\{\begin{array}{l}[\alpha_{1k}+\delta_{1k}I^L+\delta_{1k}q(I^U-I^L)][\alpha_{2k}+\delta_{2k}I^L+\delta_{2k}q(I^U-I^L)]\\+[\beta_{1k}+\rho_{1k}I^L+\rho_{1k}q(I^U-I^L)][\beta_{2k}+\rho_{2k}I^L+\rho_{2k}q(I^U-I^L)]\\+[\gamma_{1k}+\eta_{1k}I^L+\eta_{1k}q(I^U-I^L)][\gamma_{2k}+\eta_{2k}I^L+\eta_{2k}q(I^U-I^L)]\end{array}\right\}\le$$

$$\left\{\begin{array}{l}\sqrt{\sum_{k=1}^{n}\left([\alpha_{1k}+\delta_{1k}I^L+\delta_{1k}q(I^U-I^L)]^2+[\beta_{1k}+\rho_{1k}I^L+\rho_{1k}q(I^U-I^L)]^2+[\gamma_{1k}+\eta_{1k}I^L+\eta_{1k}q(I^U-I^L)]^2\right)}\\\times\sqrt{\sum_{k=1}^{n}\left([\alpha_{2k}+\delta_{2k}I^L+\delta_{2k}q(I^U-I^L)]^2+[\beta_{2k}+\rho_{2k}I^L+\rho_{2k}q(I^U-I^L)]^2+[\gamma_{2k}+\eta_{2k}I^L+\eta_{2k}q(I^U-I^L)]^2\right)}\end{array}\right\};$$

$$\sum_{k=1}^{n}\left\{\begin{array}{l}[\alpha_{1k}+\delta_{1k}I^L+\delta_{1k}q(I^U-I^L)][\alpha_{2k}+\delta_{2k}I^L+\delta_{2k}q(I^U-I^L)]\\+[\beta_{1k}+\rho_{1k}I^L+\rho_{1k}q(I^U-I^L)][\beta_{2k}+\rho_{2k}I^L+\rho_{2k}q(I^U-I^L)]\\+[\gamma_{1k}+\eta_{1k}I^L+\eta_{1k}q(I^U-I^L)][\gamma_{2k}+\eta_{2k}I^L+\eta_{2k}q(I^U-I^L)]\end{array}\right\}\le$$

$$\max\left\{\begin{array}{l}\sum_{k=1}^{n}\left([\alpha_{1k}+\delta_{1k}I^L+\delta_{1k}q(I^U-I^L)]^2+[\beta_{1k}+\rho_{1k}I^L+\rho_{1k}q(I^U-I^L)]^2+[\gamma_{1k}+\eta_{1k}I^L+\eta_{1k}q(I^U-I^L)]^2\right),\\\sum_{k=1}^{n}\left([\alpha_{2k}+\delta_{2k}I^L+\delta_{2k}q(I^U-I^L)]^2+[\beta_{2k}+\rho_{2k}I^L+\rho_{2k}q(I^U-I^L)]^2+[\gamma_{2k}+\eta_{2k}I^L+\eta_{2k}q(I^U-I^L)]^2\right)\end{array}\right\}.$$

Then, there are $0\le R_1^q(P_1,P_2)\le 1$ and $0\le R_2^q(P_1,P_2)\le 1$, namely $R_1^q(P_1,P_2), R_2^q(P_1,P_2)\in[0,1]$.
Therefore, this proof is finished.

If the importance of each SNIN s_{jk} (j = 1, 2; k = 1, 2, …, n) in P_1 and P_2 is considered and specified by the weight v_k for $v_k \in [0, 1]$ and $\sum_{k=1}^{n} v_k = 1$, the q-indeterminate weighted correlation coefficients of SNISs P_1 and P_2 can be given as follows:

$$W_1^q(P_1,P_2)=\frac{\sum_{k=1}^{n} v_k \begin{Bmatrix}[\alpha_{1k}+\delta_{1k}I^L+\delta_{1k}q(I^U-I^L)][\alpha_{2k}+\delta_{2k}I^L+\delta_{2k}q(I^U-I^L)]\\ +[\beta_{1k}+\rho_{1k}I^L+\rho_{1k}q(I^U-I^L)][\beta_{2k}+\rho_{2k}I^L+\rho_{2k}q(I^U-I^L)]\\ +[\gamma_{1k}+\eta_{1k}I^L+\eta_{1k}q(I^U-I^L)][\gamma_{2k}+\eta_{2k}I^L+\eta_{2k}q(I^U-I^L)]\end{Bmatrix}}{\begin{Bmatrix}\sqrt{\sum_{k=1}^{n} v_k\left([\alpha_{1k}+\delta_{1k}I^L+\delta_{1k}q(I^U-I^L)]^2+[\beta_{1k}+\rho_{1k}I^L+\rho_{1k}q(I^U-I^L)]^2+[\gamma_{1k}+\eta_{1k}I^L+\eta_{1k}q(I^U-I^L)]^2\right)}\\ \times\sqrt{\sum_{k=1}^{n} v_k\left([\alpha_{2k}+\delta_{2k}I^L+\delta_{2k}q(I^U-I^L)]^2+[\beta_{2k}+\rho_{2k}I^L+\rho_{2k}q(I^U-I^L)]^2+[\gamma_{2k}+\eta_{2k}I^L+\eta_{2k}q(I^U-I^L)]^2\right)}\end{Bmatrix}}; \tag{8}$$

$$W_2^q(P_1,P_2)=\frac{\sum_{k=1}^{n} v_k \begin{Bmatrix}[\alpha_{1k}+\delta_{1k}I^L+\delta_{1k}q(I^U-I^L)][\alpha_{2k}+\delta_{2k}I^L+\delta_{2k}q(I^U-I^L)]\\ +[\beta_{1k}+\rho_{1k}I^L+\rho_{1k}q(I^U-I^L)][\beta_{2k}+\rho_{2k}I^L+\rho_{2k}q(I^U-I^L)]\\ +[\gamma_{1k}+\eta_{1k}I^L+\eta_{1k}q(I^U-I^L)][\gamma_{2k}+\eta_{2k}I^L+\eta_{2k}q(I^U-I^L)]\end{Bmatrix}}{\max\begin{Bmatrix}\sum_{k=1}^{n} v_k\left([\alpha_{1k}+\delta_{1k}I^L+\delta_{1k}q(I^U-I^L)]^2+[\beta_{1k}+\rho_{1k}I^L+\rho_{1k}q(I^U-I^L)]^2+[\gamma_{1k}+\eta_{1k}I^L+\eta_{1k}q(I^U-I^L)]^2\right),\\ \sum_{k=1}^{n} v_k\left([\alpha_{2k}+\delta_{2k}I^L+\delta_{2k}q(I^U-I^L)]^2+[\beta_{2k}+\rho_{2k}I^L+\rho_{2k}q(I^U-I^L)]^2+[\gamma_{2k}+\eta_{2k}I^L+\eta_{2k}q(I^U-I^L)]^2\right)\end{Bmatrix}}. \tag{9}$$

It is obvious that the q-indeterminate weighted correlation coefficients of Eqns (8) and (9) also contain these properties:

(P1) $W_1^q(P_1,P_2)=W_1^q(P_2,P_1)$ and $W_2^q(P_1,P_2)=W_2^q(P_2,P_1)$;

(P2) $W_1^q(P_1,P_2)=W_2^q(P_1,P_2)=1$ if $P_1 = P_2$;

(P3) $W_1^q(P_1,P_2), W_2^q(P_1,P_2)\in[0,1]$.

3. MDM approach based on the q-indeterminate weighted correlation coefficients of SNISs

Regarding the q-indeterminate weighted correlation coefficients of SNISs, this section presents a MDM approach with decision makers' risk attitudes in SNIS setting.

For a MDM problem in indeterminate decision-making setting, suppose that a set of m alternatives is represented by $P = \{P_1, P_2, …, P_m\}$ and evaluated by a set of n criteria $H = \{H_1, H_2, …, H_n\}$. Then, the weight vector of H is specified as $\boldsymbol{V} = (v_1, v_2, …, v_n)$ by decision makers. Thus, when decision makers give the satisfactory evaluations of each alternative P_i (i = 1, 2, …, m) over criteria H_k (k = 1, 2, …, n), their evaluation values are expressed by the truth NN $A_{ik}(I)=\alpha_{ik}+\delta_{ik}I\subseteq[0,1]$, the indeterminacy NN $B_k(I)=\beta_k+\rho_kI\subseteq[0,1]$, and the falsity NN $C_k(I)=\gamma_k+\eta_kI\subseteq[0,1]$ for $I \in [I^L, I^U]$, which are constructed as SNIN $p_{ik}=\langle A_{ik}(I),B_{ik}(I),C_{ik}(I)\rangle=\langle\alpha_{ik}+\delta_{ik}I,\beta_{ik}+\rho_{ik}I,\gamma_{ik}+\eta_{ik}I\rangle$ (k = 1, 2, …, n; i = 1, 2, …, m). Consequently, the decision matrix of SNINs $P=\left(p_{ik}\right)_{m\times n}$ can be established in SNIS setting.

In this MDM problem with SNIS information, we can develop a MDM approach by the q-indeterminate weighted correlation coefficients of SNISs, along with the decision makers' risk attitudes, such as the small risk (q = 0), the moderate risk (q = 0.5), and the large risk (q = 1), and give the following decision steps:

Step 1: The ideal solution/alternative $P^*=\{p_1^*,p_2^*,...,p_n^*\}$ is yielded from the decision matrix P by the following formula p_k^* (k = 1, 2, …, n):

$$p_k^*=\langle a_k^*,b_k^*,c_k^*\rangle=\left\langle\max_i(\alpha_{ik}+\delta_{ik}I^U),\min_i(\beta_{ik}+\rho_{ik}I^L),\min_i(\gamma_{ik}+\eta_{ik}I^L)\right\rangle \text{ for } I\in[I^L, I^U]. \tag{10}$$

Step 2: By using Eqn (8) or Eqn (9) regarding one of decision makers' risk attitudes, such as the small risk for q = 0, the moderate risk for q = 0.5, and the large risk for q = 1, the q-indeterminate weighted correlation coefficient between P_i (i = 1, 2, …, m) and P^* is given by the following formula:

$$W_1^q(P_i,P^*)=\frac{\sum_{k=1}^{n}v_k\left\{[\alpha_{ik}+\delta_{ik}I^L+\delta_{ik}q(I^U-I^L)]a_k^*+[\beta_{ik}+\rho_{ik}I^L+\rho_{ik}q(I^U-I^L)]b_k^*+[\gamma_{1k}+\eta_{1k}I^L+\eta_{1k}q(I^U-I^L)]c_k^*\right\}}{\sqrt{\sum_{k=1}^{n}v_k\left([\alpha_{1k}+\delta_{1k}I^L+\delta_{1k}q(I^U-I^L)]^2+[\beta_{1k}+\rho_{1k}I^L+\rho_{1k}q(I^U-I^L)]^2+[\gamma_{1k}+\eta_{1k}I^L+\eta_{1k}q(I^U-I^L)]^2\right)}\sqrt{\sum_{k=1}^{n}v_k\left((a_k^*)^2+(b_k^*)^2+(c_k^*)^2\right)}},\quad(11)$$

or

$$W_2^q(P_i,P^*)=\frac{\sum_{k=1}^{n}v_k\left\{[\alpha_{1k}+\delta_{1k}I^L+\delta_{1k}q(I^U-I^L)]a_k^*+[\beta_{1k}+\rho_{1k}I^L+\rho_{1k}q(I^U-I^L)]b_k^*+[\gamma_{1k}+\eta_{1k}I^L+\eta_{1k}q(I^U-I^L)]c_k^*\right\}}{\max\left\{\sum_{k=1}^{n}v_k\left([\alpha_{1k}+\delta_{1k}I^L+\delta_{1k}q(I^U-I^L)]^2+[\beta_{1k}+\rho_{1k}I^L+\rho_{1k}q(I^U-I^L)]^2+[\gamma_{1k}+\eta_{1k}I^L+\eta_{1k}q(I^U-I^L)]^2\right),\sum_{k=1}^{n}v_k\left((a_k^*)^2+(b_k^*)^2+(c_k^*)^2\right)\right\}}.\quad(12)$$

Step 3: Ranking order of the alternatives and the best one are given regarding the values of the q-indeterminate weighted correlation coefficient corresponding to one of decision makers' risk attitudes.

Step 4: End.

4. MDM example with SNIS information

This section presents a MDM example regarding the selection problem of open pit slope design schemes to indicate the practicability and flexibility of the proposed MDM method with decision makers' risk attitudes in SNIS setting.

Open pit slope design for an open pit mine is a critical problem in the process of mine design and development. Then, an optimal excavation configuration is provided in the context of safety, ore recovery and financial return (Read & Stacey, 2009). However, the open pit slope stability, the economic benefit, and the environmental requirements of mining should be considered as important factors by mining owners. To choose a suitable slope design scheme (alternative) for an open pit mine, assume that a set of four potential design schemes (alternatives) $P = \{P_1, P_2, P_3, P_4\}$ is given for the open pit mine, which must be satisfactorily evaluated by the three critical factors (criteria): the safe factor (R_1), the economical factor (R_2), and the environmental factor (R_3). Regarding the importance of the three critical factors, the weight vector of the three criteria is given by $\boldsymbol{V} = (0.3, 0.4, 0.3)$.

Then, a group of experts/decision makers is invited to satisfactorily evaluate the four alternatives over the three criteria by the truth NN $A_{ik}(I)=\alpha_{ik}+\delta_{ik}I\subseteq[0,1]$, the indeterminacy NN $B_{ik}(I)=\beta_{ik}+\rho_{ik}I\subseteq[0,1]$, and the falsity NN $C_{ik}(I)=\gamma_{ik}+\eta_{ik}I\subseteq[0,1]$ for the specified indeterminacy $I \in [0, 1.5]$ in the simplified neutrosophic indeterminate MDM problem, and then their valuation values can be constructed as SNINs $p_{ik}=\langle A_{ik}(I),B_{ik}(I),C_{ik}(I)\rangle=\langle\alpha_{ik}+\delta_{ik}I,\beta_{ik}+\rho_{ik}I,\gamma_{ik}+\eta_{ik}I\rangle$ (k = 1, 2, 3; i = 1, 2, 3, 4) and the following decision matrix of SNISs:

$$P=\begin{bmatrix}P_1\\P_2\\P_3\\P_4\end{bmatrix}=\begin{bmatrix}<0.7+0.2I,0.1+0.3I,0.1+0.1I> & (0.7+0.2I,0.2+0.1I,0.2+0.2I> & <0.6+0.2I,0.2+0.2I,0.2+0.2I>\\<0.8+0.1I,0.1+0.2I,0.1+0.3I> & <0.7+0.2I,0.2+0.1I,0.3+0.1I> & <0.7+0.1I,0.2+0.2I,0.1+0.1I>\\<0.7+0.1I,0.2+0.1I,0.1+0.2I> & <0.8+0.1I,0.2+0.1I,0.1+0.2I> & <0.7+0.2I,0.3+0.1I,0.2+0.1I>\\<0.8+0.1I,0.1+0.2I,0.2+0.1I> & <0.7+0.1I,0.1+0.2I,0.2+0.1I> & <0.7+0.1I,0.2+0.1I,0.2+0.2I>\end{bmatrix}.$$

In this case, the proposed approach is applied to the indeterminate MDM problem with SNISs for $I \in [0, 1.5]$ and depicted by the following calculation procedures.

Firstly, the ideal solution/alternative $P^*=\{p_1^*,p_2^*,p_3^*\}$ is yielded by using Eqn (10) for the decision matrix P as follows:

$$P^*=\{p_1^*,p_2^*,p_3^*\}=\{<1,0.1,0.1>,<1,0.1,0.1>,<1,0.2,0.1>\}.$$

Then, the values of the q-indeterminate weighted correlation coefficient between SNISs P_i and P^* (i = 1, 2, 3, 4) are calculated by Eqn (11) or (12) regarding to the small risk for q = 0 or the moderate risk for q = 0.5 or the large risk for q = 1 of the decision makers in the indeterminate range of $I \in [I^L, I^U]$ = [0, 1.5] and tabulated in Tables 1 and 2.

In Tables 1 and 2, the ranking orders of alternatives and the best slope design schemes regarding $W_1^q(P_i, P^*)$ and $W_2^q(P_i, P^*)$ along with the small risk (q = 0) or the moderate risk (q = 0.5) or the large risk (q = 1) of the decision makers denominate their difference. However, the ranking orders regarding $W_1^q(P_i, P^*)$ indicate small sensitivity with respect to

Table 1. Decision results regarding $W_1{}^q(P_i, P^*)$ along with the small risk (q = 0) or the moderate risk (q = 0.5) or the large risk (q = 1) of the decision makers

q	$W_1{}^q(P_i, P^*)$	Ranking order	The best one
q = 0	0.9772, 0.9748, 0.9796, 0.9836	$P_4 > P_3 > P_1 > P_2$	P_4
q = 0.5	0.9346, 0.9398, 0.9359, 0.9516	$P_4 > P_2 > P_3 > P_1$	P_4
q = 1	0.8920, 0.9010, 0.8948, 0.9123	$P_4 > P_2 > P_3 > P_1$	P_4

Table 2. Decision results regarding $W_2{}^q(P_i, P^*)$ along with the small risk (q = 0) or the moderate risk (q = 0.5) or the large risk (q = 1) of the decision makers

q	$W_2{}^q(P_i, P^*)$	Ranking order	The best one
q = 0	0.6900, 0.7493, 0.7629, 0.7473	$P_3 > P_2 > P_4 > P_1$	P_3
q = 0.5	0.8715, 0.9308, 0.9275, 0.9288	$P_2 > P_4 > P_3 > P_1$	P_2
q = 1	0.7556, 0.7299, 0.7112, 0.7497	$P_1 > P_4 > P_2 > P_3$	P_1

the three risk attitudes of the decision makers and the best one is always P_4, while the ranking orders and the best one regarding $W_2{}^q(P_i, P^*)$ indicate large sensitivity with respect to the three risk attitudes of the decision makers. Clearly, either the different q-indeterminate weighted correlation coefficients or the different risk attitudes of the decision makers can affect the ranking orders of alternatives. Then, the final decision result depends on one of decision makers' three risk attitudes in the indeterminacy $I \in [0, 1.5]$, which shows the flexibility and practicability of the proposed MDM method in SNIS setting.

Especially, existing MDM methods using the weighted correlation coefficients of SvNSs (Ye, 2013a, 2013b) are only the special cases of the proposed MDM method using the q-indeterminate weighted correlation coefficients of SNISs when q is only a specified value. In the MDM process, furthermore, existing MDM methods (Ye, 2013a, 2013b) cannot indicate decision makers' risk attitudes due to a lack of changeable interval values (NNs) in SvNSs, while the proposed MDM method can indicate decision makers' risk attitudes with q-indeterminate SvNSs in the setting of SNISs and demonstrate the advantage of flexible decision-making in the indeterminate MDM process. Therefore, existing MDM methods only give unique decision result without decision makers' risk attitudes and also cannot handle such an indeterminate MDM problem with decision makers' risk attitudes, such as the small risk for q = 0, the moderate risk for q = 0.5, and the large risk for q = 1, in SNIS setting. It is obvious that the proposed MDM method is superior to existing ones (Ye, 2013a, 2013b).

Conclusions

Regarding the indeterminacy of simplified neutrosophic information in indeterminate decision-making setting, this study presented SNIS to express the hybrid information of both SNS and NN, which is depicted by the truth, falsity and indeterminacy NNs in indeterminate and inconsistent situations. Then based on the de-neutrosophication technology using the parameterized SvNSs of SNISs, we introduced the q-indeterminate correlation coefficients of SNISs with a parameter $q \in [0, 1]$. Next, a simplified neutrosophic indeterminate MDM method using the q-indeterminate correlation coefficients of SNISs was established corresponding to decision makers' risk attitudes, such as the small risk for q = 0, the moderate risk for q = 0.5, and the large risk for q = 1, to carry out MDM problems in SNIS setting. Eventually, the proposed MDM method was applied to a MDM example of selecting a satisfactory slope design scheme for an open pit mine in SNIS setting to indicate the practicability and flexibility of the proposed MDM method. Regarding the decision results, we discussed how different q-indeterminate correlation coefficients with various risk attitudes of decision makers affect the ranking order of alternatives and the best one. Then, the main advantage of this study is the flexibility and practicability of the proposed MDM method in indeterminate MDM problems. In the future work, this study will be further extended to image processing, clustering analysis, and pattern recognition under SNIS environment.

Conflicts of interest

The authors declare no conflict of interest.

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