

## OPTIMAL OUTCOME SHARING WITH A CONSORTIUM OF CONTRACTORS

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**Abstract.** Where a consortium of contractors is involved, there exist no guidelines in the literature on what the outcome sharing arrangement should be. The paper addresses this shortfall. It derives the optimal outcome sharing arrangement for risk-neutral and risk-averse contractors within the consortium, and between the consortium and a risk-neutral owner. Practitioners were engaged in a designed exercise in order to validate the paper's propositions. The paper demonstrates that, at the optimum: the proportion of outcome sharing among contractors with the same risk-attitude should reflect the levels of their contributions; the proportion of outcome sharing among contractors with the same level of contribution should be lower for contractors with higher levels of risk aversion; a consortium of risk-neutral contractors should receive or bear any favourable or adverse project outcome respectively; and the proportion of outcome sharing to a consortium of risk-averse contractors should reduce, and the fixed component of the consortium fee should increase, when the contractors become more risk-averse or the level of the project outcome uncertainty increases. The paper proposes an original solution to the optimal sharing problem in contracts with a consortium of contractors, thereby contributing to current practices in contracts management.

**Keywords:** outcome sharing, consortium, contractors, risk attitude, utility function, contribution, construction contracts.

### Introduction

Engaging a consortium of contractors is regarded as a useful way to undertake large-scale complex projects, where extensive resources, skills and expertise are required (Ross 2006). Entering foreign markets has also motivated the formation of consortia, combining foreign and local firms. The consortium approach enables the owner to engage a team, rather than individual companies that may not be a good cultural fit (Ross 2006).

Within a consortium, the contractors perform effort (at cost) that leads to some project outcome, which is observable both to the contractors and owner (Petersen 1993). The outcome is not only dependant on the contractors' effort but is also affected by events which are outside of the contractors' influence. That is, there exists outcome uncertainty. A detailed review of uncertainties in construction projects can be found in Barnes (1983) and El-Sayegh (2008). The contractors' effort cannot be fully monitored by the owner. That is, there exists information asymmetry (Laffont, Martimort 2002; Puddicombe 2009). Due to the existence of the outcome uncertainty and asymmetric information, an opportunist contractor may act in its own interests instead of the owner's interest (Petersen 1993). Because effort is at cost to the contractor, the contractor may not give the effort that the owner desires (Eisenhardt 1989). This

may lead to conflict between the consortium contractors and the owner (Holmstrom 1979; Bartling, Von Siemens 2010), and it may affect the success of the project work (Harmon 2003; Hughes *et al.* 2012).

One way of addressing this is for the owner to provide an incentive to the contractors based on the consortium's actual performance as measured by project outcome expressed relative to a target (Shavell 1979; Eisenhardt 1989; Zhao 2005). Typically, such incentive approaches are exemplified by cost reimbursable contracts, with an outcome sharing arrangement or formula, based on a target (Carmichael 2000). The contracts align the contractors' interests with those of the owner, but at the price of transferring risk to the contractors. Eisenhardt (1989) points out that the outcome uncertainty introduces risk, which must be borne by someone. Eisenhardt (1989) argues that, as uncertainty increases, it becomes increasingly expensive to shift risk to the contractor. The trade-off between incentives and risk in determining the sharing arrangement is central to the design of contracts with an outcome sharing arrangement (Weitzman 1980).

In the construction literature, although the notion of outcome sharing is well established, there is no consensus as to what is the optimal sharing arrangement, or the proper model to adopt. No guidance is available for practitioners. Hughes *et al.* (2012) conclude that further research is

required on the level of outcome sharing that is ideal for a cost-focused infrastructure project. Tang *et al.* (2008) give that there is a clear need to explore sharing models that are appropriate to the construction industry. Badenfelt (2008) argues that the current practice of choosing sharing arrangements is arbitrary and not based on scientifically sound evidence or mathematical calculation. Hosseinian and Carmichael (2013) address single contractor-owner sharing problems in contracts with a risk-neutral contractor. They show that the optimum contract transfers to the contractor all potential monetary underruns/overruns (expressed relative to a target) associated with the contractor effort and events beyond the contractor's influence. Hosseinian and Carmichael (2012b) discuss the outcome-sharing problem between a single contractor and an owner in contracts with cooperative behaviour. They show that, at the optimum, the sharing arrangement is linear in the project outcome and is affected by the parties' risk aversion, and the outcome uncertainty has no influence on the sharing arrangement.

No publications exist investigating the outcome-sharing problem in contracts with a consortium of contractors, even though consortia are commonly engaged for delivering large-scale projects. This paper addresses this gap in knowledge. The paper derives the optimal outcome sharing arrangement within a consortium of contractors, which may be either risk-neutral or risk-averse, and between the consortium and a risk-neutral owner. It is noted that owners may be risk-neutral or risk-averse (Uher, Toakley 1999; Lyons, Skitmore 2004). See Carmichael (2004, 2006) and Clemen and Reilly (2001) for meanings of 'risk' and risk attitudes. The term 'outcome' refers to a project's equivalent monetary outcome expressed relative to a target that is desired by the owner. An outcome might, for example, be expressed with respect to: cost underruns/overruns relative to a target cost; late completion cost or early completion saving relative to a target duration; or monetary value of quality of work done compared with a target level of quality. The paper provides new guidance to owners and contractors as to what is the best way to reward a consortium of contractors through the terms of a contract. The paper will be of interest to owners and contractors.

The paper first outlines the optimization results leading to the paper's propositions, which are subsequently tested in a designed exercise. This is followed by the conclusions.

It is noted that the sharing problem involving a consortium of contractors exists in two components (Shavell 1979; Holmstrom, Milgrom 1987; Lambert 2001). The first is how to share the project outcome between the owner and the consortium. The second is how to distribute the consortium's share of the project outcome among the contractors within the consortium. For presentation purposes, this study focuses on outcome sharing with a consortium of two contractors, but the results are applicable to more than two contractors.

## 1. A basis for determining contractors' fees

A possible sharing arrangement in contracts with a consortium of two contractors,  $i = 1$  (Contractor A) and  $i = 2$  (Contractor B), may determine the contractors' fees based on a target cost according to:

$$Fee_i = F_i + n_i (Tc - Ac) \quad i = 1, 2, \quad (1a)$$

where:  $F_i$  is a fixed component of a contractor's fee;  $n_i$  is a sharing ratio for contractor  $i$ , taking values in the range 0 to 1;  $Tc$  is a target cost estimate of the work; and  $Ac$  is the actual cost of the work. A similar expression can be written for a project duration-based incentive.

The sum of the contractors' fees is the total fee paid by the owner to the consortium. Accordingly, the consortium's fee is given by:

$$Fee = \sum_{i=1}^2 F_i + \sum_{i=1}^2 n_i (Tc - Ac) = F + n(Tc - Ac), \quad (1b)$$

where  $F$  is a fixed component of the consortium's fee, and  $n$  is the consortium's sharing ratio, taking values in the range 0 to 1:

$$n = \sum_{i=1}^2 n_i \leq 1. \quad (2)$$

Carmichael (2000, 2002) argues that the target cost and target duration estimates can be agreed by the parties, or established by a third independent party. Love *et al.* (2011) point out that the target should be set at a very high performance standard but should also be achievable.

This paper establishes the optimum form in target relationships such as Eqns (1a) and (1b).

## 2. Theoretical results

For risk-neutral assumptions on the contractors within the consortium, Appendix A establishes the optimum parameters of Eqn (1a). In particular, using \* to denote the optimum form:

$$n_1^* = \frac{1}{1 + (k_2/k_1)^2} \quad n_2^* = \frac{1}{1 + (k_1/k_2)^2}; \quad (3)$$

$$F_1^* = \text{Min fee}_1 - \frac{k_1^2}{2b} n_1^{*2} - \frac{k_2^2}{b} n_1^* n_2^*; \quad (4a)$$

$$F_2^* = \text{Min fee}_2 - \frac{k_2^2}{2b} n_2^{*2} - \frac{k_1^2}{b} n_1^* n_2^*; \quad (4b)$$

where  $\text{Min fee}_i$  is the minimum fee required by contractor  $i$  to motivate it to agree to the contractual arrangement,  $b$  is a coefficient converting units of effort<sup>2</sup> to monetary units, and  $k_i$  is a constant coefficient representing the contribution of contractor  $i$  towards the outcome. A contractor's contribution level is based on that contractor's expenditure as a proportion of the total consortium cost (Ross 2003, 2006; Love *et al.* 2011). However,

where expenditure does not properly reflect the relative influence of each contractor on the outcome, the value used for contribution level can be adjusted (Ross 2006).

For risk-averse assumptions on the contractors within the consortium, Appendix B establishes the optimum parameters of Eqn (1a):

$$n_i^* = \frac{1}{1 + r_i \sigma^2 b / k_i^2} \quad i = 1, 2; \quad (5)$$

$$F_1^* = \text{Min fee}_1 - \frac{k_1^2}{2b} n_1^{*2} - \frac{k_2^2}{b} n_1^* n_2^* + \frac{1}{2} n_1^{*2} r_1 \sigma^2; \quad (6a)$$

$$F_1^* = \text{Min fee}_2 - \frac{k_2^2}{2b} n_1^{*2} - \frac{k_1^2}{b} n_1^* n_2^* + \frac{1}{2} n_2^{*2} r_2 \sigma^2, \quad (6b)$$

where  $r_i$  is the level of risk aversion of contractor  $i$ ; and  $\sigma^2$  is the variance of the outcome.

Eqn (3) shows that  $n_1^* \geq n_2^*$  if  $k_1 \geq k_2$  and  $n_1^* < n_2^*$  if  $k_1 < k_2$ , and more generally  $n_i^* \geq n_j^*$  if  $k_i \geq k_j$  and  $n_i^* < n_j^*$  if  $k_i < k_j$ , where  $i$  and  $j$  represent any two of the  $m$  contractors within the consortium. Similarly Eqn (5) generally shows that  $n_i^* \geq n_j^*$  if  $k_i / r_i \geq k_j / r_j$  and  $n_i^* < n_j^*$  if  $k_i / r_i < k_j / r_j$ .

### 3. Propositions

#### 3.1. Outcome sharing among contractors

An examination of Eqns (3) and (5) leads to the following propositions:

1. *The proportion of outcome sharing among risk-neutral contractors (in a consortium) should reflect the levels of their contributions.*
2. *The proportion of outcome sharing among risk-averse contractors (in a consortium), with the same level of risk aversion, should reflect the levels of their contributions.*
3. *The proportion of outcome sharing among contractors (in a consortium), with the same level of contribution towards the project outcome, needs to be lower for contractors with higher levels of risk aversion.*

#### 3.2. Outcome sharing between owner and consortium

An examination of Eqns (3), (5) and (6) leads to the following propositions:

4. *The proportion of outcome sharing to a consortium of risk-averse contractors needs to reduce with (4a) increasing uncertainty level in the outcome, or with (4b) increasing the risk aversion levels of the contractors.*
5. *With a consortium of risk-averse contractors, the consortium's fixed fee needs to increase with (5a) increasing outcome uncertainty, or with (5b) increasing contractors' risk aversion.*
6. *A consortium of risk-neutral contractors wishes to receive/bear all potential monetary underrun/overrun associated with the project outcome.*

## 4. Proposition testing

In order to test the above propositions, an empirical study was conducted on sixty experienced contractor practitioners.

### 4.1. Risk attitude measurement

In a designed exercise, the practitioners were interviewed to measure their risk attitudes (utility functions and levels of risk-aversion), based on certainty equivalence (Clemen, Reilly 2001).

Of the 60 contractors in the sample, the majority (48 or 80%) were found to be risk-averse, 12 (20%) to be risk-neutral, and none to be risk-seekers. The tendency to risk aversion may be because contractors are sensitive to going out of business, and contractors' diversification and opportunity to spread their exposures are low.

For contractors, who were found to be risk-averse, their levels of risk aversion were then established. These ranged from 5/Tc (least averse) to 33.3/Tc (most averse).

### 4.2. Outcome sharing among contractors

For a fee defined as in Eqn (1a), the contractors were asked to choose the desired value of sharing ratio,  $n_i$ , where they play the role of Contractor A in a consortium of two contractors. Three scenarios were designed, based on the contribution level of Contractor A towards the actual cost, compared with that for Contractor B: namely  $k_1 = k_2$ ;  $k_1 > k_2$ ; and  $k_1 < k_2$ . In the cases where  $k_1 > k_2$  and  $k_1 < k_2$ , the contractors were told to make decisions based on ratios of  $k_1$  to  $k_2$  of 60/40 and 40/60, respectively.

The results shown in Table 1 imply that a risk-neutral contractor prefers a higher (compared to the other contractor) sharing ratio as a member of a consortium, where it has a higher level of contribution towards the actual cost. The opposite occurs when a contractor has a lower contribution level. This supports the validity of Proposition 1.

The results of Figure 1 imply that a risk-averse contractor prefers a high sharing ratio value in a consortium of two contractors where it has a higher contribution level towards the actual cost, compared to the other contractor. This supports the validity of Proposition 2.

Figure 1 also supports the validity of the theoretical results in terms of the relationship between a contractor's sharing ratio and level of risk aversion in a consortium (Eqn (5)). It shows that an increase in the value of  $n_1$  follows from a decrease in level of risk aversion. This supports the validity of Proposition 3.

Table 1. Contractor A's sharing ratio selected by the risk-neutral contractors

Contribution comparison	Mean	St dev	Range
$k_1 > k_2$	0.48	0.03	0.4 – 0.5
$k_1 = k_2$	0.40	0.04	0.3 – 0.45
$k_1 < k_2$	0.31	0.04	0.25 – 0.35

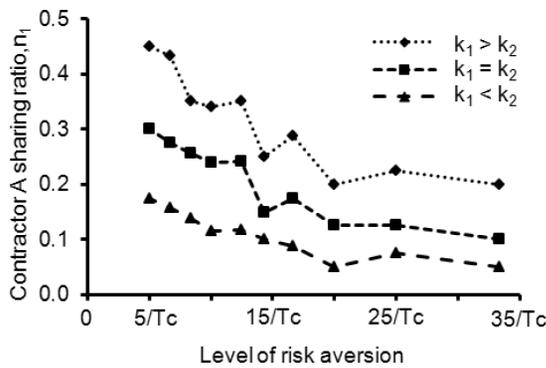


Fig. 1. Contractor A's sharing ratio selected by risk-averse contractors

Table 2. Coding for indicator variables  $x_1$  and  $x_2$

Contribution comparison	$x_1$	$x_2$
$k_1 > k_2$	1	0
$k_1 = k_2$	0	0
$k_1 < k_2$	0	1

For level of risk aversion  $r$ , indicator variables  $x_1$  and  $x_2$  (coding defined in Table 2), and constant coefficients  $\beta_0, \beta_1, \beta_2$  and  $\beta_3$ , a regression equation of the form:

$$n_1 = \beta_0 + \beta_1 (r \times Tc) + \beta_2 x_1 + \beta_3 x_2, \quad (7)$$

can be described. The best fit results using Figure 1 data are shown in Table 3. For the overall regression, the coefficient of determination,  $R^2$ , equals 0.83 with a p-value of 0.000. The low p-values in Table 3 indicate statistical significance, and the results are unlikely to have occurred by chance, thereby supporting the reliability of the regression results (Montgomery *et al.* 2012). Accordingly, the effects of risk aversion and contribution on  $n_1$  are significant. This provides further support for Propositions 2 and 3.

### 4.3. Outcome sharing between owner and consortium

For a fee defined as in Eqn (1b), the contractors were asked to choose a desired consortium sharing ratio,  $n$ , and a desired consortium tendered fee,  $F$ , assuming they were Contractor A in a consortium of two similar contractors. The contractors were required to make their decisions in three different uncertainty scenarios. Table 4 gives different actual cost intervals for different levels of uncertainty, assuming costs are normally distributed.

Table 3. Regression results for Eqn (7)

	Coefficient			
	$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$
Value	0.31	-0.008	0.108	-0.103
Standard error	0.010	0.001	0.009	0.009
p-value	0.000	0.000	0.000	0.000

Table 4. Intervals in actual cost for different scenarios due to events outside of the contractors' influence

Probability of occurrence	Uncertainty level		
	Low	Usual	High
68%	(0.97Tc, 1.03Tc)	(0.95Tc, 1.05Tc)	(0.92Tc, 1.08Tc)
95%	(0.93Tc, 1.07Tc)	(0.90Tc, 1.10Tc)	(0.83Tc, 1.17Tc)
99.7%	(0.90Tc, 1.10Tc)	(0.85Tc, 1.15Tc)	(0.75Tc, 1.25Tc)
Standard deviation	0.03Tc	0.05Tc	0.08Tc

*Risk-averse contractors.* Figure 2 shows the consortium's average sharing ratio values selected by the risk-averse contractors. An increase in the level of actual cost uncertainty leads to a decrease in the value of  $n$  selected by the contractors, reflecting the contractors' concerns. This supports the validity of Proposition 4a. Figure 2 also shows that an increase in the value of  $n$  follows from a decrease in the level of risk aversion. This supports the validity of Proposition 4b.

Using a regression equation of the form:

$$n = \alpha_0 + \alpha_1 (r \times Tc) + \alpha_2 \left( \frac{\sigma}{Tc} \right), \quad (8)$$

where  $\sigma$  is actual cost standard deviation, and  $\alpha_0, \alpha_1$  and  $\alpha_2$  are constant coefficients, the best fit regression results using Figure 2 data are shown in Table 5. For the overall regression, the coefficient of determination,  $R^2$ , equals 0.79 with a p-value of 0.000. The low p-values in Table 5 show that the effects of outcome uncertainty and

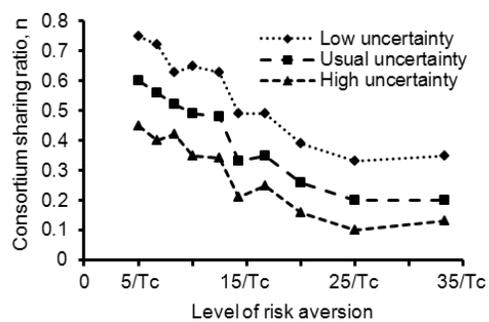


Fig. 2. Consortium's average sharing ratio values selected by the risk-averse contractors

Table 5. Regression results for Eqn (8)

	Coefficient		
	$\alpha_0$	$\alpha_1$	$\alpha_2$
Value	0.93	-0.02	-5.2
Standard error	0.022	0.001	0.315
p-value	0.000	0.000	0.000

risk aversion on consortium sharing ratio are significant (Montgomery *et al.* 2012). This provides further support for Propositions 4a and 4b.

The consortium’s average fixed fee values are shown in Figure 3, and present an opposing trend to Figure 2. An increase in the fixed fee follows from an increase in the level of cost uncertainty, reflecting the contractors’ concerns about uncertainty in the actual cost of the work. This supports the validity of Proposition 5a.

Figure 3 also supports the validity of the theoretical results in terms of the relationship between the consortium’s fixed fee,  $F$ , and the contractors’ levels of risk aversion (Eqns (6a) and (6b)). It shows that an increase in the value of  $F$  follows from an increase in the contractors’ levels of risk aversion. This supports Proposition 5b.

Using a regression function of the form:

$$F = \delta_0 + \delta_1 (r \times T_c) + \delta_2 \left( \frac{\sigma}{T_c} \right), \quad (9)$$

where  $\delta_0$ ,  $\delta_1$  and  $\delta_2$  are constant coefficients, the best fit regression results using Figure 3 data are shown in Table 6. For the overall regression,  $R^2$  equals 0.70 with a p-value of 0.000. The low p-values in Table 6 show that the effects of outcome uncertainty and risk aversion on the consortium’s fixed fee are significant (Montgomery *et al.* 2012). This provides further support for Propositions 5a and 5b.

*Risk-neutral contractors.* The results of Table 7 show that the risk-neutral contractors selected high sharing ratios. This supports Proposition 6, although the proposition suggests that the preferred value of  $n$  for risk-neutral contractors is 1. The falling short of the value 1 might imply that no contractors are exactly risk-neutral.

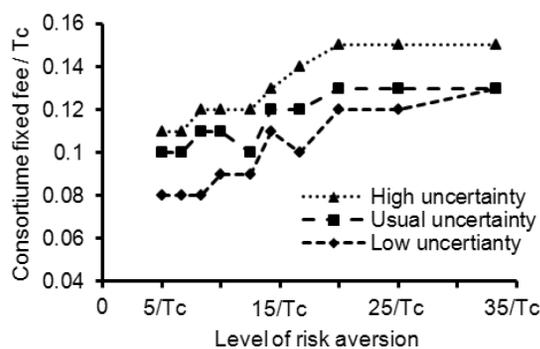


Fig. 3. Consortium’s average fixed fee values selected by the risk-averse contractors

Table 6. Regression results for Eqn (9)

	Coefficient		
	$\delta_0$	$\delta_1$	$\delta_2$
Value	0.054	0.002	0.625
Standard error	0.003	0.000	0.047
p-value	0.000	0.000	0.000

Table 7. Average values (standard deviations) of consortium sharing ratio and fixed fee/ $T_c$  identified by the risk-neutral contractors

Cost uncertainty level	Sharing ratio	Fixed fee / $T_c$
Low	0.89 (0.06)	0.068 (0.01)
Usual	0.86 (0.06)	0.070 (0.01)
High	0.83 (0.06)	0.073 (0.01)

A comparison between Table 7 and Figure 3 reveals that the risk-neutral contractors selected lower values (than the risk-averse contractors) for the consortium’s tendered fixed fee, being between 0.068 $T_c$  and 0.073 $T_c$ , with an average of 0.07 $T_c$ . This may be because risk-neutral contractors are not sensitive to any risk associated with cost savings/overruns compared to risk-averse contractors.

### 5. Discussion

A sharing arrangement should ensure that the contracting parties receive an equitable share of any actual outcomes which are better/worse than the targets agreed by the parties (Ross 2006; Love *et al.* 2011). Perceptions of equity and fairness in sharing arrangements play a role in contractor behaviour (Bresnen, Marshall 2000; Davis, Walker 2003; Love *et al.* 2011). If the sharing arrangement is judged inappropriate, then performance may be reduced (Bresnen, Marshall 2000). Disagreements, claims, and disputes eventually distort relationships among the parties, and these can be influenced by inappropriate sharing arrangements (Rahman, Kumaraswamy 2005). Previous research has stressed that the sharing arrangement should be fair (Cook, Hancher 1990; Bower *et al.* 2002; Love *et al.* 2011; Badenfelt 2008), though how ‘fair’ and ‘unfair’ are defined is undetermined in contracts with a consortium of contractors. This paper’s findings address this gap in knowledge. Owners and contractors might have a general idea about risk sharing and contractor motivation, however their decisions are not based on any model or theory (Badenfelt 2008). This paper’s findings assist owners and contractors establish an optimal sharing arrangement. Contracting parties, at tender time, might negotiate any sharing arrangement (Al-Subhi Al-Harbi 1998); the paper’s results should assist the contracting parties in this negotiation. Ignoring the optimal solution may lead to translating unattractive risk to contractors, resulting in a conflict of interest between the contracting parties, and perhaps putting the project success at stake. Barnes (1983) claims that owners in such situations have problems in getting their work completed to a satisfactory standard and without substantial additional costs.

### 6. Recommendations

Based on the paper’s analysis and data, recommendations are given here for sharing ratio values. The recommended values are based on the data collected from a group of medium-sized contractors, and hence their validity is limited to similar situations, until further data are assembled.

**6.1. Consortium with risk-averse contractors**

The level of risk aversion of the sample risk-averse contractors is first classified into three groups – low, medium and high. The results that follow are relatively insensitive to the choice of boundaries between these groups.

By calculating the average sharing ratios presented in Figure 2, recommended values of sharing ratios can be obtained. These are summarized in Table 8.

From Table 8 the highest sharing ratio ( $n = 0.75$ ) needs to be used in work with low cost uncertainty where a consortium of low risk-averse contractors is engaged. By contrast, the lowest sharing ratio ( $n = 0.20$ ) needs to be offered to a consortium with high risk-averse contractors in high cost uncertainty work. These recommendations are consistent with Turner and Simister (2001) who, based on transaction cost theory, argue that fixed price contracts (the special case of Eqn (1b) where  $n = 1$ ) should be used where uncertainty of the project outcome is low, and cost plus fixed fee contracts (the special case of Eqn (1b) where  $n = 0$ ) should be used where uncertainty of the project outcome is high.

In the situation where there is little or no information about both the cost uncertainty and the contractors' levels of risk aversion, the average value of the sharing ratio is recommended. This average equals 0.45, which is obtained from data presented in Table 8. This value is close to the opinion of Ross (2003), McGeorge and Palmer (2002) and Love *et al.* (2011), who suggest that any outcome should be spread 50:50 in construction projects.

Table 9, based on the data presented in Figure 1, provides guidance for sharing the outcome between two

Table 8. Recommended values of consortium sharing ratios for different levels of uncertainty and contractor average risk aversion

Cost uncertainty	Contractor average level of risk aversion		
	Low	Medium	High
Low	0.75	0.60	0.40
Usual	0.55	0.45	0.25
High	0.40	0.35	0.20

Table 9. Comparing two ( $i = 1, 2$ ) risk-averse contractors' sharing ratios within a consortium

Contribution comparison	Risk aversion comparison	Sharing ratio comparison
$k_1 > k_2$	$r_1 > r_2$	$n_1 > n_2$
	$r_1 = r_2$	
	$r_1 < r_2$	
$k_1 = k_2$	$r_1 > r_2$	$n_1 < n_2$
	$r_1 = r_2$	$n_1 = n_2$
	$r_1 < r_2$	$n_1 > n_2$
$k_1 < k_2$	$r_1 > r_2$	$n_1 < n_2$
	$r_1 = r_2$	
	$r_1 < r_2$	

Table 10. Comparing two ( $i = 1, 2$ ) risk-neutral contractors' sharing ratios within a consortium

Contribution comparison	Sharing ratio comparison
$k_1 > k_2$	$n_1 > n_2$
$k_1 = k_2$	$n_1 = n_2$
$k_1 < k_2$	$n_1 < n_2$

contractors,  $i = 1, 2$ , within a consortium in terms of contractor contribution and risk aversion.

Table 9 shows that between two contractors with different contribution levels, regardless of the risk aversion level, the contractor should receive/bear any monetary underrun/overrun associated with the project outcome in proportion to its contribution. In the case where both contractors have the same contribution, the contractor with a lower risk aversion level should receive/bear a higher monetary underrun/overrun.

**6.2. Consortium with risk-neutral contractors**

For a consortium with risk-neutral contractors, the paper's findings are that all monetary underrun/overrun (expressed relative to a target) associated with the project outcome should be received/borne by the consortium. Table 10, based on Table 1 data, provides guidance for sharing the outcome between two risk-neutral contractors,  $i = 1, 2$ , within a consortium. This table shows that the contractor with the higher contribution level should receive/bear a higher monetary underrun/overrun associated with the project outcome.

**Conclusions**

This paper derived the optimal outcome sharing arrangement in contracts with a consortium of contractors. The paper demonstrated:

1. The proportion of outcome sharing to a consortium of risk-averse contractors should reduce and the fixed component of the consortium fee should increase when the contractors within the consortium become more risk-averse or the level of the outcome uncertainty increases.
2. A consortium of risk-neutral contractors should wholly receive or wholly bear any favourable or adverse outcome, respectively.
3. The proportion of outcome sharing among contractors with the same risk-attitude should reflect the level of contribution.
4. The proportion of outcome sharing among contractors with the same level of contribution should be lower for contractors with higher levels of risk aversion.

The empirical study provided strong persuasive evidence and support for the paper's propositions.

A number of studies have discussed sharing problems in contracts. However no study has addressed outcome-sharing issues with a consortium of contractors.

This paper's findings address this gap in knowledge. The paper gives an original solution to the optimal sharing problem with a consortium of contractors, contributing to current practices in contracts management. The paper's results provide guidance to those involved in designing contracts as to what is the best way to reward a consortium of contractors through the terms of a contract. Where the sharing ratio and fixed fee are the subject of negotiation, the insight from this paper should assist the contracting parties in this negotiation.

*Future research.* It is acknowledged that there may exist parameters affecting the selection of the optimal sharing ratio and fixed fee other than those considered in this paper, and this could be the subject of future research.

A number of extensions to the paper are possible, for example to derive the optimal multiple outcomes sharing arrangement, and to expand to a more diverse sample of contractors in empirical studies. Another extension possibility is to consider the sharing problem in contracts where the effort is undertaken cooperatively by the contractors, that is the contractors work in the owner's interests without any need for incentives.

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## Appendix A

### Risk-neutral contractors

Consider an owner contemplating engaging a consortium of two contractors,  $i = 1, 2$ . The owner is not able to fully monitor the contractors' efforts, but the owner is able to measure the outcome of the collective contractors' efforts. The owner desires an optimal outcome-sharing contract that maximizes the owner's expected utility, while ensuring that the contractors agree to the contractual arrangement, and the contractors select their effort levels acceptable to the owner.

Consider the outcome, denoted by  $x$  and measured in monetary units, which depends on the contractors' efforts, denoted by  $e_i$ , and events which are outside of the contractors' control,  $\varepsilon$ .  $\varepsilon$  is assumed to be normally distributed with a mean of zero and variance  $\sigma^2$  (Holmstrom, Milgrom 1987). Although the contractors' skills influence the outcome, it is assumed here that all potential contractors have equivalent skills. It is also assumed here that the outcome varies linearly with effort, giving:

$$x = \sum_{i=1}^2 k_i e_i + \varepsilon. \quad (\text{A1})$$

The linearity assumption is not critical; rather it simplifies the mathematical manipulation. The constant coefficient  $k_i$  represents the contribution of a contractor towards the outcome. In contracts based on a target cost, such as Eqn (1a), the outcome may be interpreted as cost savings/overruns.

Let the contractors' fees, denoted by  $Fee_i$ , be also linear functions of the outcome:

$$Fee_i = F_i + n_i x, \quad i = 1, 2. \quad (\text{A2})$$

The sum of the contractors' fees is the total fee paid by the owner to the consortium.

The owner's utility (or payoff), in monetary units, is the difference between the outcome received and the consortium's fee:

$$U_o = x - \sum_{i=1}^2 Fee_i. \quad (\text{A3})$$

The owner is assumed here to be risk-neutral. That is, the owner simply wishes to maximize its expected utility. Using Eqns (A1) and (A2), and the notation  $E[ ]$  to denote expected value, the expected utility of the owner is given by:

$$E[U_o] = \left( 1 - \sum_{i=1}^2 n_i \right) \left( \sum_{i=1}^2 k_i e_i \right) - \sum_{i=1}^2 F_i. \quad (\text{A4})$$

Expression (A4) incorporates the special case of a fixed expected utility only, where  $\left(1 - \sum_{i=1}^2 n_i\right) \left(\sum_{i=1}^2 k_i e_i\right) = 0$ . In this case the owner's expected utility equals  $-\sum_{i=1}^2 F_i$ . In compensation for this negative expected utility, the owner receives the end-product of the work.

The contractors receive their fees, and apply effort, but incur costs. These effort costs are not included in the actual cost of the work, namely  $A_c$  in Eqn (1a), because the contractors' efforts are not observable by the owner (Shavell 1979; Feltham, Xie 1994). Let  $C_i(e_i)$  be the dollar amount necessary to pay a contractor for inducing a particular effort level,  $e_i$ . Let the effort  $e_i = 0$  be the effort a contractor would select without any incentive; that is  $C_i(0) = 0$ .

The contractors' utilities (or payoffs), in monetary units, are the difference between their fees received and the cost of their efforts:

$$U_i = Fee_i - C_i(e_i), \quad i=1,2. \tag{A5}$$

The contractors are assumed to be risk-neutral. That is, they simply wish to maximize their expected utilities. Using Eqns (A1) and (A2), each contractor's expected utility is given by:

$$E[U_i] = F_i + n_i(k_1 e_1 + k_2 e_2) - C_i(e_i), \quad i=1,2. \tag{A6}$$

The contractor cost function,  $C_i(e_i)$ , is assumed to increase with effort,  $e_i$ , at an increasing rate. The simplest function that meets this requirement can be written as:

$$C_i(e_i) = \frac{b}{2} e_i^2, \quad i=1,2. \tag{A7}$$

Here  $b$  is a constant coefficient converting units of effort<sup>2</sup> to money. This effort cost function has been popularly adopted (Holmstrom, Milgrom 1987, 1991).

Substituting Eqn (A7), into Eqn (A6):

$$E[U_i] = F_i + n_i(k_1 e_1 + k_2 e_2) - \frac{b}{2} e_i^2, \quad i=1,2. \tag{A8}$$

The owner desires an optimal outcome-sharing contract that maximizes its expected utility:

$$\text{Max}_{n_1, n_2, F_1, F_2} \left(1 - \sum_{i=1}^2 n_i\right) (k_1 e_1 + k_2 e_2) - \sum_{i=1}^2 F_i, \tag{A9}$$

subject to two constraints for each contractor.

Constraints, occur because the contract needs to provide the contractors with their minimum expected utilities (Min fee<sub>*i*</sub>,  $i = 1, 2$ ) to motivate the contractors to accept the contract:

$$F_i + n_i(k_1 e_1 + k_2 e_2) - \frac{b}{2} e_i^2 \geq \text{Min fee}_i, \quad i=1,2. \tag{A10}$$

Other constraints occur because the contractors will select their efforts so as to maximize their expected utilities. In order to motivate the contractors to put their efforts in the owner's interests, the outcome-sharing contract needs to maximize the contractors' expected utilities:

$$\text{Max}_{e_i} F_i + n_i(k_1 e_1 + k_2 e_2) - \frac{b}{2} e_i^2, \quad i=1,2. \tag{A11}$$

Expressions (A9), (A10) and (A11) constitute the optimization problem.

Differentiating expression (A11) with respect to  $e_i$  and setting to zero provides the optimal level of effort (denoted with \*):

$$e_i^* = \frac{k_i}{b} n_i, \quad i = 1, 2. \tag{A12}$$

The optimal value of  $f_i$  would be such that expression (A10) holds as an equality, that is:

$$F_i^* = \text{Min fee}_i - n_i (k_1 e_1 + k_2 e_2) + \frac{b}{2} e_i^2, \quad i = 1, 2. \quad (\text{A13})$$

Substituting Eqns (A12) and (A13) into (A9), the owner's problem can be restated as:

$$\text{Max}_{n_1, n_2} E[U_0] = \text{Max}_{n_1, n_2} \left( \sum_{i=1}^2 \frac{k_i^2}{b} n_i \right) - \sum_{i=1}^2 \left( \text{Min fee}_i + \frac{k_i^2}{2b} n_i^2 \right). \quad (\text{A14})$$

Differentiating expression (A14) with respect to  $n_i$ ,  $i = 1, 2$ , and setting to zero:

$$n_i = 1, \quad i = 1, 2. \quad (\text{A15})$$

This result does not satisfy Eqn (2). Accordingly the maximum of expression (A14) lies on the line  $n_1 + n_2 = 1$  which is the boundary of the admissible region of the maximization problem.

Introducing a Lagrange multiplier  $\lambda$ , the maximization becomes:

$$\text{Max}_{n_1, n_2} \left( \sum_{i=1}^2 \frac{k_i^2}{b} n_i \right) - \sum_{i=1}^2 \left( \text{Min fee}_i + \frac{k_i^2}{2b} n_i^2 \right) + \lambda \left( \sum_{i=1}^2 n_i - 1 \right). \quad (\text{A16})$$

Differentiating expression (A16) with respect to  $n_i$ ,  $i = 1, 2$ , and  $\lambda$ , setting to zero, and simplifying leads to the optimal sharing ratios of Eqns (3).

Substituting Eqn (A12) into (A13), leads to the optimal fixed fees of Eqns (4a) and (4b).

## Appendix B

### Risk-averse contractors

The development of Appendix A may be extended to incorporate outcome sharing between a risk-neutral owner and a consortium of two risk-averse contractors. The outcome, contractors' fees, the expected utility of owner and contractors' cost functions remain as in Eqns (A1), (A2), (A4) and (A7).

Risk aversion is characterized by a concave utility function. Here, the exponential utility function, because it has been popularly adopted (Holmstrom, Milgrom 1987; Kirkwood 2004), is used. The contractors wish to maximize their expected utilities. Maximizing a contractor's expected utility is equivalent to maximizing its certainty equivalent; a contractor's certainty equivalent is its expected fee minus its cost of effort and its risk premium (Clemen, Reilly 2001). In obtaining the risk premium, a suitable approximation is provided by Pratt (1964) and also discussed in Clemen and Reilly (2001), and is given by:

$$\text{Risk Premium}_i = \frac{n_i^2 r_i \sigma^2}{2}, \quad i = 1, 2, \quad (\text{B1})$$

where  $r_i$  is a contractor's level of risk aversion.

The expected fees to the contractors can be obtained by substituting Eqn (A1) into Eqn (A2), while noting that  $E[\varepsilon] = 0$ :

$$E[\text{Fee}_i] = F_i + n_i k_i e_i, \quad i = 1, 2. \quad (\text{B2})$$

Using Eqns (B1), (B2) and (A7), the certainty equivalents corresponding to the contractors' expected utilities are given by:

$$CE_i = F_i + n_i (k_1 e_1 + k_2 e_2) - \frac{b}{2} e_i^2 - \frac{n_i^2 r_i \sigma^2}{2}, \quad i = 1, 2. \quad (\text{B3})$$

The contractors select their efforts,  $e_i$ ,  $i = 1, 2$ , that maximize their expected utilities:

$$\text{Max}_{e_i} F_i + n_i (k_1 e_1 + k_2 e_2) - \frac{b}{2} e_i^2 - \frac{n_i^2 r_i \sigma^2}{2}, \quad i = 1, 2. \quad (\text{B4})$$

However, the contractors will only agree to the contractual arrangement if the contractors' certainty equivalents exceed their minimum utilities (Min Fee<sub>i</sub>,  $i = 1, 2$ ):

$$F_i + n_i (k_1 e_1 + k_2 e_2) - \frac{b}{2} e_i^2 - \frac{n_i^2 r_i \sigma^2}{2} \geq \text{Min fee}_i, \quad i = 1, 2. \quad (\text{B5})$$

Expressions (A9), (B4) and (B5) constitute the optimization problem.

Differentiating Eqn (B4) with respect to  $e_i$  and setting to zero provides the optimal level of effort, and this leads to Eqn (A12).

The optimal value of  $f_i$  would be such that expression (B5) holds as an equality, that is:

$$F_i^* = \text{Min fee}_i - n_i (k_1 e_1 + k_2 e_2) + \frac{b}{2} e_i^2 + \frac{n_i^2 r_i \sigma^2}{2}, \quad i = 1, 2. \quad (\text{B6})$$

Substituting Eqns (A12) and (B6) into (A9), the owner's problem can be restated as:

$$\text{Max}_{n_1, n_2} \left( \sum_{i=1}^2 \frac{k_i^2}{b} n_i \right) - \sum_{i=1}^2 \left( \text{Min fee}_i + \frac{k_i^2}{2b} n_i^2 + \frac{n_i^2 r_i \sigma^2}{2} \right). \quad (\text{B7})$$

Differentiating expression (B7) with respect to  $n_i$ ,  $i = 1, 2$ , and setting to zero, leads to the optimal sharing ratio of Eqn (5).

Substituting Eqn (A12) into (B6), leads to the optimal fixed components of Eqns (6a) and (6b).

Where the contractors' levels of risk aversion approach zero and the contractors become risk neutral, the solutions of expression (B7) lie on the line  $n_1 + n_2 = 1$ , and the optimal sharing ratios are obtained by Eqns (3).

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