STRENGTH ASSESSMENT OF HISTORIC BRICK MASONRY

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Abstract. The identification of residual service life of a structure is an exceptionally demanding task in the case of reconstructed or newly modernised buildings. The identification of residual service life requires the study and knowledge of the mutual interaction of a building with its external environment, in particular, the time variable effects and impacts which lead to degradation processes and phenomena affecting and, in the absolute majority of cases, degrading the physical, mechanical and other properties of materials and structures. The article presents the results of in-situ and laboratory research of strength parameters of masonry from the start of the 20th century applying destructive and non-destructive tests. Besides, a probabilistic model and a procedure for the determination of masonry strength are described. It appears that the probabilistic approach leads to a design value by ca 5% higher than the deterministic approach.

Keywords: masonry, compressive strength, brick, mortar, destructive test, non-destructive test, probability, lognormal distribution, normal distribution, histogram.

Introduction

Despite a relatively extensive research into masonry structures, the issue of a reliable determination of the load-bearing capacity of existing, mainly historic stone masonry structures is still waiting for a satisfactory solution (Witzany *et al.* 2006). The underlying reason is the exceptionally high indeterminacy of input parameters (Kotlík *et al.* 2000; Šrámek 1990), a large spatial variability of materials of masonry structures used e.g. in the masonry composition of one building, one storey or in one masonry member (e.g. a wall, pier) (Witzany *et al.* 2006; Heidingsfeld 1997). Numerous local practices and specific characteristics of component parts, extraction and treatment methods of natural masonry units of which the masonry was made, the composition of masonry and masonry structures going back to a respective time frame and architectural style, all this significantly affects the mechanical properties of masonry. Figure 1 presents an example of the relationship of the compressive strength of arenaceous marl to the sampling depth (Kotlík *et al.* 2000) and the compressive strength of CP 20 bricks coming from different periods and localities in the Czech Republic (Witzany *et al.* 2006). It is noted that the loss of strength in arenaceous marl at the depth of sampling of 14–15 m may be caused by different ground pressures during the deposition of marl layers. Figure 2 presents an example of the relationship of the ultimate compressive strength f_{ubexp} to the water content w_{hm} of bricks.
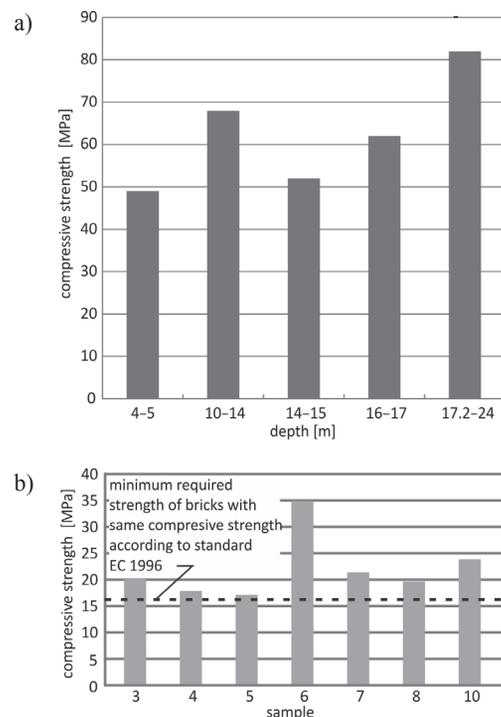

Fig. 1. a – Compressive strength of arenaceous marl in relation to sampling depth in Zeměchy quarry (Kotlík *et al.* 2000); b – Experimentally identified bricks with same compressive strength from different periods and sampling sites (3, 4 – 21st century, Prague; 5–7 – early 20th century, Kutna Hora; 8,10 – early 20th century, Humpolec)

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Compressive strength of historic masonry may be significantly affected by degradation processes that influence the surface and close-to-surface layers of overground masonry (processes of chemical, physical and microbiological corrosion).

Chemical degradation processes are an inseparable component of degradation processes caused by moisture. Chemical corrosion of building materials is an action or a set of actions where as a consequence of the effects of an aggressive environment the principal physical and mechanical characteristics of materials fall below the values necessary for preserving their utility value.

The actions involve chemical reactions, reactions between solid and liquid, or gaseous phases. The co-participants in the reactions acting at the phase interface, are, apart from the chemical reaction itself, transport phenomena; as a consequence of their action reacting substances are brought in and reaction products carried away. So that a reaction may be running, permanent transfer – transport of the mass of a reacting liquid phase and its efficient components must be ensured.

Due to salt crystallisation in pores, or due to hydration pressures, pressures arise inside the structure of building materials which gradually impair this structure causing so-called degradation processes. The growing volumes of some salts which are transformed into hydrates (increased water content) cause crystallisation hydration pressures reaching values in the order of tens of MPa, which exceed the common actual tensile strengths of building materials. The growth of crystals is limited by small pore spaces and crystals develop considerable expansive pressures which grow with temperature. The evaporation of water causes the dehydration of crystals and their disintegration. With a repetitive increase in moisture, hydroscopic salts newly absorb water and recrystallise. By this repetitive process (crystallisation and recrystallisation), together with the washout of binder components, the structure tends to gradually disintegrate and fall apart (Witzany *et al.* 2006, 2008).

The mechanisms of degradation processes, their intensity and the velocity of their passage in time are related to the material structure, e.g. the pore system, specific surfaces, etc. These parameters are, in a decisive way, affected mainly by transport processes in materials, primarily by moisture (in the liquid as well as gaseous phase), which is the principal carrier of various aggressive substances transported into the interior structure of building materials and structures which, as a rule, change their chemical, physical and mechanical properties by their action (Fig. 3). Partial results of the chemism of sampled specimens have pointed out some correlations between the chemical characteristics of investigated materials and the amounts of chemical substances contained in their pore system, or bonded to the material itself. As Figure 3 clearly shows, the growth in the amount of
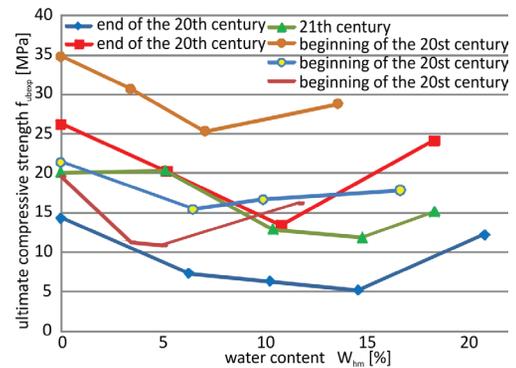

Fig. 2. An example of the relationship of the ultimate compressive strength f_{ubexp} to the water content w_{hm} of bricks (Čejka 2009)
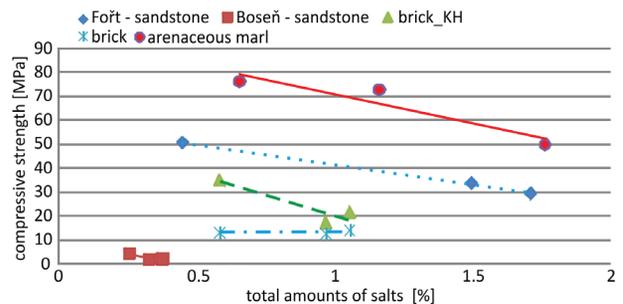

Fig. 3. The effect of total amounts of salts on the compressive strength of masonry units (Čejka 2009)

salts is accompanied by the reduction of the compressive strength of masonry units.

A precondition for the identification of residual mechanical properties of historic masonry and its load-bearing capacity is a detailed description, mapping and analysis of all mechanical failures, cracks, the condition of surface layers, masonry heterogeneity, the thickness and the quality of binder in bed joints, the type and dimensions of masonry units and masonry bonds.

Special care is required for the identification of the load-bearing capacity of stone or mixed masonry composed of irregular walling units (undressed quarry stone) or of various types of natural stone (arenaceous marl, fine-grained sandstone, coarse-grained sandstone, limestone, granite, etc.), and multi-layer masonry (so-called emplecton). The predominantly positive effect of the triaxial compressive stress state of mortar which applies in classic brickwork cannot be applied to masonry in which chippings and sharp-edged walling units of undressed quarry stone are found. The points of the occurrence of vertical tensile cracks splitting a masonry block into individual parts ("stanchions") are most frequently the cross sections with inefficient masonry bonding in several layers. Local stress states characterised by the tensile component arising around masonry units with a relatively higher modulus of elasticity against the surrounding units may be unfavourably demonstrated only at higher values of stone or mixed masonry.

1. Experimental research of compressive strength of historic brick masonry

Experimental research involved the analysis of the heterogeneity (homogeneity) of the brick masonry of a printing works in Humpolec from the start of the 20th century. The research manifested a relatively large dispersion of identified properties within the selected section of a masonry wall with dimensions of 9×4 m. A virtual network with dimensions of 9×4 m was "laid" onto the investigated brick masonry. The intersections of this virtual network (the total of 18 intersections, Table 1) were the points for the destructive and non-destructive verification of compressive strength and water content (% mass).

The values of standardised compressive strength obtained by a destructive test on test cores with a diameter of 35 mm and a length of approximately 50–70 mm at points of the virtual network intersections serve as the basis for the comparison with the values obtained by non-destructive methods. The compressive strength values obtained from test cores in a press manifest a non-uniform random distribution of the identified characteristics of masonry (Fig. 4). The measured values of compressive strength range in the interval from 40 to 230% of the average compressive strength of a walling unit in the investigated structure. The sampling of test cores cooled by water requires the identification of the moisture content of walling units at the site of presumed sampling (before sampling) and a subsequent modification of the moisture content of a sampled specimen to the level corresponding to the initial masonry moisture at the site of the sampled specimen. The water content of masonry at the points of individual samplings ranged from 0.38% to 15.66%.

Figure 5 shows experimentally identified strengths of masonry units in compression and their moisture content at the time of sampling in the intersections of a virtual network of 9×4 m. The figure clearly indicates the effect of moisture on compressive strength of masonry units for the analysed structure. Trend similar to those indicated in Figure 2 is observed. Averaged absorbability obtained from about a hundred of specimens (18 locations, 5–6 tests at each location) is 16.1% (with standard deviation 24.6%). Averaged moisture, needed in the following analysis to assess the partial factor for masonry strength, can be thus estimated as 2.51%.

Figure 6 shows spatial variability of water content (% mass) and compressive strength of masonry units (bricks) and mortar in the intersections of the virtual network plotted onto the investigated masonry.
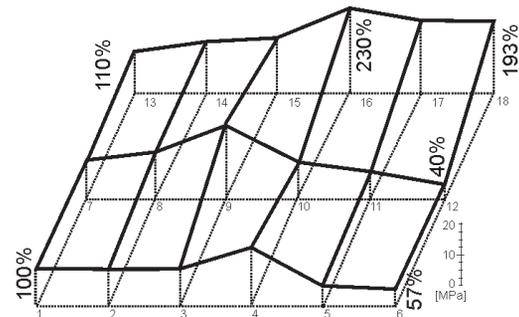

Fig. 4. Comparison of compressive strengths of masonry units in the points of intersection of the virtual network 9×4 m identified by a destructive method (compression test in a press, reference value (100%) is the strength at the point #1)

Table 1. Results of tests of masonry units and mortar (MPa)

| Probe | Masonry units | | | Mortar |
	Press	Schmidt hammer type L	drill with an indenter	drill with an indenter
1	13.9	16.5	24.3	1.09
2	15.9	16.4	28.0	~~1.75~~
3	8.9	19.2	34.4	0.92
4	~~29.0~~	16.8	~~21.6~~	0.73
5	6.7	12.3	15.2	0.97
6	4.1	0.6	1.0	X
7	9.8	18.9	23.7	0.73
8	14.2	16.3	23.7	0.91
9	18.6	13.9	28.0	0.89
10	9.4	13.4	X	0.79
11	6.9	18.9	12.0	0.96
12	4.0	18.0	11.8	X
13	18.7	18.3	26.1	1.09
14	11.9	17.1	24.8	0.73
15	16.2	19.7	23.5	1.01
16	19.1	14.0	33.7	0.75
17	17.6	17.6	29.6	0.8
18	16.7	12.5	26.9	0.93

Note: X – not measured, crossed out digits – outliers.

Humpolec – compressive strength x absorbability

Legend:
    1–18 – compressive strengths of masonry units in the points of intersection of the virtual network 9 x 4 m identified by a destructive method (compression test in a press) with moisture detected in situ
    A – the influence of moisture on capacity of compresive strength of bricks, in the point 1 of intersection of the virtual network
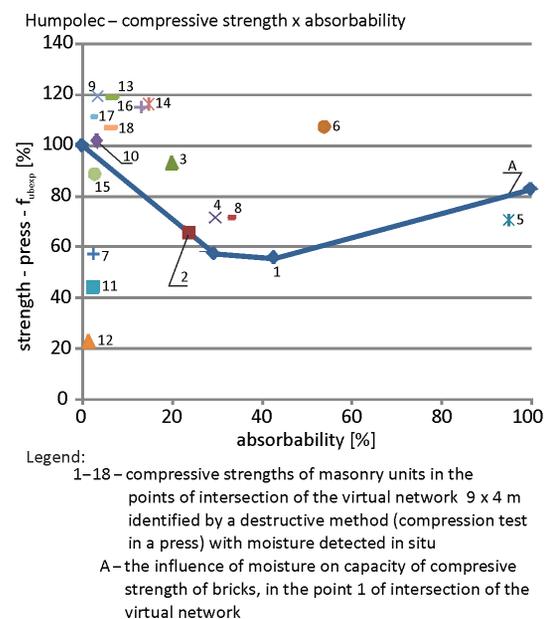
Fig. 5. Experimentally obtained relationships of the compressive strength and the water content $f_{u,exp} \times w_{hm}$ for a brick (Čejka 2009)
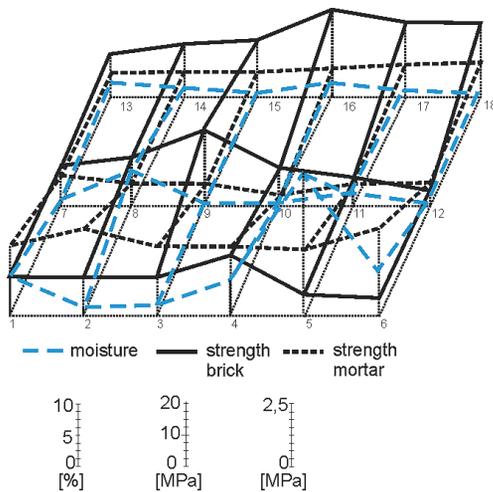
Fig. 6. The distribution of water content and compressive strength of masonry units (bricks) and mortar along the area of an experimentally investigated masonry structure (Humpolec) in the points of intersection of the virtual network 9×4 m
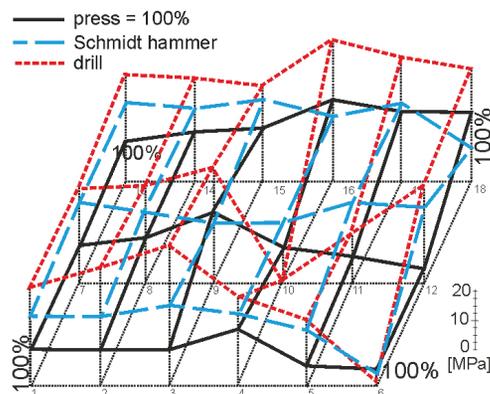

Fig. 7. Comparison of compressive strengths of masonry units identified by non-destructive methods with compressive strengths of masonry units identified in a press (100% = compressive strength identified in a press)

The comparison of the results obtained by destructive and non-destructive methods (Fig. 7) shows visible differences. The values obtained by non-destructive measurement using a modified percussion drill with an indenter range from 25 to 300% as compared to the values obtained by a non-destructive method (100%). The differences found for the Schmidt hardness tester ranged from 50 up to 230%. The strength analysis of mainly masonry units based on non-destructive methods indicates their limited credibility (Fig. 7).

Note that a modified percussion drill with an indenter complemented by a revolution counter and a meter measuring the magnitude of acting force (force in the magnitude of 150 N) is fitted with a bit of 6 mm diameter. The depth of drilled boreholes at a specified number of revolutions (varying for individual materials) is measured. The compressive strength of masonry units or binder is identified from an average depth from minimally three valid boreholes at one point using a calibration relationship.

2. Experimentally identified compressive strengths of bricks using destructive and non-destructive methods

2.1. Destructive tests

Based on long-term experience, the values obtained by testing masonry units in a press are considered more credible than the results obtained with the Schmidt hammer and drill with an indenter.

The data from the tests in a press are, therefore, considered as initial data, whereas the results obtained with the Schmidt hammer and a drill with an indenter are only informative. Initially, it is verified whether the data set includes outliers. Such observations may arise e.g. by a measurement error or by measurement at a point of a local non-homogeneity. Outliers may distort the assessment results, which, in turn, may lead to erroneous conclusions. Grubbs' test for outliers (Grubbs 1969; Ang, Tang 2007; Holický 2013) indicates that the hypothesis that a data set contains outliers can be denied. Note that the significance level considered in this study in tests for outliers is always 0.05 – a common value for engineering applications. An outlier in the measurement in a press, however, was identified based on the comparison with the results obtained with a drill with an indenter (see text below).

The basic sample characteristics of tests in a press are provided in Table 2. The method of moments (Ang, Tang 2007; Holický 2013) for which the knowledge of the underlying probability distribution of the variable is not needed (unlike e.g. the maximum likelihood method) is used; the outlier is excluded from the sample.

Note that in the statistical assessment of the mean value, the statistical uncertainty resulting from a limited number of tests is also considered ($n = 17$ after the elimination of the outlier). Assuming a normal distribution the mean $m_{fb,\text{test}}$ and the standard deviation $s_{fb,\text{test}}$ can be bounded by the intervals:

$$11.0 \text{ MPa} < m_{fb,\text{test}} < 14.0 \text{ MPa};$$

$$4.36 \text{ MPa} < \sigma_{fb,\text{test}} < 6.61 \text{ MPa}.$$

A 75% confidence – standard value for civil engineering applications according to ISO 2394:1998 *General principles on reliability for structures* – is considered. More details about the interval estimation of the mean and the standard deviation may be found e.g. in Ang and Tang (2007), Holický (2013).

As the present-day EN and ISO codes provide no guidance for the interval estimation, the analysis below only considers the sample mean $m_{fb,\text{test}} = 12.5$ MPa.

A lognormal distribution is usually assumed for the probability description of masonry units. Due to a small size of the sample, this distribution is considered and goodness-of-fit tests (Ang, Tang 2007; Holicky 2013), which may otherwise serve for the selection of a suitable probability distribution, are not carried out.

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Table 2. Statistical characteristics of variables affecting masonry strength

Variable	Symbol	Distribution	Mean μ	Coefficient of variation V
Strength of masonry units	$f_{b,test}$	LN*	12.5 MPa	0.41
Conversion factor – masonry units	η_b	LN	1	0.2
Strength of mortar	$f_{m,test}$	LN	0.89 MPa	0.14
Conversion factor – mortar	η_m	LN	1	0.2
Coefficient for calculation of f	K	LN	~1.25K	0.2

Note: *Lognormal distribution with the origin at zero (hereinafter simplified as a "lognormal distribution").

Fig. 8. Strengths of masonry units measured with the Schmidt hammer or the drill and a press

2.2. Non-destructive tests – Schmidt hammer (type L)

Non-destructive testing was conducted by the Schmidt hammer (type L), commonly used for ceramics. Calibration scale proposed and verified for tests of bricks by a testing laboratory was applied. Grubbs' test does not detect any outlying observations. Figure 8 presents the strengths of masonry units measured by the Schmidt hammer and in a press. Using the results of linear regression analysis (Ang, Tang 2007; Holický 2013), the relationship of the strengths identified by the press and the Schmidt hammer and the prediction interval for the 75% confidence are plotted by grey curves.

It is evident that both variables do not significantly depend on each other, and the prediction interval is considerably wide. If we, for example, identify the strength of 20 MPa with the Schmidt hammer, it may be assumed that the strength identified by a press will lie with a 75% probability in the interval of 7 to 23 MPa. The credibility of the measurement with the Schmidt hammer thus seems very low in this case.

2.3. Non-destructive tests – the drill with an indenter

For the drill with an indenter, Grubbs' test detected one outlying observation. Figure 8 shows the strengths of masonry units measured with the drill and in the press, including the outlier. The figure also plots the relationship of the strength identified in the press and by the drill and the prediction interval for the 75% confidence (black curves).

It is apparent that based on the results obtained with the drill the strength of masonry units may be estimated slightly better. If we, for example, identify the strength of 20 MPa with the drill, there is a 75% probability that the strength obtained by a press will range in the interval of 7 to 15.5 MPa. The credibility of the measurement, however, is still quite low for this method in this specific case.
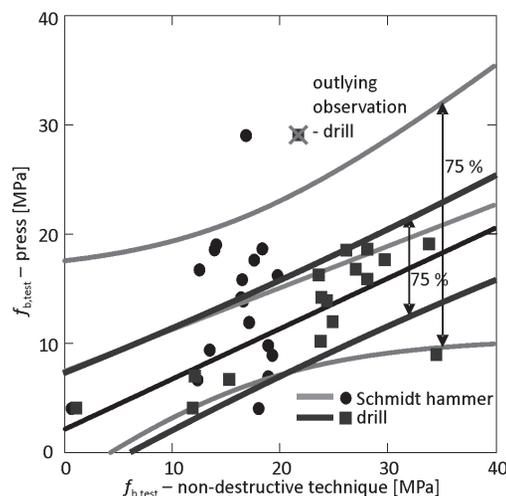
It is thus recommended to exploit the data obtained with the drill for the identification of potential outliers in the press results. The detected observation (#4 in Table 1) is considered as an outlier for the press and is eliminated. The sample mean for the press data then decreases by 7% and the coefficient of variation by 13%.

2.4. Assessment of compressive strength of binder identified with the drill with an indenter

Grubbs' test for outliers indicates that strength measurements of mortar obtained with the drill with an indenter contain an outlier (#2 in Table 1). The basic sample characteristics of the tests are summarised in Table 2 (excluding the outlying observation). The probabilistic description of mortar strength usually assumes a lognormal distribution. Due to a very small size of the sample, this distribution is considered and goodness-of-fit tests are not carried out.

2.5. Evaluation of masonry strength

The evaluation of masonry strength on the basis of tests is usually a key task in the assessment of historic masonry structures. Previous studies pointed out that the partial factor method tends to be conservative. It may suffice for the design of new structures where reliability may easily be increased. In the verification of historic structures, however, it may lead to costly repairs and, potentially, also to the loss of a heritage value (ICOMOS 2003). Holický *et al.* (2009), Mojsilovic and Faber (2009), Stewart and Lawrence (2007), Sykora and Holicky (2010) recommended using probabilistic methods for the assessment of masonry strength. In comparison with deterministic methods, the probabilistic approach allows a rational consideration of:

– Randomness of material characteristics of masonry units and mortar;

– Statistical uncertainties due to a limited number of tests;

– Inaccuracies of testing methods;

– Simplifications adopted in the model for masonry strength (model uncertainty).

The obtained experimental results are further applied in the probabilistic assessment of masonry strength. It is based on the procedure for the assessment of compressive strength of masonry in the direction perpendicular to bed joints pursuant to EN 1996-1-1:2005 *Design of masonry structures – Part 1-1: General rules for reinforced and non-reinforced masonry structures* and the Czech National Annex to ISO 13822:2010 *Bases for design of structures – Assessment of existing structures*. Characteristic and design values obtained by the deterministic approach are compared to the corresponding fractiles of the probabilistic model.

3. Probabilistic models of basic variables

3.1. Conversion factors for the strength of masonry units and mortar

Statistical characteristics of the variables affecting the strength of masonry are summarised in Table 2. The assessment of the compressive strength of masonry according to EN 1996-1-1:2005 and ISO 13822:2010 requires the knowledge of the mean value of the standardised strength of masonry units f_b. The strengths from testing in a press $f_{b,test}$ are converted into the standardised strength using the conversion factor η_b:

$$\eta_b = f_b / f_{b,test}. \tag{1}$$

It is assumed that the testing on small specimens in a press is unbiased ($\mu_{\eta b} = 1$) and thus:

$$\mu_{fb} \approx \mu_{\eta b} \mu_{fb,test} = 12.5 \text{ MPa.} \tag{2}$$

Due to insufficient data for the identification of the variability of the conversion factor for performed tests, the analysis below assumes the coefficient of variation $V_{\eta b} = 0.2$ (a common value for non-destructive testing) and a lognormal distribution (see for instance Ang, Tang 2007; Holický 2009, 2013).

The conversion factor η_m for the assessment of the mortar strength f_m on the basis of test results $f_{m,test}$ is:

$$\eta_m = f_m / f_{m,test}. \tag{3}$$

Based on experience, the conversion factor is characterised by a lognormal distribution ($\mu_{\eta m} = 1$ and $V_{\eta m} = 0.2$). Hence, the mean value of mortar strength is:

$$\mu_{fm} \approx \mu_{\eta m} \mu_{fm,test} = 0.89 \text{ MPa.} \tag{4}$$

Coefficient *K*

Pursuant to the Czech National Annex NF to ISO 13822:2010, the calculation of the characteristic strength of non-reinforced masonry requires the determination of the coefficient *K*, see Eqn (5) below. According to this Annex, it is further assumed that *K* = 0.5. The probabilistic model of the coefficient *K* describes model uncertainties, covering the lack of experimental data, the model simplification and the effect of the unknown workmanship quality.

Unlike the strength of masonry units and mortar, it is usually impossible to obtain experimental data for the identification of the coefficient *K* in the assessment of a specific structure. That is why the probabilistic model of this coefficient is based on previous experience and the assessment of experimental data described in available literature (JCSS 2011; Sykora, Holicky 2010; Brehm 2011). It can approximately be assumed that $\mu_K / K = 1.25$ and $V_K = 0.2$; a lognormal distribution is an appropriate probabilistic model.

3.2. Compressive strength of masonry

Characteristic value

Pursuant to ISO 13822:2010, the characteristic compressive strength f_k of non-reinforced masonry made with general purpose mortar is given by:

$$f_k = K f_b^{\alpha} f_m^{\beta} = K (\mu_{\eta b} \mu_{f b,test})^{0.65} (\mu_{\eta m} \mu_{f m,test})^{0.25} =$$
$$0.5 \times (1 \times 12.5)^{0.65} \times (1 \times 0.89)^{0.25} = 2.5 \text{ MPa.} \tag{5}$$

If outliers are not eliminated, the characteristic strength is $f_k = 2.6$ MPa, i.e. a value by about 5% higher. If outliers are eliminated and the lower limits for the mean values $f_{b,test}$ and $f_{m,test}$ are considered, we obtain $f_k = 2.3$ MPa (by about 10% lower than according to Eqn (5)). The difference, however, may be greater, particularly for small sample sizes.

Note that the empirical Eqn (5) need not always describe the actual compressive strength of masonry with adequate accuracy. In some cases, other models may be more suitable, such as the exponential function similar to Eqn (5) with general values of exponents (JCSS 2011) or more advanced methods described in Stewart and Lawrence (2002).

Design value

The design value of masonry strength is derived from the characteristic value by means of the partial factor γ_M:

$$f_d = f_k / \gamma_M = 2.5 / 2.0 = 1.25 \text{ MPa.} \tag{6}$$

According to ISO 13822:2010, the partial factor is identified as the product of the following coefficients:

γ_{m1} – the basic value of the partial factor (2.0 for masonry of full bricks laid on general purpose mortar).

$0.85 \leq \gamma_{m2} \leq 1.2$ – accounting for the effect of the regularity of the masonry bond and the filling of joints with mortar; the lower limit of the interval applies to a regular bond and perfect filling of joints with mortar (here $\gamma_{m2} = 1.0$).

$1.0 \leq \gamma_{m3} \leq 1.25$ – accounting for the effect of increased moisture, in the interval from 4% to 20% interpolation is used ($\gamma_{m3} = 1.0$ for the averaged moisture 2.51%, see Section 1).

$1.0 \leq \gamma_{m4} \leq 1.4$ – accounting for the effect of vertical and inclined cracks in masonry, for non-degraded masonry without cracks ($\gamma_{m4} = 1.0$).

Target reliability for existing structures

In accordance with EN 1990:2002 *Basis of structural design*, the design strength of masonry f_d is considered as a fractile of a corresponding probability p_d:

$$p_d = \Phi(-\alpha_R \times \beta_t) = \Phi(-0.8 \times 3.1) = 0.0066, \quad (7)$$

where $\Phi(\cdot)$ is the distribution function of a standardised normal distribution, the sensitivity coefficient of the FORM method α_R is estimated by the value 0.8 (EN 1990:2002) and the target reliability index $\beta_t = 3.1$ is accepted from Sykora and Holicky (2013) for high costs of safety measures and high failure consequences. Discussion concerning the assessment of the target reliability level for existing (historic) structures is out of the scope of this paper, for details see Schueremans and Van Gemert (2004), Holický (2012), Sykora and Holicky (2012).

3.3. Probabilistic analysis

Probabilistic model of masonry strength

The probabilistic model of masonry strength is developed to enhance the accuracy of the characteristic and design value estimates. For simplification, statistical uncertainties related to a limited number of tests are not considered. The probabilistic model of masonry strength f is given as:

$$f = K (\eta_b f_{b,test})^{0.65} (\eta_m f_{m,test})^{0.25}, \quad (9)$$

where all variables are considered as lognormal variables (Table 2). This can be accepted in the majority of practical cases. It may then be easily proved that the resultant masonry strength also has a lognormal distribution, which is in agreement with previous studies (Ellingwood, Tallin 1985; Stewart, Lawrence 2007; Mojsilovic, Faber 2009). The logarithm $\ln(f)$ has a normal distribution with the mean value and the standard deviation:

$$\mu_{\ln(f)} = \mu_{\ln(K)} + 0.65[\mu_{\ln(\eta b)} + \mu_{\ln(fb,test)}] + 0.25[\mu_{\ln(\eta m)} +$$

$$\mu_{\ln(fm,test)}]; \quad \sigma_{\ln(f)} = \sqrt{\{\sigma_{\ln(K)}^2 + 0.65^2[\sigma_{\ln(\eta b)}^2 +}$$

$$\sigma_{\ln(fb,test)}^2] + 0.25^2[\sigma_{\ln(\eta m)}^2 + \sigma_{\ln(fm,test)}^2]\}, \quad (10)$$

where $\mu_{\ln(X)}$ and $\sigma_{\ln(X)}$ refer to the mean value and the standard deviation of $\ln(X)$:

$$\mu_{\ln(X)} = \ln(\mu_X) - 0.5\ln[1 + V_X^2];$$

$$\sigma_{\ln(X)} = \sqrt{\{\ln[1 + V_X^2]\}}, \quad (11)$$

where μ_X and $V_X = \sigma_X / \mu_X$ denote the mean value and the coefficient of variation of the variable X according to Table 2. The characteristics of masonry strength, therefore, may be identified on the basis of analytical relationships without the use of specialised software.

Results of probabilistic analysis

Characteristic and design values are usually defined as a fractile of the probability distribution of a material parameter. Note that a fractile corresponding to the probability p is such a value of a random variable which is exceeded with a probability $(1-p)$. The fractile of a lognormal distribution x_p is identified from the formula:

$$x_p = \mu \times \exp\{u_p \sqrt{[\ln(1 + V^2)]}\} / \sqrt{(1 + V^2)}, \quad (12)$$

where $u_p = \Phi^{-1}(p)$ is the fractile of a standardised normal distribution of a corresponding probability p. The fractile u_p is identified as the value of the inverse distribution function of a standardised normal distribution Φ^{-1}, which is available e.g. in MS Excel. Thus, the fractile can be assessed without specialised statistical software.

Equations (10) and (11) yield the mean value of masonry strength of 3.05 MPa and the coefficient of variation of 0.37. The probability density of masonry strength, the characteristic and design values are shown in Figure 9.
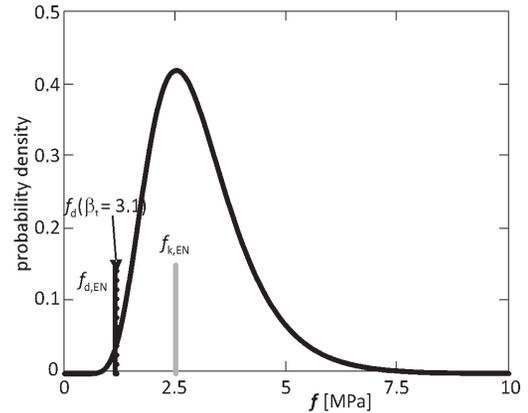
Fig. 9. The probability density function of masonry strength, the characteristic value $f_{k,EN}$ and design values identified deterministically $f_{d,EN}$, and probabilistically $f_d(\beta_t = 3.1)$

Table 3 presents characteristic and design values of masonry strength calculated pursuant to different approaches and partial factors γ_M. It appears that the characteristic value identified as a 5% fractile is nearly by 40% smaller than the characteristic value according to ISO 13822:2010, which rather corresponds to the mean value. Similar findings were also made in the previous studies (Holický *et al.* 1997; Sykora, Holicky 2010).

Table 3. Characteristic and design values of masonry strength (in MPa) and partial factors

Approach	f_k	f_d	γ_M
deterministic	2.5	1.25	2.0
deterministic including uncertainties in identification of $f_{b,test}$ and $f_{m,test}$	2.3	1.15	2.0
probabilistic ($\beta_t = 3.1$)	1.6[*]	1.2	1.4
probabilistic ($\beta_t = 3.8$)	1.6[*]	1.0	1.6

Note: [*]Specified as a 5% fractile.

Considering the target reliability index $\beta_t = 3.8$ the probabilistic approach yields a value by 15% lower than the deterministic approach, which may be an acceptable agreement for practical applications. For $\beta_t = 3.1$, which may be more suitable for historic structures, the calculated design value is by ca 5% higher as compared to the deterministic approach. In this case, the contribution of the probabilistic approach application is insignificant. In the previous study by Sykora and Holicky (2010), however, the probabilistic approach resulted in a significant increase in the design value by 25%.

The partial factors identified by the probabilistic method are significantly lower as compared to the considered value $\gamma_M = 2.0$, which is caused by differences in characteristic values.

4. Discussion and recommendations

The experimental part of the study focusing on general aspects of the assessment of the strength of historic brick masonry, the sampling of specimens of masonry units and mortar and the assessment of tests has resulted in the formulation of the following recommendations:

- a detailed description, mapping and analysis of all mechanical failures, cracks, the condition of surface layers, masonry heterogeneity, the thickness and quality of binder in bed joints, the type and dimensions of masonry units and masonry bonds must be made;
- inaccuracies in the identification of physical and mechanical properties of historic masonry (mainly when using non-destructive methods) must be considered in the specification of these properties;
- due to the variability of masonry properties, information about its mechanical properties must be obtained on the basis of tests; the identification of masonry strength may then have a key role in the evaluation of historic and other existing structures;
- outlying observations, which may be caused by a measurement error or by measurement at a point of local non-homogeneity, must be verified. Such observations may distort the sample characteristics of the set and lead to erroneous conclusions;
- the application of strengthening materials – strips and fibres – based on high-strength carbon fibres belongs to progressive methods of strengthening and stabilisation of historic brick masonry (Witzany *et al.* 2011). A relatively high modulus of elasticity of carbon fibres as compared to the modulus of deformation of masonry (1:4–1:100) allows the capturing of tensile forces at the initial phase of their appearance and thus the elimination of crack appearance and development – i.e. at strain values smaller than the strain at the crack appearance limit.

Conclusions

Uncertainties in the identification of physical and mechanical properties of historic masonry must be compensated by the growing ratio between the experimentally identified ultimate strength and the actual strain of a masonry structure. The issue of ultimate or admissible load-bearing capacity is usually raised in cases of the occurrence of extensive masonry degradation, or during a reconstruction involving a change in loading or major interventions in the existing masonry. Any reconstruction project should always respect the principle of the preservation of the original structural concept and design of the whole structure, or the removal of all former insensitive interventions and incongruous structural members. This principle, at the same time, also delimits the concept for the design of a prepared reconstruction. Each major increase in the loading of the existing historic masonry structure or any related interventions or modifications must be subjected to a detailed qualitative assessment, or, if enough accurate input values are available, to numerical assessment to preventatively avoid any potential subsequent appearance of cracks and masonry degradation.

In the numerical part of the paper, the probabilistic model of masonry strength is developed; it appears that:

- As compared to deterministic methods, the probabilistic approach allows a better description of the randomness of basic variables, statistical and model uncertainties and inaccuracies of testing methods, and thus leads to less conservative results.
- The relationship between the strengths of masonry units and mortar assessed by a test and their actual strength must be considered by introducing a suitable conversion factor.
- Suitable theoretical models of the variables affecting masonry strength must be selected on the basis of previous experience and test results.
- A lognormal distribution with the origin at zero is a suitable model for masonry strength in the majority of cases; the presented probabilistic approach is suitable for practical applications as it does not require specialised software.
- The probabilistic analysis of the strength of investigated historic masonry reveals that a 5% fractile of masonry strength is by nearly 40% lower than the characteristic value according to ISO 13822:2010. For the target reliability index of 3.1 the design value based on the probabilistic approach is by about 5% greater than that obtained by the deterministic approach.

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