SIMPLIFIED FORMULATIONS FOR THE DETERMINATION OF ROTATIONAL SPRING CONSTANTS IN RIGID SPREAD FOOTINGS RESTING ON TENSIONLESS SOIL

Konuralp GİRGİN

Department of Civil Engineering, Civil Engineering Faculty, Istanbul Technical University,
34469 Maslak, Istanbul, Turkey

Abstract. In spread footings, the rotational spring constants, which represent the soil-structure interaction, play an important role in the structural analysis and design. To assign the behaviour of soil, which is generally represented via Winkler-type tensionless springs, necessitates time consuming iterative computing procedures in practice. In this study, a straightforward approach is proposed for the soil-structure interaction of rigid spread footings especially subjected to excessive eccentric loading. By considering the uplift of footing, the rotational spring constants of those type footings under axial load and biaxial bending are easily attained through the proposed simplified formulations. Since these formulations enable manual calculation, iterative computer efforts are not required. The formulations under consideration can be applicable to symmetric and non-symmetric rigid spread footings. The numerical results of this study are verified with SAP2000.

Keywords: spread footing, rotational spring constant, tensionless Winkler-type soil, axial load, bending, biaxial bending, soil-structure interaction.

Introduction

A footing is a sub-structural component transferring the loads from the superstructure to soil. Spread footings are widely used in various types of civil engineering structures such as bridges, wind towers, large-span and low-rise structures. The soil-structure interaction has a major importance on the deflections, second-order effects and stability for the analysis and design of those structures.

There are numerous studies in the current literature to design and analyses of spread footings. However, no significant work on the determination of rotational spring constant in spread footings regarding the uplifting from soil. A literature survey on this subject mentioned above is summarized below.

Wilson (1997) proposed a practical solution tool that may be used by bridge engineers to compute design components of rectangular footings. Aristizabal-Ochoa (2002) presented the criteria to assess the base rotational spring restraint for the second-order analysis of a cantilevered precast concrete column on an isolated reinforced concrete footing and its anchorages with piling in elastic soil. Allotey and Naggar (2003) investigated the moment-rotation response of rigid spread footings resting on Winkler soil model. Uplift-yield foundation conditions were derived in their study. The full definition of entire static moment-rotation response can be handled by the devel-oped solutions for uplift-and yield-only conditions. Wang *et al.* (2005) presented a paper for comprehensive review of state-of-art on the analysis of beams and plates resting on elastic foundations considering the interaction of between structure and supporting soil media. Z. C. Girgin and K. Girgin (2005) represented the numerical method focusing non-uniform beam-columns on resting on variable one-or two-parameter elastic foundations or supported by no foundation; a variable iterative algorithm is developed for computer application of the method. Z. C. Girgin and K. Girgin (2006) proposed a generalized numerical method for non-uniform Timoshenko beam–columns subjected to several effects through a unified approach based on the Mohr method. In their study, Winkler-type elastic foundation modulus and shear foundation modulus can be considered. Gerelymos and Gazetas (2006a) studied on a Winkler model with four types of generalized springs for the lateral static and dynamic response of rigid caissons in a homogeneous elastic soil. Gerelymos and Gazetas (2006b) developed a nonlinear Winkler-spring method for the static, cyclic, and dynamic response of caisson foundations. The nonlinear behaviour of soil can be modeled realistically by using suitable couple translational and rotational nonlinear interaction springs and dashpots. Apostolou *et al.* (2007) investigated the rocking and overturning response of slender rigid structures

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Corresponding author: Konuralp Gïrgïn
E-mail: *girgink@itu.edu.tr*

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allowed to uplifting. The structure rests on the surface of either a rigid base or a linearly elastic continuum. In their study, a large-displacement approach is adopted. Özmen (2011) developed a general method to obtain the base pressures of rectangular footings under biaxial bending. In this study, since position of the neutral axis is not known initially, a process of successive approximations is developed. Orakdöğen *et al.* (2008) presented a case study on the performance evaluations of a 3D building strengthened by additional shear-walls. The soil-structure interaction is included in the push-over analysis by considering the tensionless elastic-plastic Winkler soil model. Different foundation types such as mat foundation, continuous or single footings are considered in their study. Rodriguez-Gutierrez and Aristizabal-Ochoa (2013a) introduced an analytical method capable of determining the axial load and biaxial moment capacities concerning rigid footing of arbitrary shape resting on soil. Uniform, linear and parabolic types of soil pressure distributions are considered to design spread footings. Rodriguez-Gutierrez and Aristizabal-Ochoa (2013b) proposed design nomograms for isolated rectangular and symmetrical trapezoidal rigid spread footings subjected to biaxial bending and axial load to directly determine maximum axial load and biaxial moment capacities without exceeding the bearing capacity of the supporting soil. Gazetas *et al.* (2013) developed formulas and charts in the dimensionless format to obtain nonlinear effective rotational stiffness of foundations of any (reasonable) shape. Anastasopoulos and Kontoroupi (2014) offered a simplified method to analyze the seismic performance of rocking systems, taking account of soil inelasticity and foundation uplifting. The soil–foundation system is replaced by springs and dashpots. There exists some design standards which are Fema 273 (1997) and FHWA-RC (2014) about this subject. In addition, half-space theory may be considered as an alternative approach for the determination of rotational spring constants of rigid spread footings (Smoltczyk 2002).

If the uplift of spread footings subjected to excessive eccentric loading is disregarded, their rotational spring constants can be easily assigned through multiplying the moment of inertia of footing with the subgrade reaction of soil. However, as the eccentricity increases, the uplift from Winkler-type tensionless soil becomes prominent, thus above simple approach leads to improper results. To solve this problem, the non-linear solution methods are required that generally necessitates time consuming iterative procedures. In this study, the simplified formulations enabling the manual calculation are proposed to avoid from the iterative computer procedures. These formulations are developed to assess the rotational spring constants of circular and rectangular rigid spread footings subjected to axial load and biaxial bending by taking the foundation uplift from the soil into consideration. Since only the soil pressure distribution is regarded, these formulations can be applicable to the non-symmetric rigid spread footings as well as symmetric ones.

1. Rigid spread footings subjected to axial force and bending moment

In this section the rigid spread footings under the effect of axial force and bending moment are addressed. The types of spread footing under consideration in this study are circular and rectangular. The assumptions to compute the rotational spring constant in these footings are as follows:

a) The footing is circular or rectangular;
b) The footing behaves as a rigid body;
c) Distribution of soil pressure and the location of zero pressure are linear;
d) Tensionless Winkler-type soil is considered under the footing;
e) Static loading case is under consideration.

The rotational spring constant K_r is defined as the bending moment to rotate the rigid footing one degree around the specified global direction and it is expressed as:

$$K_r = \frac{M}{\theta},\qquad(1)$$

where M and θ are the bending moment and rotation of rigid footing, respectively.

1.1. Circular spread footings

A typical circular spread footing under axial force and bending moment is illustrated in Figure 1. Herein R, M, N, σ_1, d_1, x and K_s signify the diameter, bending moment, axial force, maximum soil pressure, maximum displacement, depth of compressive zone from the neutral axis and coefficient of subgrade reaction of soil, respectively. The eccentricity (e) is defined as:

$$e = \frac{M}{N}.\qquad(2)$$

If the eccentricity (e) is equal and less than $R/8$, a tension zone is not under consideration. For this case, the rotational spring constant of circular spread footing K_r is written:

$$K_r = K_s I = K_s \frac{\pi R^4}{64},\qquad(3)$$

where I is the moment of inertia of footing.

If the eccentricity (e) is greater than $R/8$, the tension zone will occur (Fig. 1). In this case, through the equations given in İnan (1988), maximum soil pressure and depth of the compressive zone in circular spread footing can be written by using following equations:

$$\sigma_1 \cong 4 \left\{ \frac{0.372 + 0.056\left(1 - 2\dfrac{e}{R}\right)}{\left(1 - 2\dfrac{e}{R}\right)^{3/2}} \right\} \frac{N}{R^2};\qquad(4)$$

$$x \cong \frac{1}{2}\left\{ 2.33\left(1 - 2\frac{e}{R}\right) + 0.58\left(1 - 2\frac{e}{R}\right)^3 \right\} R.\qquad(5)$$
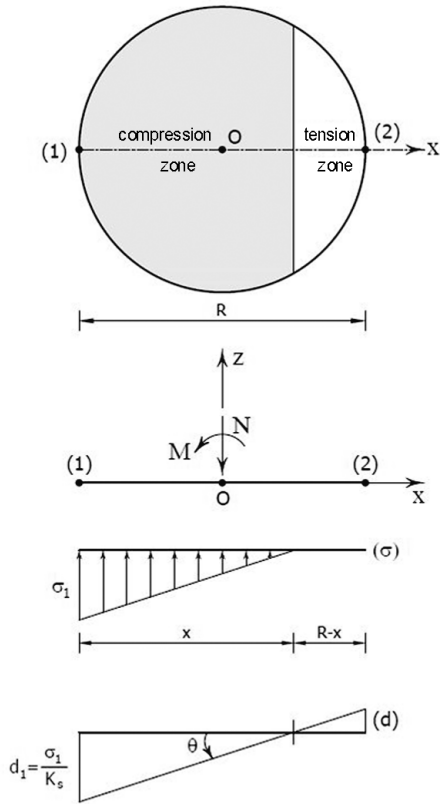
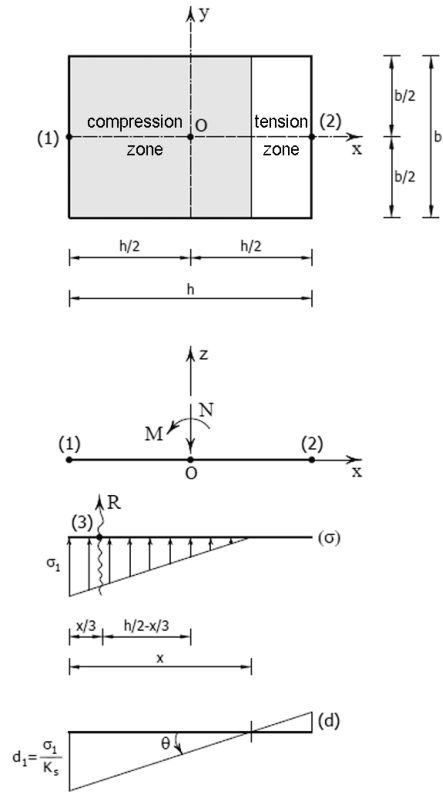Fig. 1. Soil pressure distribution and displacement profile of soil

Fig. 2. Soil pressure distribution and displacement profile of soil

In the case of uplift, the rotation of spread footing (θ) can be expressed by using the Eqns (4) and (5):

$$\theta = \frac{d_1}{x} = \frac{8N}{K_s R^3 \gamma^{5/2}} \left\{ \frac{0.372 + 0.056\gamma}{2.33 + 0.58\gamma^2} \right\}, \qquad (6)$$

where:

$$\gamma = 1 - 2\frac{e}{R}. \qquad (6a)$$

The rotational spring constant K_r can be written in terms of the eccentricity and μ:

$$K_r = \mu \frac{K_s \pi R^4}{64}; \qquad (7)$$

$$e \le \frac{R}{8} \;\rightarrow\; \mu = 1; \qquad (7a)$$

$$e > \frac{R}{8} \;\rightarrow\; \mu = \frac{8e\gamma^{5/2}}{\pi R} \left\{ \frac{2.33 + 0.58\gamma^2}{0.372 + 0.056\gamma} \right\}, \qquad (7b)$$

where μ is a coefficient corresponding to the variation of the eccentricity.

1.2. Rectangular spread footings

Rectangular spread footing is another footing type under consideration. A typical footing subjected to the axial load and bending moment is displayed in Figure 2.

If the eccentricity is equal and less than $h/6$, a tension zone does not exist. The rotational spring constant of footing can be defined as:

$$K_r = K_s I = K_s \frac{bh^3}{12}, \qquad (8)$$

where h represents the length of footing in the direction of bending moment, b is the perpendicular length to h.

If the eccentricity is greater than $h/6$, the vertical and moment equilibrium equations for the point 3 can be written, the maximum soil pressure σ_1 can be handled:

$$\sigma_1 = \frac{2N}{3b\left(\frac{h}{2} - e\right)} \qquad (9)$$

and maximum displacement at point 1 is defined as:

$$d_1 = \frac{\sigma_1}{K_s} = \frac{2N}{3K_s b\left(\frac{h}{2} - e\right)}, \qquad (10)$$

then the rotation of rigid footing θ is calculated:

$$\theta = \frac{d_1}{x} = \frac{\sigma_1}{3K_s\left(\frac{h}{2} - e\right)} = \frac{2N}{9K_s b\left(\frac{h}{2} - e\right)^2} \qquad (11)$$

and through Eqn (1), the rotational spring constant is:

$$K_r = \frac{9K_s be}{2}\left(\frac{h}{2} - e\right)^2. \tag{12}$$

Briefly, using Eqns (8) and (12), the rotational spring constant of rectangular footing K_r can be calculated via the eccentricity and μ coefficient:

$$K_r = \mu K_s \frac{bh^3}{12}; \tag{13}$$

$$e \leq \frac{h}{6} \rightarrow \mu = 1.0; \tag{13a}$$

$$e > \frac{h}{6} \rightarrow \mu = \frac{54e}{h^3}\left(\frac{h}{2} - e\right)^2. \tag{13b}$$

The variation between distribution of soil pressure and rotational spring constant K_r is shown depending on the eccentricity in Figure 3.

1.3. Rectangular rigid spread footing subjected to axial force and biaxial bending

In this section, it is addressed how to be described the rotational spring constants of rectangular spread footing subjected to axial force and biaxial bending. There are five possible cases corresponding to five zones for rotational spring constants in Figure 4.
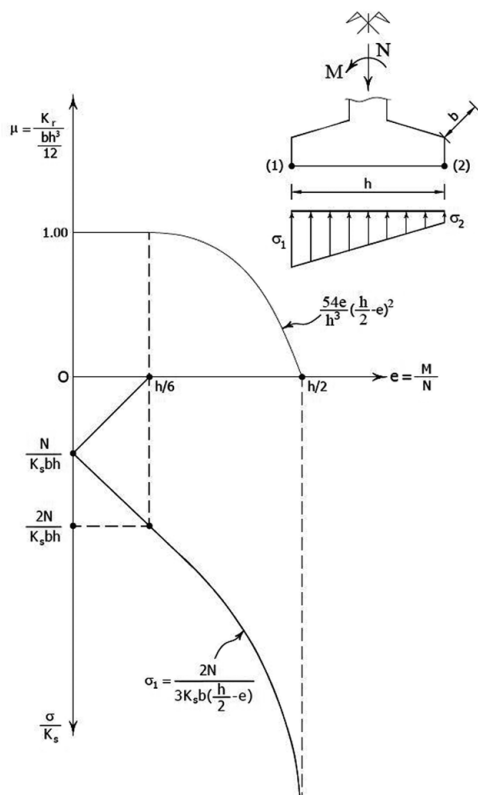
Fig. 3. The variation of soil pressures (σ_1, σ_2) and the rotational spring constant K_r
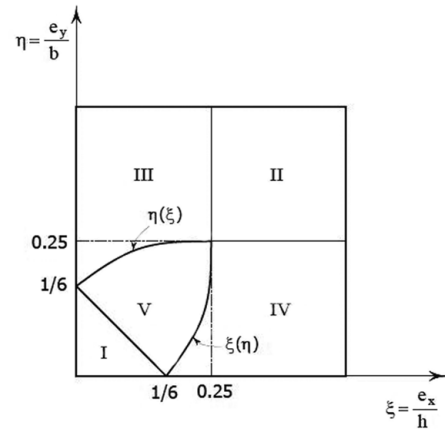
Fig. 4. Five different zones corresponding to five cases for the determination of K_{rx}, K_{ry}

First the following basic definition are made:

$$e_x = \frac{M_y}{N}, \quad e_y = \frac{M_x}{N}; \tag{14}$$

$$\xi = \frac{e_x}{h}, \quad \eta = \frac{e_y}{b}; \tag{15}$$

where e_x and e_y are the eccentricities, ξ and η signify the dimensionless ones.

Case I ($\xi \leq 1/6$ and $\eta \leq 1/6$)

In Zone I, rotational spring constants of spread footing K_{rx} and K_{ry} are defined as:

$$K_{rx} = K_s \frac{b^3 h}{12}, \quad K_{ry} = K_s \frac{bh^3}{12}. \tag{16}$$

The procedure explained below is followed for the calculation of rotational spring constants regarding to the cases of II, III, IV and V that tension zone occurs at the foundation soil:

1) The maximum compressive stresses and the locations of neutral axes can be expressed through the equations in Trupia and Saygun (2009).
2) The displacements due to the corresponding soil stresses are calculated on the principal axes of the footing foundation.
3) x_o and y_o distances in the Figures 5 to 8 are determined.
4) θ_x and θ_y rotations of the footing are achieved by dividing the end displacements by x_o and y_o respectively.
5) θ_x and θ_y rotations in principal axes of spread footing are calculated, K_{rx} and K_{ry} rotational spring constants can be determined through the following expressions:

$$K_{rx} = \frac{M_x}{\theta_x}, \quad K_{ry} = \frac{M_y}{\theta_y}. \tag{17}$$
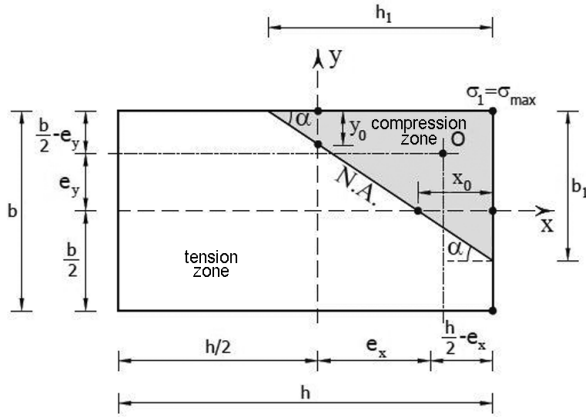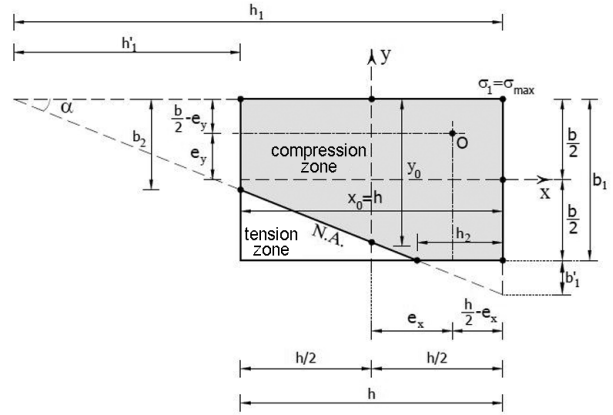
Fig. 5. Soil pressure distribution in rigid rectangular footing for the Case II

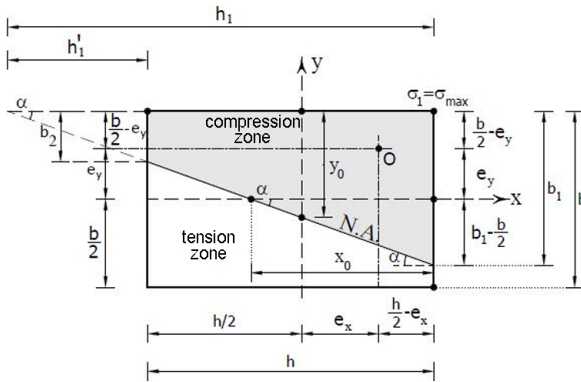Fig. 8. Soil pressure distribution in rigid rectangular footing for the Case V

Case II ($\xi \geq 0.25$ and $\eta \geq 0.25$)

This case is represented by Zone II in Figure 5. The following expressions are used for the determination of K_{rx} and K_{ry} defined in Eqn (17):

$$h_1 = 2h(1-2\xi), \quad b_1 = 2b(1-2\eta) ; \qquad (18)$$

$$\sigma_1 = \sigma_{max} = \frac{6N}{b_1 h_1} ; \qquad (19)$$

$$\theta_x = \frac{\sigma_{max}}{K_s b_1}, \quad \theta_y = \frac{\sigma_{max}}{K_s h_1} . \qquad (20)$$

Case III ($\xi < 0.25$ and $\eta \geq \eta(\xi) = 0.25 + \frac{16}{3}(\xi-0.25)^3$)

This case corresponds to Zone III in Figure 6. Footing rotations θ_x and θ_y in principal axes are calculated by leading expressions:

$$k_1 = \frac{-2\xi}{1+4\xi} + \frac{\sqrt{1-12\xi^2}}{1+4\xi} ; \qquad (21)$$

$$h_1 = \frac{h}{1-k_1} \ , \ b_1 = (1-2\eta)\frac{1+k_1+k_1^2}{1+k_1+k_1^2+k_1^3}2b ; \quad (22)$$

$$\sigma_1 = \sigma_{max} = \frac{3}{1-2\eta}\frac{1+k_1+k_1^2+k_1^3}{\left(1+k_1+k_1^2\right)^2}\frac{N}{bh} ; \qquad (23)$$

$$\theta_x = \frac{\sigma_{max} h_1}{K_s b_1}\frac{1+k_1}{2h_1-h}, \quad \theta_y = \frac{\sigma_{max}}{K_s h_1} . \qquad (24)$$

Case IV ($\xi \geq \xi(\eta) = 0.25 + \frac{16}{3}(\eta-0.25)^3$ and $\eta < 0.25$)

Zone IV corresponds to this case shown in Figure 7. The following way is tracked step by step for the determination of θ_x and θ_y :

$$k_2 = \frac{-2\eta}{1+4\eta} + \frac{\sqrt{1-12\eta^2}}{1+4\eta} ; \qquad (25)$$

Fig. 6. Soil pressure distribution in rigid rectangular footing for the Case III

Fig. 7. Soil pressure distribution in rigid rectangular footing for the Case IV
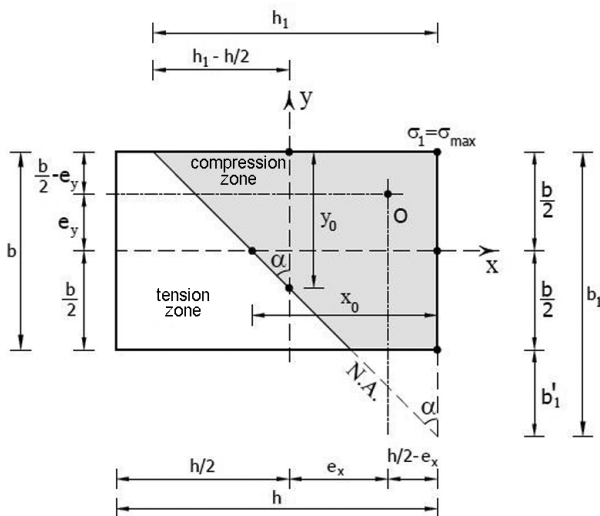
$$h_1 = (1-2\xi)\frac{1+k_2+k_2^2}{1+k_2+k_2^2+k_2^3}2h, \quad b_1 = \frac{b}{1-k_2} ; \quad (26)$$

$$\sigma_1 = \sigma_{max} = \frac{3}{1-2\xi}\frac{1+k_2+k_2^2+k_2^3}{\left(1+k_2+k_2^2\right)^2}\frac{N}{bh} ; \quad (27)$$

$$\theta_x = \frac{\sigma_{max}}{K_s\,b_1} , \qquad \theta_y = \frac{\sigma_{max}\,b_1}{K_s\,h_1}\frac{1+k_2}{2b_1-b} . \quad (28)$$

Case V ($\xi \le 0.25+16/3(\xi-0.25)^3$ and

$\eta \le 0.25+16/3(\eta-0.25)^3$)

This case is represented by Zone V in Figure 4, soil pressure distribution for the Case V is given in Figure 8. After moment equilibrium equations are written for the top and right side of the footing, leading fourth order equations can be derived as:

$$\begin{aligned}&(1-k_1)^3(1+k_1)-4\xi(1-k_1^3)(1-k_1)-\\&2(1-2\xi)(1-k_1)k_2^3+k_2^4 = 0;\end{aligned} \quad (29a)$$

$$\begin{aligned}&(1-k_2)^3(1+k_2)-4\eta(1-k_2^3)(1-k_2)-\\&2(1-2\eta)(1-k_2)k_1^3+k_1^4 = 0.\end{aligned} \quad (29b)$$

The minimum roots of equations k_1 and k_2 are obtained from Eqn (29a) and Eqn (29b). Then θ_x and θ_y are expressed as the follows:

$$h_1 = \frac{h}{1-k_1}, \quad b_1 = \frac{b}{1-k_2}, \quad b_2 = k_1 b_1 ; \quad (30)$$

$$\sigma_1 = \sigma_{max} = \frac{6N(1-k_1)(1-k_2)}{bh(1-k_1^3-k_2^3)} ; \quad (31)$$

$$\theta_x = \frac{\sigma_{max}\,h_1}{K_s\,b_1}\frac{1+k_1}{2h_1-h} ;$$

$$\theta_y = \frac{\sigma_{max}}{2K_s\,h\,b_2}\left\{(1+k_2)b_2-k_1(2b_2-b)\right\} . \quad (32)$$

2. Numerical examples

In this section, the numerical examples about the rotational spring constants of rigid spread footings are presented to reveal the efficiency of formulation developed herein. The coefficient of subgrade reaction for soil (K_s) is given 20,000 kN/m^2 in all examples.

2.1. Circular footing

Herein, a circular rigid footing of 6.00 m diameter resting on tensionless Winkler-type soil is exemplified. By using Eqn (6) and Eqn (1), the rotational spring constants of footing are calculated under the effect of axial force and bending moments. The results are tabulated in Table 1. The values of K_r illustrated in Table 1 are directly calculated by Eqns (7), (7a) and (7b) as well.

2.2. Rectangular footing subjected to axial force and bending moment

Rectangular rigid footing resting on tensionless Winkler-type soil and analysis model of SAP2000 are illustrated in Figures 9 and 10, respectively. Rotational spring constants of footing have been calculated under the effect of axial force and bending moments. The results can be seen in Table 2 by using the Eqns (11) and (1). The values of K_r illustrated in Table 2 are also directly determined by Eqns (13), (13a) and (13b). All the results are compared with those ones of SAP2000. It is seen that the results close to each other by dividing footing into smaller pieces in SAP2000.

Table 1. Rotational spring constants of circular footing

e (m)	M (kNm)	N (kN)	x (m)	σ_1 (kN/m^2)	σ_2 (kN/m^2)	θ (rad)	K_r (kNm)
0.3750	375.0	1000.0	6.000	53.052	17.684	0.00029473	1272351
0.7500	750.0	1000.0	6.000	70.735	0	0.00058946	1272351
0.9375	937.5	1000.0	5.371	80.013	–	0.00074486	1258630
1.1250	1125.0	1000.0	4.794	91.523	–	0.00095465	1178442
1.3125	1312.5	1000.0	4.242	106.272	–	0.00125274	1047701
1.5000	1500.0	1000.0	3.713	125.708	–	0.00169304	885983
1.6875	1687.5	1000.0	3.204	152.242	–	0.00237593	710248
1.8750	1875.0	1000.0	2.713	190.153	–	0.00350447	535031
2.0625	2062.5	1000.0	2.237	247.736	–	0.00553607	372557
2.2500	2250.0	1000.0	1.775	343.111	–	0.00966680	232755
2.4375	2437.5	1000.0	1.322	523.464	–	0.01979678	123126
2.6250	2625.0	1000.0	0.877	952.866	–	0.05431610	48328

Table 2. Rotational spring constants of rectangular footing

			This study		SAP2000						
					n = 20		n = 50		n = 500		
e (m)	M (kNm)	N (kN)	θ (rad)	K_r (kNm)	θ (rad)	K_r (kNm)	θ (rad)	K_r (kNm)	θ (rad)	K_r (kNm)
0.50	500	1000.00	0.00188	265957	0.00187	267380	0.00188	265957	0.00188	265957
0.60	500	833.33	0.00188	265957	0.00187	267380	0.00188	265957	0.00188	265957
0.70	500	714.28	0.00188	265957	0.00188	265957	0.00188	265957	0.00188	265957
0.80	500	625.00	0.000193	259200	0.00192	260417	0.00193	259067	0.00193	259067
0.90	500	555.56	0.000204	245025	0.00204	245098	0.00204	245098	0.00204	245098
1.00	500	500.00	0.000222	225000	0.00221	226244	0.00223	224215	0.00222	225225
1.10	500	454.55	0.000249	200475	0.00249	200803	0.0025	200000	0.00250	200000
1.20	500	416.67	0.000289	172800	0.00287	174216	0.0029	172414	0.00290	172414
1.30	500	384.62	0.000349	143325	0.00346	144509	0.00349	143266	0.00350	142857
1.40	500	357.14	0.000441	113400	0.00433	115473	0.00441	113379	0.00442	113122
1.50	500	333.33	0.000593	84375	0.0058	86207	0.00591	84602	0.00593	84317
1.60	500	312.50	0.000868	57600	0.00831	60168	0.00862	58005	0.00868	57604
1.70	500	294.12	0.001452	34425	0.01366	36603	0.01438	34771	0.01453	34412
1.80	500	277.78	0.003086	16200	0.02672	18713	0.03019	16562	0.03086	16202
1.90	500	263.16	0.011696	4275	0.06579	7600	0.10278	4865	0.11683	4280

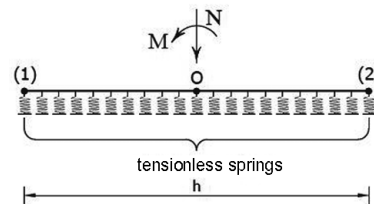*n* represents the number of dividing members.

Fig. 10. Analysis model of footing in SAP2000

2.3. Rigid rectangular footing subjected to axial force and biaxial bending

Rotational spring constants of rectangular rigid footing (Fig. 11) resting on tensionless Winkler-type soil have been achieved under the external forces corresponding to five different zone (Fig. 4) seen in Table 3.

Table 3. The forces and bending moments on rectangular footing

Zone	N (kN)	M_x (kNm)	M_y (kNm)	e_x/h	e_y/b
I	1000	250	500	0.0833	0.0625
II	1000	1200	1800	0.300	0.300
III	1000	1200	1200	0.200	0.300
IV	1000	800	1800	0.300	0.200
V	1000	800	1200	0.200	0.200

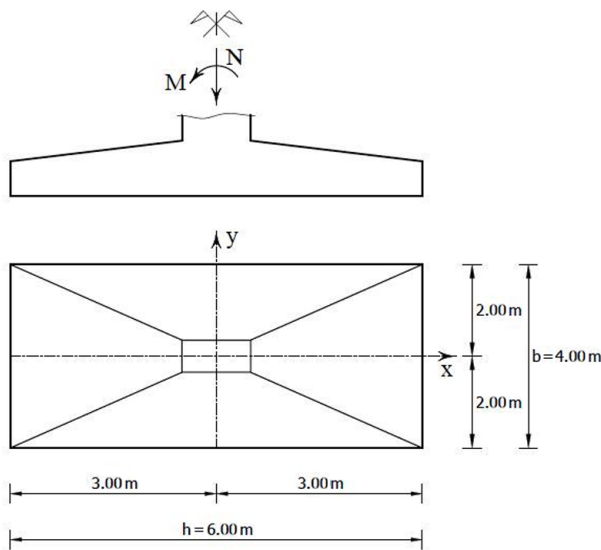Fig. 9. Dimensions of rigid rectangular footing (Example 3.2)

Fig. 11. Dimensions of rigid rectangular footing (Example 3.3)

The rotational spring constants of footing are obtained in detail by the simplified formulations presented in this study, then the findings are verified with SAP2000. The footing has been meshed into 12×8 square elements in the analysis with SAP2000. The results are tabulated and compared with the ones handled with SAP2000 in Table 4. The results confirm the suitability and efficiency of the method.

2.4. Calculation of rotational spring constants of rigid rectangular footing

Case I: $N = 1000$ kN, $M_x = 250$ kNm, $M_y = 500$ kNm.

$$e_x = \frac{M_y}{N} = \frac{500}{1000} = 0.50 \text{ m}, \quad e_y = \frac{M_x}{N} = \frac{250}{1000} = 0.25 \text{ m};$$

$$\xi = \frac{e_x}{h} = \frac{0.5}{6.0} = 0.0833, \quad \eta = \frac{e_y}{b} = \frac{0.25}{4.0} = 0.0625;$$

$$\xi = 0.0833 \leq 1/6 \text{ and } \eta = 0.0625 \leq 1/6 \rightarrow \textbf{Zone I};$$

$$K_{rx} = 20\,000 \frac{4^3.6}{12} = 640\,000 \text{ kNm};$$

$$K_{ry} = 20\,000 \frac{4.6^3}{12} = 1\,440\,000 \text{ kNm}.$$

Case II: $N = 1000$ kN, $M_x = 1200$ kNm, $M_y = 1800$ kNm.

$$e_x = \frac{M_y}{N} = \frac{1800}{1000} = 1.80 \text{ m}, \quad e_y = \frac{M_x}{N} = \frac{1200}{1000} = 1.20 \text{ m};$$

$$\xi = \frac{e_x}{h} = \frac{1.8}{6.0} = 0.30, \quad \eta = \frac{e_y}{b} = \frac{1.20}{4.0} = 0.30;$$

$$\xi = 0.30 \geq 0.25 \text{ and } \eta = 0.30 \geq 0.25 \rightarrow \textbf{Zone II}.$$

By using Eqns (18), (19), and (20) θ_x and θ_y are calculated:

$$h_1 = 2*6.00(1-2*0.30) = 4.80 \text{ m};$$

$$b_1 = 2*4.00(1-2*0.30) = 3.20 \text{ m};$$

$$\sigma_1 = \sigma_{max} = \frac{6*1000}{3.20*4.80} = 390.625 \text{ kN/m}^2;$$

$$\theta_x = \frac{390.625}{20\,000*3.20} = 0.006104 \text{ rad};$$

$$\theta_y = \frac{390.625}{20\,000*4.80} = 0.004069 \text{ rad, then } K_{rx} \text{ and } K_{ry}$$

are obtained via Eqn (17):

$$K_{rx} = \frac{1200}{0.006104} = 196\,592 \text{ kNm};$$

$$K_{ry} = \frac{1800}{0.004069} = 442\,369 \text{ kNm}.$$

Case III: $N = 1000$ kN, $M_x = 1200$ kNm, $M_y = 1200$ kNm.

$$e_x = \frac{M_y}{N} = \frac{1200}{1000} = 1.20 \text{ m}, \quad e_y = \frac{M_x}{N} = \frac{1200}{1000} = 1.20 \text{ m};$$

Table 4. The rotational spring constants of rectangular footing under axial force and biaxial bending

Zone	This study					SAP2000			
	σ_{max} (kN/m²)	θ_x (rad)	θ_y (rad)	K_{rx} (kNm)	K_{ry} (kNm)	θ_x (rad)	θ_y (rad)	K_{rx} (kNm)	K_{ry} (kNm)
I	78.125	–	–	640000	1440000	–	–	640000	1440000
II	390.625	0.00610	0.004069	196592	442369	0.00599	0.00403	200334	446650
III	259.429	0.00407	0.001776	294623	675676	0.00400	0.00176	300000	681818
IV	259.429	0.00266	0.002715	300300	662983	0.00264	0.00270	303030	666667
V	172.387	0.00178	0.001187	449438	1010952	0.00176	0.00118	454545	1016949

$\xi = \dfrac{e_x}{h} = \dfrac{1.2}{6.0} = 0.20$, $\quad \eta = \dfrac{e_y}{b} = \dfrac{1.20}{4.0} = 0.30$;

$\xi = 0.20 < 0.25$ and

$\eta = 0.30 \geq 0.25 + \dfrac{16}{3}(0.20 - 0.25)^3 = 0.2493 \rightarrow$ **Zone III**;

θ_x and θ_y are calculated by using Eqns (21), (22), (23) and (24):

$k_1 = \dfrac{-2*0.20}{1+4*0.20} + \dfrac{\sqrt{1-12*0.20^2}}{1+4*0.20} = 0.1784$;

$h_1 = \dfrac{6.00}{1-0.1784} = 7.3028$ m;

$b_1 = (1-2*0.30)\dfrac{1+0.1784+0.1784^2}{1+0.1784+0.1784^2+0.1784^3}*$

$\quad 2*4.00 = 3.185$ m;

$\sigma_1 = \sigma_{max} = \dfrac{3}{1-2*0.30}\dfrac{1+0.1784+0.1784^2+0.1784^3}{\left(1+0.1784+0.1784^2\right)^2}$

$\quad \dfrac{1000}{4.00*6.00} = 259.429$ kN/m^2;

$\theta_x = \dfrac{259.429*7.3028}{20\,000*3.185}\dfrac{1+0.1784}{2*7.3028-6.00} = 0.004073$ rad;

$\theta_y = \dfrac{259.429}{20\,000*7.3028} = 0.001776$ m,

then K_{rx} and K_{ry} are obtained Eqn (17):

$K_{rx} = \dfrac{1200}{0.004073} = 294623$ kNm;

$K_{ry} = \dfrac{1200}{0.001776} = 675\,676$ kNm.

Case IV: $N = 1000$ kN, $M_x = 800$ kNm, $M_y = 1800$ kNm.

$e_x = \dfrac{M_y}{N} = \dfrac{1800}{1000} = 1.80$ m, $\quad e_y = \dfrac{M_x}{N} = \dfrac{800}{1000} = 0.80$ m;

$\xi = \dfrac{e_x}{h} = \dfrac{1.8}{6.0} = 0.30$, $\quad \eta = \dfrac{e_y}{b} = \dfrac{0.80}{4.0} = 0.20$;

$\xi = 0.30 \geq 0.25 + \dfrac{16}{3}(0.45 - 0.25)^3 = 0.293$ and

$\eta = 0.20 < 0.25 \rightarrow$ **Zone IV**.

By using Eqns (25), (26), (27) and (28) θ_x and θ_y are calculated:

$k_2 = \dfrac{-2*0.20}{1+4*0.20} + \dfrac{\sqrt{1-12*0.20^2}}{1+4*0.20} = 0.1784$;

$h_1 = (1-2*0.30)\dfrac{1+0.1784+0.1784^2}{1+0.1784+0.1784^2+0.1784^3}*$

$\quad 2*6.00 = 4.7776$ m;

$b_1 = \dfrac{4.00}{1-0.1784} = 4.8685$ m;

$\sigma_1 = \sigma_{max} = \dfrac{3}{1-2*0.30}\dfrac{1+0.1784+0.1784^2+0.1784^3}{\left(1+0.1784+0.1784^2\right)^2}$

$\quad \dfrac{1000}{4.00*6.00} = 259.428$ kN/m^2;

$\theta_x = \dfrac{259.428}{20\,000*4.8685} = 0.002664$ rad;

$\theta_y = \dfrac{259.428}{20\,000*4.7776}\dfrac{1+0.1784}{2*4.8685-4.00} = 0.002715$ rad,

then using Eqn (17), K_{rx} and K_{ry} are calculated:

$K_{rx} = \dfrac{800}{0.002664} = 300300$ kNm;

$K_{ry} = \dfrac{1800}{0.002715} = 662983$ kNm.

Case V: $N = 1000$ kN, $M_x = 800$ kNm, $M_y = 1200$ kNm.

$e_x = \dfrac{M_y}{N} = \dfrac{1200}{1000} = 1.20$ m, $\quad e_y = \dfrac{M_x}{N} = \dfrac{800}{1000} = 0.80$ m;

$\xi = \dfrac{e_x}{h} = \dfrac{1.2}{6.0} = 0.20$, $\quad \eta = \dfrac{e_y}{b} = \dfrac{0.80}{4.0} = 0.20$;

$\left.\begin{cases} \xi = 0.20 \leq 0.25 + 16/3(0.20-0.25)^3 = 0.2493 \\ \eta = 0.20 \leq 0.25 + 16/3(0.20-0.25)^3 = 0.2493 \end{cases}\right\} \rightarrow$

Zone V.

The above fourth order equations are written by using Eqns (29a) and (29b):

$\left(1-k_1\right)^3\left(1+k_1\right) - 0.8\left(1-k_1^3\right)\left(1-k_1\right) -$

$1.2\left(1-k_1\right)k_2^3 + k_2^4 = 0$;

$\left(1-k_2\right)^3\left(1+k_2\right) - 0.8\left(1-k_2^3\right)\left(1-k_2\right) -$

$1.2\left(1-k_2\right)k_1^3 + k_1^4 = 0$.

Since ξ is equal to η, k_2 is replaced with k_1 in the first equation then minimum root of fourth order equation is found as:

$k_2 = k_1 = 0.173996 \cong 0.174$;

θ_x and θ_y are calculated by using Eqns (30), (31) and (32):

$$h_1 = \frac{6.00}{1-0.174} = 7.264 \text{ m}, \quad b_1 = \frac{4.00}{1-0.174} = 4.843 \text{ m};$$

$$b_2 = 0.174 * 4.843 = 0.843 \text{ m};$$

$$\sigma_1 = \sigma_{max} = \frac{6*1000(1-0.174)(1-0.174)}{4.00*6.00(1-0.174^3-0.174^3)} = $$
$$172.385 \text{ kN/m}^2;$$

$$\theta_x = \frac{172.385*7.264}{20\,000*4.843} \frac{1+0.174}{2*7.265-6.00} = 0.00178 \text{ rad};$$

$$\theta_y = \frac{172.385}{2*20000*6.00*0.843}$$
$$\{(1+0.174)*0.843 - 0.174*(2*0.843-4.00)\} = $$
$$0.001187 \text{ rad},$$

then K_{rx} and K_{ry} are obtained by Eqn (17).

$$K_{rx} = \frac{800}{0.00178} = 449\,438 \text{ kNm};$$

$$K_{ry} = \frac{1200}{0.001187} = 1\,010\,952 \text{ kNm}.$$

Conclusions

In this study, the simplified formulations to consider the soil-structure interaction in rigid spread footings are proposed especially for the large-span and low-rise structures. The following results can be drawn from this study:

– Soil-structure interaction can be achieved by these formulations enabling manual calculations. The formulations under consideration especially address a non-linear contact problem of footing subjected to excessive eccentric loading. This approach will provide a fast and reliable tool to the engineers in practice.

– Rotational spring constant by multiplying the moment of inertia of footing with subgrade reaction of soil is not valid by considering uplift case. The proposed formulations were developed to overcome this difficulty.

– It should be mentioned that the exact results via SAP2000 necessitate many springs and time consuming computational efforts via iterative procedure. Whereas the proposed formulations represent a fast and straightforward approach.

– Time consuming iteration steps are not necessary by presented formulations. The formulations under consideration coincide with the results of SAP2000 exactly that a large number of tensionless springs are utilized.

Notation

h – Dimension of rectangular spread footing in bending direction;

b – Dimension of rectangular spread footing perpendicular to h;

K_r – Rotational spring constant K_r;

K_{rx} – Rotational spring constant around the x axis;

K_{ry} – Rotational spring constant around the y axis;

K_s – The coefficient of subgrade reaction of soil;

R – Diameter of circular footing;

x_o, y_o – Compression zone lengths of footing on principal x and y axis;

σ_1 – Maximum soil pressure;

θ – The rotation of rigid spread footing;

θ_x – The rotation of rigid spread footing around x axis;

θ_y – The rotation of rigid spread footing around y axis;

μ – is a coefficient corresponding to the variation of the eccentricity.

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Konuralp GİRGİN. Dr, Professor at Structural Division of Civil Engineering Department of Istanbul Technical University. His research fields are structural and earthquake engineering. He is the author of many papers published in various journals and proceedings.