A CREDIBILISTIC MEAN-SEMIVARIANCE-PER PORTFOLIO SELECTION MODEL FOR LATIN AMERICA

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Abstract. Many real-world problems in the financial sector have to consider different objectives which are conflicting, for example portfolio selection. Markowitz proposed an approach to determine the optimal composition of a portfolio analysing the trade-off between return and risk. Nevertheless, this approach has been criticized for unrealistic assumptions and several changes have been proposed to incorporate investors' constraints and more realistic risk measures. In this line of research, our proposal extends the mean-semivariance portfolio selection model to a multiobjective credibilistic model that besides risk and return, also considers the price-to-earnings ratio to measure portfolio performance. Uncertain future returns and PER ratio of each asset are approximated using L-R power fuzzy numbers. Furthermore, we consider budget, bound and cardinality constraints. To solve the constrained portfolio optimization problem, we use the algorithm NSGA-II. We assess the proposed approach generating a portfolio with shares included in the Latin American Integrated Market. Results show that this new approach is a good alternative to solve the portfolio selection problem when multiple objectives are considered.

Keywords: fuzzy portfolio selection, credibility theory, L-R power fuzzy numbers, mean-semi-variance-PER, evolutionary multiobjective optimization.

JEL Classification: C61, G11, G17.

Introduction

Stock exchange investors have a variety of strategies available to allocate their wealth. Short-term investors mainly employ technical analysis (Sobreiro et al., 2016; Zhu, Atri, & Yegen, 2016) and chartist analysis (Gerritsen, 2016; Schmitt & Westerhoff, 2017). Long-term inves-

tors widely use fundamental analysis (De Oliveira, Nobre, & Zárate, 2013; Shen, Yan, & Tzeng, 2014) or passive investment strategies (García, Guijarro, & Moya, 2013; García, Guijarro, & Oliver, 2018). Basically, all these strategies concentrate on the expected return of the assets, which are analyzed individually, not as a portfolio. Therefore, asset returns are considered to be independent. Nevertheless, when a portfolio is built, it is necessary to take into account the return correlation of the assets in the portfolio. Markowitz (1952) was the first who changed the focus of investment analysis away from individual assets selection towards the concept of diversification and shed light on the portfolio selection problem.

The portfolio selection problem deals with the selection of assets among a group of candidate assets to create a portfolio which best maximizes investor's goals regarding several criteria such as return and risk. Markowitz (1952) was the one to tackle this problem in a quantitative way and proposed as selection criteria the mean and the variance of the returns of the assets in order to account for the risk-return trade-off. Nevertheless, the model by Markowitz has been criticized for several reasons. A main drawback is assuming multivariate normality, which is not the case. Furthermore, integer constraints that limit a portfolio to have a specified number of assets, or to impose limits on the proportion of the portfolio held in a given asset cannot be easily applied (Chang, Meade, Beasley, & Sharaiha, 2000). Finally, regarding the objectives which are to be simultaneously optimized, only return and risk are considered, which is not realistic. Furthermore, using the variance of returns in order to quantify the risk has been identified as a shortcoming, as well.

In this context, many researches have tackled the portfolio selection problem mainly from three perspectives: i) dealing with the uncertainty of assets return; ii) including new criteria for estimating portfolio risk; iii) adding new constraints faced by investors.

Regarding the uncertainty of assets return, assets' rate of returns are assumed to be random variables and probability theory is used to select the optimal portfolio (Huang, 2009). However, as noted by Huang (2010), randomness is not the only sort of uncertainty in real life, particularly when persons are involved. Furthermore, the information available in the financial markets for investment decision making is often incomplete, ambiguous and vague (Gupta, Mittal, & Mehlawat, 2013, 2014). Due to the above considerations, as from the 90's of last century researchers have applied fuzzy set theory (Zadeh, 1965) to describe and study the fuzziness contained in portfolio investment. There is a large body of research describing how to use possibility distributions to model the uncertainty on returns (Carlsson, Fullér, & Majlender, 2002; Saborido, Ruiz, Bermúdez, Vercher, & Luque, 2016; Vercher, Bermúdez, & Segura, 2007; Yue & Wang, 2017). However, the widely-employed possibility measure is not self-dual, which is an important drawback. In other words, it is not consistent with the law of excluded middle and the law of contradiction. For example, a fuzzy event with possibility value 1 may still not occur. Furthermore, it is possible that two fuzzy events with different chances of occurrence may have the same possibility value (Gupta et al., 2013). Therefore, the possibility value gives little information to the investor and may confuse her/him. To solve this limitation, B. Liu and Y. K. Liu (2002) suggested a self-dual credibility measure. Note that the fuzzy event will surely happen if its credibility value is 1 and fail if its credibility value is 0 (Gupta, Mehlawat, Inuiguchi, & Chandra, 2014a). Thus, credibility value is consistent with investors' judgement and the confusion will disappear. Since then, different researchers have

used credibility distributions to approximate the uncertainty on returns (Barak, Abessi, & Modarres, 2013; Huang, 2006; Jalota, Thakur, & Mittal, 2017b; Vercher & Bermúdez, 2015). For the above reasons, this paper extends the literature on portfolio selection model by assuming that the return on each asset is an L-R power fuzzy variable whose moments are assessed employing their credibility distributions.

As for the criteria to evaluate portfolio's performance, many practitioners and academics use the variance as the risk measure to solve the portfolio selection problem (Metaxiotis & Liagkouras, 2012), despite its deficiencies. Among them, it is important to underline that not just downside deviations from the expected return, that is, losses, but gains as well (Gupta et al., 2013). Additionally, it is not an appropriate risk measure if return distributions are asymmetric (Chunhachinda, Dandapani, Hamid, & Prakash, 1997). To solve this limitation of the mean-variance model, several downside risk measures were proposed by Fishburn (1977), Morgan (1996), Markowitz (1959), Rockafellar and Uryasev (2000), and Speranza (1993), to just consider the negative deviations from a reference return level. In this context, semivariance is probably the most popular downside risk measure. In contrast to variance, semivariance is direct, clear and can easily reflect investors' intuition about risk (Huang, 2008). Additionality, it is a more appropriate risk measure when an investor is worried about underperformance rather than over performance of portfolio (Markowitz, Todd, Xu, & Yamane, 1993). Therefore, this paper applies the semivariance to measure the risk in actual stock markets.

Finally, in the classical portfolio selection problem, the main decision criteria employed by investors are return and risk. However, other criteria might generate an equal or greater satisfaction level for the investor. Following Omidi, Abbasi, and Nazemi (2017), when other criteria are considered, it may be possible to obtain portfolios in which lower return or higher risk are compensated by other criteria, which may produce more satisfaction to investors seeking, not just to maximize return and minimize risk, but to consider other variables. A review of the literature shows that numerous papers have been devoted to develop the original mean-variance model by Markowitz into a new multicriteria framework in order to account for additional decision criteria (Fang, Chen, & Fukushima, 2008; García et al., 2013; Li, Zhu, Sun, Aw, & Teo, 2018; Xia, Liu, Wang, & Lai, 2000). Following this trend, the price-to-earnings ratio (P/E ratio) is one important criterion applied by practitioners to select stocks, due to its ability to capture the current expectations of the market about the companies (Pouya, Solimanpur, & Rezaee, 2016). As far as we know, no previous research has evaluated the performance of the portfolio price to earnings ratio (PER) in a credibility environment. Thus, this study makes a contribution to the literature by proposing a fuzzy multiobjective model, where besides return and risk, also PER ratio is included to measure the performance of a portfolio.

The aim and contribution of our research is to extend the mean-semivariance portfolio selection model from a stochastic environment into a multi-criteria portfolio model in a fuzzy environment. Besides return and risk, the proposed approach also considers the PER ratio to measure portfolio performance. Uncertain future returns and PER of each asset is calculated by means of L-R power fuzzy numbers. To make a more realistic model, budget, bound and cardinality constraints are included, as well. The inclusion of these constraints

in the portfolio optimization model change it into a constrained NP-hard multi-objective problem, for which traditional optimization methods cannot be used to find efficient portfolios. In order to solve this problem we apply the Non-dominated Sorting Genetic Algorithm II (NSGA-II), which is the most commonly used multiobjective evolutionary algorithm (MOEAs) for solving similar portfolio optimization problems. The performance of this approach is assessed using the stocks included in the Latin American Integrated Market, which integrates the stock exchange markets of Chile, Colombia, Mexico, and Peru for the period from June 2011 to December 2016.

The remainder of the research is organized as follows: First, the basics of L-R fuzzy numbers and the credibility theory are introduced. Then, the multiobjective credibilistic mean-semivariance-PER portfolio selection model is described and the methodology to solve the proposed approach is discussed. The next section presents an actual empirical study to stress the advantages of our model. Finally, Section 5 concludes.

1. L-R fuzzy number and the credibility theory: basic background

In this section, some basic definitions from the relevant literature on L-R fuzzy number and the credibility theory are presented so the model introduced in section 2 can properly be understood.

1.1. L-R power fuzzy number

Following Jalota, Thakur, and Mittal (2017a), using L–R fuzzy numbers for individual stocks makes it possible to capture the information about their behavior more precisely, so their proper contribution to the portfolio is calculated in a more appropriate way.

Definition 1. The functions L, R:[0,1]→[0,1] are reference functions of a fuzzy number they satisfy the following conditions $A = (x, \mu A(x))$, they satisfy the following conditions (Dubois & Prade, 1987):
  i) $L(1) = R(1) = 0$, $L(0) = R(0) = 1$;
  ii) $L(x)$ and $R(x)$ are strictly decreasing and upper semicontinuous functions.

Definition 2. A fuzzy number $M = (a, b, c, d)L_\pi R_\rho$ is said to be an LR-type fuzzy number if its membership function has the following form (Dubois & Prade, 1980):

$$\mu_M(x) = \begin{cases} L_\pi\left(\dfrac{b-x}{b-a}\right), & Si \, a \leq x < b \\ 1, & Si \, b \leq x \leq c \\ R_\rho\left(\dfrac{x-c}{d-c}\right), & Si \, c < x \leq d \\ 0, & \text{Otherwise} \end{cases}$$

where $(b - a)$ and $(d - c)$ show the left and right spreads of M, respectively. L_π and R_ρ are

the reference functions that define the left and right shapes of M, respectively. In this paper, left and right shapes of L-R fuzzy numbers are defined by $L_\pi(k) = 1 - x^\pi$, and $L_\rho(k) = 1 - x^\rho$, respectively. Throughout this study, L-R power fuzzy numbers will be denoted by $M = (a, b, c, d)_{\pi\rho}$.

In the case of LR-fuzzy numbers with linear reference functions or with the same shape for L and R, the aggregation provides fuzzy numbers of the same shape (Vercher et al., 2007). However, when the shape of the fuzzy numbers is not the same, their aggregation will not result in fuzzy numbers with the same shape (Inuiguchi, Ichihashi, & Tanaka, 1990; León & Vercher, 2004). Throughout this paper, the LR-fuzzy numbers employed will have the same reference functions L and R. In this way, following arithmetical rules hold according to the extension principle of Zadeh:

Theorem 1. Let $A = (a_1, b_1, c_1, d_1)_{LR}$ and $B = (a_2, b_2, c_2, d_2)_{LR}$ be two LR-fuzzy numbers and $\lambda \in \Re$ be a real number (Vercher et al., 2007). Then,

a) $A + B = \left(a_1 + a_2, \quad b_1 + b_2, \quad c_1 + c_2, \quad d_1 + d_2\right)_{LR}$;

b) $\lambda A = \begin{cases} \left(\lambda a_1, \lambda b_1, \lambda c_1, \quad \lambda d_1\right)_{LR}, & if \ \lambda \geq 0, \\ \left(\lambda a_1, \lambda b_1, |\lambda| c_1, |\lambda| d_1\right)_{LR}, & if \ \lambda < 0, \end{cases}$

where the addition and multiplication by a scalar is defined by means of the sup-min extension principle.

1.2. The credibility theory

In the capital markets investors deal with different fuzzy phenomena, not just randomness. In order to describe fuzziness, Zadeh (1965) proposed for the first time the concept of fuzzy set via membership function. Additionally, to measure a fuzzy event, Zadeh (1978) proposed the possibility measure. This measure has been widely accepted and applied, but it is not consistent with the law of excluded middle and the law of contradiction, that is, it has no the self-duality property. Following Huang (2010), by using possibility, investors knowing the possibility level of a portfolio reaching a target return, cannot know the possibility level of the opposite event. Therefore, this situation may be confusing for investors. To avoid this situation, B. Liu and Y. K. Liu (2002) defined a self-dual measure, a credibility measure. The credibility theory was introduced later by Liu (2004) and extended in Liu (2007).

Definition 3. Let ξ be a fuzzy variable with membership function μ, and x a real number. The credibility measure of a fuzzy event, characterized by $\xi \leq x$, is defined by B. Liu and Y. K. Liu (2002):

$$Cr\{\xi \leq x\} = \frac{1}{2}\left(\sup_{y \leq x}\mu(y) + 1 - \sup_{y > x}\mu(y)\right), \quad \forall x \in R.$$

The value of credibility takes values in [0, 1] (Liu, 2004). It is easy to verify that the credibility is self-dual, that is,

$$Cr\{\xi \leq x\} + Cr\{\xi > x\} = 1.$$

Definition 4. Let ξ be a fuzzy variable, then the expected value of ξ is defined by B. Liu and Y. K. Liu (2002):

$$E(\xi) = \int_{0}^{+\infty} Cr\{\xi \geq x\} dx - \int_{-\infty}^{0} Cr\{\xi \leq x\} dx$$

provided that at least one of the two integrals is finite.

The crisp equivalent expression for credibilistic expected value of an L-R power fuzzy $\xi = (a, b, c, d)_{\pi\rho}$, it is obtained by deriving the expected value of a fuzzy variable (Jalota et al., 2017b):

$$E(\xi) = \frac{1}{2}\left[b + c + \frac{(d-c)\rho}{\rho+1} - \frac{(b-a)\pi}{\pi+1} \right]. \tag{1}$$

Definition 5. Let ξ be a fuzzy variable with finite expected value $e = E[\xi]$. Then the semivariance of ξ is defined by B. Liu and Y. K. Liu (2002):

$$SV[\xi] = E\left[\left([\xi - e]^{-} \right)^{2} \right],$$

where

$$[\xi - e]^{-} = \begin{cases} \xi - e, & Si \, \xi \leq e \\ 0, & Si \, \xi > e. \end{cases}$$

The crisp equivalent expression for the credibility measure based semivariance value of an L-R power fuzzy number $\xi = (a, b, c, d)_{\pi\rho}$, was derived by (Jalota et al., 2017b):

$$SV(\xi) = \begin{cases} \dfrac{(e-a)^2}{2} - \dfrac{(e-a)(b-a)}{\pi+1} + \dfrac{(b-a)^2}{(\pi+1)(\pi+2)} + \dfrac{(e-c)^{\rho+2}}{(d-c)^{\rho}(\rho+1)(\rho+2)}, & \text{if } c < e \leq d \\[4mm] \dfrac{(e-a)^2}{2} - \dfrac{(e-a)(b-a)}{\pi+1} + \dfrac{(b-a)^2}{(\pi+1)(\pi+2)}, & \text{if } b < e \leq c \\[4mm] \dfrac{(e-a)^2}{2} - \dfrac{(e-a)(b-a)}{\pi+1} - \dfrac{(b-e)^{\pi+2}}{(b-a)^{\pi}(\pi+1)(\pi+2)} + \dfrac{(b-a)^2}{(\pi+1)(\pi+2)}, & \text{if } a < e \leq b \\[4mm] 0 & \text{otherwise} \end{cases} \tag{2}$$

2. Multiobjective credibilistic mean-semivariance-P/E portfolio selection model

This section discusses the proposed credibilistic multi-objective portfolio selection model and the methodology used to solve it. The parameters and decision variables used in the mathematical model are the following:

Parameters

ξ_{ri}: fuzzy rate of return of the asset i expressed as L-R power fuzzy number
$\xi_{ri} = (a_{ri}, b_{ri}, c_{ri}, d_{ri})\pi_{ri}\rho_{ri}$, i: 1, 2, ..., n.

$\xi_{P/Ei}$: fuzzy *P/E* of the asset i expressed as L-R power fuzzy number

$\xi_{P/Ei} = (a_{P/Ei}, b_{P/Ei}, c_{P/Ei}, d_{P/Ei})\pi_{P/Ei}\rho_{P/Ei}$, *i*: 1, 2, ..., *n*.

ξ_{rp}: fuzzy expected return of the portfolio expressed as L-R power fuzzy number

$\xi_{rp} = (a_{rp}, b_{rp}, c_{rp}, d_{rp})\pi_{rp}\rho_{rp}$,

e: expected return of the portfolio,

u_i: maximal fraction of the capital budget allocated to asset *i*, *i*: 1, 2, ..., *n*.

l_i: minimal fraction of the capital budget allocated to asset *i*, *i*: 1, 2, ..., *n*.

k: number of assets held in the portfolio.

Decision variables

ω_i : proportion of the total funds invested in the asset *i*, *i*: 1, 2, ..., *n*,

y_i: binary variable indicating whether the asset *i* is contained in the portfolio, *i*: 1, 2, ..., *n*, that is

$$y_i = \begin{cases} 1, & \text{if asset } i \text{ is contained in the portfolio,} \\ 0, & \text{otherwise.} \end{cases}$$

2.1. Objective functions

i) Return

Considering that in the capital markets information available for the decision maker is often incomplete, ambiguous and vague, this paper assumes that an investor allocates his capital among n assets that have fuzzy returns. The return on the i-th asset is expressed by L-R power
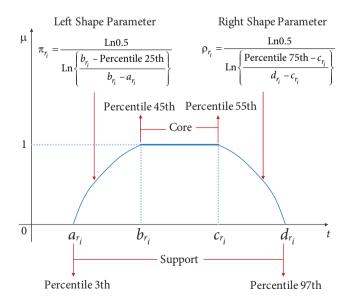

Figure 1. Parameters of the membership function of ξ_{ri} by using the sample percentiles of the returns of the asset *i*

fuzzy numbers (i.e. $\xi_{ri} = (a_{ri}, b_{ri}, c_{ri}, d_{ri})\pi_{ri}\rho_{ri}$, i: 1, 2, ..., n., T: 1, 2, ..., n), where its α-level cuts are $[\xi_{ri}]^{\alpha} = [\{b_{ri} - (b_{ri} - a_{ri})\}(1 - \alpha), \{c_{ri} - (d_{ri} - c_{ri})\}(1 - \alpha)]$ for $\alpha \in [0, 1]$.

The core, support and shape parameters of the fuzzy return of every asset are obtained from the empirical percentiles of its historical returns (Vercher & Bermúdez, 2012, 2013, 2015), as shown in the Figure 1. Anoter way to obtain the mentioned parameter values is from the expertise of professional investors.

The maximization of the expected return of the portfolio can be expressed by the following crisp objective:

$$Max\, F_1(\omega_i) = \sum_{i=1}^{n} \left[\left[\frac{1}{2} \left[b_{r_i} + c_{r_i} + \frac{(d_{r_i} - c_{r_i})\rho_{r_i}}{\rho_{r_i} + 1} - \frac{(b_{r_i} - a_{r_i})\pi_{r_i}}{\pi_{r_i} + 1} \right] \right] \omega_i \right].$$

ii) Risk

Semivariance is better suited to properly capture risk than the variance (Markowitz, 1959), as semivariance is a downside risk, which means that it only deals with adverse deviations.

The minimization of the semivariance of the portfolio can be expressed by the following crisp objective:

$MinF_2(\omega_i) =$

$$\begin{cases} \dfrac{(e-a_{r_\rho})^2}{2} - \dfrac{(e-a_{r_\rho})(b_{r_\rho}-a_{r_\rho})}{\pi_{r_\rho}+1} + \dfrac{(b_{r_\rho}-a_{r_\rho})^2}{(\pi_{r_\rho}+1)(\pi_{r_\rho}+2)} + \dfrac{(e-c_{r_\rho})^{\rho_{r_\rho}+2}}{(d_{r_\rho}-c_{r_\rho})^{\rho_{r_\rho}}(\rho_{r_\rho}+1)(\rho_{r_\rho}+2)}, & if\; c_{r_\rho} < e < d_{r_\rho}; \\[4mm] \dfrac{(e-a_{r_\rho})^2}{2} - \dfrac{(e-a_{r_\rho})(b_{r_\rho}-a_{r_\rho})}{\pi_{r_\rho}+1} + \dfrac{(b_{r_\rho}-a_{r_\rho})^2}{(\pi_{r_\rho}+1)(\pi_{r_\rho}+2)} & if\; b_{r_\rho} < e < c_{r_\rho} \\[4mm] \dfrac{(e-a_{r_\rho})^2}{2} - \dfrac{(e-a_{r_\rho})(b_{r_\rho}-a_{r_\rho})}{\pi_{r_\rho}+1} - \dfrac{(b_{r_\rho}-e)^{\pi_{r_\rho}+2}}{(b_{r_\rho}-a_{r_\rho})^{\pi_{r_\rho}}(\pi_{r_\rho}+1)(\pi_{r_\rho}+2)} + \dfrac{(b_{r_\rho}-a_{r_\rho})^2}{(\pi_{r_\rho}+1)(\pi_{r_\rho}+2)}, & if\; a_{r_\rho} < e < b_{r_\rho}; \\[4mm] 0, & otherwise. \end{cases}$$

iii) Price-to-earnings ratio

Price-to-earnings (PER) is a valuation tool of market's confidence in the shares of a firm (Masa'deh, Tayeh, Al-Jarrah, & Tarhini, 2015). The practical common way to compute the PER is dividing the current stock's price by its current earnings per share. Generally, most investors consider the PER criterion in their analysis and investment decisions (Pouya et al., 2016). Therefore, it is reasonable to include the PER as an additional criterion in the mean-semivariance base model to adapt it to investors' requirements.

However, because of incomplete information in the capital markets, *P/E* ratios are only vague estimates. Therefore, this paper considers that the *P/E* ratio of the *i*-th asset is expressed by an L-R power fuzzy number (i.e. $\xi_{P/Ei} = (a_{P/Ei}, b_{P/Ei}, c_{P/Ei}, d_{P/Ei})\pi_{P/Ei}\,\rho_{P/Ei}$, i: 1, 2, ..., n. T: 1, 2, ..., n), where its α-level cuts are $[\xi_{P/Ei}]\alpha = [\{b_{P/Ei} - (b_{P/Ei} - a_{P/Ei})\}(1 - \alpha), \{c_{P/Ei} - (d_{P/Ei} - c_{P/Ei})\}(1 - \alpha)]$ for $\alpha \in [0, 1]$.

The parameters of the L-R power fuzzy numbers are calculated with the information from the empirical percentiles of the historical *P/E* data, as previously explained.

Generally speaking, higher values of the *P/E* ratio indicate that market participants are expecting that the earnings of the company increase (Lu et al., 2011). In the light of the above consideration, the purpose of the following crisp objective is to maximize the portfolio's *P/E* ratio:

$$Max\, F_3\left(\omega_i\right) = \sum_{i=1}^{n}\left[\left[\frac{1}{2}\left[b_{P/E_i} + c_{P/E_i} + \frac{\left(d_{P/E_i} - c_{P/E_i}\right)\rho_{P/E_i}}{\rho_{P/E_i}+1} - \frac{\left(b_{P/E_i} - a_{P/E_i}\right)\pi_j}{\pi_{P/E_i}+1}\right.\right.$$

2.2. Constraints

In the classical mean variance model selling short is not permitted, it is possible to buy and sell assets in any fraction and investors have no limitation regarding the number of assets included in the portfolio. Moreover, investors have no preferences over assets in their portfolio and there is no problem if they hold different assets types (Lwin, Qu, & Kendall, 2014). However, the above situation does not match reality. Therefore, it is necessary to incorporate into the formulation of the problem different types of restrictions to consider investor preferences and market conditions. In this research, we broaden the original mean variance model in order to include following five real-world constraints:

i) Capital budget constraint on the assets is denoted by $\sum_{i=1}^{n}\omega_i = 1$,

ii) Short selling of shares banning constraint is denoted by $\omega_i \geq 0$, $i = 1, 2, \ldots, n$,

iii) The maximal fraction of wealth invested in asset i is denoted by $\omega_i \leq u_i y_i$, $i = 1, 2, \ldots, n$,

iv) The minimal fraction of wealth invested in asset i is denoted by $\omega_i \geq l_i y_i$, $i = 1, 2, \ldots, n$,

v) Cardinality constraint of the portfolio is denoted by $\sum_{i=1}^{n} y_i = k$, $y_i \in \{0,1\}$, $i = 1, 2, \ldots, n$.

2.3. The decision problem

The multiobjective credibilistic mean-semivariance-PER portfolio selection model is formulated as

$$Max\, F_1\left(\omega_i\right) \quad Min\, F_2\left(\omega_i\right) \quad Max\, F_3\left(\omega_i\right)$$

subject to
$$\sum_{i=1}^{n}\omega_i = 1, \qquad i = 1, 2, \ldots, n,$$
$$\omega_i \geq 0, \qquad i = 1, 2, \ldots, n,$$
$$\omega_i \leq u_i y_i, \qquad i = 1, 2, \ldots, n,$$
$$\omega_i \geq l_i y_i, \qquad i = 1, 2, \ldots, n,$$
$$\sum_{i=1}^{n} y_i = k \qquad y_i \in \{0,1\}, \quad i = 1, 2, \ldots, n.$$

For this study, a feasible portfolio P is defined as efficient or Pare-to-optimal if no other feasible portfolio P' exists such that,

$$P_{F_1(\omega_i)} \geq P'_{F_1(\omega_i)}, \ P_{F_2(\omega_i)} \leq P'_{F_2(\omega_i)} \ \text{and} \ P_{F_3(\omega_i)} \leq P'_{F_3(\omega_i)}$$

with strict inequality for at least one of them. The efficient solutions set is the Pareto optimal set in the decision space, and each of their three objective function values constitute the Pareto optimal frontier in the criterion space. The solutions of the Pareto optimal frontier are non-dominated.

2.4. Solution methodology

In the above paragraph, a multiobjective credibilistic mean-semivariance-PER portfolio selection model has been introduced. The model has three objective functions for the crisp goals of the return, risk and PER, respectively. The model also considers, budget, bound and cardinality constraints to make it more suitable for real use, as well. However, the inclusion of these new constraints converts the problem into a quadratic mixed-integer problem, which is NP-hard. In order to overcome this inconvenience, multiobjective evolutionary algorithms (MOEAs) can be applied.

The most widely-used MOEA for solving the constrained portfolio optimization problem is the Non-dominated Sorting Genetic Algorithm II (NSGA-II) (Liagkouras & Metaxiotis, 2015), first presented by Deb, Agrawal, Pratap, and Meyarivan (2002), which is the one applied in our research.

The procedural steps of this algorithm are those described by R. Arora, Kaushik, and R. Arora (2015) and Deb et al. (2002).

The experimental parameter configuration for testing this algorithm are presented in Table 1 below.

Table 1. Experimental parameters NSGA-II

Parameters	Value
Population Size	400
Distribution Index for Crossover	10
Probatility of Crossover	0.9
Distribution Index for Mutation	50
Probatility of Mutation	0.01
Maximum Number of Generations	500

Figure 2 shows the overall structure of the multiobjective credibilistic mean-semivariance-PER portfolio selection model.
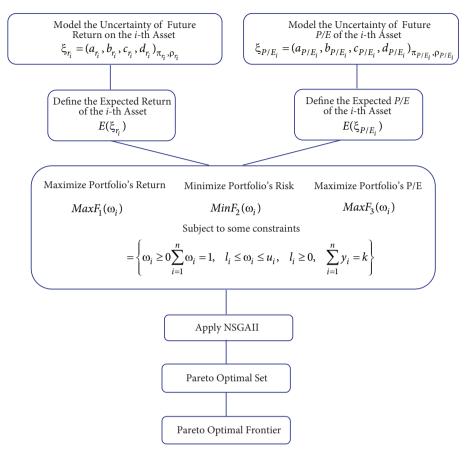
Figure 2. Overall structure of the multiobjective credibilistic mean-semivariance-PER portfolio selection model

3. Results

In order to show the usefulness of the model presented, we apply it on a real-world empirical study using the data set extracted from the MILA market. The Latin American Integrated Market began operating in 2011 and it is the result of an agreement signed between the Santiago Stock Exchange (BVS), Colombian Stock Exchange (BVC), and Mexican Stock Exchange (BMV) to trade equities from the four countries. While the individual MILA markets are illiquid and segmented, with trading and capitalization concentrated on few firms (Torre & Schmukler, 2007), their combination provides investors with a larger set of securities to choose. In this way, they can extend their possibilities of diversification and potentially improve the risk-return trade-off in their portfolios. Candidate stocks to be included in the investment portfolio are 46 listed stocks ($n = 46$) on the MILA Market: 23 Mexican, 18 Chilean, 3 Colombian and 2 Peruvian stocks. Weekly closing adjusted prices and their PER are employed, ranging t = 344 periods from June 03, 2011 to December 29, 2017.

The sample returns of individual assets are evaluated as for $r_{it} = (p_{it} - p_{it-1})/(p_{it-1})$, $i = 1, 2, \ldots, 46$ and $t = 1, 2, \ldots, 344$, where p_{it} is the closing adjusted price of the asset i on Friday of week t. Next, the L-R power fuzzy numbers' membership functions of the return, ξ_{ri} and ratio P/E, $\xi_{P/Ei}$ are obtained as functions of the sample percentiles of the data in the corresponding columns of the returns $\{r_{ti}\}_{t=1}^{344}$, and of the P/E ratios $\{P/E_{ti}\}_{t=1}^{344}$, respectively, as explained in Section 3.

In the multiobjective credibilistic mean-semivariance-P/E portfolio selection model, following diversification conditions are assumed: $l_i = 0$ and $u_i = 0.3$ for every $i = 1, 2, \ldots, 46$. Following Gupta, Mehlawat, Inuiguchi, and Chandra (2014b), it is not advisable to have very few or too many assets in the portfolio so as to achieve diversification. Usually, investors use between 3 to 10 assets to diversify their portfolio. Therefore, following this suggestion, it is considered to set k = 10 assets for an admissible portfolio. Furthermore, investing in just a few assets also reduces the transaction costs of the portfolio.

Figure 3 illustrates a three-dimensional representation of the approximated Pareto-front obtained by NSGA-II for the proposed model. Note that the group of points represents the set of efficient portfolios (or non-dominated solutions) whose objective functions (return-risk-PER) cannot be improved simultaneously by any other solution. It can be observed that NSGA-II provides a widely-distributed set of efficient portfolios.

Additionally, it is convenient to study the influence of the introduction of the PER as an additional portfolio selecting criterion. Figure 4 illustrates the relationship between expected PER and expected return of the selected portfolios in the two-dimensional space. In order to interpret the figure correctly, it must be noted that the PER has two different, contradictory, interpretations. The first one is related with future expectations. The idea below this interpretation is that investors buy stocks of companies that are supposed to increase their benefits in the future. As a result of this behaviour, price of such stocks increase and also their PER. Following this interpretation, a high PER is a signal that a company will increase its benefits and therefore it should be included in the portfolio. The second interpretation states that a
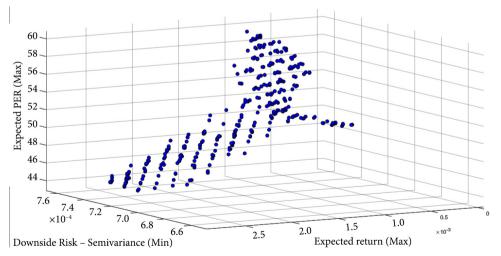

Figure 3. Approximations of the Pareto optimal front of the multiobjective credibilistic mean-semivariance-P/E portfolio se-lection model

high PER is related with "expensive" stocks. The investment required to buy the stocks will only be payed-back by the benefits of the company after a longer period. Furthermore, the probability of shares with high PER to continue increasing their stock price is smaller than for those stocks with a low PER. Therefore, companies with high PER should not be included in the portfolio.

Figure 4 shows that selected portfolios with high expected PER value obtain lower expected returns, whereas low expected PER values are associated with higher expected returns. This outcome is in line with the second interpretation of the PER.

Portfolio risk versus expected PER is analysed in Figure 5. This figure shows that the riskiest portfolios can both have high and low expected PER. That is, extreme expected PER
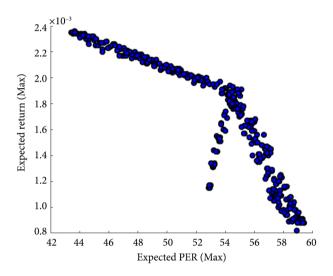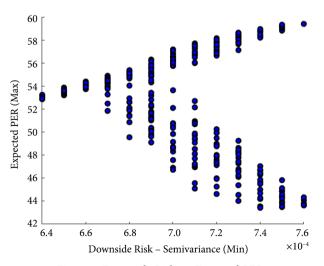

Figure 4. Expected PER vs. Expected Return

Figure 5. Downside Risk vs. Expected PER

values are associated with more risk. This can be explained by the different interpretations of the price to earnings ratio by market participants. On the one hand, when the PER is relatively high, it is more likely that such good earnings expectations are not met by companies, so price evolution will be negative. On the other hand, a relatively low PER value does not mean that the company is undervalued. It rather means that company's earnings will be reduced in the future, which will have a negative impact on its stock price, as well.

Finally, it must be stressed that portfolios in Figures 4 and 5 are the same portfolios which are introduced in Figure 3. Therefore, all of them are efficient portfolios, as no one is dominated by another portfolio.

Conclusions

The portfolio selection problem was first introduced by Markowitz. The first model proposed dealt with the problem of selecting the assets to include in the portfolio, maximizing return and minimising risk. Mean and variance of returns were used to capture the risk-return trade-off. Since the seminal work by Markowitz, different approaches have been proposed to solve the limitations of the so-called mean-variance model. The new approaches tackle problems such as the uncertainty of assets return, the criteria for estimating portfolio risk and the incorportaion of actual constraints faced by investors. This paper continues this line of research.

Based on Liu's credibility theory, this paper extends the mean-semivariance portfolio selection model to a multiobjective credibilistic model. The research uses return, risk and the price-to-earnings ratio (PER) to measure portfolio performance. L-R power fuzzy numbers are employed to model the uncertainty of stocks' future return and PER ratio. The membership functions of these fuzzy numbers are build using the sample percentiles of the historical dataset of the return and PER, respectively. With the aim of making the model more realistic, cardinality constraint and upper and lower bound constraints are considered. NSGA-II is employed to solve the resulting constrained multiobjective optimization problem. The computational results of this novel model provide sets of non-dominated portfolios widely distributed over the Pareto front which provide a true picture of trade-offs.

The proposed approach has several benefits over previous researches as we use: (i) the fuzzy framework over the probabilistic framework; (ii) the credibilistic environment over the possibilistic environment; iii) the fuzzy semivariance over fuzzy variance to measure portfolio risk. Furthermore, besides return and risk, the proposed approach also considers PER to measure portfolio performance. This ratio is widely employed by investors when selecting portfolios' components, so it is important to include it in the model in order to improve its practical implementation.

The feasibility and performance of the proposed model is presented creating a portfolio with companies included in the MILA Market, which is a stock market where shares from Mexican, Chilean, Colombian and Peruvian companies are traded. This is the first time that this market has been studied in the frame of portfolio selection strategies. One important aim of this stock market is to offer investors a way to diversify their investments in Latin America. This paper shows that this objective has been reached, as it is possible to generate a number of portfolios that fulfil many real-world conditions demanded by investors.

Results show that the model can be apply under real world conditions. The model generates a three-dimensional Pareto-front that illustrates the trade-off between return, risk and PER. Portfolios on this frontier cannot be improved by any other portfolio in terms of return, risk and PER simultaneously. It is interesting to underline that the different interpretations of PER by market participants, as a "buy" or "sell" signal, are captured by the model. This lack of consensus in the interpretation leads portfolios with extreme PER values to being riskier.

Finally, several future research possibilities exist to overcome the limitations of our study. Such research lines include the use of other selection criteria. In fact, the research context could be expanded in order to include other criteria considered by contemporary investors, who also care about companies' performance evaluation issues, especially after losses caused by the financial crisis (Ahmed, Ali, Ejaz, & Ahmad, 2018; Narkunienė & Ulbinaitė, 2018; Zemguliene & Valukonis, 2018).

Additionally, transaction costs are one of the main concerns for investors and portfolio managers as the utility of the portfolio depends heavily on them. Therefore, a future research opportunity would be to extend the proposed portfolio optimization model to incorporate transaction cost constraints.

Another limitation of this research is to treat portfolio optimization as a single-period problem, that is, assuming that the allocation decision made at the beginning is static until the end of an investment. The single-period framework is a myopic portfolio strategy which does not capture intertemporal effects and hedging demands. Thus, a future research option is to develop an approach to consider a multi-period portfolio optimization model.

Moreover, in this research, we consider the semivariance to measure portfolio risk. However, portfolio risk quantification has been widely studied in the literature and several downside risk measures have been proposed. In this sense, a future interesting research would be to consider other coherent risk measures, such as value at risk and mean-absolute semi-deviation.

Author contribution

FG and JGB conceived the study and the methodology, JGB obtained the data, JO was responsible for the data curation, analysis and editing, FG, JGB and JO wrote the first draft of the article and RT made proofreading and supervision.

Disclosure statement

Authors declare that they do not have any competing financial, professional, or personal interests from other parties.

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