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INCENTIVES IN HIERARCHICAL ORGANIZATIONS

Vladimir Balashov, Vladimir Burkov, Dmitrij Novikov, Aleksei Zalojnev

Institute of control sciences, Profsojuznaya st., 65, Moscow, 117997, Russia.
Phone: +(095)3349051, fax: +(095)3348911. E-mail: vlab@ipu.rssi.ru

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Abstract. Game-theoretical models of the incentive mechanisms are considered for the hierarchical organization.

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1. Introduction

The paper contains the consideration of management (particularly – incentive) problems in hierarchical organizations. The main method of exploration is the game-theoretical models analysis [1]; [2]), which allows describing adequately the preferences of systems' elements. Essential attention is paid to the model of multi-level organizational system (OS). Results of other models' exploration are briefly discussed in the conclusion (see [3] for the details).

2. Basic Incentive Problem in Multi-Level OS

Consider three-level OS, which consists of one meta-principal (P) on the highest level of hierarchy, n principals (P_j) on the middle level, $j = \overline{1,n}$, and N agents (A_{ij}) on the lowest level, $i = \overline{1,n_i}$, n_j – number of agents in j-th subsystem (subordinated to j-th principal), $j = \overline{1,n}$, $\sum_{j=1}^{n} n_j = N$. The set of meta-principal and n subordinated to him principals will be referred to as the metasystem. Each agent A_{ij} chooses his action $y_{ij} \in A_{ij}$, receives the reward $\sigma_{ij}(y_j) \in M_{ij}$ from j-th principal and bears costs $c_{ij}(y_{ij})$, where $y_j = (y_{1j}, y_{2j}, ..., y_{n_j j}) \in A_j = \prod_{i=1}^{n_j} A_{ij}$ – a vector of j-th subsystem agents' actions. Thus the goal function of A_{ij} takes the form:

$$f_{ij}(y_j) = \sigma_{ij}(y_j) - c_{ij}(y_{ij}), \quad i = \overline{1,n_j}, \quad j = \overline{1,n} \qquad (1)$$

P_j is characterized by the income $H_j(y_j)$, bears incentive costs $\sum_{i=1}^{n_j} \sigma_{ij}(y_j)$ and receives the reward $\sigma_j(Y^j) \in M_j$ from meta-principal, where $Y^j = Q_j(y_j) \in A^j$ is the aggregated output of j-th subsystem, $Q_j: A_j \to A^j$, i.e. his goal function is:

$$\Phi_j(y_j) = H_j(y_j) - \sum_{i=1}^{n_j} \sigma_{ij}(y_j) + \sigma_j(Y^j), j = \overline{1,n}. \qquad (2)$$

Meta-principal is characterized by his income $H(Y)$, where $Y = (Y^1, Y^2, ..., Y^n) \in A = \prod_j A^j$, and bears incentive costs $\sum_{j=1}^{n}\sigma_j(Y^j)$ i.e. his goal function is:

$$\Phi(Y) = H(Y) - \sum_{j=1}^{n}\sigma_j(Y^j). \quad (3)$$

The sequence of OS functioning is as following: meta-principal chooses the reward functions $\{\sigma_j(Y^j)\}$ and reveals it to principals, then each principal chooses incentive functions $\{\sigma_{ij}(y_j)\}$ for his subordinates, then the latter choose their actions. Assume that all the elements of the OS (meta-principal, principals and agents) choose their strategies independently (coalitions are not considered) to maximize corresponding goal functions. The presence of aggregation (of subsystem outputs and principals' preferences) is essential, so from meta-principal's point of view P_j's goal function is:

$$\Phi_j(Y^j) = h_j(Y^j) - c_j(Y^j) + \sigma_j(Y^j),\ j = \overline{1,n}, \quad (4)$$

$h_j(Y^j)$: $A^j \to \Re^1$, $c_j(Y^j)$: $A^j \to \Re^1$ are such that $\forall\, Y \in A^j$: $\forall\, y_j \in A_j$: $Q_j(y_j)=Y^j \quad h_j(Y^j)=H_j(y_j)$,

$$c_j(Y^j) = \sum_{i=1}^{n_j}\sigma_{ij}(y_j). \quad (5)$$

The difference between (2) and (4) is explained by the fact that the meta-principal generally has aggregated conception about the models of subsystems, coordinated with their "detailed" models in the sense of (5).

Denote $P_j(\{\sigma_{ij}\}) \subseteq A_j$ – a set of Nash equilibriums for the agents of j-th subsystem (the set of actions, implemented by the incentive functions $\{\sigma_{ij}\}$):

$$P_j(\{\sigma_{ij}\}) = \{\, y_j \in A_j \mid \forall\, i = \overline{1,n_j}\ \forall\, t_{ij} \in A_{ij}\ \sigma_{ij}(y_{ij}, y_{-ij}) - c_{ij}(y_{ij}) \geq \sigma_{ij}(t_{ij}, y_{-ij}) - c_{ji}(t_{ij})\ \}, \quad (6)$$

$y_{-ij} = (y_{1j}, y_{2j}, \ldots, y_{i-1j}, y_{i+1j}, \ldots, y_{n_j j})$ – is a situation for i-th agent in j-th subsystem. If the hypothesis of independent behavior (HIB) is valid (there are no joint restrictions on the collection of agents' choices), then

$$P_j(\{\sigma_{ij}\}) = \prod_{i=1}^{n_j} P_{ij}(\sigma_{ij}),$$

$$P_{ij}(\sigma_{ij}) = Arg \max_{y_{ij} \in A_{ij}} f_{ij}(y_{ij}). \quad (7)$$

Denote $R_j(\sigma_j)$ – the set of j-th subsystem equilibriums in the framework of the meta-system:

$$R_j(\sigma_j) = \{Y^j \in A^j \mid \forall\, t^j \in A^j\ \ h_j(Y^j) - c_j(Y^j) + \sigma_j(Y^j) \geq h_j(t^j) - c_j(t^j) + \sigma_j(t^j)\}, \quad (8)$$

$R(\{\sigma_j\}) = \prod_{j=1}^{n} R_j(\sigma_j)$ – the set of principals' equilibriums.

In two-level OS an incentive problem is formulated as following ([2]; [4]): find a feasible incentive function, which maximizes principal's goal function on the set of agents' equilibriums. When truing to generalize this problem to the multi-level OS, one meets some difficulties. Despite that the operator of aggregation $Q_j(\cdot)$ is defined such that $A^j = Q_j(A_j)$, i.e. $\forall\, y_j \in A_j$ $\exists\, Y^j \in A^j$ and $\forall\, Y^j \in A^j$ $\exists\, y_j \in A_j$: $Y^j = Q_j(y_j)$, restrictions M on the incentive functions may turn out to be such that for some j and/or for some $Y^j \in R_j(\sigma_j)$ there exists no incentive $\{\sigma_{ij} \in M_{ij}\}$, which implement the required actions ($\exists\, y_j \in P_j(\{\sigma_{ij}\})$: $Q_j(y_j) = Y^j$). In other words, choosing some incentive function, the meta-principal could not be sure that the implemented output of subsystem (ones, which maximize goal function of the principal) may be attained by some implementable in the framework of the subsystem combination of agents actions.

Denote $P_j = \bigcup_{\sigma_{ij} \in M_{ij}} P_j(\{\sigma_{ij}\})$ $R = \bigcup_{\sigma_j \in M_j} R(\{\sigma_j\})$

and introduce the following assumption (once introduced the assumption is considered to be valid hereafter), which guarantee the coordinatability of the model's restrictions.

A1. $\forall\, Y \in R\ \ \forall\, j = \overline{1,n}\ \exists\ \ y_j \in P_j\ :\ Y^j = Q_j(y_j)$.

Under the assumption A.1 the incentive problem for the metasystem is:

$$H(Y^*) - \sum_{j=1}^{n}\sigma_j(Y^{*j}) \to \max_{\{\sigma_j \in M_j\}}, \quad (9)$$

$$Y^{*j} \in R_j(\sigma_j),\quad j = \overline{1,n}, \quad (10)$$

Assumption A.1 guarantees that the aggregates, defined in (10) may be implemented by the principals as the results of the following incentive problems solving:

$$H_j(y_j^*) - \sum_{i=1}^{n_j}\sigma_{ij}(y_j^*) + \sigma_j(Y^{*j}) \to \max_{\sigma_{ij} \in M_{ij}}, \quad (11)$$

$$y_j^* \in P_j(\{\sigma_{ij}\}),\ j = \overline{1,n}. \quad (12)$$

Denote $\sigma = \{\sigma_j\}$ and define the efficiency of management in metasystem (efficiency of incentive mechanism) as

$$K(\sigma) = \max_{Y \in R(\sigma)} \Phi(Y). \quad (13)$$

Denote $\sigma^j = \{\sigma_{ij}\}$, then the efficiency of management in j-th subsystem is:

$$K_j(\sigma^j) = \max_{y_j \in P_j(\sigma^j)} \Phi_j(y_j). \quad (14)$$

Note, that (13) and (14) imply that the hypothesis of benevolence (HB) is valid, i.e. principals and agents choose from the set of the corresponding goal function maximums the strategy, which is the most preferable by the upper level authority (in multi-level OS the HB makes the sense only under assumption A.1).

Introduce the following assumptions about goal functions and feasible sets.

A2. $A_{ij} = A^j = A = [0, +\infty)$.
A3. $c_{ij}(y_{ij})$, $c_j(y_j)$ – nondecreasing, bounded below functions.
A3'. A3, $c_{ij}(y_{ij})$, $c_j(y_j)$ are continuos, monotone increasing and $c_{ij}(0)=c_j(0)=0$.
A3''. A3', $c_{ij}(y_{ij})$, $c_j(y_j)$ are convex, continuos differentiable and $c'_{ij}(0) = c'_j(0) = 0$.
A4. $M_{ij} = M_j$ are the sets of non-negative valued piece-wise continuos functions.
A4'. $M_{ij} = \{\sigma_{ij} \mid \forall\ y_{ij} \in A_{ij}\ 0 \le \sigma_{ij}(y_{ij}) \le C_{ij}\}$;
$M_j = \{\sigma_j \mid \forall\ Y^j \in A^j\ 0 \le \sigma_j(Y^j) \le c_j\}$;

A4''. $\{M_{ij}\}: = \{\sigma_{ij} \mid \forall\ y \in A\ \sum_i \sigma_{ij}(y_{ij}) \le C_j\}; \{M_{ij}\}: =$

$\{\sigma_j \mid \forall\ Y \in A\ \sum_{j=1}^{n} \sigma_j(Y^j) \le c\}$.

Constants $\{C_{ij}\}$, $\{c_j\}$, c are upper restrictions of the corresponding incentive functions.

Lemma 1. If the assumptions A.2, A.3 and A.4 are valid, then the assumption A.1 is also valid.

Define the two-level analog of the considered multi-level OS. If: i) the number of principals equals to the number of agents (i.e. that one agent is subordinated to one and only one principal: $n = N$; such an OS may be called trivial); ii) the aggregation does not occur ($A^j = A_j$, $Q_j(y_j) = y_j$) and iii) principals are passive (they are not interested in the results of agents' activity and play the role of the ideal information transmitters), then one obtain two-level OS with the efficiency of management the same as in the original three-level OS with $H_j(y_j) \equiv 0$, $j = \overline{1,n}$.

3. Incentives in Multi-Level OS Without Aggregation of Information

In this section the case of no aggregation is considered (the principal is assumed having full information). Results are presented inductively – from the simplest one-agent two-level OS to multi-agent multi-level OS.

Consider OS, which consists of one principal and one agent (if $n = 1$ and/or $N = 1$, then indexes are omitted). Goal functions of the principal and the agent are: $\Phi(y) = H(y) - \sigma(y)$, $f(y) = \sigma(y) - c(y)$. It is well-known ([4]; [5]), that under assumptions A.2 and A.3' minimal incentive costs of the principal to implement an action $y^* \in A$ equals $c(y^*)$. Hence, the efficiency of management is:

$$K_0(C) = \max_{y \in P(C)} [H(y) - c(y)], \quad (15)$$

$$P(C) = \{y \in A \mid c(y) - \min_{y \in A} c(y) \le C\}. \quad (16)$$

Introduce one intermediate principal with the following goal function

$$\Phi_1(y) = H_1(y) + \sigma_1(y) - \sigma(y). \quad (17)$$

Principal's goal function will change to: $\Phi(y) = H(y) - \sigma_1(y)$. The set of implementable actions of the agent will remain the same (see (16), while the set of actions, implementable in the metasystem is:

$$R(c) = \{y \in A \mid c(y) - \min_{y \in A} c(y) - H_1(y) \le c\}. \quad (18)$$

It is obvious that minimum of incentive costs is achieved when the restrictions on incentives are coordinated: $P(C) = R(c)$, hence

$$C - c = H_1(y^*), \quad (19)$$

where $y^* = arg \max_{y \in P(C)} [H(y) + H_1(y) - c(y)]$. The efficiency of management under the condition (19) is

$$K_1(C) = \max_{y \in P(C)} [H(y) + H_1(y) - c(y)]. \quad (20)$$

Comparing (15) and (20), one can see that the correspondence between the efficiencies of incentives depend on the sign of principal's income function. If $\forall\ y \in A\ H_1(y) \ge 0$, then $\forall\ C \ge 0\ K_1(C) \ge K_0(C)$. If $\forall\ y \in A\ H_1(y) \le 0$, then $\forall\ C \ge 0\ K_1(C) \le K_0(C)$. If $H_1(\cdot)$ is an alternating function, then additional exploration is required.

Thus if some intermediate level (principal) is added to the two-level OS, then if the additional income of this principal is positive, then the efficiency of management increases, in the opposite case – decreases (it is worth noting that the restrictions on information processing were not considered). So, adding one passive principal (without taking into account information processing) does not change the efficiency of management.

Consider multi-agent system with N agents. If this agents do not interact, then under assumptions A.4 or A.4' all the results, presented above, still will be valid (the problem may be decomposed on the set of N single-agent incentive problems). The efficiency of management in the OS with homogenous agents will be $NK_0(C)$ or $NK_1(C)$, where C is a restriction on individual incentives.

General results on multi-agent OS management problems are presented in [6]. Below the case of weakly-connected agents is considered (i.e. incentives and costs of each agents depend on his own action, while there exists general restriction on the total reward – see assumption A.4'').

In the absence of aggregation under the assumption A.4'' the set of actions, implementable in two-level OS, is:

$$P(C) = \{y \in A \mid \sum_{i=1}^{N} c_i(y_i) \le C\}, \tag{21}$$

and the efficiency of management is:

$$K_3(C) = \max_{y \in P(C)} [\, H(y) - \sum_{i=1}^{N} c_i(y_i)\,]. \tag{22}$$

Introduce n intermediate principals, then the goal functions will take the form:

$$\Phi(y) = H(y) - \sum_{j=1}^{n} \sigma_j(y_j), \tag{23}$$

$$\Phi_j(y_j) = H_j(y_j) - \sigma_j(y_j) - \sum_{i=1}^{n_j} \sigma_{ij}(y_{ij}) \tag{24}$$

$$f_{ij}(y_{ij}) = \sigma_j(y_{ij}) - c_{ij}(y_{ij}). \tag{25}$$

Let the total incentive fund of the meta-principal is limited by positive constant c. Suppose, that he has fixed some distribution $\{C_j\}$ of this fund between th subsystems: $C_j \ge 0,\ \sum_{j=1}^{n} C_j = c$. Then the set of the actions, implementable in j-th subsystem, is

$$P_j(C_j) = \{y_j \in A_j \mid \sum_{i=1}^{n_j} c_{ij}(y_{ij}) - H_j(y_j) \le C_j\}. \tag{26}$$

The efficiency of management in three-level OS under the HB is

$$K_4(c) = \max_{\sum_j C_j \le C} \ \max_{y_j \in P_j(C_j)} [H(y) + \sum_{j=1}^{n} \{H_j(y_j) - \sum_{i=1}^{n_j} c_{ij}(y_{ij})\}]. \tag{27}$$

Suppose that $C = c$. If $H_j(y_j) \equiv 0$, then $\forall\ C \ge 0\ \ K_4(C) \le K_3(C)$, i.e. if the principals are passive, then the efficiency of management in multi-level OS with weakly-connected agents is less or equals to the efficiency of management in the corresponding two-level OS. If $H_j(y_j) < 0$, then the efficiency is strictly lower; if $H_j(y_j) > 0$, then the efficiency may be strictly higher.

Note, that, when defining $K_4(c)$, principles of incentive fund allocation between the subsystems was not fixed (the first maximum in (27) corresponds to the solution of this allocation problem). If the allocation principles are fixed a'priori, then the efficiency may only decrease.

4. Incentives in Multi-Level OS With Aggregation of Information

Define for any $Y^j \in A^j$ the following set:

$$A_j(Y^j) = \{y_j \in A_j \mid Q_j(y_j) = Y^j\}. \tag{28}$$

Let $y_{ij}^{\min}(Y^j)$ is a solution of the following problem:

$$\sum_{i=1}^{n_j} c_{ij}(y_{ij}) \to \min_{y_j \in A_j(Y^j)}; \tag{29}$$

$y_{ij}^{\max}(Y^j)$ is a solution of the following problem:

$$\sum_{i=1}^{n_j} c_{ij}(y_{ij}) \to \max_{y_j \in A_j(Y^j)}. \tag{30}$$

Denote

$$c_j^{\min}(Y^j) = \sum_{i=1}^{n_j} c_{ij}(y_{ij}^{\min}(Y^j)), \tag{31}$$

$$c_j^{\max}(Y^j) = \sum_{i=1}^{n_j} c_{ij}(y_{ij}^{\max}(Y^j)). \tag{32}$$

Obviously, (31) and (32) satisfy (5), i.e. the real model of the subsystem corresponds to the model of this subsystem, operated by the principal. Moreover, for any $Y \in A$ any principal's costs function $c_j(Y^j)$ satisfies:

$$c_j^{\min}(Y^j) \le c_j(Y^j) \le c_j^{\max}(Y^j). \qquad (33)$$

Aggregated costs function $c_j^{\min}(Y^j)$ of the principal minimizes his incentive costs to implement the aggregated output Y^j and corresponds to the "ideal" aggregation. Interval of costs, defined by (33) reflects incomplete information of the meta-principal about the models of subsystems. Hence the following statements are valid.

Theorem 1. Under assumptions A.1 and A.4 maximal guaranteed efficiency of management in multi-level OS equals

$$K_g^{\max} = \max_{Y \in A} [H(Y) - \sum_{j=1}^{n} c_j^{\max}(Y^j)]. \qquad (34)$$

Theorem 2. Under assumptions A.1 and A.4 maximal efficiency of management in multi-level OS equals corresponds to the full information of the meta-principal and equals

$$K^{\max} = \max_{Y \in A} [H(Y) - \sum_{j=1}^{n} c_j^{\min}(Y^j)]. \qquad (35)$$

Theorem 3. i) "Ideal" aggregation takes place iff the aggregated principals' costs satisfy (31); ii) Without taking into account information traffic and information processing limits of the management authorities, the aggregation of information in incentive problems does not increase the efficiency of management.

Qualitatevely, the diffenence between (35) and (34), on one hand, characterizes losses of the meta-principal, caused by his uninforativeness, and, on the other hand, joint profit of principals, gained due to the fact that they are better informed, then the meta-principal.

5. Conclusion

Limits, imposed on this paper, allowed to describe in some details only the incentive problems in multi-level OS with aggregation of information. Detailed analysis of other, specific for multilevel OS, management models may be found in [3]; [6]; [7]. The results of the game theoretical models' exploration allow to claim the **principle of rational centralization (RC) of management**, which states that the rational degree of centralization is achieved if the following factors are taken into account.

The *factor of aggregation* results in the aggregation of information about system's element, subsystems, etc., with the increase of the level of hierarchy.

The *economic factor* comprises changes in the resources (financial, material, organizational and so on), caused by the changes of staff under centralization or/ and decentralization.

The *factor of uncertainty* reflects the dependence of the information, possessed by the elements of the organization from the mechanisms of management.

The *organizational factor* corresponds to the authority relations, i.e. to the institutional ability of certain elements to establish "the rules" for other elements.

The *informational factor* embraces the changes of the traffic values and elements' limits of information processing.

Application of the RC principle to certain organizational systems allows to formulate conditions, sufficient for arbitrary decomposition of such management mechanisms as resource allocation mechanisms, incentive problems [5]; [8] expert decision-making mechanisms, etc.

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