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# MULTIOBJECTIVE OPTIMIZATION (MOO) IN PRIVATIZATION

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**Abstract.** Deregulation of public enterprises and services by privatization is very fashionable nowadays. The aim of privatization is mainly to increase effectiveness, while the government itself likes to maximize its revenue at the occasion of the takeover. Most of these public enterprises show a shortage in investment while maintenance of a reasonable employment level in the new private firm is also strongly desirable, not to mention the ecological obligations imposed on the new private firm. It means that takeover bids have to face multiple objectives and different stakeholders, i.e., all the parties interested in the issue. Traditionally the optimization of all these objectives is then judged upon in a rather subjective way. Consequently, there is a need for a more general and objective, not to say scientific, method which can compare several takeover bids for privatization optimizing multiple objectives sometimes with different units of measurement. With that purpose, a method is developed, which takes into consideration upper limits, lower bounds, dominating and nondominating effects, ending up with a set of nondominated takeover bids, which are ranked by using ratio analysis and Reference Point Theory, whereas objectivity and decreasing marginal utility are fully respected [1]. A simulation on several takeover bids for a public enterprise given multiple objectives follows the theoretical explanation.

**Keywords:** takeover bids, decreasing marginal utility, ratio analysis, reference point theory, simulation

## 1. The Problem

Nowadays a general tendency in the world exists towards privatization of public enterprises and services. Privatization means that government services or state enterprises are turned over to private ownership. The purpose is mainly to create a better management for these institutions. Better management means efficiency and effectiveness. Efficiency stands for production with the lowest cost; effectiveness for the satisfaction of the consumer, requiring the necessary new investments for quality amelioration and innovation. The ultimate goal, however, is much broader, as multiple agents such as the government and the eventual buyer, but also the employees of these institutions and even the whole population, given the repercussions on the general tax level, are interested in the issue. All these agents, called nowadays with the modern word of "Stakeholders" may pursue different, mostly independent and even contradictory, objectives [2].

For the government the employment goal has to be an independent objective separated from an optimal takeover price. This price could include a compensation for unemployment, but the unemployed may not be satisfied with a monetary compensation alone. The government may also ask for pollution abatement and for new investments as most of these public enterprises show a shortage in investment. As already said before, different agents will try to influence the decision or even multiple stakeholders ould

be present. Sometimes under the decision influencing agents pressure groups may be lobbying such as trade unions, ecologists and consumer organizations. It means that in fact *Social Welfare* and even *Social Well-Being*, think of pollution, has to be optimized given multiple objectives.

## 2. Conditions for Privatization

The obligation to come to free competition on the market place forms an essential and primary condition for privatization. At that moment, a firm, which is not efficient and effective, will go bankrupt, whereas a public enterprise is there, so to say for eternity. This condition, nevertheless, has some exceptions.

First, some utility sectors, such as electricity and public transport, operate with decreasing costs and ask for huge investments at the origin. At that moment, a Public-Private Partnership (PPP) is more appropriate. PPP is linked to the total investment project: design, construction, operation, maintenance and financing. In addition, the risks of the project are shared..

Secondly, a fundamental right is recognized for a minimum of some social needs, such as for electricity, postal services and public transport. At that moment, a Contractual System between the government and the firm is signed, whereby the government provides a subsidy to finance this minimum of social needs.

## 3. Forms of Privatization

Privatization can be partial or total. With partial privatization the government keeps at least 50% of the shares plus one. As the government keeps an absolute majority, the result is the same as without privatization. Different forms are possible for total privatization. In a first case, all shares are thrown on the market. At that moment, Multiobjective Optimization is not possible, as the government has no control anymore on the individual shareholders. In a second case, the majority of the shares goes to one private investor. In a final form, one shareholder acquires all the shares. In the second and the third cases, MOO is applicable.

## 4. The Choice of Objectives

The government, as the representative of the general interest, would prefer:

- maximization of Value Added;
- maximization of new investments;
- maximization of employment;
- minimization of pollution.

The government, in its position to equilibrate the budget, prefers:

- maximization of the Internal Rate of Return (IRR);
- maximization of the takeover price;
- minimization of the Payback Period, as taxes increase with increase in profits [3].

The potential investor likes:

- minimization of the takeover price;
- minimization of additional requirements, such as on labor and investments.

### 4. 1. Maximization of Value Added

Maximization of Value Added will maximize national wealth on the long term.

### 4. 2. Maximization of new investments

Most of nationalized firms neglected to invest. Increase in investment deepening presents a necessary condition for privatization.

### 4. 3. Maximization of employment

The government is satisfied already, if the existing employment is maintained, which is called “rigidity of the labor requirements” [4]. The lack of rigidity in the labor requirements by privatization has caused important social unrest in many countries, which did not take into account this labor rigidity.

As no maximization of employment is maintained, rigidity of the labor requirements will be used as a filter, when preliminary the propositions for takeover are screened.

### 4. 4. Maximization of the takeover price for government and minimization of the takeover price for the potential investor

Due to this dual situation, a Fair Price is proposed. In Brazil, two separate consulting firms estimated the value of the takeover. In case of strong differences in the estimation, a third consulting firm was approached [5].

In case of appeal on the stock market for minority shareholders, the situation on the stock market will influence the moment of the takeover. A consortium of banks will estimate a logical price for launching on the stock market. This price per share will influence the fixation of the fair price for the investor. In this way the fair price will act as a filter, when preliminary the propositions for takeover are screened.

### 4. 5. The danger of a “shock therapy”

In all situations, privatization has to occur one by one and not with all public firms together. Applications in Russia, Eastern Germany and in other countries with transition economies have shown the danger of such “shock therapy” [6, 7].

All these conditions on filtering, optimization and timing will create different situations, which will be characterized as models.

## 5. Different Models for Privatization

### 5. 1. First model with complete rigidity in the labor requirements

Few takeover candidates will accept at the same time: optimum Value Added and investment, a fair takeover price and complete rigidity in the labor requirements. These conditions have certainly to be weakened as shown in the following model.

### 5. 2. Second model with relative rigidity in the labor requirements

In this model, a natural outflow of labor occurs by death, illness or age. The government could interfere with a system of pre-pensions. For instance, a pre-pension is allotted at the age of 50, with pension at the age of 65.

### 5. 3. Third model with unemployment allowances for an unlimited time

In this model, the government interferes with unemployment allowances for an unlimited time. This model with the promotion of laziness is socially not acceptable and in contradiction with a human right on labor.

### 5. 4. What model to choose?

The second model is the most acceptable economically and socially. However, the model has to be assisted by an obligation to recruit new personnel after the needs of the new private firm. In this way, the maximization of employment enters the picture again.

Finally, the objectives for takeover are changed as follows:

1. a filter on a relative rigidity of the takeover of the existing personnel;
2. a filter in accepting the fair takeover price;
3. maximization of discounted Value Added in constant prices, e.g., over a ten year period;
4. the maximization of the discounted Value Added is assisted by a maximization of a surplus in the balance of payments current account e.g., over a ten year period;
5. maximization of new investment, e.g., over a ten year period. The obligation not to shut down the firm in that period is also included;
6. maximization of new employment;
7. minimization of the Payback Period;
8. minimization of pollution.

## 6. Some Techniques for Multiobjective Optimization (MOO)

### 6. 1. Cost-Benefit Analysis

Cost–Benefit Analysis is a method with a monetary unit, such as the EURO, as the common unit of measurement. Indeed even benefits are expressed in the chosen monetary unit, either in a direct or indirect way. In fact, Cost–Benefit works with only one objective, viz. benefits minus costs. Additionally Cost–Benefit represents a materialistic approach as even unemployment is only expressed in monetary terms.

### 6. 2. Cost-Effectiveness Analysis

Cost–Effectiveness is limited to two objectives: the costs expressed in a common monetary unit on the one side and a single effectiveness indicator on the other. However, optimality is absent in cost–effectiveness.

### 6. 3. Fractional Programming

$$\max.\frac{E}{C}=\frac{\max.effectiveness}{\min.costs}.$$

For privatization research the fractional programming formula has to be reversed

$$\max.\frac{C}{E}=\frac{\max.takeover\ price\ at\ the\ government\ side}{\min.unemployment}.$$

Other objectives for privatization are excluded such as: investment commitments for the new firm, the change in the general tax level, the expected value added, the influence on the balance of payments and other financial, monetary, regional and ecological repercussions.

### 6. 4. The Linear Method with Weights

The objectives are added up by using weights. Weights are defined, as the importance, which is attached to each of the objectives in a set of mutually exclusive and collectively exhaustive objectives and the sum of the weights, assigned to the elements of this set, has to be one:

$$u\left(x_j\right)=\sum_{i=1}^{n} w_i x_{ij}, \qquad (1)$$

with: $u(x_j)$ as total utility for the multiple objectives of alternative $j$;

$w_i$ = weight for objective $i$ with: $\Sigma\ w_i = 1$;

$i$ = 1,2, ..., $n$; $n$ the number of objectives;
$x_{ij}$ = response of alternative $j$ on objective $i$.

A negative sign in the formula represents a minimization.

A convex linear combination with the use of weights leads to extreme results with few chances for an in–between alternative [8]. A simple weighting of the data and any convex linearity has to be excluded being in contradiction with the economic law of diminishing marginal utility.

#### 6. 5. A Nonconvex Multi-Objective Utility Function has to be assumed

Neither Cost-Benefit Analysis, Cost-Effectiveness Analysis, Fractional Programming nor Linear Method with Weights seems to satisfy the conditions for multiobjective optimization. Rather a Nonconvex Multi–Objective Utility Function has to be assumed. Therefore, Reference Point Theory with Maximal Criterion Values and Ratio Analysis are proposed.

## 7. Reference Point Theory with Maximal Criterion Values

#### 7. 1. Maximal Criterion Values

Reference point theory with maximal criterion values responds to decreasing marginal utility, to nonconvex linearity and to an objective choice of the reference point. Suppose for instance two alternatives, A with as co-ordinates respectively 20, 10 and 5 and B with as co-ordinates 12, 15, 7, then the co-ordinates of the reference point will be respectively 20, 15 and 7.

#### 7. 2. Normalization by Scores

Due to different units of measurement for the objectives, normalization has to occur. Normalization means that a score is given to each objective. Scores, as contrasted with weights, are nonadditive.

$$x_j = [s_1x_{1j} , s_2x_{2j} ,..., s_ix_{ij} ,... s_nx_{nj}], \tag{2}$$

with: $s_i$ = the score of objective $i$ as determined by the normalization process;
($i$ = 1,2,..., $n$, $n$ the number of objectives);
$x_{ij}$ = response of alternative $j$ on objective $i$;
$x_j$ = the row vector of utility for alternative $j$;
($j$ = 1, ..., $m$; $m$ the number of alternatives).

Minima are set over in maxima by allocating the highest scores to the alternatives with the lowest figures of an objective.

In this way a normalized matrix is obtained:

$$[{}_Nx_{ij}]. \tag{3}$$

$N$ indicates the normalization;

${}_N\mathrm{x}_{\mathrm{ij}}$ = the normalized objective $i$ of alternative $j$.

#### 7. 3. The Formula of the Min–Max Metric [9]

If the following matrix is given:

$$[r_i - {}_Nx_{ij}], \tag{4}$$

with: $i$ = 1, 2,..., $n$ as the objectives ;
$j$ = 1, 2,..., $m$ as the alternatives;
$r_i$ = the $i$-th co-ordinate of the reference point;
${}_Nx_{ij}$ = the normalized objectives $i$ of alternative $j$;

then this matrix is subject to:

$$\underset{(j)}{\mathrm{Min}} \ldots \{\max_{(i)} (\mathrm{r}_i - {}_Nx_{ij})\}. \tag{5}$$

This distance function forms the basis of Reference Point Theory.

The choice of the scores remains a problem. A decision is taken in favor of a mechanical system, namely ratio analysis.

## 8. Ratio Analysis

Elsewhere, a critical study was made of different kind of ratio systems [10]. Mostly, they got negative comments, but not for the Van Delft-Nijkamp Method of Square Roots:

$$\frac{x_{ij}}{z_{ij}}, \text{ with: } z_{ij} = \sqrt{\sum_{j-1}^{j-m} x_{ij}^2}, \tag{6}$$

with: $j$ = 1,2,...,$m$; $m$ the number of alternatives
$n$ being the number of objectives;
$x_{ij}$ = response of alternative $j$ on objective $i$.

## 9. Ratio Analysis as an Independent System

For a Ratio Analysis as an Independent System we suggest two methods: an additive approach with ratios and ratio analysis combined with Reference Point Theory with Maximal Criterion Values.

### 9. 1. Additive Approach with Ratios

Elsewhere we have proven that an additive approach with ratios can function as an independent method for Multiobjective Optimization [11].

Therefore, we shall add up Van Delft-Nijkamp ratios in the following way:

$$y_j = \sum_{i=1}^{i=n} \frac{x_{ij}}{z_{ij}}, \qquad (7)$$

with: $y_j$ = alternative $j$ valued after the ratio system for all objectives;

$j$ = 1, 2,..., $m$; $m$ the number of alternatives;

$n$ being the number of objectives;

$y_j$ = alternative $j$ valued after the ratio system.

All ratios are smaller or equal to one, but contrary to weights their sum can be larger than one in the additive method with ratios.

### 9. 2. Ratio Analysis combined with Reference Point Theory with Maximal Criterion Values

Here also it is proven that Ratio Analysis combined with Reference Point Theory with Maximal Criterion Values can function as an independent method for Multiobjective Optimization. Once again the Van Delft-Nijkamp ratios will be used.

## 10. Simulation with Additive Ratios Analysis and with Ratios combined with Reference Point Theory

In a simulation exercise the Ratios of the Van Delft-Nijkamp Method of Square Roots are used for the Additive Method with Ratios and for the Reference Point Method.

Anyway, no weights or scores are used. All attributes are treated equally. If more importance has to be given to an objective, the stakeholders or their delegates will agree to add one or more subobjectives to the objective in question.

After a filtering process, assume that three projects, *A*, *B* and *C*, are maintained, with their different objectives as depicted in the following table.

In the additive system with ratios, all ratios are smaller than one, but contrary to weights their sum is larger than one.

The additive ratio system gives full preference to project A. Project A is excellent for the Internal Rate of Return, the Payback Period and for New Investments, on the average for the creation of Value Added and for generating a surplus on the current account of the Balance of Payments, but rather poor in generating new employment.

In the Reference Point Theory with Ratios, all co-ordinates of the reference point and the deviations from the reference point are smaller than one. The ranking of the Reference Point Theory is made after the smallest size of the deviations. At that moment the ranking for the best choice is as follows:

1. takeover bid A,
2. takeover bid B,
3. takeover bid C.

This means the same as in the additive ratio system.

In both systems, all attributes are treated equally. On the contrary, one may argue to allocate weights to the objectives, now represented by their ratios:

$$u\left(x_j\right) = \sum_{i=1}^{n} w_i x_{ij}, \quad \text{(see formula (I))}$$

e.g.:

weight ratio IRR: 0.10
weight ratio Payback Period: 0.10
weight ratio New Investment: 0.10
weight ratio New Employment: 0.40
weight ratio VA: 0.20
weight ratio Balance of Payments (current account): 0.10

At that moment, all arguments against weights are taken up again:

- creation of a superobjective,
- the given formula leads to a convex linearity, which has to be excluded being in contradiction with the economic law of diminishing marginal utility,
- a convex linear combination with the use of weights leads to extreme results with few chances for an in-between alternative.

To these arguments is added: the difficulty to agree on weights between many stakeholders. Unanimity among the stakeholders is difficult for weights with their hundredfold possibility.

In order to avoid the disadvantages of weights, importance can be given by introduction of subobjectives related to an objective. Of course, inferior objectives have to be avoided. Let us give an example related to the simulation.

To denote its importance “new employment” is substituted for instance by:

**Table I. Multiple Objectives Optimization with Additive and Reference Point Ratio Analysis**

I a – The Data

| Projects | 1. IRR (in %) MAX. | 2. Payback Period in years MIN. | 3. New Inv. (in bill.) MAX. | 4. New Employm. (in jobs) MAX. | 5. V. A. in m. (discounted) MAX. | 6. Bal. of Paym. curr. acc. (in m.) MAX. |
|---|---|---|---|---|---|---|
| Project A | **12** | **5** | **4.5** | **750** | **800** | **150** |
| Project B | **12** | **7** | **3** | **800** | **600** | **200** |
| Project C | **10** | **9** | **2.5** | **900** | **850** | **150** |
| Totals | **34** | **21** | **10** | **2,450** | **225** | **500** |

I b – Sum of squares and their square roots

| Projects | | | | | | |
|---|---|---|---|---|---|---|
| Project A | 144 | 25 | 20.25 | 562500 | 640000 | 22500 |
| Project B | 144 | 49 | 9 | 640000 | 360000 | 40000 |
| Project C | 100 | 81 | 6.25 | 810000 | 722500 | 22500 |
| Sum of squares | 388 | 155 | 35.5 | 2012500 | 1722500 | 85000 |
| Square roots | 19.6977156 | 12.4498996 | 5.9581876 | 1418.6261 | 1312.4405 | 291.5475947 |

I c – Objectives divided by their square roots and Additive Method with Ratios

| | | | | | | | **sum** | **rank** |
|---|---|---|---|---|---|---|---|---|
| Project A | 0.609207699 | 0.401610 | 0.7552632 | 0.52868053 | 0.60955 | 0.514495755 | 2.61559 | **1** |
| Project B | 0.609207699 | 0.562254 | 0.5035088 | 0.563926 | 0.4571636 | 0.685994341 | 2.2575 | **2** |
| Project C | 0.507673083 | 0.722897 | 0.4195907 | 0.63441664 | 0.6476484 | 0.514495755 | 2.1560 | **3** |

I d – Reference Point Theory with Ratios: co-ordinates of the reference point equal to the maximal criterion values

| | | | | | | |
|---|---|---|---|---|---|---|
| $r_i$ | 0.609207699 | 0.401610 | 0.7552632 | 0.63441664 | 0.64765 | 0.685994341 |

I e – Reference Point Theory: deviations from the reference point

| | | | | | | | **max.** | **rank min.** |
|---|---|---|---|---|---|---|---|---|
| Project A | 0 | 0 | 0 | 0.10573611 | 0.03810 | 0.171498585 | 0.17150 | **1** |
| Project B | 0 | 0.160644 | 0.2517544 | 0.070491 | 0.19048 | 0 | 0.251754 | **2** |
| Project C | 0.101534617 | 0.321288 | 0.3356725 | 0 | 0.00000 | 0.171498585 | 0.33567 | **3** |

1. direct new employment in jobs,
2. indirect new employment in jobs,
3. number of hours worked per man-year,
4. average wage per hour.

To denote its importance "Value Added" is substituted for instance by:

1. absolute Value Added,
2. Value Added divided by the total number of man-years.

In this way an additional advantage is present. Some alternatives may be strong in direct employment and another in indirect one.

Anyway, the choice of objectives has to be the result of a consensus between all the stakeholders involved; which is also the case with weights.

## 11. Some Practical Consequences

A rigidity in application is one of the objections, which could be raised. In order to answer this remark the several phases of the model will be reconsidered.

(1) The government has to fix its objectives. This phase has always to be present though many governments forget to state clearly their objectives.

(2) The private proposals for takeover have to be filed which is also necessary for privatization without a model.

(3) Output will be immediately when new propositions are introduced.

In this way, it is shown that the model is very flexible.

## 12. Summary and Conclusion

If a model for privatization has to be developed it has to take into consideration Social Welfare and even Social Well-Being. At that moment, several stakeholders are interested in the issue, but may postulate several different objectives sometimes expressed in different units of measurement. It is true, however, that the economic axiom is accepted whereby all objectives are measurable. Moreover, the different units of measurement

cannot be reduced to a single unit. At that moment traditional methods such as cost-benefit, cost-effectiveness or weighting in order to satisfy "as much as possible" all stakeholders and all objectives cannot be applied. Therefore a new model was developed, which first with a filtering stage, then with ratio analysis and with the use of a distance function, produced a ranking, which brought a univocal ranking of all the privatization proposals which passed the filtering stage. The model is flexible enough to face all peculiar problems of privatization.

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