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DEVELOPMENT AND APPLICATION OF A MATHEMATICAL MODEL FOR EVALUATING EXTRA ENERGY COST DUE TO DISPROPORTIONAL ENERGY CONSUMPTION

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Abstract. In this investigation, a scientific approach is presented in quantifying energy losses associated with production facilities. The corresponding analytical approach in estimating energy requirement of equipment is shown. In particular, mathematical information on how extra energy cost due to extra energy losses that occur in equipment is measured has been shown. The premise of the study is based on the fact that the value of the efficiency of the equipment determines the value of the energy that can be lost for a time frame. Similarly, the heat generated in some equipment, like lamps and air conditioner, can constitute energy lost in equipment. The paper then further shows that the energy cost paid for the energy losses that occur in equipment can be determined by using the energy cost rate used for calculating the cost paid for the useful work done. As manufacturing companies strive to meet and exceed the expected needs of the customer with cost saving manufacturing processes, a major hurdle is the losses in energy transfer as a result of inefficiencies in operations. This paper is geared towards achieving effective and efficient manufacturing processes by researching into trends in energy losses. The results obtained show the feasibility of the applied procedure. There is no previous documentation that has addressed the current problem using the approach presented. This is therefore a new way of viewing energy consumption.

Keywords: Energy consumption, energy losses, mathematical model, energy conservation, energy cost, manufacturing system, equipment, profit improvement

1. Introduction

Research into socio-economic dimensions of energy offers a challenge for researchers, energy planners, and policy makers. Many efforts have addressed the important areas of energy-efficiency improvement, evaluating energy conservation programs, etc. The main thrust of these research areas is the careful energy use, understanding the effects of socio-economics and behaviour of people on efficient energy use both at home and in industrial settings. Jaber et al. (2005) concluded that there is an evidence of excessive energy use and wasteful energy consumption behaviour, even in poor households, due to lack of public awareness and lack of state-sponsored energy conservation programs.

Interest in energy-efficiency improvements has been reinvigorated by concerns ranging from the environmental effects of fossil fuel combustion to energy price volatility and natural security (Anderson and Newell, 2004). Energy expenditure in manufacturing is a major component of the unit production cost (Mackulak et al., 1980; Liberator and Miller, 1985; Lin and Moodie, 1989). It is essentially important in energy intensive industries such as steel, tile and glass-ware where adequate planning is made concerning the acquisition of energy resources and its utilization.

To a large extent the energy utilization in a manufacturing plant strongly depends on technology, complexity of operations and the age of equipment. Researchers and practitioners have established the relationship between energy utilization and the age of machines and equipment in manufacturing to be an increasing linear function. Thus, as the facilities become older, the more the energy requirements and consumption of the plant. However, the energy utilization in manufacturing systems will determine the profitability of such manufacturing activities and the ability to sustain production in the long term. In view of the current profit improvement drive by many organizations, there is an intensive effort by manufacturers to reduce avenues for operational inefficiencies, such as energy losses or inefficient energy utilization. New methods are continually devised for minimizing energy losses through the adoption of various technologies. It is obvious that high energy losses in the whole production system translate into increased energy cost for the company with the associated increase in product price which may warrant low demand and low return on investment (ROI).

Ozdamar and Birbil (1999) reported that energy costs constitute more than 90% of the unit production cost in the tile manufacturing company. This has stimulated top management towards the optimal design of energy consuming centers. Recently, intensive efforts have been focused on the proper maintenance of facility towards minimizing energy losses in equipment. Although in practice, energy losses are unavoidable, successful efforts could be made towards minimal energy losses in order to achieve an optimum energy cost in production activities. To this end, manufacturing companies are constantly embarking upon a number of programmes and initiatives. Such initiatives are tagged with some ‘action-driven words’ such as ‘operation energy cuts’, ‘action against energy losses’, ‘energy wastes reduction by 10%’, etc. Energy losses may be broadly classified as mechanical and electrical losses, among others. Mechanical losses relate to friction, power transmission on belts, gear systems, etc. For electrical losses, energy losses in copper and iron in motors and drives are possible.

Since the energy expended due to heating effect of the parts and any other non-needed effects are not useful, we term them as energy wasted or lost. Basically, energy expended is divided into two parts – the useful work done and the energy loss. In the energy literature it is not clearly stated how data relevant to energy losses in terms of evaluating extra energy cost due to disproportional energy consumption should be collected. This paper aims at discussing how the organization could gather information on energy losses in manufacturing.

The section that follows is a detailed description of the methodology adopted in carrying out this research. The section contains a number of assumptions that makes the application of the mathematical model proposed thereafter feasible. In section 3, a case example is presented to especially address the various ways of collecting and analyzing information in practice, based on the model developed. The solutions are presented in section 4. Section 5 is a discussion of results. In section 6, conclusions are drawn from the study.

2. Methodology

This model is based on some assumptions, which are as follows:

i. The disproportionality in energy consumption is assumed to be the energy losses that occur in all equipment used either in a company or in a house of residence.
ii. The rate of energy consumption for a specific time interval in a day is periodic, having a fundamental period of 24 hours.
iii. The source of energy is immaterial.
iv. Energy can be consumed throughout the whole 24 hours in a day or otherwise.
v. The power output of all the equipment used in the company or house are known also as the power input.
vi. The company implements a good maintenance policy such that the efficiency of the equipment used have little or no change in value (very negligible change) with time and mode of usage.
vii. There is no fluctuation in the power supplied to all the equipment except that the power supply may either be stepped down or up for some equipment for easy and safe usage.

Rate of Energy Consumption – Rate of Useful Work done = Rate of Energy Loss:

$$\frac{dE}{dt} - \frac{dW}{dt} = \frac{dE_L}{dt}. \quad (1)$$

For an equipment I; $\frac{dE_i}{dt} - \frac{dW_i}{dt} = \frac{dE_{Li}}{dt}$, (2)

but, $\frac{dE_i}{dt} = \frac{d}{dt}\left(E_o \pm E_{vi}\right)$, (3)

where E_o is the main energy supplied.

E_{vi} is the energy increase or decrease due to the use of a transformer:

Also, $\frac{dW_i}{dt} = \eta_i \frac{dE_i}{dt} = \eta_i \frac{d\left(E_o - E_{vi}\right)}{dt}$, (4)

where η_i symbolizes the overall efficiency of machine i. Substituting equations (3) and (4) into equation (5) gives:

$$\frac{d\left(E_o \pm E_{vi}\right)}{dt} - \eta_i \frac{d}{dt}\left(E_o \pm E_{vi}\right) = \frac{dE_{Li}}{dt}, \quad (5)$$

$$\frac{d}{dt}\left(E_o \pm E_{vi}\right)\left(1 - \eta_i\right) = \frac{dE_{Li}}{dt}. \quad (6)$$

Therefore, the total loss for a particular period of time.

$$\left(\frac{dE_L}{dt}\right)_9 = \sum_{i=1}^{N} \frac{dE_{Li}}{dt} + \sum_{j=1}^{M} \frac{dL_j}{dt} = K_9, \quad (7)$$

$$\left(\frac{dE_L}{dt}\right)_9 = \sum_{i=1}^{N} \frac{dE_{Li}}{dt}\left(E_o \pm E_{vi}\right)\left(1-\eta_i\right) + \sum_{j=1}^{M} \frac{dL_j}{dt} = K_9 \,, \tag{8}$$

where, $\frac{dL_j}{dt}$ represents the rate of loss of energy consumed in other equipment.

The magnitude of the stepped up or down power $\frac{d}{dt}E_{vi} = I \times \delta_{vi}$,

where I is current and δvi is the small change in voltage. G is a counter, (g = 1, 2, 3, …, n) signifying the period of time concerned.

Equation (8) can be represented in Fourier series to aid the study of this paper.

$$\left(\frac{dE_L}{dt}\right)_g = f(t) = \begin{cases} K_1 & 0 < t < \frac{24}{r} \\ K_2 & \frac{24}{r} < t < \frac{48}{r} \\ & . \\ & . \\ & . \\ & . \\ K_n & \frac{24(r-1)}{r} < t < 24 \end{cases} \tag{9}$$

r can be any number either a mixed fraction or a positive integer but not a fraction,

where f(t) = f(t + 24y)

y represents the number of days, y = 0, 1, 2, 3, …

It should be noted that K_1 or K_2 or K_n can be equal to one another or not. This depends on the situations at the company concerned. From Fourier series:

$$f(t) = a_o + \sum_{n=1}^{\infty}\left(a_n \text{Cos}nt + b_n \text{Sin}nt\right), \tag{10}$$

$$a_o = \frac{1}{24}\int_0^{24} f(t)\,dt \tag{11}$$

$$a_o = \frac{1}{24}\left[\int_0^{\frac{24}{r}} K_1 dt + \int_{\frac{24}{r}}^{\frac{48}{r}} K_2 dt + \ldots\ldots + \int_{\frac{24(r-1)}{r}}^{24} K_n dt\right], \tag{12}$$

$$a_o = \frac{1}{24}\left[\left(K_1 t\right)_o^{\frac{24}{r}} + \left(K_2 t\right)_{\frac{24}{r}}^{\frac{48}{r}} + \ldots\ldots + \left(K_n t\right)_{\frac{24(r-1)}{r}}^{24}\right], \tag{13}$$

$$a_o = \frac{1}{24}\left[K_1\frac{24}{r} + K_2\left(\frac{48}{r} - \frac{24}{r}\right) + \ldots.. + K_n\left(24 - \frac{24(r-1)}{r}\right)\right], \tag{14}$$

$$a_o = \frac{1}{24}\left[\frac{24K_1}{r} + \frac{24K_2}{r} + \ldots.. + \frac{24K_n}{r}\right], \tag{15}$$

$$a_o = \frac{K_1}{r} + \frac{K_2}{r} + ... + \frac{K_n}{r}. \tag{16}$$

From equation (8),

$$a_o = \frac{1}{r}\left[\left(\frac{dE_L}{dt}\right)_1 + \left(\frac{dE_L}{dt}\right)_2 + ... + \left(\frac{dE_L}{dt}\right)_n\right]. \tag{17}$$

Determining the value of a_n

$$a_n = \frac{1}{12}\int_0^{24} f(t)\,\mathrm{Cos}\frac{n\pi}{12}tdt, \tag{18}$$

$$a_n = \frac{1}{12}\left[\int_0^{\frac{24}{r}} K_1\mathrm{Cos}\frac{n\pi}{12}tdt + \int_{\frac{24}{r}}^{\frac{48}{r}} K_2\mathrm{Cos}\frac{n\pi}{12}tdt + ... + \int_{\frac{24(r-1)}{r}}^{24} K_n\mathrm{Cos}\frac{n\pi}{12}tdt\right], \tag{19}$$

$$a_n = \frac{1}{12}\left[\frac{12K_1}{n\pi}\left(\mathrm{Sin}\frac{n\pi t}{12}\right)_0^{\frac{24}{r}} + \frac{12K_2}{n\pi}\left(\mathrm{Sin}\frac{n\pi t}{12}\right)_{\frac{24}{r}}^{\frac{48}{r}} + ... + \frac{12K_n}{n\pi}\left(\mathrm{Sin}\frac{n\pi t}{12}\right)_{\frac{24(r-1)}{r}}^{24}\right], \tag{20}$$

$$a_n = \frac{1}{12}\left(\frac{12K_1}{n\pi}\left(\mathrm{Sin}\frac{24n\pi}{12r} - \mathrm{Sin}0\right) + \frac{12K_2}{n\pi}\left(\mathrm{Sin}\frac{48n\pi}{12r} - \mathrm{Sin}\frac{24n\pi}{12r}\right) + \right.$$

$$\left.\frac{12K_n}{n\pi}\left(\mathrm{Sin}\frac{24n\pi}{12r} - \mathrm{Sin}\frac{24(r-1)n\pi}{12r}\right)\right), \tag{21}$$

$$a_n = \frac{1}{n\pi}\left[K_1\mathrm{Sin}\frac{2\pi n}{r} + K_2\left(\mathrm{Sin}\frac{4\pi n}{r} - \mathrm{Sin}\frac{2\pi n}{r}\right) + ... + Kn\left(\mathrm{Sin}2\pi n - \mathrm{Sin}\frac{2\pi(r-1)n}{r}\right)\right], \tag{22}$$

$$a_n = \frac{1}{n\pi}\left[K_1\mathrm{Sin}\frac{2\pi n}{r} + K_2\left(\mathrm{Sin}\frac{4\pi n}{r} - \mathrm{Sin}\frac{2\pi n}{r}\right) + ... + K_n\mathrm{Sin}\frac{2\pi(r-1)n}{r}\right]. \tag{23}$$

Since $\mathrm{Sin}2\pi n = 0$,

where n = 1, 2, ….

when n = r, then $a_n = 0$

Similarly, $$b_n = \frac{1}{12}\int_0^{24} f(t)\,\mathrm{Sin}\frac{n\pi t}{12}dt, \tag{24}$$

$$b_n = \frac{1}{12}\left[\int_0^{\frac{24}{r}} K_1\mathrm{Sin}\frac{n\pi t}{12}dt + \int_{\frac{24}{r}}^{\frac{48}{r}} K_2\mathrm{Sin}\frac{n\pi t}{12}dt + ... + \int_{\frac{24(r-1)}{r}}^{24} K_n\mathrm{Sin}\frac{n\pi t}{12}dt\right], \tag{25}$$

$$b_n = \frac{1}{12}\left[-\frac{12K_1}{n\pi}\left(\mathrm{Cos}\frac{n\pi t}{12}\right)_0^{\frac{24}{r}} - \frac{12K_2}{n\pi}\left(\mathrm{Cos}\frac{n\pi t}{12}\right)_{\frac{24}{r}}^{\frac{48}{r}} + ... + \frac{12K_n}{n\pi}\left(\mathrm{Cos}\frac{n\pi t}{12}\right)_{\frac{24(r-1)}{r}}^{24}\right], \tag{26}$$

$$b_n = \frac{1}{12}\left(-\frac{12K_1}{n\pi}\left(\text{Cos}\frac{24n\pi}{12r} - \text{Sin}\frac{n\pi(0)}{12}\right) - \frac{12K_2}{n\pi}\left(\text{Cos}\frac{48n\pi}{12r} - \text{Cos}\frac{24n\pi}{12r}\right) - \frac{12K_n}{n\pi}\left(\text{Cos}\frac{24n\pi}{12r} - \text{Cos}\frac{24(r-1)n\pi}{12r}\right)\right), \tag{27}$$

$$b_n = \frac{1}{12}\left(-\frac{12K_1}{n\pi}\left(\text{Cos}\frac{2n\pi}{r} - 1\right) - \frac{12K_2}{n\pi}\left(\text{Cos}\frac{4n\pi}{r} - \text{Cos}\frac{2n\pi}{12r}\right) - \frac{12K_n}{n\pi}\left(\text{Cos}2\pi n - \text{Cos}\frac{2(r-1)n}{r}\right)\right), \tag{28}$$

but $\cos 2\pi n = 1$, when $n = 1, 2, \ldots$

and $\cos 4\pi n = \cos 2\pi n = 1$

Similarly,

$$b_n = \frac{1}{\pi n}\left[-K_1\left(\text{Cos}\frac{2\pi n}{r} - 1\right) - K_2\left(\text{Cos}\frac{4\pi n}{r} - \text{Cos}\frac{2\pi n}{r}\right)\ldots\ldots - K_n\left(1 - \text{Cos}\frac{2\pi(r-1)n}{r}\right)\right], \tag{29}$$

$$b_n = \frac{1}{\pi n}\left[K_1\left(\text{Cos}\frac{2\pi n}{r} - 1\right) + K_2\left(\text{Cos}\frac{2\pi n}{r} - \text{Cos}\frac{4\pi n}{r}\right) + \ldots\ldots - K_n\left(\text{Cos}\frac{2\pi(r-1)n}{r} - 1\right)\right]. \tag{30}$$

Hence, substituting equations (16), (23) and (30) into equation (10) gives:

$$f(t) = \left(\frac{K_1}{r} + \frac{K_2}{r} + \ldots + \frac{K_n}{r}\right) + \sum \frac{1}{n\pi}\left(K_1\text{Sin}\frac{2\pi n}{r} + K_2\left(\text{Sin}\frac{4\pi n}{r} - \text{Sin}\frac{2\pi n}{r}\right) + K_n\left(-\text{Sin}\frac{2\pi(r-1)n}{r}\right)\right)$$
$$\text{Cosnt} + \frac{1}{\pi n}\left(K_1\left(1 - \text{Cos}\frac{2\pi n}{r}\right) + K_2\left(\text{Cos}\frac{2\pi n}{r} - \text{Cos}\frac{4\pi n}{r}\right) + \ldots + K_n\left(\text{Cos}\frac{2\pi(r-1)n}{r} - 1\right)\right)\text{Sinnt}. \tag{31}$$

Equation (31) is the general formula to represent the rate of total energy loss for different periods in a day or more. The total energy lost can be determined as:

$$E_L = \int_o^t \left(\frac{dE_L}{dt}\right)_g dt = t\left(\frac{dE_L}{dt}\right)_g, \tag{32}$$

where t is time in hours.

The cost of energy consumed in one hour s represented as U hence, the cost of energy loss for t hours in period g is:

$$C_g = U \times t\left(\frac{dE_L}{dt}\right)_g. \tag{33}$$

Hence, the total cost of energy loss for the whole periods.

$$C = \sum_{g=1}^{\hbar} C_g = \sum_{g=1}^{\hbar} U \times t\left(\frac{dE_L}{dt}\right)_g \tag{34}$$

Thus, equation (34) is the mathematical model for evaluating extra energy cost due to disproportional energy consumption.

3. Case study

The workers in Ajasco Company resume by 800hrs GMT and close by 1800hrs GMT. During the working period, all the machines function considerably well. The company has all its machines driven by induction motors. In the factory, there is an overhead crane which is driven horizontally by means of two similar induction motors, whose rotors are the two steel I-beams, on which the crane rolls. The 3-phase, 4-pole linear stators, which are mounted on opposite sides of the crane have a pole-pitch of 6mm and are energized by a variable-frequency electronic source. One of the motors has the following data:

Stator frequency = 25 Hz   Power to stator = 6 KW

Stator copper and iron loss – 1.2 KW   Crane speed = 2.4 m/s

Also, the company has other machines which are also driven by induction motors and whose data are as follows:

Power Input (KW)	Efficiency (%)
Machine 2	89
Machine 3	83
Machine 4	84
Machine 5	75
Machine 6	88
Machine 7	83
Machine 8	93
Machine 9	94.0
Machine 10	95.0

After the end of the working periods, all the machines are switched off, also fans and Air-conditioner while lamps are put on to lighten the company premises for security purpose. The total number of lamps are put on between the period from 1800hrs GMT to 600hrs GMT the next day. The losses due to the heat generated by the lamps and the energy absorbed by the over globe for each of the lamps sum up to be 0.5KW when the company has 20 of such lamps. From 600hrs to 1800hrs GMT only the lamps, fans and air-conditioners are put on for the convenience of the workers. The loss due to fans and air-conditioners sum up to be 0.3KW. There are 30 office lamps, for which the loss due to one of them is 0.25KW. Determine: (i) the total power loss for a day in the company, (ii) the total cost that would have been paid for the total energy loss for a whole month, if there is no work on Saturdays and Sundays. (Note: The security lamps are put on at that fixed time while all machines, fans and AC are switched off also with office lights). Predict what will happen at the transient periods if (iii) the equipment were not switched off (iv) the equipment have been switched off before the period (Note: 1Kwh cost N4.00).

4. Solutions

1. The total loss for a day:

From 000hrs to 600hrs GMT, $\left(\frac{dE_L}{dt}\right)_1 = 0.5 \times 20 = 10$KW.

From 600hrs to 800hrs GMT, $\left(\frac{dE_L}{dt}\right)_2 = 0.25 \times 30 + 0.3 = 7.8$KW.

From 800hrs to 1800hrs GMT,

$$\left(\frac{dE_L}{dt}\right)_3 = \sum_{i=1}^{10} \frac{dE_i}{dt}(1-\eta_i) + \frac{dL_1}{dt} + 0.3 + (0.25 \times 30),$$

where $\frac{dL_1}{dt}$ is loss in the crane = 1.2 KW,

but solving for: $\sum_{i=1}^{10} \frac{dE_i}{dt}(1-\eta_i)$.

Machine i	Power loss (KW)
2	8.25
3	13.6
4	12
5	18.75
6	10.8
7	14.79
8	7
9	5.64
10	2.5
Total	92.16

Hence, $\left(\frac{dE_L}{dt}\right)_3 = 92.16 + 1.2 + 0.3 + 0.25 \times 30 = 101.16$ KW.

Power loss from 1800hrs to 2400hrs; $\left(\frac{dE_L}{dt}\right)_4 = \left(\frac{dE_L}{dt}\right)_1 = 10$ KW.

Therefore, the total power loss for a day is:

$$\left(\frac{dE_L}{dt}\right)_1+\left(\frac{dE_L}{dt}\right)_2+\left(\frac{dE_L}{dt}\right)_3+\left(\frac{dE_L}{dt}\right)_4 = 10 + 7.8 + 101.16 + 10 = 121.16 \text{ KW.}$$

2. Let us consider a month with:

The total number of work days = 30 – 8 = 22 days.

The energy loss for 30 days during the hours from 000hrs to 600hrs GMT = $(E_L)_1$

$$(E_L)_1 = 6\times 30\times\left(\frac{dE_L}{dt}\right)_1 = 6 \times 30 \times 10 = 1800 \text{ KWh.}$$

The energy loss for 22 days during the working hours from 800hrs to 1800hrs GMT is $(E_L)_3$:

$$(E_L)_3 = 10\times 22\times\left(\frac{dE_L}{dt}\right)_3 = 10 \times 22 \times 101.16 = 22255.2 \text{ KWh.}$$

The energy loss for 22 days during the hours from 600hrs to 8000hrs GMT is $(E_L)_2$:

$$(E_L)_2 = 2\times 22\times\left(\frac{dE_L}{dt}\right)_2 = 2 \times 22 \times 7.8 = 343.2 \text{ KWh.}$$

The energy loss for 30 days during the hours from 1800hrs to 2400hrs GMT is $(EL)_4$:

$$(E_L)_4 = 6\times 30\times\left(\frac{dE_L}{dt}\right)_4 = 6 \times 30 \times 10 = 1800 \text{ KWh.}$$

Therefore, total energy loss for a whole month having thirty days:

$$E_L = (E_L)_1 + (E_L)_2 + (E_L)_3 + (E_L)_4 = 1800 + 343.2 + 22255 + 1800.$$

Hence, total cost for the energy loss = 26198.4 × 4 = N104793.6.

3. If the equipment used in the preceding periods are not needed in the succeeding periods and they were not switched off before the transition, the value of the energy loss increases.

4. If, on the other hand they were switched off, the energy loss will remain constant as long as the efficiencies of the machines are constant.

5. Discussion of results

The case study gives us an insight into what can cause the extra cost of energy consumption. From the example cited, some of the causes are due to low efficiencies of the machine, use of high-resistive lamps and fans with low efficiencies. Summing up the individual effects of these causes for a whole month gave an amount of money that is so high. Companies have to check the efficiencies of the machines. They have to see that the efficiencies of the machines are improved by buying of goods spare parts and implementing good maintenance policy. Losses in machines (especially heavy equipment) constitute the greatest cause of extra cost due to disproportional energy consumption.

6. Conclusions

The quantification of energy losses in production facilities offers a challenge to researchers, energy planners, and policy makers. This is because enormous amounts of energy are consumed periodically on material processing, power generation (through electricity and fuel consumption), in-plant transportation, etc. This usually results in high production cost, and reduced profits for the organization. Since expenditure on energy runs into several millions of dollars annually, the optimal utilization of energy is therefore an urgent solution to the problem of energy losses in order to reduce wastes. Surely, this would enhance business profitability. The proposed methodology in this study seems to offer convincing explanations for energy losses in industries. Significant savings in finance could be achieved if organizations take advantage of the potential savings that this study suggests.

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