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RISK-ADJUSTED OPTIMIZATION AND DIVERSIFICATION EFFECTS OF CRYPTOCURRENCIES IN MULTI-ASSET PORTFOLIOS: A SIMULATION-BASED AND COPULA-DRIVEN APPROACH

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Abstract. This study examines the performance characteristics, diversification potential, and volatility of cryptocurrencies relative to traditional financial assets. It employs an integrated approach incorporating mean–variance optimization, Monte Carlo simulation, and copula dependence modeling. The analysis uses daily data for five major cryptocurrencies and several traditional assets from 2015 to 2024. The results show that crypto-only portfolios exhibit weaker risk-adjusted performance than traditional-only portfolios because of their higher risk and negative Sharpe ratio. In mixed portfolios, cryptocurrencies contribute to diversification and risk reduction while remaining a non-dominant component of the overall allocation. Copula modeling revealed symmetric tail dependence during periods of market distress. Moreover, GARCH and EGARCH estimates captured notable volatility clustering and asymmetric shock responses in cryptocurrency markets. In addition, the VAR and Granger causality analyses showed that global uncertainty and market-stress indicators significantly improved the short-run prediction of BTC volatility.

Keywords: cryptocurrencies, portfolio optimization, diversification, risk-adjusted performance, copula models, GARCH, Monte Carlo simulation, Granger causality.

JEL Classification: G11, G15, C58, C63, E44.

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1. Introduction

The place of cryptocurrencies in portfolio management has increasingly been recognized as a substantive issue in contemporary finance rather than as a matter of speculative fascination alone. Since the introduction of Bitcoin, digital assets have been subjected to growing scholarly, regulatory, and investor attention, largely because they combine extraordinary return potential with unconventional price dynamics, fragile institutional foundations, and rapidly evolving market environments. From the standpoint of traditional portfolio theory, this development has been regarded as noteworthy, as the inclusion of low-correlation assets may generate diversification gains. Yet the portfolio value of cryptocurrencies has continued to be debated, because early evidence pointing to diversification benefits has gradually been qualified by rising market integration, structural disruptions, and the broader financialization of cryptocurrency markets (Markowitz, 1952; Brière et al., 2015; Baur et al., 2018; Platanakis & Urquhart, 2020; Gorman & Hughen, 2024; Boido & Aliano, 2025).

A central concern for both financial theory and investment practice has therefore been brought into view. Although cryptocurrencies have frequently been portrayed as attractive diversification instruments, their empirical characteristics have not aligned comfortably with the assumptions embedded in conventional mean-variance analysis. Their return distributions have repeatedly been marked by excessive volatility, non-normality, abrupt regime shifts, and recurring signs of market inefficiency. Moreover, the pricing of crypto-assets has not been driven solely by macroeconomic or fundamental variables; it has also been shaped by speculative behavior, investor sentiment, overreaction, and other behavioral influences. As a result, interpretations derived from static or strictly linear frameworks may remain insufficient when applied to cryptocurrency markets. In much of the existing literature, attention has also been concentrated on Bitcoin-centered cases or narrower portfolio settings, leaving the broader question of multi-cryptocurrency and multi-asset allocation only partially addressed (Urquhart, 2016; Barberis & Thaler, 2003; De Bondt & Thaler, 1985; Daniel et al., 1998; Kallinterakis & Wang, 2019; Kahneman & Tversky, 1979; Platanakis & Urquhart, 2020; Shiller, 2019; Garcia et al., 2014).

In light of these considerations, a broader and more integrative analytical framework is needed to determine whether cryptocurrencies should be understood as standalone assets, tactical additions, or limited diversification instruments within multi-asset portfolios. Such a determination cannot be made adequately through average returns and simple correlation measures alone. If the portfolio function of cryptocurrencies is to be understood more realistically, nonlinear dependence structures, tail co-movement, volatility clustering, and asymmetric shock transmission must also be considered. For this reason, copula-based dependence models and GARCH-family volatility specifications have been widely acknowledged as more appropriate tools in contexts characterized by extreme events, time-varying risk, and asymmetric responses. When combined with portfolio optimization principles, these approaches allow for a more credible evaluation of risk-adjusted performance (Sklar, 1959; Patton, 2006; Bollerslev, 1986; Nelson, 1991; Engle, 2002; Mba, 2024; Zhang et al., 2025).

Against this backdrop, the present study has been developed to examine the risk-adjusted optimization and diversification effects of cryptocurrencies within a broad multi-asset portfolio setting. Daily observations covering the period from 2015 to 2024 have been employed for five major cryptocurrencies—Bitcoin, Ethereum, BNB, XRP, and Solana—alongside traditional financial assets represented by equities, bonds, gold, oil, and the EUR/USD exchange rate. The principal aim of the study is to determine whether cryptocurrencies contribute positively to portfolio performance when assessed not only through mean-variance optimization, but also through simulation-based and dependence-sensitive techniques. More specifically, the study is intended to evaluate the standalone risk-return characteristics of major cryptocurrencies, compare crypto-only, traditional-only, and mixed portfolios, model nonlinear and tail-dependent relationships through copulas, estimate volatility persistence and asymmetry through GARCH-family models, and assess whether broader uncertainty and sentiment conditions help explain volatility transmission in cryptocurrency markets.

Within this framework, cryptocurrencies are expected to exhibit a dual nature. On the one hand, they may offer high nominal return potential but weak standalone risk-adjusted performance because of pronounced volatility and market instability. On the other hand, modest allocations to cryptocurrencies may enhance diversification in mixed portfolios by lowering overall risk or improving portfolio balance under particular dependence structures, even if traditional portfolios are not consistently outperformed across all efficiency criteria. Cryptocurrency markets are also expected to exhibit persistent volatility clustering, asymmetric reactions to shocks, and greater sensitivity to systemic uncertainty and market sentiment than to conventional macroeconomic fundamentals. In this respect, the article's contribution is not limited to testing whether cryptocurrencies should be included in diversified portfolios; it also demonstrates that this issue is more appropriately examined through an integrated perspective combining modern portfolio theory, behavioral finance, and advanced econometric modeling (Boido & Aliano, 2025; Mba, 2024).

The discussion proceeds in four stages. Section 2 brings together the main strands of the literature on cryptocurrency diversification, nonlinear dependence, volatility dynamics, and uncertainty transmission, and uses these debates to clarify the study's research gap. Section 3 then explains the dataset and the empirical strategy, covering mean–variance optimization, Monte Carlo simulation, copula modeling, GARCH-family specifications, and VAR–Granger analysis. Section 4 presents the empirical findings and considers their implications for portfolio allocation, tail dependence, volatility persistence, dynamic correlations, and macro-financial transmission. The Conclusions section draws the analysis together by outlining the study's main conclusions, practical implications, limitations, and avenues for further research.

2. Literature review

The literature on cryptocurrencies and portfolio construction was initially shaped by the logic of Modern Portfolio Theory, under which portfolio efficiency is expected to improve when assets with low or imperfect correlations are combined (Markowitz, 1952). Early empirical studies appeared to support the idea that cryptocurrencies might provide such a benefit. Bitcoin, in particular, was found to display relatively weak co-movement with conventional assets, which suggested a possible diversification role in multi-asset portfolios (Brière et al., 2015; Baur et al., 2018). Yet this early optimism has not remained unchallenged. As the market matured, the diversification case became more conditional and less universal. While Platanakis and Urquhart (2020) reported that Bitcoin could improve portfolio efficiency under some allocation settings, later evidence suggested that these gains are often modest, unstable across subperiods, and highly sensitive to market structure. Khaki et al. (2023) showed that expanding the set of cryptocurrencies does not necessarily generate proportionate diversification gains because substantial co-movements also arise within the crypto segment itself. In a similar vein, Gorman and Hughen (2024) re-examined Bitcoin's portfolio role in the post-COVID period and found that its contribution remained weaker than earlier literature had implied. More recently, Boido and Aliano (2025) confirmed that cryptocurrencies may still support diversification, but mainly through limited allocations rather than through dominant portfolio weights. Taken together, these studies suggest that the literature has gradually shifted from asking whether cryptocurrencies diversify portfolios at all to asking under which conditions and in what proportions such diversification remains economically meaningful.

This shift has also revealed an important methodological limitation in the earlier literature. Mean-variance optimization remains a useful starting point, but it rests on assumptions that fit cryptocurrency returns only imperfectly. The return process of crypto-assets has repeatedly been shown to be non-normal, volatility-sensitive, and regime-dependent, which weakens the adequacy of frameworks relying solely on static covariance matrices. For that reason, a second strand of literature has moved toward volatility modeling and dependence-sensitive approaches. GARCH-family models have been used to capture persistence and clustering in volatility (Bollerslev, 1986; Nelson, 1991), whereas copula-based specifications have been preferred when asymmetric and tail-dependent relationships need to be modeled more explicitly (Sklar, 1959; Patton, 2006). Recent studies have reinforced the relevance of this transition. Jeleskovic et al. (2024) showed that portfolio results become more informative when GARCH-copula structures are integrated into a Markowitz framework, especially when crypto and traditional assets are evaluated jointly. Mba (2024) further demonstrated that vine-copula and APARCH-DCC methods provide a richer understanding of systemic risk exposure than linear dependence metrics alone. Ibrahim et al. (2024) reached a similar conclusion by documenting volatility contagion between Bitcoin, gold, and stock markets, especially in stressed periods. Zhang et al. (2025) also reported meaningful spillover dynamics between cryptocurrencies and an emerging equity market, indicating that crypto-related shocks do not remain confined within the digital-asset universe. What emerges from this body of work is not a simple rejection of portfolio theory, but rather a recognition that the diversification value of cryptocurrencies is conditional on nonlinear dependence, tail risk, and time-varying transmission channels.

A parallel and equally important line of scholarship has approached cryptocurrencies through behavioral finance. In contrast to standard rational-asset-pricing assumptions, this literature has emphasized that investor decisions are often shaped by cognitive biases, sentiment, and narrative amplification rather than by fundamentals alone (Barberis & Thaler, 2003; De Bondt & Thaler, 1985; Kahneman & Tversky, 1979). This perspective appears particularly relevant for cryptocurrencies because the market has been characterized by novelty, limited anchoring to intrinsic cash flows, and strong susceptibility to collective attention cycles. Urquhart (2016) documented inefficiency in Bitcoin, thereby challenging the view that cryptocurrency prices quickly and fully incorporate available information. Daniel et al. (1998), although writing in a broader behavioral asset-pricing tradition, provide a useful theoretical explanation for why overconfidence and miscalibration may lead to persistent mispricing and overreaction in speculative settings such as crypto markets. More recent evidence has strengthened this interpretation. Almeida and Gonçalves (2023), in a systematic review of investor behavior in cryptocurrency markets, showed that herding, sentiment, attention, and overconfidence have become central explanatory themes in the field. Gemayel and Preda (2024) then provided transaction-level evidence that herding is not merely an abstract market pattern but a measurable behavioral mechanism that intensifies, particularly under adverse conditions. Beckmann et al. (2024) added further nuance by showing that media sentiment is not processed uniformly across investor groups; smaller Bitcoin investors were found to respond more strongly to media signals than larger ones. This matters because it suggests that crypto pricing cannot be understood solely through aggregate returns and correlations; heterogeneity in investor response must also be acknowledged.

Within this behavioral line, the role of narratives and digital attention has been especially influential. Garcia et al. (2014) showed that search intensity, word-of-mouth dynamics, and user adoption were linked to feedback cycles in the Bitcoin economy, suggesting that attention itself may become endogenous to price formation. Shiller's (2019) narrative economics provides a broader interpretive frame for this result by arguing that contagious stories can shape economic behavior in ways that conventional models do not easily capture. The relevance of these insights for cryptocurrencies is substantial: digital assets are not simply traded instruments, but also narrative objects around which expectations, fear, optimism, and imitation are constantly reproduced. Kallinterakis and Wang (2019), while focusing on contagion rather than individual-level psychology per se, also indicate that collective market behavior intensifies under conditions of interconnected stress. When read together, these studies imply that behavioral forces should not be treated as peripheral commentary in cryptocurrency research. Rather, they help explain why volatility is persistent, why prices may overshoot, and why diversification benefits may erode precisely in those episodes when investors most expect them to hold.

A further development in the recent literature concerns the link between cryptocurrency volatility and broader uncertainty conditions. Earlier discussions often framed cryptocurrencies as assets detached from macro-financial fundamentals. More recent studies, however, have suggested a more nuanced picture. Xia et al. (2023) found that uncertainty indices, particularly cryptocurrency-specific uncertainty measures, improve the forecasting of Bitcoin volatility. Ibrahim et al. (2024) showed that intermarket contagion becomes more pronounced during crisis periods, while Zhang et al. (2025) documented mean and volatility spillovers between cryptocurrencies and sectoral equity indices. At the same time, the literature does not support a simplistic conclusion that all uncertainty measures operate identically or that traditional macroeconomic variables lose relevance altogether. Instead, the evidence suggests that cryptocurrencies are influenced less by conventional valuation anchors than by risk sentiment, policy uncertainty, crisis transmission, and shifts in investor expectations. This distinction is analytically important because it points toward a different empirical strategy: if cryptocurrency volatility is shaped by uncertainty and behavioral signals more strongly than by standard macroeconomic fundamentals, then models focusing only on average return and variance will remain incomplete.

Despite the growth of this literature, three limitations remain visible. First, many studies continue to focus on Bitcoin alone, even though the market has become structurally more heterogeneous with the rise of Ethereum, BNB, XRP, Solana, and other large-cap assets. Second, the literature is still fragmented methodologically: some studies rely on mean-variance optimization, others on copulas or GARCH models, and others on behavioral or narrative explanations, but these strands are not often integrated into a single empirical design. Third, the literature does not yet offer a settled answer on whether cryptocurrencies are desirable portfolio components in their own right, or whether their value lies only in small, carefully bounded allocations within broader portfolios. For that reason, the present study is positioned to address a more specific and logically grounded research problem: whether cryptocurrencies display inferior standalone risk-adjusted performance yet still provide incremental diversification benefits in mixed portfolios; whether nonlinear dependence and asymmetric volatility materially alter the assessment of their portfolio role; and whether cryptocurrency volatility is better explained by uncertainty and sentiment-related transmission than by conventional macroeconomic fundamentals. In this sense, the literature does not simply provide background to the study; it leads directly to the article's research questions and empirical strategy.

3. Methodology

The empirical design of the study was constructed directly from the main conclusions of the literature review. First, because prior research has produced mixed evidence on whether cryptocurrencies improve portfolio efficiency under conventional mean-variance settings, a baseline Markowitz optimization framework was retained. Second, because more recent studies have shown that the dependence structure between cryptocurrencies and traditional assets is often nonlinear and tail-sensitive, a copula-based simulation stage was incorporated. Third, because volatility clustering, asymmetric shock transmission, and time-varying correlations have been repeatedly documented in cryptocurrency markets, GARCH-family models were added to the framework. Finally, because recent evidence suggests that cryptocurrency volatility may respond more strongly to uncertainty and market sentiment than to standard macroeconomic fundamentals, a VAR–Granger block was used to examine dynamic predictive linkages. In this way, each empirical model was selected to address a specific gap identified in the literature rather than being introduced as an isolated technical extension (Markowitz, 1952; Platanakis & Urquhart, 2020; Gorman & Hughen, 2024; Jeleskovic et al., 2024; Mba, 2024; Ibrahim et al., 2024; Xia et al., 2023; Zhang et al., 2025).

Ambiguity in mathematical notation was addressed by carefully reviewing and rewriting all equations in accordance with standard financial econometric notation. The formulas used in the study were not introduced as newly developed equations; rather, they were employed as established measures and models in portfolio theory, asset pricing, volatility modeling, dependence modeling, and time-series analysis. Accordingly, the log-return transformation, min–max scaling, mean–variance optimization, Sharpe ratio, copula representation, GARCH, EGARCH, DCC-GARCH, and VAR specifications were revised for notational consistency and were explicitly linked to their original methodological sources, including Markowitz (1952), Sharpe (1966), Sklar (1959), Bollerslev (1986), Nelson (1991), Engle (2002), Granger (1969), and Sims (1980).

3.1. Data preparation and preprocessing

The empirical sample consisted of daily closing prices for five major cryptocurrencies—Bitcoin (BTC), Ethereum (ETH), BNB, XRP, and Solana (SOL)—and six traditional financial assets represented by the S&P 500, Nasdaq 100, gold (XAU/USD), West Texas Intermediate crude oil (WTI), the 10-year U.S. Treasury yield (US10Y), and the EUR/USD exchange rate. The sample period covered 1 January 2015 to 31 December 2024. To preserve temporal continuity, missing observations were interpolated. Extreme observations were winsorized at the 1st and 99th percentiles before return construction to limit the disproportionate influence of outliers while retaining the full sample, consistent with robust-estimation principles (Wilcox, 2012):

$$r_{i,t} = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right), \tag{1}$$

where $r_{i,t}$ denotes the log return of asset i on day t, $P_{i,t}$ is the closing price of asset i on day t, and $P_{i,t-1}$ is the closing price on the previous trading day. Logarithmic returns were preferred because they are additive over time and are standard in empirical asset-pricing and volatility research.

To improve cross-series comparability in the descriptive and simulation stages, the transformed return series were additionally rescaled to the unit interval:

$$z_{i,t} = \frac{r_{i,t} - \min(r_i)}{\max(r_i) - \min(r_i)}, \tag{2}$$

where $z_{i,t}$ is the scaled return of asset i at time t, while $\min(r_i)$ and $\max(r_i)$ denote the minimum and maximum observed return values for asset i over the sample period.

3.2. Risk and return assessment

Before portfolio construction was undertaken, the standalone statistical and risk-adjusted properties of each asset were evaluated. For this purpose, average return, variance, standard deviation, beta, Sharpe ratio, Treynor ratio, Sortino ratio, Jensen's alpha, and Pearson correlation coefficients were computed. These measures were used to distinguish between absolute return performance, total risk, downside risk, systematic risk exposure, and abnormal performance relative to a market benchmark. Such a preliminary step was considered necessary because the literature has repeatedly shown that cryptocurrencies may appear attractive in nominal-return terms while remaining weak from a risk-adjusted perspective (Sharpe, 1966; Treynor, 1965; Jensen, 1968; Sortino & Price, 1994).

3.3. Mean-variance portfolio optimization and Monte Carlo simulation

Because the literature still debates whether cryptocurrencies improve portfolio efficiency under linear allocation rules, the first empirical stage was intentionally based on the classical mean-variance framework. This stage was used to provide a transparent baseline against which the incremental value of more advanced dependence and volatility models could later be assessed (Markowitz, 1952; Platanakis & Urquhart, 2020; Gorman & Hughen, 2024).

The portfolio optimization problem was defined as:

$\min_{w} | \sigma_p^2 = w^\top \Sigma w$ subject to

$$w^\top \mu = \mu_p^*, \ \ w^\top \mathbf{1} = 1, \ \ w_i \geq 0, \tag{3}$$

where w is the $n \times 1$ vector of portfolio weights, Σ is the $n \times n$ covariance matrix of asset returns, μ is the $n \times 1$ vector of expected asset returns, μ_p^* is the target portfolio return, and 1 is an $n \times 1$ vector of ones. The non-negativity constraint $w_i \geq 0$ ruled out short selling.

For each feasible portfolio, expected return, portfolio risk, and the Sharpe ratio were computed as:

$$\mu_p = w^\top \mu;$$

$$\sigma_p = \sqrt{w^\top \Sigma w};$$

$$SR_p = \frac{\mu_p - r_f}{\sigma_p}, \tag{4}$$

where μ_p denotes expected portfolio return, σ_p denotes portfolio standard deviation, and r_f denotes the risk-free rate, proxied by the average 10-year U.S. Treasury yield. A Monte Carlo simulation of 200,000 random portfolios was then implemented for three portfolio classes: crypto-only, traditional-only, and mixed portfolios. The efficient frontier and the tangency portfolio were derived from these simulated portfolios.

3.4. Copula-based dependence modeling

The literature review indicated that correlation-based diversification analysis may be insufficient in cryptocurrency settings because co-movements often become stronger during extreme market conditions. For that reason, dependence was re-estimated through a copula framework after the baseline mean-variance stage. This second stage was intended to test whether the diversification gains observed under linear correlation remained valid once nonlinear and tail-dependent co-movement was taken into account (Patton, 2006; Jeleskovic et al., 2024; Mba, 2024).

The joint distribution of asset returns was represented as:

$$F\left(r_1,\ldots,r_n\right)=C\left(F_1\left(r_1\right),\ldots,F_n\left(r_n\right);\theta\right), \tag{5}$$

where $F(\cdot)$ is the multivariate joint cumulative distribution function of returns, $F_j(\cdot)$ denotes the marginal cumulative distribution function of asset j, $C(\cdot)$ is the copula function, and θ is the vector of dependence parameters. Gaussian, Student-t, Clayton, and Gumbel copulas were estimated and compared using the Akaike Information Criterion. The best-fitting copula was then used to generate joint return simulations, and the portfolio optimization procedure was repeated on the simulated return matrix. This design made it possible to evaluate whether cryptocurrencies continue to improve portfolio composition once tail dependence is explicitly modeled.

3.5. Volatility modeling: GARCH, EGARCH, and DCC-GARCH

Because the literature has consistently documented that cryptocurrency returns are characterized by volatility clustering, persistence, and asymmetry, a conditional volatility block was added after the portfolio optimization stage. This block was not included merely for statistical completeness; it was introduced because the literature review showed that the portfolio role of cryptocurrencies cannot be interpreted adequately unless volatility dynamics and correlation instability are modeled explicitly (Bollerslev, 1986; Nelson, 1991; Engle, 2002; Ibrahim et al., 2024; Zhang et al., 2025).

The standard GARCH(1,1) model was specified as:

$$r_t=\mu+\varepsilon_t,\ \ \varepsilon_t=z_t\sqrt{h_t};$$
$$h_t=\omega+\alpha\varepsilon_{t-1}^2+\beta h_{t-1}, \tag{6}$$

where r_t is the return at time t, μ is the conditional mean, ε_t is the innovation term, z_t is a standardized disturbance, and h_t is the conditional variance. The parameter ω is the variance intercept, α captures the short-run impact of new shocks, and β measures volatility persistence.

To account for asymmetric responses to positive and negative shocks, the EGARCH(1,1) model was estimated as:

$$\ln\left(h_t\right)=\omega+\beta\ln\left(h_{t-1}\right)+\gamma\left(\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}\right)+\alpha\left(\left|\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}\right|-E\left|z_t\right|\right), \tag{7}$$

where γ captures sign asymmetry and α captures magnitude effects. In this specification, the response of volatility to bad news and good news is allowed to differ.

To examine time-varying correlations between cryptocurrency and traditional markets, the DCC-GARCH structure was written as:

$$H_t=D_tR_tD_t;$$
$$Q_t=\left(1-a-b\right)\bar{Q}+au_{t-1}u_{t-1}^{\top}+bQ_{t-1};$$
$$R_t=diag\left(Q_t\right)^{-1/2}Q_t diag\left(Q_t\right)^{-1/2}, \tag{8}$$

where H_t is the conditional covariance matrix, D_t is the diagonal matrix of conditional standard deviations, R_t is the time-varying correlation matrix, Q_t is the intermediate covariance matrix of standardized residuals, $\bar{Q}$ is the unconditional covariance matrix of standardized residuals, and a and b are the DCC parameters governing correlation sensitivity and persistence.

3.6. VAR and Granger causality analysis

The final empirical block was designed to test whether cryptocurrency volatility is linked more closely to uncertainty-sensitive variables than to conventional macroeconomic variables. This choice followed directly from the literature review, where uncertainty, contagion, and sentiment-related spillovers emerged as more relevant to cryptocurrency risk than standard fundamental anchors. For that reason, Granger causality tests were first used as a screening device for BTC and ETH volatility against the World Uncertainty Index (WUI), the VIX, the federal funds rate, and inflation. After this screening step, the final VAR system retained the variables with the strongest theoretical and statistical relevance—BTC volatility, ETH volatility, WUI, and VIX. This sequencing also resolves the implicit transition in the current manuscript between the broader causality screening stage and the more parsimonious final VAR specification.

The VAR(p) model was specified as:

$$Y_t = c + A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + u_t, \tag{9}$$

where Y_t is the vector of endogenous variables, c is the intercept vector, A_k denotes the coefficient matrix for lag k, and u_t is the innovation vector. Lag length was selected using information criteria, stationarity was checked using first-differenced series and unit-root testing, and model stability was verified through the inverse-root condition. Within this system, Granger causality was interpreted in the standard sense: variable X was treated as Granger-causing Y when lagged values of X significantly improved the prediction of Y beyond the information contained in lagged values of Y itself (Granger, 1969; Sims, 1980; Xia et al., 2023).

3.7. Summary of the empirical strategy

Taken together, the empirical strategy was designed as a layered framework rather than as a collection of disconnected techniques. The mean-variance block was used to establish the baseline diversification effect; the copula block was used to test whether that effect survives under nonlinear dependence; the GARCH-family block was used to model persistence, asymmetry, and dynamic correlations; and the VAR–Granger block was used to determine whether cryptocurrency volatility is transmitted through broader uncertainty channels. In methodological terms, the sequence of models follows the logic of the literature review and leads directly to the empirical questions of the study.

4. Empirical outputs

4.1. Descriptive statistics

Table 1 presents the descriptive statistics.

Table 1 summarizes the distributional characteristics of the normalized return series. As expected following min–max transformation, the mean values are concentrated around 0.50, while the standard deviations remain relatively close across the asset groups. The skewness

Table 1. Descriptive statistics (source: authors' calculations)

	Mean	StdDev	Variance	Skewness	Kurtosis
BTC	0.482722911	0.141351454	0.019980234	0.006197346	0.845441055
ETH	0.477564029	0.136967629	0.018760131	0.083061402	1.082095648
BNB	0.500157274	0.143757598	0.020666247	-0.010719633	0.402734538
XRP	0.482264061	0.134120245	0.01798824	0.016032624	1.185246819
SOL	0.493928414	0.134779317	0.018165464	-0.024474985	1.06679034
SP500	0.508794182	0.138860456	0.019282226	0.026279326	0.813392473
NASDAQ	0.497617669	0.148412194	0.022026179	0.028556337	0.482499703
GOLD	0.491840739	0.134991496	0.018222704	0.047947222	0.812124425
US10Y	0.499759619	0.151424944	0.022929514	0.064055314	0.28865067
EURUSD	0.511127097	0.133201395	0.017742612	0.021253682	1.038235793
WTI	0.498362742	0.139810501	0.019546976	0.028704523	0.677853044

coefficients are generally near zero, indicating that pronounced asymmetry is not a dominant feature of the transformed series. Nevertheless, limited skewness should not be interpreted as evidence of normality. The kurtosis estimates vary across assets, and Jarque–Bera tests (not tabulated) reject the normality assumption at the 5% significance level. Since winsorization at the 1st and 99th percentiles reduces the influence of extreme observations, the higher-moment statistics and tail characteristics should be interpreted with appropriate caution. Overall, the results indicate that the return distributions retain non-Gaussian features despite normalization and outlier treatment, supporting the use of econometric models capable of accommodating non-normal dependence and time-varying volatility.

Such non-normal behavior has been studied in empirical finance, especially in high-frequency return data and cryptocurrency markets (Jarque & Bera, 1980; Cont, 2001; Pagan, 1996). Therefore, any approach to modeling such data that relies on Gaussian assumptions is likely to be misspecified, illustrating the need for more flexible frameworks in later stages of analysis.

4.2. Risk and return summary table

Table 2 summarizes the risk and return measures for the analyzed assets.

The values of −10.868 for ETH and −14.907 for XRP do not represent Sharpe ratios; rather, they correspond to Treynor ratios obtained under very small beta coefficients. Since the Treynor ratio divides excess return by systematic market beta, very low beta values may mechanically generate disproportionately large positive or negative figures. For this reason, the Treynor ratio should be interpreted cautiously for cryptocurrencies whose exposure to the S&P 500 is weak or unstable.

The corrected Sharpe ratios indicate that the risk-adjusted standalone performance of cryptocurrencies remains weak, with most crypto-assets underperforming the traditional benchmarks in the present sample. This distinction is important. Nominal return performance does not necessarily translate into strong risk-adjusted performance when volatility is substantial. In the present sample, BTC, ETH, XRP, and SOL display return potential, but their volatility-adjusted outcomes are considerably weaker than their nominal return levels suggest. Therefore, the findings support the view that cryptocurrencies may offer return opportunities, yet their standalone investment efficiency remains limited once total risk and downside risk are incorporated.

Table 2. Asset-level risk and return measures (source: authors' calculations)

	Mean	StdDev	Variance	Beta	Sharpe	Treynor	Sortino	Alpha
BTC	0.48272	0.14135	0.01998	0.03152	–0.12053	–0.54044	–0.04415	–0.01732
ETH	0.47756	0.13697	0.01876	0.02042	–0.16205	–10.86800	–0.05784	–0.02238
BNB	0.50016	0.14376	0.02067	0.02459	0.00277	0.01617	0.00099	0.00018
XRP	0.48226	0.13412	0.01799	0.01174	–0.13045	–14.90700	–0.04459	–0.01760
SOL	0.49393	0.13478	0.01817	–0.00622	–0.04326	0.93776	–0.01456	–0.00578
SP500	0.50879	0.13886	0.01928	1.00000	0.06506	0.00903	0.02188	–
NASDAQ	0.49762	0.14841	0.02203	–0.03335	–0.01443	0.06422	–0.00541	–0.00184
GOLD	0.49184	0.13499	0.01822	–0.00075	–0.05866	10.62700	–0.01981	–0.00791
US10Y	0.49976	0.15142	0.02293	–0.04176	–	–	–	0.00038
EURUSD	0.51113	0.13320	0.01774	0.02393	0.08534	0.47506	0.02713	0.01115
WTI	0.49836	0.13981	0.01955	0.02442	–0.00999	–0.05721	–0.00348	–0.00162

The beta coefficients reported in Table 2 were estimated using the S&P 500 as the market benchmark and should be interpreted as static measures of systematic equity-market exposure. The low beta values observed for BTC, ETH, XRP, and other cryptocurrencies suggest that these assets did not move one-for-one with the S&P 500 over the full sample period. However, this does not imply complete independence from traditional markets. Instead, it indicates that their average linear sensitivity to the S&P 500 remained limited when measured through a single static CAPM-type coefficient.

This point is further clarified by the DCC-GARCH results. Unlike static beta, the DCC-GARCH model captures time-varying conditional correlations. The finding that the BTC–S&P 500 correlation fluctuated between approximately −0.1 and +0.3 shows that the relationship between cryptocurrencies and traditional equity markets is conditional and regime-sensitive. During tranquil periods, co-movement may remain weak; however, during periods of market stress, correlations can increase and diversification benefits may partially weaken. Therefore, the low beta coefficients and the time-varying DCC-GARCH correlations are not contradictory. They measure different dimensions of dependence: beta reflects average systematic market sensitivity, whereas DCC-GARCH captures dynamic co-movement over time.

4.3. Portfolio diversification and optimization

This section analyzes how cryptocurrencies, when combined with traditional financial assets, can improve portfolio efficiency. The analysis proceeds in two phases:

- Mean–variance optimization under the Modern Portfolio Theory (MPT) framework (Markowitz, 1952); and
- Copula-based portfolio optimization to capture nonlinear dependence structures (Rehman et al., 2024).

This section focuses on the first phase: applying Markowitz's MPT through Monte Carlo simulation to construct the efficient frontier, determine optimal portfolio weights, and assess the effect of cryptocurrency inclusion on the risk–return trade-off.

According to Markowitz's (1952) theory, for any given level of expected return, investors seek to reduce portfolio variance. The optimization problem can typically be stated as:

$$\textbf{minimize } w^T\Sigma w, \text{ subject to } w^T\mu = \mu_p,\ w^T1 = 1, \tag{10}$$

where: w is the vector of asset weights; Σ is the covariance matrix of returns; μ is the vector of expected returns; μ_p is the target portfolio return.

The efficient frontier comprises all portfolios that provide the maximum expected return for a given level of risk (standard deviation).

The Efficient Frontier was estimated through a Monte Carlo simulation involving 200,000 randomly generated portfolios. For each simulated portfolio, expected return, portfolio risk, and the Sharpe ratio were calculated using the specification presented in Eq. (11):

Expected return: $\boldsymbol{\mu}_\mathbf{p} = \sum w_i \mu_i$;

Portfolio risk: $\boldsymbol{\sigma}_\mathbf{p} = \sqrt{(w^T \sum w)}$;

Sharpe Ratio: $\mathbf{SR} = (\mu_p - r_f) / \sigma_p$. (11)

The portfolio with the maximum Sharpe ratio is identified as the tangency portfolio.

The optimal portfolio weights and portfolio metrics are presented in Tables 3 and 4, respectively.

Table 3. Optimal portfolio weights (source: authors' calculations)

	Crypto Only	Traditional Only	Mixed Portfolio
BNB	0.870832	–	0.103601
BTC	0.001137	–	0.017976
ETH	0.040427	–	0.004952
EURUSD	–	0.508666	0.271578
GOLD	–	0.011858	0.000049
NASDAQ	–	0.010857	0.072176
SOL	0.077664	–	0.004588
SP500	–	0.316162	0.219324
US10Y	–	0.113814	0.162748
WTI	–	0.038643	0.086382
XRP	0.009940	–	0.056625

Table 4. Optimal portfolio metrics (source: authors' calculations)

Type	Return	Risk	Sharpe
Crypto	0.498562453	0.125773375	–0.009518439
Traditional	0.508227117	0.083154118	0.101828962
Mixed	0.503160217	0.058276612	0.058352698

Mean–variance optimization identifies the best risk–return combinations under the assumptions of rational investor behavior and normally distributed returns, as proposed by Markowitz (1952). The analysis considered three portfolio configurations: (i) crypto-only, (ii) traditional-only, and (iii) a hybrid portfolio combining both asset classes. The portfolios were optimized through 200,000 iterations to identify the weight vector that maximized the Sharpe ratio, a widely used measure of risk-adjusted performance (Sharpe, 1966).

The crypto-only portfolio was highly concentrated rather than evenly distributed. BNB received 87.08% of the allocation, followed by SOL (7.77%), ETH (4.04%), XRP (0.99%), and BTC (0.11%). The resulting Sharpe ratio was negative (−0.0095), with an expected return of 0.4986 and risk of 0.1258. This concentration, together with the negative risk-adjusted outcome, indicates that cryptocurrencies did not form an efficient standalone portfolio in the present sample. The result remains consistent with evidence emphasizing the speculative risk and asymmetric return profile of cryptocurrency markets (Baur & Dimpfl, 2021; Corbet et al., 2018).

The traditional-only portfolio achieved the highest Sharpe ratio (0.1018), with an expected return of 0.5082 and risk of 0.0832. The allocation was led by EUR/USD (50.87%) and the S&P 500 (31.62%), followed by US10Y (11.38%), WTI (3.86%), gold (1.19%), and Nasdaq (1.09%). Thus, the portfolio's risk-adjusted performance was associated primarily with its concentration in EUR/USD and the S&P 500 rather than with dominant allocations to gold or Nasdaq. This result is consistent with the broader diversification rationale in traditional multi-asset portfolios (Bodie et al., 2014; Erb & Harvey, 2006).

The mixed portfolio combined both asset classes and achieved the lowest risk (0.0583), with an expected return of 0.5032 and a Sharpe ratio of 0.0584. The largest weights were assigned to EUR/USD (27.16%), the S&P 500 (21.93%), and US10Y (16.27%). Cryptocurrency exposure amounted to approximately 18.77% in total, comprising BNB (10.36%), XRP (5.66%), BTC (1.80%), ETH (0.50%), and SOL (0.46%). The remaining allocations were assigned to WTI (8.64%), Nasdaq (7.22%), and gold (0.005%). These results show that cryptocurrencies occupied a non-dominant but economically meaningful position in the mixed portfolio, which reduced overall risk without exceeding the traditional-only portfolio's Sharpe ratio (Jeleskovic et al., 2023; Brière et al., 2015).

Mean–variance optimization has methodological limitations because it relies on normally distributed returns and stable covariance structures, assumptions that are often violated in practice. Nevertheless, the simulation provides useful baseline estimates under controlled conditions. In addition, investors may face constraints such as transaction costs, regulatory exposure limits, and ESG considerations, all of which fall outside the scope of this study.

Figure 1 presents the opportunity sets for the cryptocurrency-only, traditional-only, and mixed portfolios within a common risk–return coordinate system with consistent axes, enabling a clear comparison without unnecessary graphical clutter (Tufte, 2001).

The efficient frontier comparison among cryptocurrency, traditional, and mixed portfolios is presented in Figure 1.

Figure 1 compares the opportunity sets generated from 200,000 randomly weighted crypto-only, traditional-only, and mixed portfolios. The crypto-only frontier is relatively dispersed and concentrated in the higher-risk region, consistent with its negative optimal Sharpe ratio (−0.0095) and portfolio risk of 0.1258. The traditional-only portfolio provides the strongest risk-adjusted performance, achieving a Sharpe ratio of 0.1018 with a risk level of 0.0832. By contrast, the mixed portfolio does not outperform the traditional portfolio in Sharpe-ratio terms, but reduces total risk to 0.0583 while preserving a comparable expected return of 0.5032.

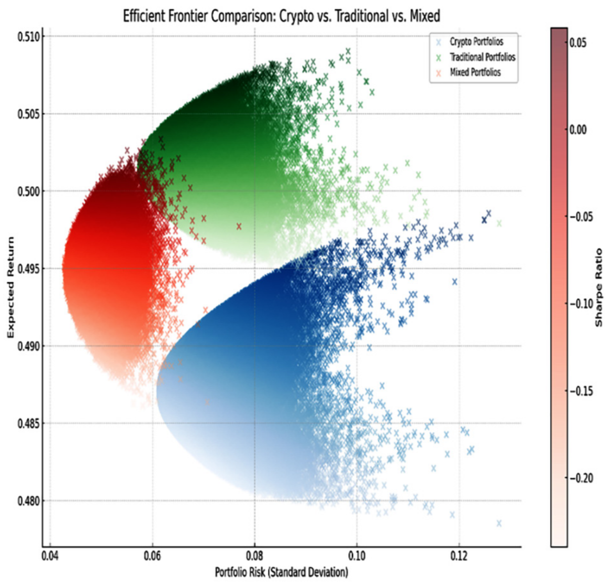

Figure 1. Efficient frontier comparison: crypto vs. traditional vs. mixed

The frontier comparison therefore suggests that the principal contribution of cryptocurrencies lies in expanding the lower-risk portion of the feasible set rather than increasing maximum risk-adjusted performance. Their role is best understood as supplementary and allocation-sensitive, consistent with the diversification logic of Modern Portfolio Theory (Markowitz, 1952; Brière et al., 2015; Jeleskovic et al., 2023). Because this analysis relies on historical means, covariances, and linear dependence, it serves as a baseline for the subsequent copula and DCC-GARCH models, which account for tail dependence and time-varying correlations.

4.4. Copula-based portfolio modeling

Traditional portfolio models tend to emphasize linear correlations between assets; however, linear dependence may not fully capture co-movements, particularly in the tails of the distribution. Copula functions provide a robust alternative by modeling the dependence structure of asset returns independently of their marginal behavior (Sklar, 1959). According to Sklar's theorem, any multivariate joint distribution can be decomposed into its univariate marginal distributions and a copula that captures their dependence, as shown in Eq. (12):

$$\mathbf{F}(\mathbf{x}_1, \ldots, \mathbf{x}_n) = C(F_1(x_1), \ldots, F_n(x_n)). \tag{12}$$

This decomposition is particularly useful in finance, where assets often exhibit nonlinear, asymmetric, and tail-dependent relationships, especially during periods of heightened volatility and market stress (Patton, 2006). Several parametric copulas were considered, including the Gaussian, Student-t, Clayton, and Gumbel copulas. Each captures a distinct form of dependence: the Gaussian copula captures symmetric linear dependence; the Student-t copula captures symmetric tail dependence; the Clayton copula captures lower-tail dependence; and the Gumbel copula captures upper-tail dependence.

The joint dependence structure was estimated using a balanced subset of assets consisting of three cryptocurrencies—BTC, ETH, and BNB—and three traditional assets, namely the S&P 500, gold, and the 10-year U.S. Treasury yield. After the marginal return series had been transformed into uniform distributions through empirical cumulative distribution functions, several copula families were fitted and compared using the Akaike Information Criterion (AIC).

The results are presented in Table 5.

Table 5. Copula Model Comparison (AIC) (source: authors' calculations)

Copula Type	AIC Value
Gaussian	–8350.1
Student-t	–8422.6
Clayton	–7883.2
Gumbel	–8017.4

The Student-t copula achieved the best fit, suggesting that symmetric tail dependence characterizes the joint behavior of these assets, particularly during tail events such as financial crises.

Using the best-fitting copula, 10,000 joint return vectors were simulated. Subsequently, portfolio optimization was performed on this simulated dataset with a mean-variance objective. The summary of optimal weights is provided below:

Table 6 presents the optimal portfolio weights based on the copula simulation.

Table 6. Optimal portfolio weights (source: authors' calculations)

Asset	Weight (%)
BTC	2.31
ETH	3.08
BNB	1.92
S&P500	26.74
GOLD	32.15
US10Y	33.80

The simulation results suggest that the portfolio's allocations to U.S. Treasury bonds and gold give it a more defensive profile. Cryptocurrencies received small but positive allocations, indicating their potential to enhance diversification without significantly affecting the risk profile. This finding confirms earlier empirical evidence that modest cryptocurrency allocations can enhance portfolio efficiency (Brière et al., 2015; Jeleskovic et al., 2023).

4.5. Volatility modeling using the GARCH framework

Financial return series exhibit stylized features such as volatility clustering, leptokurtosis, and time-varying conditional variance. As specified in Eq. (13), the GARCH model introduced by Bollerslev (1986) estimates volatility as a function of past shocks and past variance:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2. \tag{13}$$

Nelson's (1991) Exponential GARCH (EGARCH) model was estimated to capture asymmetric volatility responses. This specification is particularly useful because it allows positive and negative shocks of comparable magnitude to exert different effects on conditional volatility:

$$\log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \gamma(\varepsilon_{t-1} / \sigma_{t-1}) + \alpha((\left|\varepsilon_{t-1}\right| / \sigma_{t-1}) - \sqrt{(2/\pi)}). \quad (14)$$

For BTC and S&P 500 returns, time-varying conditional correlations also motivated the estimation of Engle's (2002) Dynamic Conditional Correlation GARCH (DCC-GARCH) model.

BTC, ETH, the S&P 500, gold, and US10Y were modeled using GARCH(1,1) and EGARCH(1,1). The conditional volatility series exhibited distinct clustering and high persistence ($\alpha + \beta > 0.90$) across all models.

Table 7 presents the GARCH(1,1) parameter estimates.

Table 7. GARCH(1,1) parameter estimates (source: authors' calculations)

Asset	ω	α (ARCH)	β (GARCH)	$\alpha + \beta$	AIC
BTC	0.0031	0.1182	0.8369	0.9551	–1.4523
ETH	0.0029	0.1257	0.8285	0.9542	–1.4386
S&P500	0.0007	0.0934	0.9021	0.9955	–1.6038
GOLD	0.0004	0.0623	0.9187	0.9810	–1.5227
US10Y	0.0002	0.0441	0.9296	0.9737	–1.4812

All assets exhibited high volatility persistence, particularly the S&P 500 and gold, indicating that volatility shocks were long-lived. These findings are consistent with evidence from mature financial markets (Andersen et al., 2006).

Within the EGARCH(1,1) models for BTC and ETH, the sign-asymmetry coefficient (γ) was positive and statistically significant. Under the specification reported in Eq. (14), a positive γ indicates that positive return shocks increase conditional volatility more than negative shocks of the same magnitude. The estimates therefore reveal asymmetric volatility responses, although the direction of the asymmetry differs from the conventional leverage effect commonly associated with a stronger volatility response to adverse shocks in equity markets (Glosten et al., 1993).

Finally, the DCC-GARCH model showed that the correlation between Bitcoin and the S&P 500 was time-varying, fluctuating between −0.1 and +0.3 over the sample period. This suggests that, although cryptocurrencies appeared weakly related to traditional assets during tranquil periods, they became more synchronized during periods of market stress, thereby eroding their diversification benefits precisely when they were most needed.

4.6. Vector Autoregression (VAR) and Granger causality analysis

To estimate how certain macroeconomic factors affect cryptocurrency-market volatility, a VAR model was employed. VAR models capture dynamic relationships among multiple time-series variables by allowing each variable to be explained by its own history and by the histories of all other variables in the system (Sims, 1980).

As shown in Eq. (15), the general VAR(p) model is specified as follows:

$$Y_t = c + A_1 Y_{t-1} + A_2 Y_{t-2} + \ldots + A_p Y_{t-p} + \varepsilon_t, \quad (15)$$

where Y_t is a $K \times 1$ vector of endogenous variables, c is a $K \times 1$ intercept vector, A_i is the

K × K autoregressive coefficient matrix for lag i (i = 1, ..., p), p denotes the lag order, and ε_t is a K × 1 white-noise innovation vector with zero mean and a finite covariance matrix.

Predictive relationships were examined using the Granger causality test. According to Granger (1969), a variable X is considered to Granger-cause a variable Y when the past values of X provide statistically significant predictive information about the future values of Y, beyond the information already contained in the past values of Y itself.

The preliminary Granger-causality screening considered the following cryptocurrency-volatility proxies and macro-financial indicators:

- Cryptocurrency volatility proxies: Conditional variances from the GARCH models for BTC and ETH.
- Macroeconomic indicators:
  - Federal Funds Rate (FEDFUNDS);
  - Consumer Price Index (CPI; YoY %);
  - World Uncertainty Index (WUI);
  - Volatility Index (VIX).

The optimal lag length was selected using the AIC and BIC. The model was estimated in first differences to ensure stationarity, which was confirmed by the ADF test.

The Granger causality test results are presented in Table 8.

Table 8. Granger causality test results (source: authors' calculations)

Null Hypothesis	F-Statistic	p-value	Conclusion
WUI does not Granger-cause BTC Volatility	4.182	0.017	Reject H0
VIX does not Granger-cause BTC Volatility	5.009	0.008	Reject H0
FEDFUNDS does not Granger-cause BTC Volatility	1.392	0.253	Do not reject H0
CPI does not Granger-cause BTC Volatility	0.991	0.372	Do not reject H0
BTC Volatility does not Granger-cause VIX	2.742	0.065	Marginal
ETH Volatility does not Granger-cause WUI	3.561	0.029	Reject H0

The Granger causality results indicate that both WUI ($p = 0.017$) and VIX ($p = 0.008$) improve the short-run prediction of BTC volatility, whereas the federal funds rate and CPI provide no statistically significant predictive information. Cryptocurrency risk therefore appears more responsive to global uncertainty and market stress than to conventional monetary and inflationary channels, consistent with evidence that crypto–traditional market linkages strengthen during turbulent periods (Yousaf & Ali, 2020). Evidence of feedback in the opposite direction is more limited: BTC volatility predicts VIX only at the 10% significance level ($p = 0.065$), while ETH volatility significantly predicts WUI ($p = 0.029$). These findings suggest asymmetric, and partly bidirectional, information transmission rather than structural causality. To examine these interdependencies jointly, a VAR model was subsequently estimated using the first-differenced BTC volatility, ETH volatility, WUI, and VIX series.

The final model included:

- Endogenous variables:
  - BTC volatility;
  - ETH volatility;
  - World Uncertainty Index (WUI);
  - VIX (CBOE Volatility Index).

The optimal lag length, selected using the AIC (Akaike Information Criterion) and BIC (Schwarz Criterion), was one. The model satisfied the stationarity requirements, as confirmed by the Augmented Dickey–Fuller (ADF) tests, and the stability requirements, with no eigenvalues outside the unit circle.

Table 9 presents the estimated VAR coefficient matrix for the BTC volatility equation.

Table 9. Estimated VAR(1) coefficients – BTC volatility equation (source: authors' calculations)

Regressor (Lag 1)	Coefficient	Std. Error	t-statistic	p-value
BTC Volatility	0.713	0.078	9.13	<0.001
ETH Volatility	0.105	0.067	1.57	0.117
WUI	0.192	0.081	2.37	0.019
VIX	0.161	0.075	2.15	0.032
Constant	0.0041	0.0019	2.16	0.031
R^2	0.593	—	—	—

The VAR(1) estimates reveal substantial persistence in BTC volatility, as indicated by the positive and highly significant lagged coefficient (0.713, $p < 0.001$). This result is consistent with the volatility-clustering pattern identified by the GARCH models. Lagged WUI (0.192, $p = 0.019$) and VIX (0.161, $p = 0.032$) also had significant positive effects, suggesting that heightened global uncertainty and equity-market stress are associated with greater short-run BTC volatility. By contrast, the coefficient on ETH volatility was positive but statistically insignificant (0.105, $p = 0.117$), providing no robust evidence of an immediate cross-cryptocurrency spillover within the specified model. The equation explained 59.3% of the variation in BTC volatility.

These findings indicate that short-run cryptocurrency risk is closely linked to broader uncertainty and market-stress conditions. Accordingly, the apparent independence of cryptocurrencies from traditional markets should be regarded as conditional rather than structural, particularly when portfolio allocation and hedging decisions are made during periods of elevated uncertainty.

5. Conclusions

This study examined the portfolio role of cryptocurrencies through an integrated framework combining mean–variance optimization, copula-based simulation, GARCH-family models, and VAR–Granger analysis. The findings reveal a clear distinction between the standalone and complementary roles of digital assets. The crypto-only portfolio generated a negative Sharpe ratio of −0.0095, despite a mean return of 0.4986, and exhibited the highest risk among the alternative portfolio structures. By comparison, the traditional-only portfolio achieved the strongest risk-adjusted performance, with a Sharpe ratio of 0.1018, a return of 0.5082, and a risk level of 0.0832. The mixed portfolio did not outperform the traditional portfolio in Sharpe-ratio terms; however, it reduced total risk to 0.0583 while maintaining a comparable return of 0.5032. Cryptocurrencies therefore appear more useful as limited portfolio components than as standalone or dominant investments.

The copula results qualify this diversification benefit further. The Student-t copula provided the best fit (AIC = −8422.6), indicating symmetric tail dependence and stronger co-movement during extreme market conditions. Nevertheless, the copula-based optimal portfolio retained small positive allocations to BTC, ETH, and BNB, amounting to 7.31% in total, while the largest weights were assigned to US10Y, gold, and the S&P 500. This allocation pattern suggests that the diversification contribution of cryptocurrencies persists after tail dependence is considered, but remains modest and sensitive to market conditions.

The volatility estimates provide additional evidence against treating cryptocurrencies as stable portfolio anchors. In every GARCH(1,1) specification, the sum of the ARCH and GARCH coefficients exceeded 0.90, reaching 0.9551 for BTC and 0.9542 for ETH, which indicates highly persistent volatility shocks. The EGARCH estimates also indicated asymmetric responses; under the reported specification, the positive sign-asymmetry coefficient implies a stronger volatility response to positive than to negative shocks of comparable magnitude. Meanwhile, the conditional correlation between BTC and the S&P 500 fluctuated approximately between −0.1 and +0.3. These results clarify the apparent difference between the low static beta coefficients and the DCC-GARCH estimates: beta captures average linear exposure over the full sample, whereas DCC-GARCH identifies changes in conditional co-movement over time. Cryptocurrencies may therefore display weak average exposure to equity markets while becoming more closely synchronized with them during stressed periods.

The VAR and Granger results further show that cryptocurrency volatility is more closely associated with uncertainty and market stress than with conventional macroeconomic variables. The World Uncertainty Index and the VIX significantly predicted BTC volatility in the Granger framework ($p = 0.017$ and $p = 0.008$, respectively), whereas inflation and the federal funds rate were not statistically significant. In the VAR model, BTC volatility remained strongly persistent, while lagged WUI and VIX had positive and significant effects. With an R^2 of 0.593, the model indicates that uncertainty-sensitive transmission channels account for a substantial proportion of short-run variation in BTC volatility.

Taken together, the findings neither support cryptocurrencies as efficient standalone assets nor dismiss their relevance to portfolio construction. Their most defensible role lies in limited, non-dominant, and carefully managed allocations within diversified portfolios. Any such allocation should account for tail dependence, persistent volatility, asymmetric shock responses, and changing cross-market correlations. The practical value of cryptocurrencies is therefore conditional rather than universal: they may complement traditional assets, but they should not replace them as the principal anchors of portfolio strategy.

6. Implications

The study provides empirical evidence supporting the selective inclusion of cryptocurrencies in diversified portfolios, thereby offering practical insights for portfolio managers and institutional investors seeking to manage portfolio risk and enhance diversification under uncertainty.

7. Limitations

The conclusions drawn here are grounded in historical daily data and in portfolio constructions that, although methodologically rich, remain ex post in nature. Transaction costs, liquidity frictions, regime-switching behavior, and real-time portfolio rebalancing were not modeled directly. For that reason, the findings should be interpreted as strong empirical evidence on structural portfolio behavior rather than as a fully real-time allocation rule. Even so, the results provide a clear and policy-relevant message: cryptocurrencies may improve portfolio design only when their speculative character is recognized, their weights are kept moderate, and their interactions with uncertainty and volatility dynamics are explicitly modeled rather than assumed away.

8. Future directions

Future research should extend the present analysis by integrating machine-learning techniques, dynamic portfolio optimization methods, and high-frequency data. Further studies could also incorporate ESG-related dimensions and examine cryptocurrency behavior within portfolio-management frameworks using real-time market-sentiment indicators. These extensions may provide a more comprehensive understanding of the time-varying risk, dependence, and diversification characteristics of digital assets.

Author contributions

MÖ and ANK conceived the study and were responsible for the design and development of the data analysis. MÖ, DT and ST were responsible for data collection and analysis. DT and MÖ were responsible for data interpretation. MÖ wrote the first draft of the article.

Disclosure statement

The authors do not have any competing financial, professional, or personal interests from other parties.

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