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WHY DOES THE BLACK-SCHOLES FORMULA NOT WORK FOR HISTORICAL VOLATILITY OR EXPECTED DRIFT?

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Abstract. The Black-Scholes (B&S) formula often fails to match market valuations of European-style options when using historical (expected) volatility. This conclusion is based on a large body of empirical evidence. We consider that the presumption of Geometric Brownian Motion, which is used for the description of the price development of the underlying asset in the B&S model, is too inaccurate, and that is a grave reason why the B&S formula, if the historical or other reasonable volatility is used in the formula, does not systematically provide results comparable with a real market valuation of options. This conclusion is also supported by the fact that if we change the assumption for the way the market price of the underlying asset develops to a process that results in a price distribution that is very close to what we empirically observe, valuation will be almost equal to the market price of the option while using historical volatility. The paper proposes an algorithm for risk-neutral option valuation, implying that there is no long-term advantage to repeatedly opening an option speculative position. The research also includes a discussion about the impact of the absence of drift of the underlying assets in the B&S formula.

Keywords: call/put option, Black-Scholes formula, risk neutrality, implied volatility, options market price, options valuation.

JEL Classification: G1, G10, G12.

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1. Introduction

There is substantial empirical evidence that the valuation of European options provided by the Black-Scholes (B&S) formula is not in accordance with the market valuation of option premiums when we input historical or other realistically expected volatility into the formula. Realistically expected volatility may include, for example, expectations due to volatility clustering and related prediction methods.

According to Haug and Taleb (2007), in the article "Why we have never used the Black-Scholes-Merton Option Pricing Formula?", whose title corresponds to their view of the B&S formula, they talk about options traders modifying the B&S formula by changing one parameter in the B&S formula, namely, the standard deviation, even if the change in the parameter is contrary to the formula. This modification actually modifies the lognormal distribution to the distribution of prices with fat tails and skewness similar to the observed historical distribution. According to Haug and Taleb (2007), options traders have been using heuristics and tricks since 1902 that allow for a wide choice of price distributions. Thus, from all of the above, it is clear that without knowledge of the implied volatility, we are not able to calculate the market price of an option using the B&S formula.

The problem, therefore, remains how a trader who does not know the implied volatility and who cannot estimate its value or calculate it using other methods than from the B&S formula will evaluate the option. Logically, he must evaluate the option without utilising the implied volatility and B&S formula, and then he may possibly calculate the value of the implied volatility. The implied volatility, therefore, looks like a side effect of the B&S formula and seems to be relevant only as a tool for communication within the options market. This is useful for market participants who do not value options using other instruments than the B&S formula, as they can easily calculate the market price of the option for all strikes and expirations using only one parameter, which is the value of the implied volatility. Also, the use of the Geometric Brownian Motion (GBM) assumption for underlying asset price development allowed to find an analytical solution, and so the whole situation became interesting for mathematicians, and subsequently the analytically obtained formulas for valuation were greatly popularised. From a certain point of view, the whole situation can be evaluated in such a way that the authors of the B&S formula probably assumed that traders would use volatility for valuation that corresponds to historical or some real expected volatility. Unfortunately, the deviation of the option price from the actual market price when using historical or realistically expected volatility is too large, in which case the B&S formula is unusable.

The research questions are: "Why does the B&S formula not provide market valuation if historical or other reasonable volatility is used?" and: "How can European options be evaluated without knowing implied volatility?"

In the financial literature, the imperfection of the model is often attributed to the assumption of constant volatility in the Black-Scholes model, and this problem has also been solved through various adjustments to the formula, which we will discuss in the Literature Review chapter.

In our research, we will further assume that by changing the type of price distribution of the underlying asset and by utilising a numerical approach, we can value options so that their valuation is in line with the market price. Details will be described in the Methodology chapter.

The primary goal of this research is to develop a valuation procedure for European-style options that results in a valuation that is closest to the current market price of the option, without the use of implied volatility and its link to the B&S formula. The paper consists of a literature review, methodology, and empirical research. The literature review discusses the historical development of European option pricing models, focusing primarily on the Black-Scholes (B&S) model. Further, the analysis addressed the limitations of the B&S model, particularly its assumptions regarding volatility and the geometric Brownian motion (GBM) process. The discussion also considered research on options trading behaviour and potential alternatives to the GBM assumption. Two hypotheses dedicated to the research questions were raised in the methodology section. To test the hypothesis, the five-step research methodology is formulated. It is based on the created algorithm and uses MATLAB. In the empirical research part, we analysed the underlying price development of the DAX index and its implications for option valuation using various pricing models. The Results and Discussion section presents and discusses the key findings in relation to previous studies. The Conclusions section summarises the main findings and highlights their practical implications. The Limitations section addresses the study's constraints and identifies directions for future research. Finally, the paper concludes with a References section.

2. Literature review

Firstly, it is important to discuss about history that concerns European option pricing. The history of the derivation of the formula, which is now well-known as Black-Scholes, is quite dramatic. Black and Scholes (1973) obtained their solution by taking advantage of previous research on option pricing that gave an idea of what the solution should look like. In brief, Luis Bachelier (1964) derived a solution using Arithmetic Brownian Motion as the process according to which the underlying asset moves. Sprenkle (1962) was the first to assume a lognormal distribution, i.e., GBM as the process of the underlying movements, but Sprenkle attempted to adjust for risk, which was neither necessary nor correct. In addition, he assumed a zero-interest rate, while Boness (1964) added a non-zero interest rate but also adjusted improperly for risk. Samuelson (1965) also attempted incorrectly to adjust for risk. Each of these formulas resembles the correct solution found by Black and Scholes. Black and Scholes, however, originally derived their equation by using the Capital Asset Pricing Model, which provides the equation for the expected return on a risky asset as a function of its risk. At approximately the same time, however, a colleague of Black's, Robert Merton (1973), wrote a paper developing the model with virtually the full mathematical solution. The solution fully respects the risk neutrality concept, and GBM is the expected process behind the price development of the underlying assets.

Many works have been published since with the aim of upgrading the formula, especially in the field of volatility. Heston and Nandi (1997) developed a closed-form option pricing formula for a spot asset whose variance follows a GARCH process. The model allows for correlation between returns of the spot asset and variance and admits multiple lags in the dynamics of the GARCH process. The single-factor (one lag) version of this model contains Heston's (1993) stochastic volatility model as a diffusion limit and thus unifies the discrete GARCH and continuous-time stochastic volatility literature on option pricing. The model provides the first option formula for a random volatility model that is solely a function of observables; all the parameters can be easily estimated from the history of asset prices observed at discrete intervals.

Some research that deals with options trading also touches on options valuation, and research shows that analytical valuation, based on the B&S formula, for example, does not give optimal results when dealing with options, which indicates that some issues are not correct. According to Abbink and Rockenbach (2006), professional traders exhibit lower performance than the student-traders. They explain this with a more intuitive and less analytic approach used by professional traders. Han-Sheng and Sabherwal (2023) draw attention to the problem of deviations in market price and valuation in the field of the effects of option trading behaviour and option prices. Guo and Loeper (2020), Augustin et al. (2023) focused on strategies concerning out-of-the-money (OTM) options that require a special approach to valuation. Carlier (2021) discussed a simple options trading strategy based on technical indicators; Brunhuemer et al. (2021) conducted research regarding option trading strategies based on the relation of implied and realised S&P500 volatilities.

Other significant contributions to the area of basic option research, which is widely known among options specialists, were made by Rubinstein (1967), Black (1989), Black and Scholes (1972), Chance (1994), Conine and Tamarkin (1984), Hull (2003, 2008), Dothan (1990), Duffie (1988), Garven (1986), Gibson (1991), Kutner (1988), Szpiro (2011).

Empirical analysis of S&P500 index options shows the single-factor version of the GARCH model to be a substantial improvement on the Black-Scholes (1973) model. The GARCH model continues to substantially outperform the Black-Scholes model even when the Black-Scholes model is updated every period while the parameters of the GARCH model are held constant. The improvement is due largely to the ability of the GARCH model to describe the correlation of volatility with spot returns. This allows the GARCH model to capture the strike price biases in the Black-Scholes model that give rise to the skew in the implied volatilities on the index options market.

As mentioned before, the valuation of European options provided by the Black-Scholes (B&S) formula is not in accordance with the market valuation of option premiums when we input historical or non-implied volatility into the formula. This experience is illustrated, for example, in Figure 1a, where there is historical volatility and implied volatility for the SP500 stock index. The implied volatility in the case of the SP500 is identical to the VIX index since this index is calculated as implied volatility using options on the SP500. Figure 1b shows the difference between these volatilities, where the implied volatility is, on average, +7% higher. In the vast majority of cases, implied volatility is greater than historical volatility, but there are also short periods when the opposite is true.

Since implied volatility is calculated as an output from the B&S formula, if we know the market price for options, it must be true that if we replace the value of the implied volatility in the B&S formula with another value, we receive a different option value. Also, if we put historical volatility into the B&S formula that is lower than the implied volatility, the calculated value of the option premium will be lower than the option market price.

a)

b)

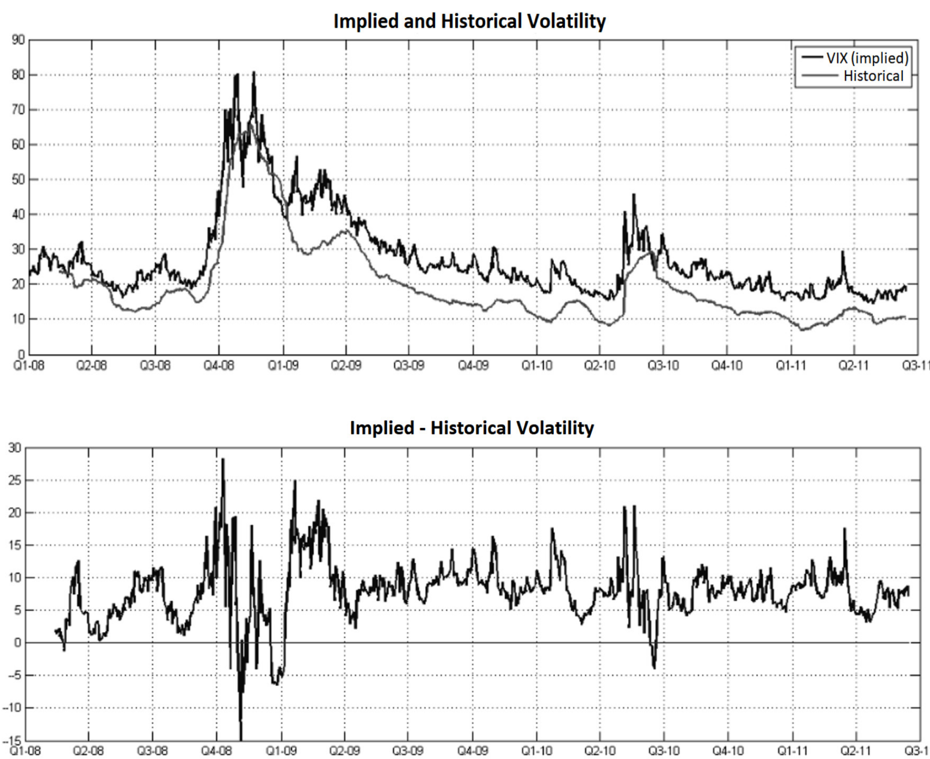


Figure 1. Typical implied volatility (VIX Index) versus historical volatility of SP5000, 2008–2011 (a), the difference between implied volatility (VIX Index) and historical volatility of SP5000 (b) (source: compiled by the authors based on Thomson Reuters, n.d.)

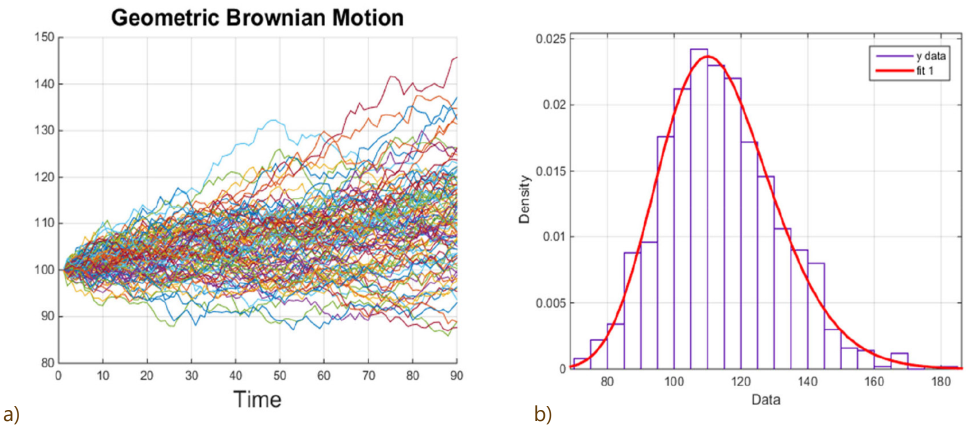


Figure 2. GBM simulation (a); price distribution of GBM (b) (source: compiled by the authors)

However, implied volatility is a specific value, usually without a reasonable financial interpretation, and it is not actually possible to obtain its value from historical volatility or using any forecast. A typical feature of the implied volatility of stock options, as Figure 1 indicates, is that it is higher than the historical volatility of the underlying stocks.

In our opinion, however, it was not possible to use the GBM assumption for the development of the price of the underlying asset in the initial derivation of the B&S formula, since it is already visually clear that this process does not create the price distribution commonly encountered in finance, i.e. distributions with deviations from normality such as general leptokurtic features with fat tails and sharpness, also extreme values, skewing, etc. To reach this conclusion, it is sufficient to compare Figure 2b, simply visually, with Figure 4, where the real historical distribution is displayed.

The illustrative simulation is in Figure 2a. An important feature of the process is the increasing changes in price when the value is higher, which causes the same value of changes in yield that arise from the financial logic. The price distribution is lognormal, and it is displayed in Figure 2b, as we have already mentioned.

In the short term, however, it seems more appropriate to use it for modelling underlying asset development, for example, a pure Random Walk model, or preferably a process that leads to an empirically observed distribution over a period equal to the time when the option expires.

Another problem that may lead to a deviation of the option premium from the market price is the inability to implement the expected drift (Eq. (1)) of the underlying assets into the B&S formula. We discuss this issue in more depth in the chapter Drift of Underlying Asset Issue.

3. Methodology

To answer precisely the two questions, "Why does the B&S formula not provide market valuation if historical or other reasonable volatility is used?" and "How can European options be evaluated without knowing implied volatility?", we formulate two research hypotheses:

- Hypothesis I: We consider that the presumption of GBM, which is used for the description of the price development of the underlying asset in the B&S Model, is too inaccurate and is the true reason why the B&S formula does not provide results comparable with the market valuation when historical or other realistically expected volatility is input into the formula.
- Hypothesis II: We may calculate a value which is very close to the market valuation of option premiums by utilising the historical or realistically expected volatility together with the presumption that the underlying asset price development does not follow GBM, but, rather, a different acceptable model for underlying asset price development (mainly a process leading to the historical price distribution).

The consequent methodology could be summarised into the following steps:

1. The valuation of ODAX options using the B&S formula, using market historical volatility instead of implied volatility. B&S formula, the analytical solution of B&S stochastic differential equations, is represented in the following Eqs. (1)–(4) (Black & Scholes, 1973):

$$C\left(S_t,t\right)=S_tN\left(d_1\right)-Ke^{-r\left(T-t\right)}N\left(d_2\right); \tag{1}$$

$$P\left(S_t,t\right)=Ke^{-r\left(T-t\right)}N\left(-d_2\right)-S_tN\left(-d_1\right); \tag{2}$$

$$d_1=\frac{\ln\left(\frac{S_t}{K}\right)+\left(T-t\right)\left(r+\frac{\sigma^2}{2}\right)}{\sigma\sqrt{T-t}}; \tag{3}$$

$$d_2=d_1-\sigma\sqrt{T-t}, \tag{4}$$

where: C – the price of a call option of European style; P – the price of a put option of European style, S_t – spot price of underlying asset, K – strike price, r – risk-free rate, σ – volatility of the underlying asset measured by standard deviation. Term $T-t$ represents the remaining time to option expiration. N equals the cumulative distribution function of the standard normal distribution with mean = 0 and standard deviation = 1. In this form of the equation, we do not assume dividend payment.

ODAX is traded on EUREX, and its underlying asset is the DAX price index. As we use the B&S formula, the market price of the underlying is assumed to be a GBM.

GBM is described by the stochastic differential equation (Dahlen et al. 2024):

$$S_t=\mu S_tdt+\sigma S_tdW_t, \tag{5}$$

where W – a Wiener process; μ – the measure of the drift of returns; and σ – the measure of volatility of returns.

Finally, we compare this valuation with the market price of ODAX options.

2. The valuation of the option price, if the market price of the underlying asset is assumed to follow a pure symmetric independent random walk (RW), and its comparison with the market price of ODAX.

3. The valuation of the option price, if the market price is assumed to be a process according to a dynamic financial market model analysed by Stádník (2014a, 2014b, 2014c). The distribution of the underlying is a leptokurtic one, and we again compare it with the market price of ODAX.

4. The valuation of the option price, if the market price is assumed to follow the ARIMA (1,1,1) process and its comparison with the market price of ODAX.

5. The valuation of the option price, if the market price distribution is taken from the historical distribution and its comparison is with ODAX.

At this point, it should be noted that for greater significance of the results, a similar procedure is realised for options (SPX) on the US stock index SP500 in the diploma thesis of Idiatullina (2019), under the guidance of Bohumil Stadnik, one of the authors of this article.

For valuation, instead of using the Black-Scholes formula, we have used a numerical approach, as such tasks (steps 2–5) are hardly solvable analytically. The option premium is determined using the market risk-neutrality principle as an equilibrium price, which ensures zero P/L on average in the long term while opening a continuously speculative position and holding the option until its expiration. This approach could be considered as a "fair valuation" for both long and short option position holders. It could be described as:

$$0 = E\left[P/L\right]0 = E\left[MAX\left(S-K,\right)0\right) - premium\right] = E\left[MAX\left(S-K,O\right) - E\left[premium\right], \qquad (6)$$

where E[] – the mean value, S is the spot price of the underlying asset at expiration, K – the strike price, and the premium is the price of the option.

As the option premium is of a constant value $E\left[premium\right] = premium$, and using Equation (3), we get:

$$premium = E\left[MAX\left(S-K,\right)0\right)\right]. \qquad (7)$$

Like the B&S formula, which does not consider a risk premium, and which is derived based on the risk neutral principle, we also avoid any discussion about the risk premium and about who and how much of the risk is does take on the long and short sides of the contract. If we evaluate more precisely, we should distinguish between a long-call and a short-call option position, as an investor in a long-call is always limited by a loss equal to the option premium, but a short-call position is not limited (theoretically) by the size of the loss. Analogically, the case of long and short-put positions.

Algorithm for valuation of option

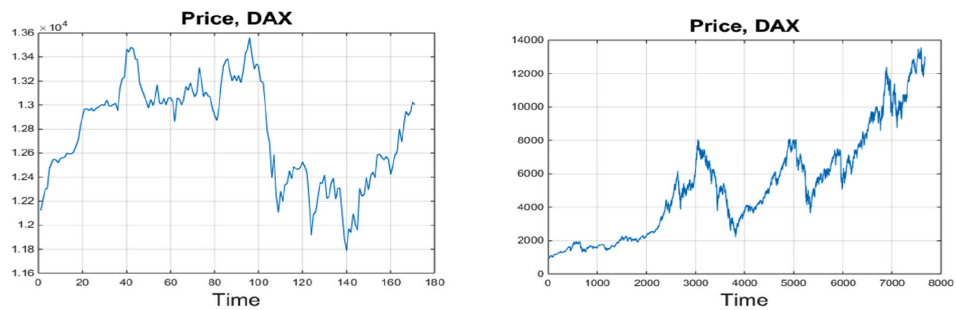
The algorithm works on the above-mentioned principle of valuation using market risk neutrality. The code checks whether we are approaching zero P/L on average as we repeatedly open the option's speculative position. In the code, there is essentially a loop system that checks which option premium value meets risk-neutral criteria.

Simulations of market price development based on the respective models are gradually incorporated into the algorithm (see Appendix). The models are designated as: "B-S" (GBM-GBM), "RW" (random walk), "ARIMA" (ARIMA (1,1,1)), "DYNAMIC" (Dynamic financial market model), "HISTORICAL" (Use of historical distribution). The model labels are in accordance with the designation of these models in the Methodology chapter. PL is calculated as the difference between the strike price and the price simulation end-points according to the above models. A risk-neutral premium is the premium that provides the smallest difference between a PL and an option premium. Then, we repeat this procedure for all strike prices. In the algorithm, the risk-neutral premium is marked as premium_call_neutral (model, strike) and premium_put_neutral (model, strike). This value is then compared with the actual market valuation, and the most appropriate model is the one where the deviation is the smallest. In the algorithm, it is marked as the best_model (model).

The algorithm is graphically illustrated in the Appendix, and it was implemented in the MATLAB programming tool in version 2018a. The algorithm consists of a total of 4 cycles, which are nested within each other. We compare the market price and the valuation of the option one month (30 days) before its expiration. We consider both types of options – put and call options. Financial data are taken from the EUREX website: http://www.eurexchange.com/ in 2020.

4. Results

According to the Methodology, we use the DAX price index as the underlying asset. Its daily price development is shown in Figure 3. The core question is: Does the price development follow GBM? From Figure 3, the answer is not positive.



a) **b)**

Figure 3. Daily Price development of the DAX index (1/2 year back) (a), (34 years back) (b) (source: compiled by the authors)

It seems that a typical feature of GBM, which is – with a higher value of the price, its increments are also higher in order to ensure the same increments of yield – does not work here.

This conclusion is also confirmed by the distribution shape in Figure 4, which is definitely not lognormal-normal, as it should be. We may observe higher values of kurtosis, which is typical for investment instruments.

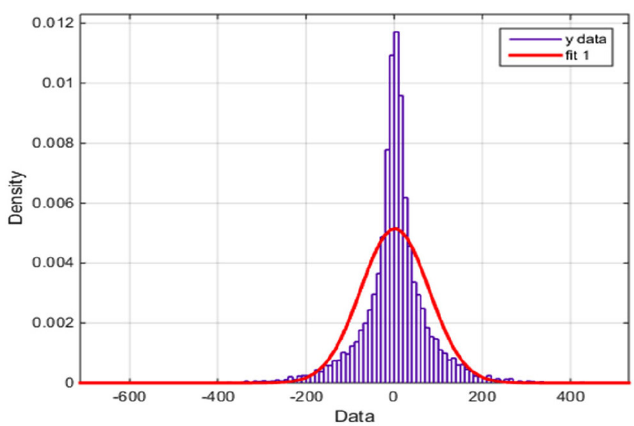


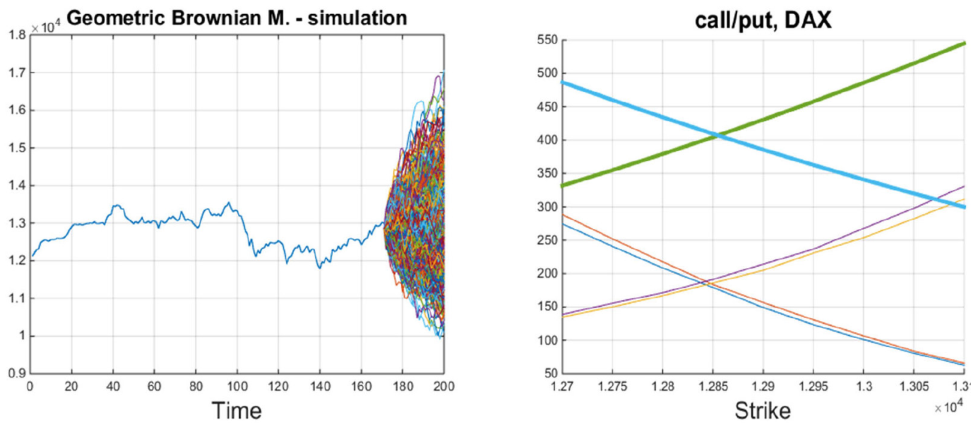
Figure 4. DAX daily price distribution (34 years back), kurtosis 8.3311 (source: compiled by the authors)

In addition, the price distribution in Figure 4 does not support GBM as the process behind the distribution when compared to Figure 2b.

From the above, it is clear that the prerequisite for GBM for the use of the B&S formula is not met.

4.1. The underlying Process is GBM (step 1 according to the methodology)

The valuation of an option using the B&S formula, into which we input historical volatility, with the market price of the option, is shown in Figure 5b.



a) **b)**

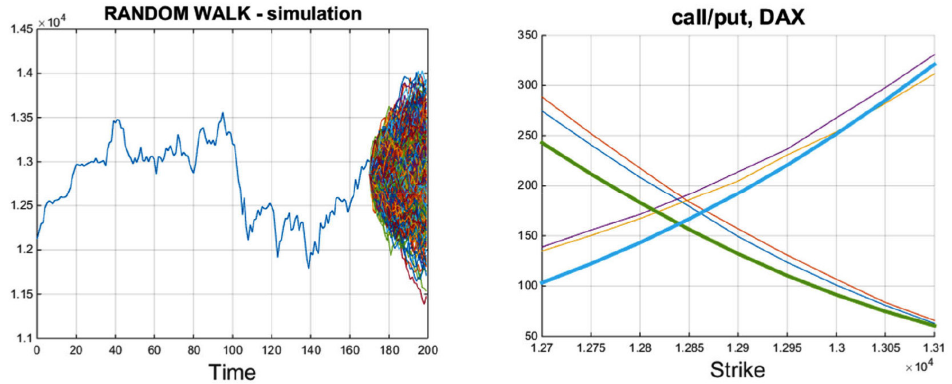
Figure 5. Simulation of GBM on DAX (a), and DAX option prices (b) (source: compiled by the authors)

Figure 5a represents the simulation for the next 30 days of DAX development if we assume that it follows GBM according to Eq. (1). The drift and volatility are calculated according to the historical data of DAX development (Figure 3a, half a year back). Figure 5b illustrates the valuation of call and put options using the Black-Scholes formula (thick lines) if we input historical volatility into the formula and market bid-ask lines (thin lines) with respect to the strike price. In the case of call options, the charts display an increase, while in the case of put options, the charts display a decrease.

We observe that the Black-Scholes formula significantly overvalues the market prices of the option premium in this case.

4.2. The underlying process is a random walk (step 2 according to the methodology)

Figure 6a represents the simulation of future DAX development if we assume that it follows a random walk. The drift and volatility are also taken from the historical time series in Figure 3a (half a year back). Figure 6b illustrates the valuation using the algorithm implemented in the MATLAB environment, and the option premium is found as an equilibrium price which ensures zero P/L on average while we open a repetitively speculative option position and hold the option till its expiration. The structure of the algorithm is described in the chapter Algorithm for Valuation of Option.



a) **b)**

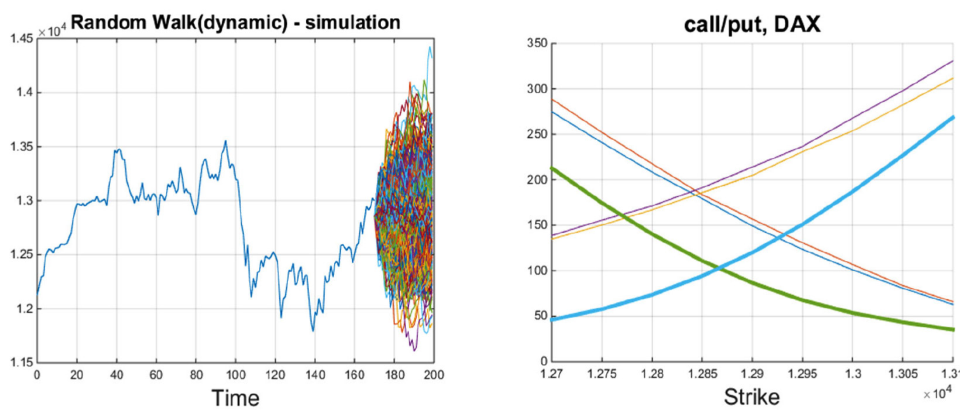
Figure 6. Simulation of a random walk on DAX (a), and DAX option prices (b) (source: compiled by the authors)

The thick lines in Figure 6b represent the valuation according to the referred model, and the thin lines are market bid-ask lines with respect to strike price.

From the results, we may conclude that the valuation is closer to the market valuation if we assume that the underlying follows a random walk than in the case of the GBM presumption.

4.3. The underlying process is based on a dynamic financial market model (step 3 according to the methodology)

In Figure 7a, we assume a dynamic financial market model developed by Stádník (2014a, 2014b, 2014c) from which future DAX development is derived.



a) **b)**

Figure 7. Simulation of a random walk with memory on DAX (a) and DAX option prices (b) (source: compiled by the authors)

The dynamic financial market model is able to model the leptokurtic features of price distribution using feedback (Figure 8), and thus, the process behind the development could be very similar to the empirically measured distribution (Figure 4).

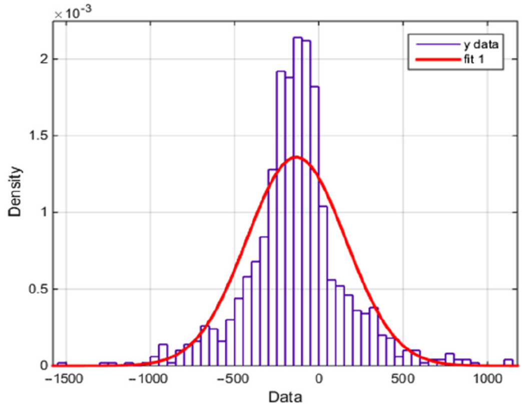
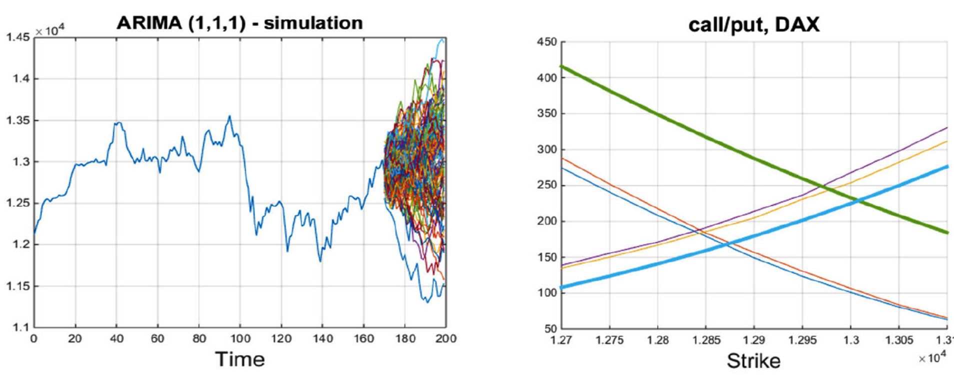


Figure 8. Simulated distribution according to a dynamic financial market model, kurtosis 5.4953 (source: compiled by the authors)

A comparison of valuation using such an underlying development is in Figure 7b. The idea of feedback processes is based on the observations that traders, investors and other market participants do not only watch present or historical data, but based on them, they also place orders to buy or sell, and thus influence future development. So, there is feedback on the financial markets, which also influences future price development and causes future directional dependency. The most usual examples are traders using technical analysis, momentum or mean-reverting trading. In the model, we work with the feedback processes regardless of whether they really help to make a profit or not. Feedback can increase or decrease the frequency of incoming buy or sell orders and, therefore, change the probability of the next price step direction from 50 % (in the case of a pure symmetric random walk) to, for example, 51%. We can, then, conclude that the probability of the next directional step depends on the past.

4.4. Underlying process is ARIMA (1,1,1) (step 4 according to the methodology)

We use ARIMA (1,1,1) to simulate DAX's future development (see Figure 9a).



a) **b)**

Figure 9. Simulation of ARIMA (1,1,1) on DAX (a), and DAX option prices (b) (source: compiled by authors)

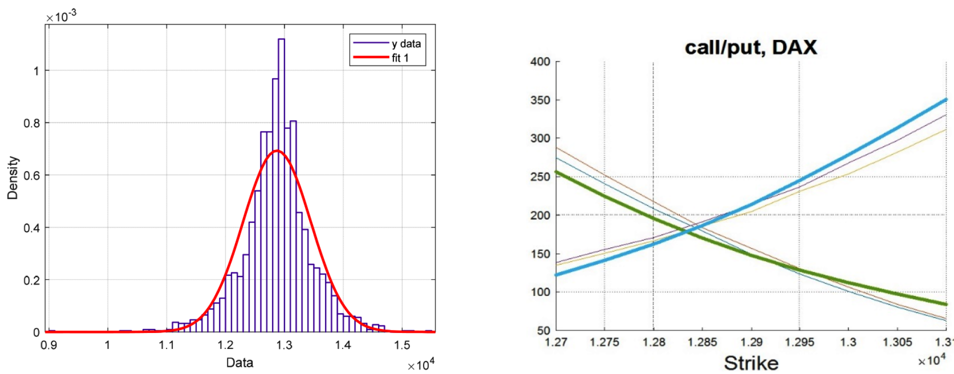
The results of the valuation are in Figure 9b. The thick lines in Figure 6b represent the valuation according to the referred model, and the thin lines are market bid-ask lines with respect to strike price. As we can see from Figure 9, the use of the ARIMA model, due to the high deviations from the market price of the option, is again not a suitable model for simulating the development of the price of the underlying asset.

4.5. Utilising historical distribution (step 5 according to the methodology)

There are basically two ways to construct a 1-month distribution based on a historical price series, as required in the Methodology chapter:

1. To separate the monthly data from the series.
2. To take the daily data and multiply the daily deviations by $\sqrt[2]{30}$ under the presumption of a random walk process and a day count convention that 1 month = 30 days.

The resulting distribution is in Figure 10a, and the resulting valuation is in Figure 10b.



a) **b)**

Figure 10. Monthly distribution. Valuation based on historical distribution (source: compiled by the authors)

The thick lines in Figure 10b represent the valuation according to the referred model and the thin lines are market bid-ask lines with respect to strike price. In this case, we see that the valuation price and the market price are very close.

5. Results and discussion

In this financial engineering research, we explain why the valuation of European-style options using the B&S formula, if we put historical or realistically expected volatility into the formula, does not correspond to the market valuation of these options. A lower value of valuation, when historical volatility is inserted into the B&S formula, comes out when the implied volatility has a higher value than the historical one, which is generally a more common case and which is well illustrated, for example, in Figure 1. In addition to great empirical evidence, this fact was verified by Idiatullina (2019) on the options of the SP500 index (SPX Weekly's – SPXW), traded on CBOE. This test of the option prices on the SP500 index was a subpart of this research.

Given the research results, it is appropriate to formulate and try to answer the following questions. What is the B&S formula actually useful for in today's financial world? Basically, it is not possible to use it for initial option valuation, but it is important as a tool for communication inside the options market. It is useful for market participants who make price options using only the B&S formula, and, in this case, they use the value of implied volatility to calculate the option price for all strikes and expirations. The implied volatility, therefore, looks like a side effect of the B&S formula and seems to be relevant only as a tool for communication within the options market.

Also, for example, Antonakakis et al. (2020) work with implied volatility in the energy market, where they consider it as a separate asset in a portfolio whose value evolves in a certain way. Implied volatility can thus be used as an additional instrument, for example, for hedging.

The second arising question can be formulated as follows: how can a trader who does not know the implied volatility evaluate the option? Logically, he must evaluate the option without utilising the implied volatility and B&S formula. Baele et al. (2019) are engaged in the development of a model that attempts to explain the market pricing of options using an approach completely different from that of Black and Scholes (1972, 1973). The issue of option prices and their valuation during trading, while it is clear that price determination does not follow the approach of Black and Scholes, appears in the Bogousslavsky and Muravyev (2024) research. Algorithmic trading, again with a different approach to option pricing, is the focus of Horambe et al. (2021). Other researchers that work with the price and volatility of an option, also distant from the classical theory of Black and Scholes, include Guo et al. (2020), Kang et al. (2021), Kedžo and Šego (2021), Muravyev and Pearson (2020), Ryu and Yang (2019), Shivaprasad et al. (2022), Fullwood et al. (2021). Marinelli and d'Addona (2017) try to find the value of implied volatility by interpolation, which would result in the corresponding market price using the B&S formula. Goswami et al. (2021) use machine learning to calculate the market price of options using a number of input parameters. Capuozzo et al. (2021) simulate the price development of an option using a non-Gaussian process, using Markov chains.

Another interesting question is why the B&S formula is so popular among academics. One possible answer could be that the Geometric Brownian Motion (GBM) assumption for underlying asset price development allowed to find an analytical solution, and so the whole situation became interesting for mathematicians, and subsequently, the analytically obtained formulas for valuation were greatly popularised.

From a certain point of view, the whole situation can be evaluated in such a way that the authors of the B&S formula probably assumed that traders would use volatility for valuation that corresponds to historical or some realistically expected volatility. Unfortunately, the deviation of the option price from the actual market price when using historical or realistically expected volatility is too large, and therefore, the B&S formula is difficult to apply in financial engineering practice.

Another problem is the absence of a trend in the price development of the underlying assets in the B&S formula. The constant that describes the drift in the price development of the underlying asset is denoted as μ in Equation (1). This constant is completely absent from Equation (2). If, for example, there is a trend assumption based on fundamental analysis or any other strategy, the equation does not take it into account at all. Thus, for example, a call option has the same value in the case of the assumption of a future increase or a decline of the underlying asset. A demonstrative example of the situation is shown in Figure 11, where two underlying assets with a predicted completely different drift have the same price of the option premium at point "0" calculated using the B&S formula. Option expiration is after 900 units of time.

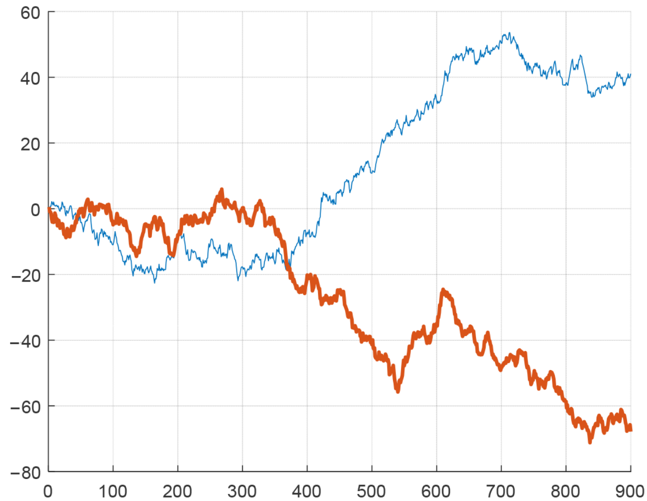


Figure 11. An example of two assets with a predicted different drift (thin and thick charts) but the same price of the option premium at "0" point, calculated using the B&S formula (source: compiled by the authors)

For example, investors who use strategies on undervalued and overvalued stocks against the market price, followed by a "buy/sell/hold" recommendation, and want to use a call option to speculate on the stock price growth, will evaluate the premium cheaper than a counterparty investor who assuming no drift and entering a short call position. Therefore, the deal will not be done.

The explanation is that the derivation of the B&S formula assumes the unpredictability of future price development, which is consistent with the theory of efficient markets, but the theory of efficient markets is a matter of discussion, as it is also in the case of the predictability of future price development in general.

6. Conclusions

The discrepancy between the market price and the result of the B&S formula is usually considered to be caused by wrong volatility or even constant volatility expectations inputted into the formula. But, based on this research, we consider that the presumption of GBM, which is used for a description of the price development of the underlying asset in the B&S Model, is too inaccurate, and, it is probably the main reason why the B&S formula does not provide results comparable with the market valuation when historical realistically expected volatility is used. This conclusion is, of course, supported by the shape of the observed historical distribution of the underlying asset price development, which does not correspond to the lognormal one, as it should be in the case of the GBM assumption. But the fundamental support of the above conclusion is supported by the fact that if the model for the price development is changed from GBM to another process, we may approach the market valuation using historical or realistically expected volatility and without using the B&S formula.

The best process in this sense is the process leading to the historical distribution of prices, and we have recognised that this process provides a valuation which is in good accordance with the market valuation. In our opinion, this conclusion is also conditioned by simple financial logic. The random walk model for underlying asset price development also seems to be sufficient. We have also checked the ARIMA model and the model with a memory (Dynamic Financial Market Model), but with problematic conclusions. Finally, we may state that we confirm both hypotheses formulated in the Methodology.

Obviously, a trader who enters the option market and does not have an implied volatility value to obtain a market valuation by inserting it into the B&S formula must accept some other assumption for the development of the market price of the underlying asset than the GBM and then value the option numerically (not by utilising the B&S formula) using the principle of risk neutrality. For these purposes, we designed and implemented the algorithm in our research, by means of which we also verified the above, i.e., that when changing the assumption about the way the underlying asset develops and using historical or realistically expected volatility, we may approach the market valuation.

7. Limitations and future research

The B&S approach provided us with a good insight into option pricing and the role of individual parameters that affect the option premium. However, if we do not have the value of implied volatility or assume a trend in the development of the underlying asset, the B&S formula in its original form is not applicable for determining the market price of the option. And in practice, it is also not used without major modifications. As a continuation of the research, we recommend a more detailed elaboration of the general framework for the valuation of options, which would take into account, in particular, a different type of distribution of the development of the underlying asset than the lognormal distribution given by GBM, as well as the possibility of including the trend in the assumption of the development of the price of the underlying asset.

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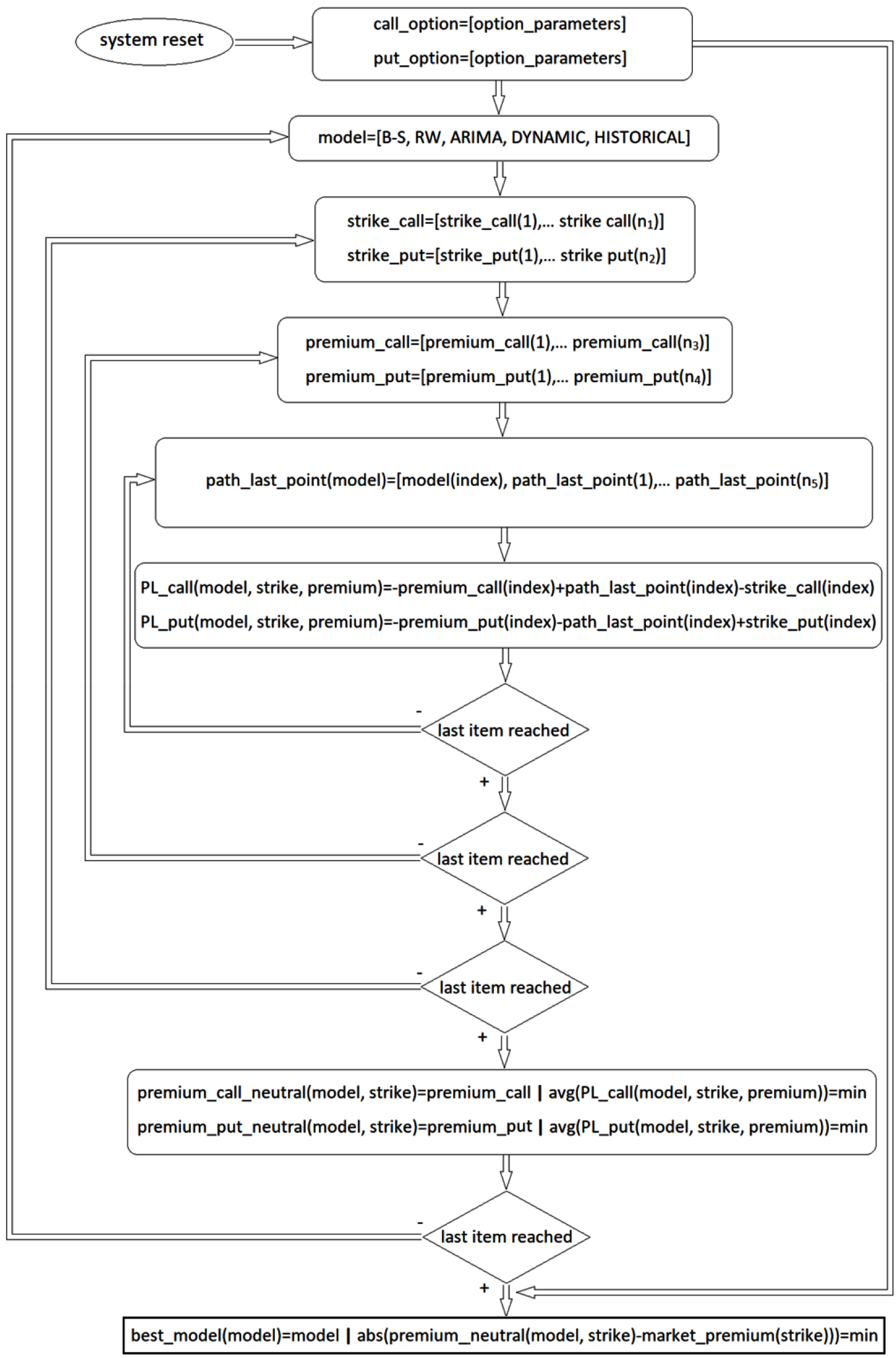
APPENDIX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put_option=[option_parameters]
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strike_put=[strike_put(1),… strike put(n2)]
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PL_call(model, strike, premium)=-premium_call(index)+path_last_point(index)-strike_call(index)
PL_put(model, strike, premium)=-premium_put(index)-path_last_point(index)+strike_put(index)
-
last item reached
+
-
last item reached
+
-
last item reached
+
premium_call_neutral(model, strike)=premium_call | avg(PL_call(model, strike, premium))=min
premium_put_neutral(model, strike)=premium_put | avg(PL_put(model, strike, premium))=min
-
last item reached
+
best_model(model)=model | abs(premium_neutral(model, strike)-market_premium(strike)))=min