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MULTIFRACTAL ANALYSIS OF BITCOIN PRICE DYNAMICS

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Abstract. This research employs Multifractal Detrended Fluctuation Analysis (MFDFA) to investigate multifractal properties in financial variables, including Bitcoin prices and economic indicators. Spanning 2019–2022, the analysis reveals multifractal scaling not only in Bitcoin prices, but also in economic indicators such as inflation rates and energy commodity prices. The non-linear singularity spectra unveil the multifaceted nature of scaling properties. Temporal analysis exposes intriguing trends in multifractality with implications for market efficiency. Furthermore, correlation analysis unveils connections among multifractal properties. For instance, a positive correlation between oil prices and Bitcoin suggests similar market forces. The log-log plot of fluctuation function F_q versus lag size demonstrates a power-law relationship, characteristic of multifractal systems. The empirical data's alignment in log-log space suggests self-similarity in the Bitcoin time series, supporting multifractality. The calculated Hurst exponents values suggest varying degrees of multifractality across the years, with 2021 exhibiting the highest degree and 2022 the lowest. Furthermore, an asymmetry index (0.5767) deviating from 0.5 indicates that the multifractal nature of the Bitcoin market is not symmetric. This research enhances risk assessment and portfolio optimization in finance. It challenges the Efficient Market Hypothesis (EMH), emphasizing the significance of MFDFA in comprehending financial market and economic factor's relationships.

Keywords: multifractal analysis, financial time series, Bitcoin, market efficiency, singularity spectra, correlation matrix.

JEL Classification: C49, C59, C69.

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1. Introduction

In financial markets, volatility is a critical factor that influences investment decisions, risk management strategies and policy formulations. Traditional models like GARCH (Generalized Autoregressive Conditional Heteroskedasticity) have been widely used for volatility modelling (Oprea et al., 2024). However, these models often assume a unifractal process and may not capture the complex, multifractal nature of financial time series. MFDFA is a technique used in time series analysis to explore the multifractal properties of a given dataset (Salat et al., 2017). It is especially useful when dealing with financial or economic data, as it helps in understanding the inherent multifractality and scaling behavior within the data. Multifractal analysis offers valuable insights into financial markets (Jiang et al., 2019). Its economic implications are substantial. In terms of market efficiency or inefficiency, traditional economic theory, particularly through the lens of the Efficient Market Hypothesis (EMH), suggests that financial markets are efficient, meaning asset prices in these markets fully incorporate all available information. This efficiency implies that achieving consistently higher-than-average returns through trading strategies is not feasible (Kantelhardt et al., 2002; Rydin Gorjão et al., 2022). However, the use of multifractal analysis in studying financial data often uncovers multifractal scaling properties, indicating that markets are not perfectly efficient. Instead, they display varying levels of inefficiency across different time scales, possibly due to factors such as information asymmetry, herding behavior or speculative bubbles (Ghosh & Kozarevic, 2019; Ke et al., 2023).

When it comes to risk assessment, multifractal analysis offers a nuanced perspective, especially in understanding the multifractal scaling of price volatility (Fernandes et al., 2022). A high level of multifractality might signal that the market is subject to extreme fluctuations, presenting higher risk. This understanding is significant for portfolio management, as it allows investors to adjust their portfolios according to the multifractal properties of assets, choosing more stable assets for risk-averse strategies or those with higher multifractality for potentially higher returns but increased risk (Wang et al., 2022). Further, multifractal analysis sheds light on herding behavior, particularly evident during market bubbles when multifractality tends to rise, suggesting that investors might be following the crowd rather than making decisions based on market fundamentals (Khan & Suresh, 2022). Additionally, this analysis is incorporated into sentiment analysis models to better understand market sentiment changes, influencing trading decisions and overall market dynamics (Cao & Ling, 2022; Telli & Chen, 2021). The implications of multifractality also extend to market regulations. Regulatory bodies might use insights from multifractal analysis to draft policies aimed at enhancing market stability and protecting investors from systemic risks. Moreover, multifractal analysis enhances market surveillance and the development of early warning systems by identifying unusual multifractal behavior that could indicate market irregularities (Huang & Tang, 2022).

For predictive modeling, incorporating multifractal features into models improves the accuracy of price forecasting by understanding how multifractality evolves over time (Segnon & Trede, 2018). This approach is also beneficial in risk management models, such as in the insurance industry, where assessing the multifractal nature of economic data aids in accurately pricing policies.

Multifractal analysis is a method used to examine the complexity and variability within time series data across different scales. Unlike traditional fractal analysis, which assumes that a system is characterized by a single scaling exponent (monofractality), multifractal analysis considers the possibility that different segments of the data may exhibit varying scaling behaviors. This allows for the detection of heterogeneity in the structure of a time series, where certain parts of the series may show more or less intense fluctuations compared to others. MFDFA quantifies how fluctuations in the data vary across different time scales and statistical moments. By calculating generalized Hurst exponents, MFDFA helps reveal whether a time series exhibits multifractality or the presence of multiple scaling behaviors, which might not be visible using simpler statistical or fractal techniques.

Bitcoin is arguably the most well-known cryptocurrency, and its price has gone through significant volatility and numerous boom-and-bust cycles since its inception in 2009 (Agnese, 2021). In the early days following its creation by an individual or group under the pseudonym Satoshi Nakamoto, Bitcoin initially had no market value (Humayun & Belk, 2018). The first known transaction was for 10,000 BTC used to purchase two pizzas in May 2010. By the end of 2012, Bitcoin's value had started to grow, reaching about $13. The year 2013 marked Bitcoin's first major peak, with its price surging to over $1,000 in November, driven by growing awareness and the broader application of blockchain technology (John et al., 2022). However, this was followed by a period of volatility and regulatory challenges between 2014 and 2016, partly due to the collapse of Mt. Gox, a major Bitcoin exchange (Cheung et al., 2015). Despite these hurdles, Bitcoin gradually recovered, supported by increased adoption and the emergence of more cryptocurrency exchanges. In 2017, Bitcoin's price experienced a meteoric rise, peaking at nearly $20,000 in December, thanks to a surge in public interest, media coverage and market speculation. This bull run was followed by a significant correction in 2018, with the price plummeting to about $3,200 by the year's end, ushering in a bear market (Selmi et al., 2018). The period from 2019 to 2020 saw Bitcoin's price begin to recover and then consolidate in the $5,000 to $10,000 range for much of the time. The end of 2020 witnessed another surge in Bitcoin's price, attributed to institutional investment, economic uncertainty, and a search for safe-haven assets amid the COVID-19 pandemic, leading to new all-time highs over $60,000 in April 2021 (Rufino, 2023; Nguyen, 2022; Sun et al., 2023). This increase was driven by institutional adoption, corporate interest, such as Tesla's investment in Bitcoin and continued retail speculation, although the price experienced significant fluctuations in 2021.

Despite these gains, Bitcoin experienced considerable volatility across the year. After reaching its mid-April peak, the price fell sharply, dipping below $30,000 in July. This downturn was influenced by a range of factors, including regulatory scrutiny across various countries, concerns over the environmental impact of Bitcoin mining and shifts in market sentiment. However, Bitcoin's price rallied once again, achieving a new all-time high above $68,000 in November. While these regulatory announcements sometimes led to price fluctuations, they also signaled a move towards more mainstream acceptance of cryptocurrencies (Mensah & Mwakapesa, 2022; Lyócsa et al., 2020).

Financial markets are complex systems exhibiting scaling behaviors that challenge conventional models of efficiency (Kristjanpoller & Bouri, 2019). In this study, we employ MFDFA to investigate the multifractal properties of a diverse set of financial variables, ranging from Bitcoin price to economic indicators. Our analysis spans multiple years: 2019–2022, offering a comprehensive view of scaling behavior dynamics. These multifractal insights have significant implications for both researchers and practitioners in finance. Understanding the multifractal nature of financial variables enhances risk assessment, portfolio diversification and decision-making (Chowdhury et al., 2023; Aslamet al., 2023a). Furthermore, findings challenge the assumption of market efficiency, emphasizing the importance of multifractal analysis in comprehending the complex dynamics of financial markets. This study offers a multidimensional view of scaling behaviors. It underscores the potential for multifractal analysis to provide deeper insights into market inefficiencies and the intricate relationships among economic factors.

The application of multifractal analysis to Bitcoin is particularly important due to the highly volatile nature of the cryptocurrency market. Traditional financial assets, such as stocks or bonds, often follow relatively stable, predictable patterns. In contrast, Bitcoin and other cryptocurrencies are characterized by frequent and extreme price fluctuations, which can be driven by a range of factors, including technological innovations, regulatory changes, speculative behavior and macroeconomic shifts. These complex dynamics make Bitcoin a prime candidate for multifractal analysis, which may reveal deeper insights into the underlying structure of its price behavior. Multifractal analysis indicates whether the Bitcoin market conforms to the Efficient Markets Hypothesis (EMH) or if it displays signs of inefficiency. The presence of multifractality suggests that Bitcoin's price movements are not entirely random and long-range correlations or non-linearities may exist.

Bitcoin is notorious for experiencing extreme price swings, which makes it difficult to model using traditional statistical methods, such as those assuming Gaussian (normal) distributions. Multifractal analysis allows for the detection of non-Gaussian distributions in Bitcoin's price fluctuations, capturing the presence of fat tails (extreme events) and long-range correlations. Furthermore, Bitcoin's multifractal properties change over time due to various external factors. Multifractal analysis, particularly when applied to different time segments, shows how the complexity of the Bitcoin market evolves (Aysan et al., 2023).

This paper is structured in several sections. In the introduction, the general context is presented. In the next Section, a brief literature review shows the most relevant and recent studies in the field. Section 3 is focused on the input data, while Section 4 presents the proposed methodology. Section 5 is dedicated to the results, in Section 6, several discussions are presented, and the conclusions are drawn in the last section.

2. Literature review

2.1. Multifractal analysis of market dynamics and volatility

A revised approach for estimating nonconcave multifractal spectra through generalized multifractal formalism was proposed in Leonarduzzi et al. (2019). An analysis utilizing the Multifractal Detrended Cross-Correlation Analysis (MFDCCA) method revealed the presence of multifractality in the cross-correlations between Bitcoin and traditional financial indicators, including major currency rates, the Dow Jones Industrial Average (DJIA), gold prices and crude oil markets (Gajardo et al., 2018; Băra et al., 2024). The authors observed that multifractality is evident across all examined cross-correlations, accompanied by asymmetry in the cross-correlation exponents under various trends of the WTI (oil price), gold and DJIA. Notably, Bitcoin exhibited a more pronounced multifractal spectrum.

A comprehensive multifractal characterization of the Bitcoin market was proposed (Zhang et al., 2019). Through a detailed examination, this research identified and analyzes the multifractal properties present in the Bitcoin market, tracing the origins of these characteristics to the fat-tailed distributions and long-range correlations in price and volume data. Further, a high-frequency analysis of Bitcoin's multifractal properties was proposed (Stavroyiannis et al., 2019). This research delved into the multifractal properties of Bitcoin prices using high-frequency data and employing techniques such as the wavelet transform modulus maxima and MFDFA. It discovered a significant degree of multifractality across various time scales. Another research investigated the non-linear interactions between Bitcoin price changes and trading volume through MFDCCA, uncovering significant multifractality (Alaoui et al., 2019). By integrating theories of fractal markets and nonlinear dynamics, the research offered deeper insights into the underlying mechanisms of the Bitcoin market, potentially enhancing the effectiveness of technical trading strategies. Moreover, analyzing 1-minute returns of Bitcoin prices, the researchers identified fat-tailed distributions and significant deviations from Gaussian expectations, with slow convergence towards Gaussianity over larger sampling periods (Takaishi, 2018). Despite the absence of daily volatility asymmetry in GARCH model analyses, the MFDFA revealed multifractality in the Bitcoin time series, influenced by temporal correlations and fat-tailed distributions.

Using MFDCCA and multivariate GARCH models, another research explored the multifractality and volatility spillovers among Bitcoin, gold and crude oil (Jin et al., 2019). The results indicated significant cross-correlations and volatility spillovers, with Bitcoin being particularly sensitive to price fluctuations in gold and crude oil markets. Furthermore, analyzing the non-linear interdependent structure and risk transmission between cryptocurrencies and China's financial market post-trading shutdown, the researchers found long memory and asymmetric multifractal characteristics in cross-correlations (Cao & Xie, 2021). Another research explored multifractal cross-correlation relationships between the S&P500, HSI stock indices and Bitcoin, revealing high non-linearity and dominant anti-persistency during market downturns (Bielinskyi et al., 2023). It provided insights into asset trading and risk diversification strategies, highlighting periods of high and low cross-correlations for effective market participation.

Investigating Bitcoin returns with MFDFA was performed (Shrestha, 2021). It revealed the multifractal nature and inefficiency of Bitcoin returns caused by autocorrelated and extreme returns. Another research developed an empirical hypothesis test to identify multifractal scaling in financial time series, particularly in the presence of heavy tails (Jiang et al., 2020). Applying this test to Bitcoin and the Nasdaq Composite Index, it found evidence of multifractal scaling, contributing to the debate on the characteristics of financial time series and offering a novel approach to analyzing the complex dynamics of technology-driven assets. Exploring daily Bitcoin returns and their multifractal properties, another research identified an inverted asymmetry in Bitcoin's volatility that changes over time (Takaishi, 2021). By employing a rolling window method, it examined the relationship between market efficiency measures and volatility asymmetry, contributing to the understanding of time-dependent characteristics in the Bitcoin market and suggesting increased market efficiency in recent periods. This paper delved into the evolving relationship between market efficiency, liquidity and the multifractal nature of Bitcoin (Takaishi & Adachi, 2020). The results indicated that prior to 2013, Bitcoin exhibited low liquidity and anti-persistent behavior as evidenced by a Hurst exponent below 0.5.

Another research combined MFDFA and chaos theory to predict market behavior, demonstrating the superiority of chaotic artificial neural network models over traditional methods (Yin & Wang, 2022). This approach underscored the predictive potential of nonlinear dynamics in financial markets, particularly for emerging digital assets like Bitcoin. (Segnon & Bekiros, 2020) investigated Bitcoin market stylized facts, proposing models to capture the dynamics of mean and variance processes.

2.2. Comparative analysis between cryptocurrencies and other assets

Another research proposed an asymmetric multifractal spectrum distribution method, employing the detrending moving average cross-correlation analysis algorithm for dynamic multifractal spectrum calculation under varying market conditions (Shen & Chen, 2023). Additionally, MFDFA was applied to assess the multifractal essence within high-frequency datasets of the Shanghai and Shenzhen stock exchanges (Fu et al., 2023). Multifractal behavior in Bitcoin price series analysis was further examined (da Silva Filho et al., 2018). It demonstrated multifractal characteristics within its historical price series, attributing these properties to long-range correlations and fat-tailed distributions. By applying MFDFA to Bitcoin and gold, this research compared the multifractal nature of their return and volatility series (Telli & Chen, 2020). It found distinct multifractal properties and higher persistence in Bitcoin, suggesting significant differences in the behavior of these assets.

The energy prices' asymmetric multifractality during COVID-19 was assessed (Khan et al., 2022), revealing stronger downside multifractality for oil products and upside for coal and natural gas. The increased asymmetry during the pandemic indicated higher market inefficiency. Improved transparency and government regulation were recommended to enhance market stability and reduce price volatility. Another research examined asymmetric multifractality and efficiency in global and Chinese renewable energy markets using A-MFDFA (Khurshid et al., 2024). Results showed increased inefficiency during COVID-19, particularly in CELS and SPGlobal prices, with herding behavior observed. SPTSX exhibited stronger upward multifractality, indicating higher efficiency, while SPIC-SH showed high multifractality and low market uncertainty. Also, (Meng & Khan, 2024) evaluated the asymmetric multifractality of cryptocurrencies during COVID-19 using a novel asymmetric MFDFA. The results showed that Bitcoin and Ripple have higher multifractality in downward movements, indicating lower efficiency and weaker safe-haven properties. Ethereum showed greater multifractality in upward trends, with improved efficiency during the pandemic. The findings suggested that cryptocurrencies were inefficient during the pandemic. Furthermore, asymmetric MFDFA was used to examine maritime shipping freight rates during upward and downward movements (Li et al., 2023). Results showed that the Baltic Dry Index and Baltic Clean Tanker Index had higher multifractality in downward trends, indicating inefficiency and unpredictability during the COVID-19 pandemic. In contrast, the Baltic Dirty Tanker Index exhibited greater multifractality in upward movements.

2.3. Impact of external factors on market dynamics

The research examined the roughness or anti-persistence characteristic of Bitcoin's volatility using MFDFA to calculate the generalized Hurst exponent of log-volatility increments (Takaishi, 2020). The results, indicating a generalized Hurst exponent below 1/2, confirm the multifractal property of log-volatility, which varies non-constantly, suggesting multifractality partially stems from distributional properties. Employing MFDCCA, another research quantified significant cross-correlations between Google Trends changes and Bitcoin market dynamics, including returns and volume changes (Zhang et al., 2018).

Utilizing MFDCCA, the researchers uncovered long-range cross-correlations between Bitcoin prices and the U.S. Economic Policy Uncertainty Index (USEPU), demonstrating significant power-law properties and multifractality (Ma et al., 2022). Investigating the relationship between Twitter-based Economic Uncertainty (TEU) and major cryptocurrencies, (Aslam et al., 2023b) applied MFDCCA to reveal significant multifractal behavior and persistent cross-correlations. Another study examined the COVID-19 pandemic's effects on fiat currencies and cryptocurrencies using asymmetric multifractal cross-correlation analysis (Fernandes et al., 2023). The results found increased complexity in cross-correlations during the pandemic, suggesting that diversifying investments across major fiat currencies and leading cryptocurrencies mitigate portfolio risk and enhance investment outcomes.

2.4. Advanced mathematical and statistical modeling techniques

Multifractal analysis of price volatility and appreciation in Bitcoin was provided (Vaz et al., 2021). Applying MFDFA and a Multifractal Regime Detecting Method (MRDM) to high-frequency Bitcoin data, this research uncovered the asset's multifractal nature, which is characterized by persistence in smaller fluctuations and anti-persistence in larger ones. In the post-COVID-19 era, another research analyzed the detrended correlations between price returns, trading volume and the number of trades in Bitcoin and Ether markets, revealing a clear multifractal structure (Wątorek et al., 2022). Another comprehensive study explored the multifractal characteristics of the cross-correlation between Bitcoin's price and trading volume, revealing asymmetric multifractal features and the impact of liquidity and volatility on this relationship (Wenhao & Guangxi, 2022). By applying the MFDCCA, the research delved into the sources of multifractality, offering perspectives on the nonlinear dependency within cryptocurrency markets and suggesting implications for investment and financial market analysis. Through asymmetric MFDFA, the researchers investigated structural breaks and asymmetric multifractality in Bitcoin and Ethereum markets (Mensi et al., 2019). Their findings revealed a nuanced multifractality gap across market trends. Exploring the multifractal behavior of daily price and volume changes across fifty cryptocurrencies, the researchers found complexity predominantly in price changes, contrasting with volume changes (Stosic et al., 2019). The absence of correlations in price changes and anti-persistent long-term correlations in volume changes were notable, with multifractality arising from broad probability distributions. Additionally, an investigation into the chaotic nature of the global cryptocurrency market was proposed (Gunay & Kaşkaloğlu, 2019), analyzing major cryptocurrencies, confirms chaos through various tests including the largest Lyapunov exponent and MFDFA.

3. Exploratory Data Analysis (EDA)

3.1. Data

In this section, a brief EDA is performed. The dataset comprises 29,794 observations and 19 variables, conforming to the initial description. A brief description for numerical variables is provided in Table 1.

Table 1. Input data description

Variable	Description
Hour	Ranges from 0 to 23, capturing the 24-hour time cycle.
Open, High, Low, BTC_USD (Close price)	The metrics related to Bitcoin prices exhibit substantial volatility, with means and medians significantly diverging from each other. For instance, the mean of the opening price is approximately 24,074.24 USD while the median is 11,604.20 USD.
Volume, qav, num_trades, taker_base_vol, taker_quote_vol	These trading-related variables also display a wide range, with Volume ranging from 0 to approximately 47,256 and qav (Quote asset volume) ranging from 0 to 1.51 billion.
Weekday	Encoded as integers from 0 to 6, following the convention where 0 represents Sunday.
El_price_DAM, Gas_price_DAM, Inflation_EU, Oil_price, Price_EUETS	These macroeconomic indicators vary substantially, with El_price_DAM (Electricity price in the Day-Ahead Market) ranging from 0.1 to 3,286, and Gas_price_DAM (Gas price in the Day-Ahead Market) ranging from 0.07 to 1,182.73.
El_quantity, Gas_quantity_DAM	These quantities also span a broad scope, with El_quantity ranging from approximately 1,244.1 to 5,052.9 and Gas_quantity_DAM ranging from 47 to about 19.88 million.

The divergence between the mean and median for several variables suggests the presence of outliers or a skewed distribution, necessitating further distributional analyses and potential transformations. Additionally, the wide range of macroeconomic and financial variables indicates the possible existence of complex temporal and cross-sectional relationships.

3.2. Statistical analysis

The statistical summary of the dataset has been represented in Figure 1, providing a comprehensive overview of the central tendency, dispersion and shape of the dataset's distribution.

The mean, median (50th percentile), standard deviation and other statistical measures for each variable provide insights into their distributions. For instance, the mean trading volume (Volume) is approximately 2487.66, with a standard deviation of 2351.42, indicating a relatively high variability.

3.3. Outlier detection and correlation

The boxplots for numerical variables have been generated (Figure 2), to serve as a tool for outlier detection. These boxplots provide insights into the quartile distribution of each variable and highlight potential outliers that lie beyond the interquartile range.

Variables such as Open, High and Low exhibit a few outliers on the higher end, as indicated by the points beyond the upper whisker. Volume shows a considerable number of outliers, particularly on the higher end. El_price_DAM and Gas_price_DAM also display outliers,

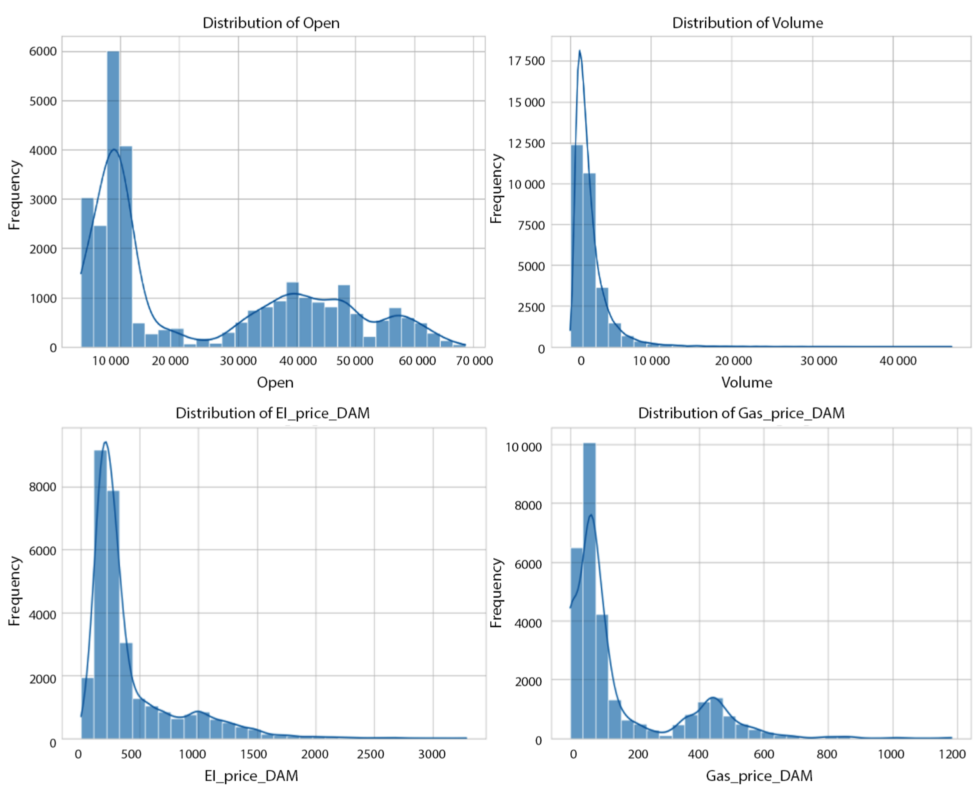

Figure 1. Distributions of the Bitcoin price and volume, electricity price and gas price

primarily on the higher end of the distribution. Inflation_EU appears to have a relatively clean distribution with no visible outliers. Next, we compute the Pearson correlation coefficients between the variables to identify potential linear relationships (Figure 3).

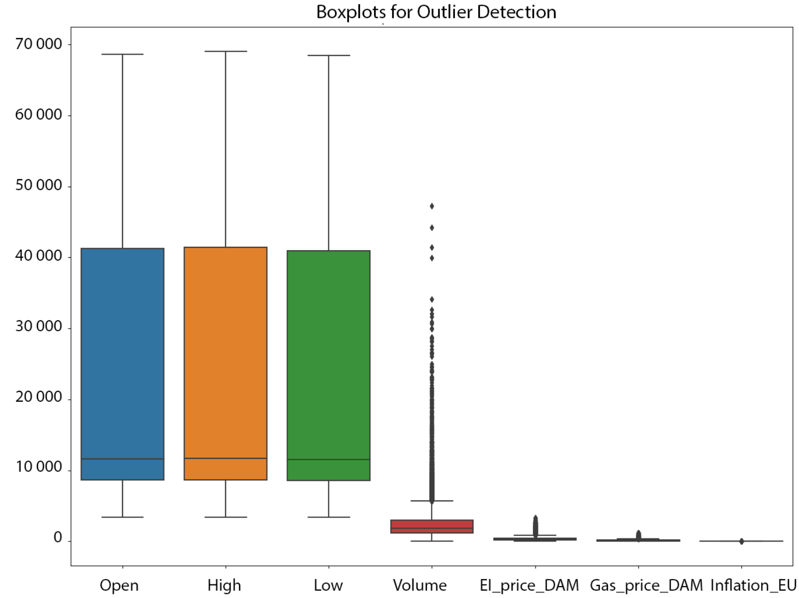

Figure 2. Boxplots of Bitcoin prices and volume, electricity price, gas price and inflation

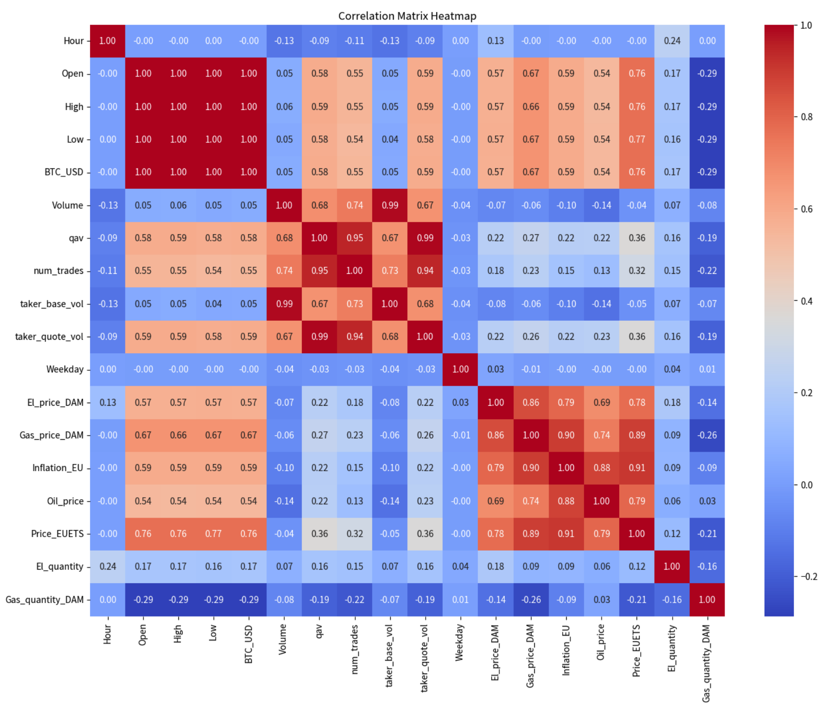

Figure 3. Correlation among input data variables

Variables such as Open, High, Low and Volume exhibit strong correlations with each other, as evidenced by the correlation coefficients close to 1 as in Figure 3. El_price_DAM and Gas_price_DAM also show a moderate positive correlation, suggesting that electricity and gas prices in the Day-Ahead Market tend to move in the same direction. The variable Inflation_EU has relatively low correlations with most other variables, indicating that it may not be a significant predictor.

4. Methodology

4.1. MFDFA methodology

MFDFA is particularly useful in the context of financial markets, such as the Bitcoin market, to assess whether they exhibit multifractality, which can imply market inefficiency. This technique helps in detecting multifractality, which often indicates inefficiency and scale-invariant structures within the data. To better understand MFDFA, we break down each step of the methodology in more detail:

Step 1. Detrending the time series. Before applying MFDFA, detrending the time series data is required. The purpose of detrending is to remove any linear trends or dependencies in the data so that we can focus on the underlying fluctuations. This step helps to isolate the short-term variations from broader, long-term movements in the data. For example, if the overall trend shows Bitcoin prices rising over time, detrending would remove this upward movement, leaving only the daily fluctuations for further analysis.

Step 2. Partitioning the time series. The detrended time series are divided into overlapping windows or segments of different sizes. These segments allow us to investigate fluctuations at various scales. Thus, the purpose of this is to study how fluctuations behave across different scales, revealing patterns at both short and long intervals. For example, one might break down the data into 10-day, 30-day and 60-day windows to observe both short-term and long-term behavior.

Step 3. Calculating the local fluctuation function. For each segment, the local fluctuation function $F(n,s)$ is calculated, which characterizes the amplitude of fluctuations within that segment. The equation for calculating $F(n,s)$ is:

$$F(n,s)=\sqrt{\frac{1}{s}\sum_{i=n}^{n+s-1}\left[x(i)-x(i)\right]^2}, \tag{1}$$

where: n is the starting point of the segment; s is the size of the segment; $x(i)$ is the detrended time series data point at index; $(x(i))$ is the mean of the data points within the segment.

Therefore, after partitioning, the local fluctuation function $F(n,s)$ is calculated, measuring the intensity of fluctuations. This fluctuation function helps quantify how much the data points deviate from their local average within each window. Higher values of $F(n,s)$ indicate larger deviations and, therefore, stronger fluctuations within that segment.

Step 4. Calculating the *qth* order moment. To characterize the multifractal nature, we calculate the *qth* order moment $F_q(s)$ for each segment size s. The equation for calculating $F_q(s)$ is:

$$F_q(s)=\left[\frac{1}{N}\sum_{n=1}^{N}\left[F(n,s)\right]^2\right]^{1/q}, \tag{2}$$

where: N is the number of segments for the given scale s; q is a parameter that can take different values. Different values of q provide information about different aspects of multifractality.

This step allows us to analyze various aspects of multifractality, as smaller values of q emphasize smaller fluctuations, while larger values of q highlight extreme deviations. For instance, if one is interested in typical fluctuations, one would choose a small value of q, whereas, for extreme price spikes or drops (like Bitcoin crashes), a larger value of q would be more useful.

Step 5. Scaling analysis. We perform a scaling analysis by plotting $\log[Fq(s)]$ against $\log(s)$ for different values of q. The slope of this plot can provide insights into the multifractal properties of the time series. This plot reveals whether the relationship follows a power-law, which would suggest multifractality in the data. If the plot shows a power-law relationship, it implies that fluctuations vary significantly across different time scales, characteristic of a multifractal time series.

Step 6. Singularity spectrum. The multifractal analysis leads to the estimation of the singularity spectrum $f(\alpha)$, which characterizes the distribution of Hölder exponents (α), where the Hölder exponent α relates to the scaling behavior of the fluctuations. The equation for singularity spectrum $f(\alpha)$ is:

$$f\left(\alpha\right)=\lim_{s\to 0}\frac{1}{q-1}\log_2\left[F_q\left(s\right)\right], \tag{3}$$

where: α is the Hölder exponent and it represents the local scaling behavior; $f(\alpha)$ is the singularity spectrum, which describes how the fluctuations are distributed across different scales.

Thus, the final key step is analyzing the singularity spectrum, which represents the distribution of Hölder exponents that describe local scaling behavior. The broader the spectrum, the more diverse the scaling behavior across different scales. A wide spectrum indicates a complex, multifractal time series with various types of fluctuations occurring at different scales. Positive values of α indicate persistent behavior (where the data continues in the same direction), while negative values indicate anti-persistence (where a reversal in direction is more likely).

The proposed methodology consists of the following stages:

- Data preparation – splitting our dataset into yearly subsets and performing MFDFA for each year;
- Detrending – apply detrending techniques to remove any linear trends from time series data. This step is essential to isolate the intrinsic fluctuations;
- Partitioning – the time series are divided into non-overlapping windows of various sizes. This helps in analyzing fluctuations at different scales;
- Calculation of fluctuation function – for each partitioned window, we calculate the fluctuation function. The fluctuation function measures the root mean square fluctuations of the data within each window;
- Scaling analysis – the fluctuation function is plotted against the window size on a log-log scale. The scaling behavior is monitored and if the relationship follows a power-law, it indicates multifractality in the data;
- Spectrum of multifractality – the spectrum of multifractality is calculated, which characterizes the distribution of scaling exponents over different scales. A wide spectrum indicates a high degree of multifractality;
- Interpretation – the results are analyzed for each year separately and for variations in the scaling behavior and multifractality across different years. We analyze the context if

there are significant changes in multifractality from year to year, if certain years exhibit stronger or weaker multifractal properties and if there are any correlations between financial events and changes in multifractality.

In the proposed methodology, the data is first prepared by splitting it into yearly subsets, allowing MFDFA to be applied separately to each year. This division is important for comparing the multifractality across different years, especially when assessing changing market dynamics. Once the data is split, detrending is applied to remove linear trends and focus on the true fluctuations. Partitioning the time series into segments of various sizes helps capture the behavior of fluctuations across short and long intervals. For each segment, the fluctuation function is calculated, measuring how much the data fluctuates within each window.

During scaling analysis, the fluctuation function is plotted against the segment size on a log-log scale to check if a power-law relationship exists. The presence of such a relationship suggests multifractality. Finally, the multifractality spectrum is computed, showing how the scaling behavior changes across different time scales. A wide multifractality spectrum indicates that the data behaves differently over short and long-time scales, signifying strong multifractal properties.

In the interpretation phase, the results are analyzed year by year to identify variations in multifractality over time. For instance, if a particular year shows a significant change in multifractality, this could be linked to specific market events, such as a major financial crisis or a Bitcoin crash. The spectrum also reveals whether some years exhibit stronger or weaker multifractality, which can offer insight into market behavior during those periods.

4.2. Multifractal spectrum

The multifractal spectrum, often represented as a plot of log(f(α)) against log(α), provides insights into how different scales contribute to the fluctuations in the data. The following aspects are interpreted in the multifractal spectrum:

(a) Scaling behavior – the x-axis of the multifractal spectrum represents the logarithm of the Hölder exponent (α). The Hölder exponent characterizes the local scaling behavior of the data. A positive α indicates a positively correlated or persistent behavior, while a negative α indicates an anti-persistent or negatively correlated behavior;

(b) Multifractal spectrum shape – the shape of the multifractal spectrum indicates the multifractal nature of the data: if the spectrum is concave (curving downward), it suggests that the data has strong multifractal properties. This means that the data has fluctuations at multiple scales and these fluctuations are not evenly distributed; if the spectrum is convex (curving upward), it suggests that the data has weaker multifractal properties, and the fluctuations are more evenly distributed across scales; whereas if the spectrum is linear, it implies a monofractal behavior, where fluctuations occur at a single scale;

(c) Width of the spectrum – the width of the multifractal spectrum indicates the range of scaling exponents present in the data. A wider spectrum implies a greater diversity of scaling behaviors across different scales;

(d) Divergence points – further we look for points on the spectrum where log(f(α)) diverges from linearity. These points correspond to the scales at which the data exhibits particularly strong or weak multifractal properties. Divergence points indicate critical scales in the data;

(e) Comparative analysis – to compare multifractal spectra across different years, we observe how the spectra for each year differ in terms of shape, width and divergence points.

Lastly, the multifractal spectrum provides critical insights into how different scales contribute to the fluctuations. The x-axis represents the Hölder exponent *alpha*, with positive values indicating persistence and negative values suggesting anti-persistence. The shape of the spectrum reveals the strength of multifractality: a concave spectrum suggests strong multifractal properties, while a convex spectrum indicates weaker multifractality. A linear spectrum points to monofractal behavior, where fluctuations occur at only one scale. The width of the spectrum indicates the range of scaling exponents, with a wider spectrum implying a more diverse range of behaviors. Divergence points on the spectrum highlight critical scales where multifractal properties are particularly strong or weak. By comparing multifractal spectra across different years, we can identify changes in market dynamics and potential correlations with economic events.

Considering the proposed methodology, the following research questions are addressed in our research:

RQ1: Are there significant changes in the multifractal properties of the Bitcoin market across these years?

RQ2: Do certain years exhibit stronger multifractality than others?

RQ3: Are there any correlations between market events or economic factors and changes in the multifractal spectrum?

The answers to these questions are given in the results section.

5. Results

5.1. Empirical results

The aim of this research is to examine the multifractal nature of the Bitcoin market using MFDFA, which, if established, could imply market inefficiency. MFDFA of BTC time series: This shows how the fluctuation function F_q scales with the lag size for q = 2 as in Figure 4.

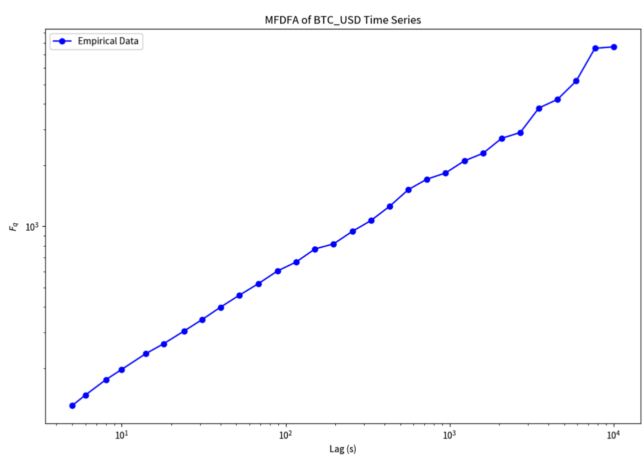

Figure 4. Plotting the MFDFA results

The log-log plot suggests a power-law relationship, which is a hallmark of fractal or multifractal systems. The fluctuation function appears to scale in a manner consistent with multifractal systems. The blue points represent the empirical data, and they seem to align well in a log-log space, suggesting self-similarity in the time series. The plot of multifractal spectrum is shown in Figure 5. This plots *h(q)* against F_q, providing insights into the multifractal nature of the time series.

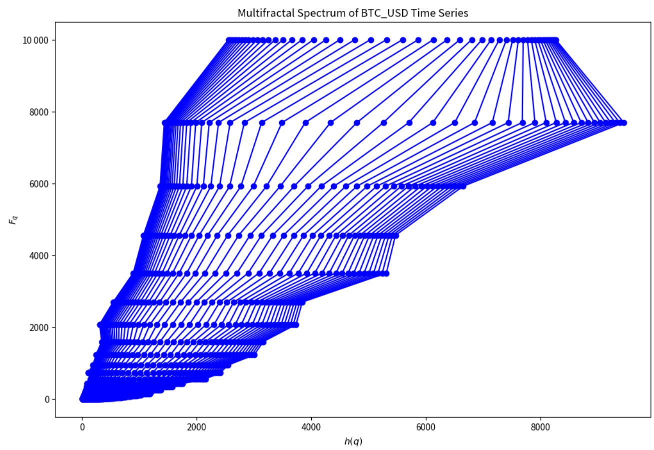

Figure 5. Plotting the multifractal spectrum

In terms of the multifractal spectrum, the plot of *h(q)* against F_q reveals a complex structure, indicative of multifractality in the BTC time series (as in Figure 6). The analysis is conducted over a range of q values, providing a comprehensive view of the multifractal nature of the time series. The results indicate that there are certain implications for market efficiency. The presence of multifractality could challenge the EMH, suggesting that the Bitcoin market may not be fully efficient.

Multifractal analysis challenges the Efficient Markets Hypothesis by revealing complex patterns and correlations that should not exist in an efficient market. The presence of multifractality implies that the market is not fully absorbing and reflecting information instantly, allowing for periods of predictability, exploitable opportunities and increased risk. By using

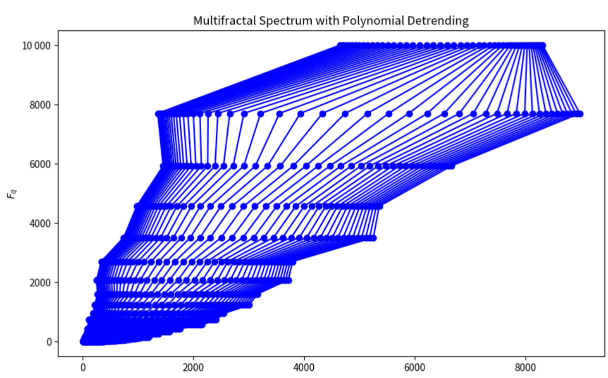

Figure 6. Plotting spectrum with polynomial detrending

multifractal analysis, we can better understand the degree of inefficiency in the market, track its temporal evolution and provide insights into potential risks and opportunities that are overlooked by traditional methods. This makes multifractal analysis an important tool for understanding and measuring the true efficiency of financial markets, particularly volatile ones like Bitcoin.

A polynomial of degree 2 is fitted to the time series and the residuals are analyzed using MFDFA. The multifractal spectrum appears to be influenced by the detrending method. This is evident from the shape and spread of the spectrum. The choice of detrending method can have significant implications for the interpretation of multifractality in financial time series. It serves as a crucial parameter that can either validate or challenge existing theories.

Exploring the full range of generalized Hurst exponents provides a more comprehensive view of the multifractal nature of the time series. We explore the full range of generalized Hurst exponents to provide a more comprehensive view of the multifractal nature of the BTC time series. This involves running MFDFA with an extended range of *q* values (Figure 7).

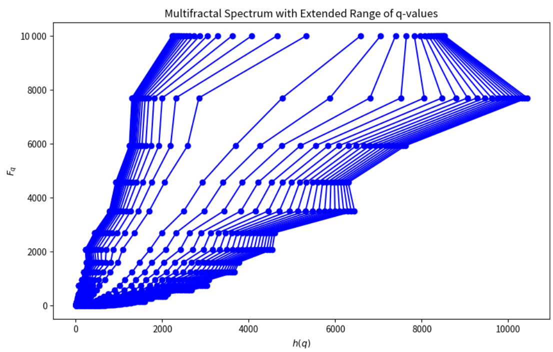

Figure 7. Multifractal spectrum with extended range of q values

An extended range of q values from –10 to 10 is employed to capture a broader spectrum of multifractality. The multifractal spectrum appears to be influenced by the choice of *q* values. The extended range provides a more comprehensive view of the multifractal nature of the Bitcoin market. The choice of *q* values serves as a significant parameter in MFDFA and has implications for the interpretation of multifractality in financial time series.

5.2. Temporal evolution of multifractality

Examining how the multifractal characteristics evolve over time offers insights into market maturity and the impact of external events on market efficiency. Thus, we partition the time series into 3 distinct temporal segments and analyze the multifractal characteristics within each segment. This allows us to observe how multifractality evolves over time, potentially revealing insights into market maturity or the impact of significant events. The analysis focusing on the temporal evolution of multifractality in the BTC time series is performed by certain intervals and the results are presented below in a series of plots, each corresponding to a distinct temporal segment of the data as in Figure 8.

The time series is partitioned into three segments: January to March, April to June and July to September. Each segment is analyzed separately to observe the evolution of multifractality. The plot reveals that the multifractal nature of the BTC time series varies across different time periods.

This is indicative of changing market conditions, external events or market maturity. However, despite the variations, a consistent pattern of multifractality is observed across all segments, reinforcing the notion that the Bitcoin market exhibits complex, multifractal behavior.

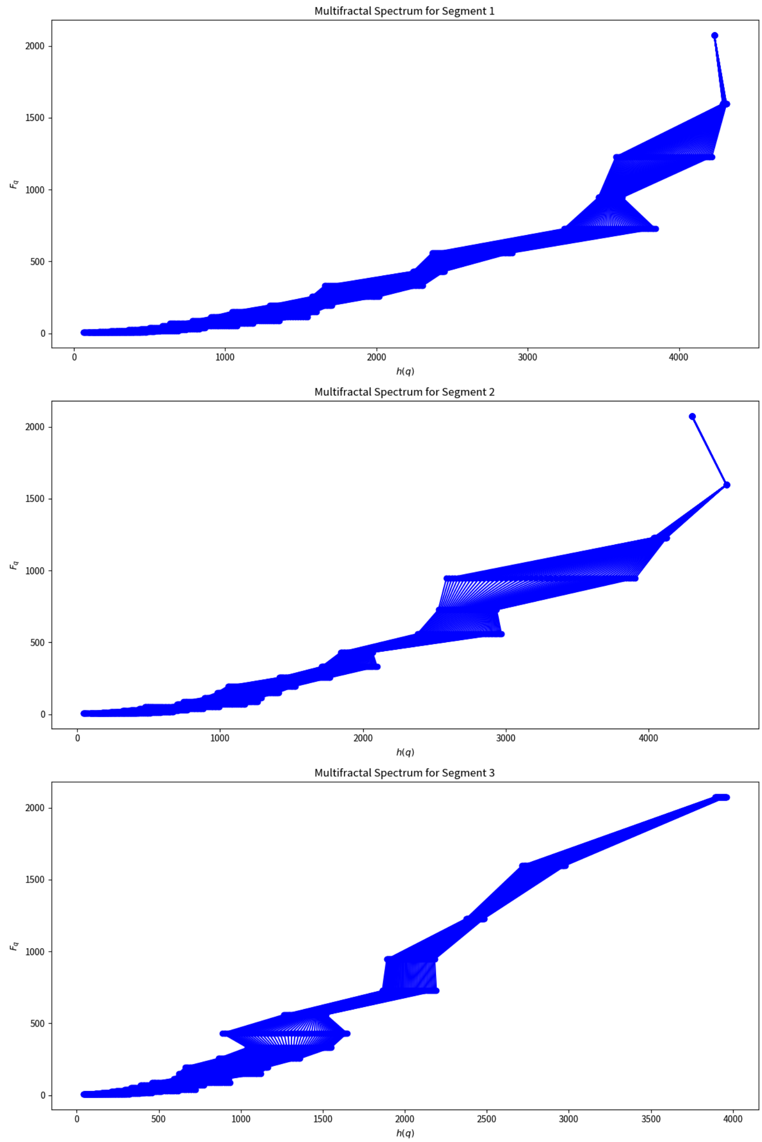

Figure 8. Multispectral spectrum for the three segments

To further elucidate the temporal evolution of multifractality in the Bitcoin market, we extend our analysis to compare the multifractal characteristics across different years (as in Figure 9). This potentially reveals long-term trends or cyclic patterns in market behavior.

The multifractal spectra for different years reveal distinct patterns, suggesting that the multifractal nature of the Bitcoin market is subject to temporal variations. The analysis potentially indicates long-term trends in market behavior, including cyclic patterns or responses to macroeconomic factors. The evolution of multifractality across years is indicative of the market's maturation process. The yearly variations in multifractality challenge or support existing financial theories, offering avenues for further academic inquiry. This longitudinal analysis adds a temporal dimension to our understanding of the multifractal characteristics of the Bitcoin market.

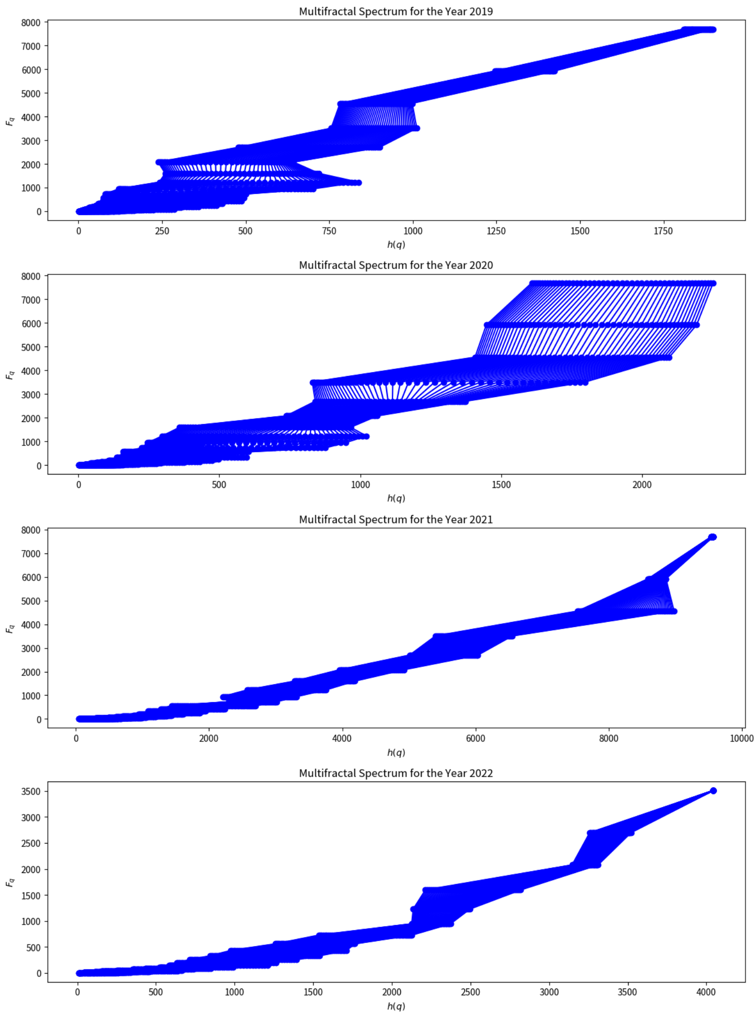

Figure 9. Multifractal spectrum at the year level

5.3. Multifractal spectrum asymmetry

Further, we analyze the asymmetry of the multifractal spectrum. This involves calculating the asymmetry index, which offers insights into the underlying mechanisms that give rise to multifractality. The multifractal spectrum is a fundamental tool utilized for discerning the scaling behavior of a system, thus unveiling underlying intricacies. The asymmetry of the multifractal spectrum is an indicative facet which potentially reflects non-uniform distribution of the scaling exponents across the spectrum. Asymmetry in the multifractal spectrum is a manifestation of multifractality and is often associated with certain inherent or imposed non-linearities within the system under observation. The calculation of an asymmetry index serves as a quantitative metric to discern this asymmetry, encapsulating nuanced information about the distribution of singularities or scaling exponents. A non-zero value of the asymmetry index signifies a deviation from symmetry in the distribution of multifractal measures.

This asymmetry stems from a plethora of sources. For instance, the underlying mechanisms generating multifractality could be inherently biased or external influences might engender a skewed distribution of scaling exponents. In either case, the asymmetry index serves as an instrumental gauge in not only quantifying this bias but also potentially unearthing the mechanisms driving multifractality. Furthermore, the precise value of the asymmetry index provides a benchmark for comparative analysis. Systems exhibiting similar asymmetry indices share comparable underlying dynamics or external influences, thus facilitating a deeper understanding of the genesis of multifractality across disparate systems. Conversely, a disparate asymmetry index usually signals a divergent underlying mechanism or influence.

Displaying the asymmetry index, we obtain 0.5767, suggesting a moderate level of asymmetry in the multifractal spectrum. An asymmetry index deviating from 0.5 indicates that the multifractal nature of the Bitcoin market is not symmetric. This could be indicative of underlying mechanisms that give rise to multifractality, such as long-range correlations or heterogeneous trading behavior.

5.4. Yearly comparison of multifractality

In this section, we aim to investigate the temporal evolution of multifractality in the Bitcoin market. Specifically, we segment the time series data into yearly intervals and perform MFDFA for each year. This allows us to observe how the multifractal nature of the market has changed over time, thereby offering insights into the market's evolving complexity and efficiency. We perform the segmentation of data into yearly intervals for the multifractal analysis for: 2019, 2020, 2021 and 2022. With this dataset, we proceed to perform MFDFA for each of these years, thereby enabling a temporal comparison of multifractality on several variables on our dataset. The comparative evaluation of multifractal characteristics across the years 2019, 2020, 2021, and 2022 are presented in Figure 10 and Figure 11. The plots employ a log-log scale to depict the fluctuation function $f(\alpha)$ against the lag parameter for each year.

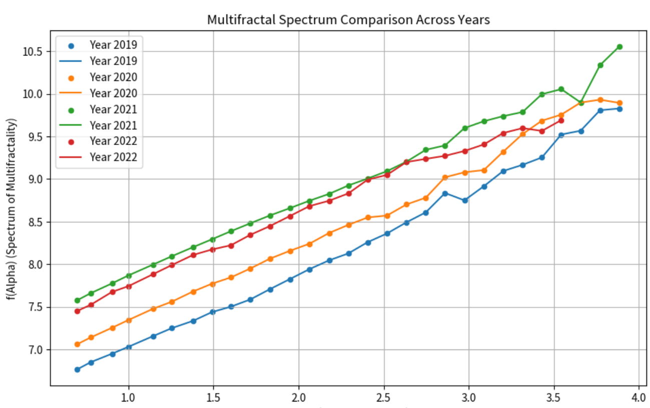

Figure 10. Multifractal spectrum comparison across years for qav

In the context of multifractal analysis, the fluctuation function $f(\alpha)$ provides insights into the scaling behavior of the time series. A steeper slope in the log-log plot indicates a higher degree of multifractality, while a flatter slope suggests a lower degree. From the plot, it is evident that the scaling behavior varies across years, indicating temporal variations in the multifractal characteristics of the Bitcoin market.

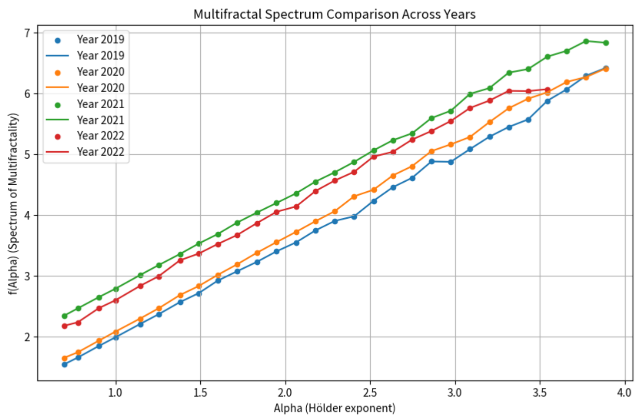

Figure 11. Multifractal spectrum comparison across years for Bitcoin

A linear multifractal spectrum suggests that the data exhibits monofractal behavior, which means that fluctuations in the data occur at a single dominant scale. In our case, one notices linear spectra for the years 2019, 2020, 2021 and 2022, indicating that the Bitcoin market during those years primarily exhibited a consistent scaling behavior across different scales.

5.5. Correlation analysis of multifractal properties of Bitcoin and economic indicators

The input dataset provides us with an opportunity to conduct a comprehensive multifractal analysis that extends beyond Bitcoin to include multiple economic indicators. The steps we perform are presented in the following subsections.

5.5.1. Multifractal scaling

We present the Hurst exponents values for different q values. These exponents are crucial for characterizing the multifractal nature of the time series. The array of Hurst exponents spans a range of values, indicating the presence of multifractality in the Bitcoin price data. Hurst exponents for different q values are:

[1.831 1.8261.820 1.814 1.808 1.801 1.794 1.786 1.779 1.771 1.763 1.755 1.746 1.738 1.729 1.72063143 1.711 1.702 1.692 1.681 1.670 1.658 1.645 1.631 1.617 1.603 1.589 1.577 1.566 1.555 1.546 1.537 1.528 1.520 1.512 1.505 1.498 1.491 1.484 1.477 1.471 1.465 1.459 1.453 1.447 1.441 1.436 1.431 1.426 1.421]

5.5.2. Singularity spectrum

Investigating the strength of singularities in the time series reveals the extent to which extreme events contribute to its multifractal nature. The spectrum provides a graphical representation of the multifractal nature of the Bitcoin price data (as in Figure 12). The x-axis represents the singularity strength (α), and the y-axis represents the singularity spectrum $f(\alpha)$). The spread of the spectrum along the α-axis indicates the degree of multifractality in the time series.

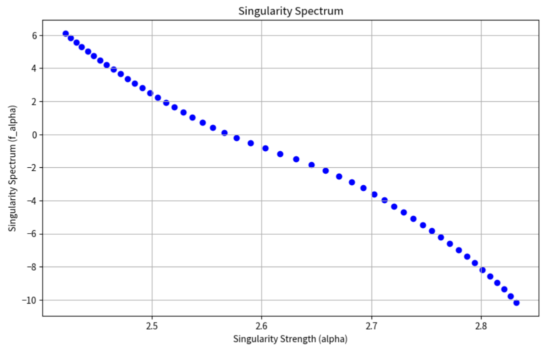

Figure 12. Singularity spectrum

5.5.3. Temporal analysis

We then proceed with the calculation of the generalized Hurst exponents for each year as in Figure 13. The calculated values are as follows: 2019: H = 1.123; 2020: H = 0.822; 2021: H = 1.421; 2022: H = 0.628.

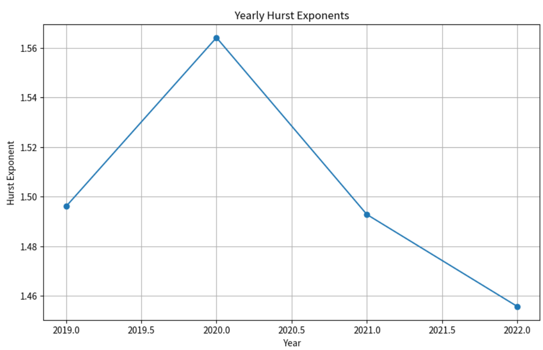

Figure 13. Yearly Hurst exponents

The generalized Hurst exponent (H) serves as a quantitative measure of the multifractality of the time series. Specifically, H > 0.5 indicates a persistent (trending) behavior, H < 0.5 indicates an anti-persistent (mean-reverting) behavior, and H = 0.5 corresponds to a Brownian motion (random walk). The calculated H values suggest varying degrees of multifractality across the years, with 2021 exhibiting the highest degree and 2022 the lowest.

5.5.4. Correlation analysis

Correlations between the multifractal properties of different economic indicators are examined, such as inflation rates, energy commodity prices and the Bitcoin market. First, we stored Hurst exponents for each of indicators: BTC_USD, Inflation_EU, Oil_price, El_price_DAM and Gas_price_DAM: 1.510, 1.521, 1.511, 0.963, 1.389. Then, a correlation matrix based on the yearly Hurst exponents for each selected economic indicator is computed as in Figure 14. The matrix reveals intriguing correlations between the multifractal properties of different economic indicators. A positive correlation of approximately 0.64 between Oil_price and Bitcoin suggests that the multifractal properties of oil prices and Bitcoin prices are somewhat aligned. The positive correlation suggests that oil prices and Bitcoin prices may be influenced by similar market forces. For instance, geopolitical tensions that drive up oil prices may also lead to increased demand for Bitcoin as a "safe haven" asset. Governments and financial institutions may need to consider the interconnectedness of these markets when implementing policies that affect oil prices, as they could inadvertently impact the Bitcoin market. For investors, this correlation could serve as a signal for portfolio diversification.

As tensions rise and oil prices increase, demand for Bitcoin may also grow, resulting in price movements that reflect similar market forces. This relationship suggests that geopolitical factors, such as conflicts, sanctions or trade disputes, could simultaneously impact both the energy and cryptocurrency markets. Investors, particularly those holding assets in either market, should consider monitoring geopolitical developments closely, as disruptions in one market could signal potential shifts in the other.

In terms of portfolio diversification, the correlation between oil prices and Bitcoin offers valuable guidance. Because these assets are influenced by common external factors, holding both in a portfolio might not provide sufficient diversification benefits during periods of geopolitical turmoil. However, an investor could use this correlation as a hedge. For instance, rising oil prices, signaling increasing risk, could prompt a shift to Bitcoin as a protective measure. Understanding this relationship allows investors to anticipate how these two markets might move in tandem and adjust their portfolios accordingly.

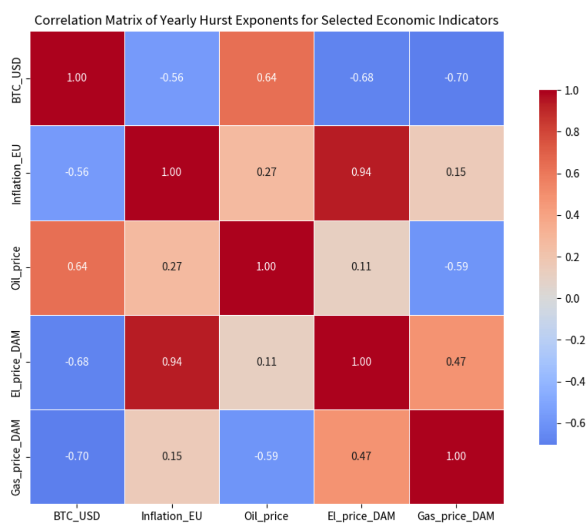

Figure 14. Correlation chart of Hurst exponents for each selected economic indicator

A very high positive correlation of approximately 0.94 between Inflation_EU and El_price_DAM indicates a strong alignment in the multifractal properties of inflation rates in the EU and electricity prices in the Day-Ahead Market. The high correlation indicates that inflation rates and electricity prices are closely tied, possibly due to the pass-through of increased production costs to consumers. Central banks and energy regulators use this information to anticipate the ripple effects of monetary policy on energy markets.

A negative correlation of approximately –0.70 between Gas_price_DAM and BTC_USD suggests that the multifractal properties of gas prices in the Day-Ahead Market and Bitcoin prices are inversely related. The negative correlation suggests that these markets are influenced by different or even opposing market forces. Policymakers need to consider how interventions in one market may have consequences in another.

One possible explanation for this inverse relationship is the distinct demand patterns and external influences on both assets. Gas prices are often driven by supply constraints, seasonal variations and regulatory factors, while Bitcoin prices are influenced by broader macroeconomic trends, including investor sentiment, institutional adoption and regulatory developments. When gas prices rise, particularly due to supply-side disruptions, the broader economy can slow, which might reduce speculative interest in riskier assets like Bitcoin. Conversely, during periods of declining gas prices, the broader economic environment may stabilize or improve, potentially encouraging more investment in cryptocurrencies as part of a diversified portfolio.

This inverse relationship provides investors with a potential tool for diversification. Holding both Bitcoin and gas-related assets in a portfolio could help balance risk, as the two markets appear to move in opposite directions under certain conditions. In this way, the inclusion of gas prices as a hedge against Bitcoin's volatility could be beneficial, particularly in managing exposure to market-specific risks.

6. Discussions

The temporal analysis of Bitcoin's multifractality is insightful, revealing how the multifractal nature of the market evolves over time. The Bitcoin market is highly influenced by several key events that significantly impact its multifractal properties.

One major factor that could explain the variations in multifractality over time is regulatory intervention. The cryptocurrency market has faced increasing regulatory scrutiny in many regions, with governments and regulatory bodies introducing policies aimed at curbing illegal activities like money laundering and tax evasion (Oprea et al., 2024). Regulatory actions, such as the banning of Bitcoin trading in certain countries or the imposition of stricter Know Your Customer (KYC) requirements, may lead to sharp changes in market behavior. These interventions often create periods of increased uncertainty and volatility, as traders react to new laws, compliance requirements or market restrictions. This could, in turn, influence the multifractal properties of Bitcoin, as periods of heightened regulation may disrupt the typical trading patterns and introduce non-linearities or irregular scaling behaviors into the time series.

Another important factor is the advent of new technologies within the cryptocurrency space. Over the past decade, technological innovations such as the introduction of Bitcoin futures, the growth of decentralized finance (DeFi) and the rise of layer-2 scaling solutions like the Lightning Network have fundamentally altered the structure of the market. These technological advancements often lead to changes in market liquidity, the behavior of market participants and even the speed at which transactions occur. Thus, such technological shifts can introduce new sources of multifractality.

The increasing institutional adoption of Bitcoin is another critical factor that likely influences the temporal evolution of its multifractal characteristics. In recent years, institutional players, including hedge funds, asset managers and publicly traded companies, have entered the Bitcoin market in increasing numbers. This shift in market composition has had a significant impact on Bitcoin's price dynamics and multifractal properties. Institutional investors often bring a different set of trading behaviors compared to retail investors, including larger trade volumes, more sophisticated risk management strategies and longer investment horizons.

External macroeconomic factors, such as global financial crises or geopolitical events, also contribute to changes in Bitcoin's multifractality over time. Bitcoin has increasingly been seen as a hedge against traditional financial systems or fiat currency instability, particularly in regions facing inflationary pressures or political uncertainty. These factors could alter the scaling behavior and introduce additional layers of complexity into the time series.

The results of our research align with and support the existing literature on multifractality and market dynamics in several key areas. Consistent with the findings of Leonarduzzi et al. (2019) and Gajardo et al. (2018), our research confirms the presence of multifractality in both cryptocurrency and traditional markets. The evidence of multifractality in Bitcoin, Ethereum, and other assets, including maritime freight rates, mirrors the multifractal nature previously identified in cross-correlations between Bitcoin and major financial assets like gold, crude oil and the DJIA. Similar to studies like Zhang et al. (2019) and Takaishi (2018), our research emphasizes the role of fat-tailed distributions and long-range correlations in creating these multifractal characteristics, particularly during periods of heightened volatility such as the COVID-19 pandemic. Furthermore, our results reinforce findings by Stavroyiannis et al. (2019) and Jin et al. (2019) on the multifractality of Bitcoin prices and volatility spillovers. The study's observation of heightened inefficiency in downward price movements for Bitcoin and Ripple, as well as inefficiency in maritime shipping indexes, supports the literature's view of increased market inefficiency during periods of stress. As suggested by Bielinskyi et al. (2023), the asymmetric multifractal behavior during downturns provides insight into how volatility impacts market dynamics. The results also validate the growing body of research on the asymmetric nature of multifractality in financial markets, as described by Alaoui et al. (2019) and Mensi et al. (2019).

One specific contribution lies in our research's focus on the COVID-19 pandemic as a specific period of heightened market inefficiency. While earlier studies (e.g., Fernandes et al., 2023) have examined the impact of the pandemic on cryptocurrencies and fiat currencies, our research deepens our understanding by specifically analyzing multifractality in both upward and downward movements during the pandemic. The temporal aspect of this analysis provides a clearer picture of how market efficiency fluctuated during a major global crisis, contributing to the ongoing debate on the resilience of financial and non-financial markets in times of uncertainty. Moreover, while existing research (Telli & Chen, 2020; Takaishi & Adachi, 2020) has compared the multifractal properties of Bitcoin and traditional assets like gold, our research expands the scope by examining multifractality in both cryptocurrencies and other assets.

Overall, the results contribute to a broader understanding of market dynamics by revealing how asymmetric multifractality affects both financial and non-financial markets, particularly during periods of heightened uncertainty like the COVID-19 pandemic. The findings emphasize the importance of asset-specific analysis when evaluating market efficiency and offer practical implications for investors, regulators and policymakers.

7. Conclusions

Multifractal analysis of financial time series data has profound economic implications. It challenges the assumptions of market efficiency, informs risk assessment and management, influences market regulations and guides predictive modelling. Understanding the multifractal nature of financial markets, including cryptocurrencies like Bitcoin, contributes to a more comprehensive and nuanced economic analysis and decision-making process. The analysis has several key implications: (1) It challenges traditional economic theories that consider markets in isolation, advocating for a more integrated approach to economic modeling and policy-making; (2) The findings have direct applications in risk management and portfolio optimization, offering investors new tools for asset diversification and hedging strategies.

Employing MFDFA, we probe the multifractal attributes of a diverse array of financial variables, encompassing Bitcoin prices and key economic indicators. Spanning multiple years, our analysis reveals the dynamic nature of scaling behaviors. Remarkably, our multifractal inquiry uncovers captivating revelations. Not only does Bitcoin price data exhibit multifractal scaling, but so do economic indicators like inflation rates and energy commodity prices.

Temporal analysis further enriches our exploration, unveiling complex multifractal trends over years. Beyond characterizing multifractality, we unravel correlations between multifractal properties across variables. Our findings hold implications for both financial researchers and practitioners. Understanding the multifractal essence of financial variables empowers risk assessment, informs portfolio diversification strategies and enhances decision-making. Our research extends a compelling challenge to the notion of market efficiency, underscoring the indispensable role of multifractal analysis.

While the research suggests applications in risk management and portfolio optimization, the practical implementation of multifractal analysis in these areas may be challenging. The complexity of multifractal models and the computational resources required could limit their use in real-time decision-making by financial practitioners. Future research could focus on the impact of extreme market events (e.g., financial crises, pandemics) on multifractal properties.

Our multifractal analysis of the Bitcoin market provides valuable insights, particularly regarding market inefficiencies in the context of the Efficient Markets Hypothesis (EMH). For investors, understanding the multifractal nature of the Bitcoin market suggests a need for more dynamic portfolio management. Since the presence of multifractality implies the possibility of extreme price fluctuations, traditional portfolio strategies may not fully account for the risks posed by Bitcoin's inherent volatility. Investors can mitigate this by diversifying their portfolios, allocating assets to safer, less volatile instruments, especially when multifractal analysis indicates periods of high market uncertainty. Risk management strategies should also be adaptive, incorporating multifractal characteristics into tools like Value-at-Risk or Expected Shortfall. They capture the risks of large, unexpected price movements, allowing investors to adjust their risk exposure accordingly.

Moreover, short-term traders might find that Bitcoin's multifractal behavior offers opportunities for exploiting price swings through strategies based on fractal-based technical indicators. These indicators could help identify optimal entry and exit points during periods of high-frequency price fluctuations. For long-term investors, recognizing the variation in Bitcoin's multifractal tendencies over time suggests adopting more conservative strategies during highly volatile periods and being more aggressive when market conditions are stable. Investors might also use hedging strategies to protect against the unpredictability of Bitcoin's behavior. For instance, the negative correlation between Bitcoin prices and gas prices, as our analysis indicates, could be leveraged to develop dynamic hedging strategies that offset losses during periods of high volatility in Bitcoin.

In practical terms, investors need to integrate multifractal characteristics into their risk models and do not rely solely on traditional methods like Gaussian distributions or random walk assumptions when forecasting price movements.

One of the primary limitations of this research is the computational complexity associated with multifractal analysis. The use of MFDFA in real-time scenarios can be challenging. The method requires significant computational resources, especially when analyzing large datasets or working with high-frequency trading data. This complexity limits the feasibility of applying multifractal analysis in fast-paced financial environments where decisions need to be made in real time. Additionally, the implementation of MFDFA in practical settings may require specialized knowledge and expertise, which can restrict its accessibility to a broader range of financial practitioners. To address these challenges, future work could focus on using parallel computing techniques to reduce processing time.

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Author contributions

SVO obtained the data and wrote the literature review, discussions and conclusions. CB analyzed the data and contributed to the methodology. BGT coordinated the authors and assisted with data collection. AB contributed to the methodology and reviewed and corrected the manuscript.

Disclosure statement

The authors report there are no competing interests to declare.

Data availability statement

The data will be made available upon request.

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