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INVESTIGATING THE HIDDEN GEOMETRY SYSTEM AT THE ENTRANCE OF THE HISTORICAL HOUSES OF KASHAN FROM THE QAJAR PERIOD TO THE BEGINNING OF THE PAHLAVI PERIOD

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Abstract. Geometry is one of the most important aspects hidden in the design of historical buildings in a way, it can be said that architecture without the presence of geometry is meaningless and lacks functional validity. The ratio and proportion that can be seen in the entrance and front facade of the historical houses of Kashan is a function of geometry, which is named hidden geometry in this research. The geometric system at the entrance of historical houses is one of the most important topics that help to better understand the historical building. The findings show that the entrance design of the houses is under the hidden geometry system, and five types of geometric systems are used to shape it, including symmetry, Fibonacci in golden triangles, Fibonacci in golden rectangles, dynamic rectangles, and regular polygons.

Keywords: historical houses, entrance gate, hidden geometry, golden proportions.

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1. Introduction

Today, the concept of beauty in shaping the urban fabric can be defined by the placement of visible elements such as the type of materials, lighting style, and, other decorative aspects. Whereas in the past architecture, practical components such as geometry, proportions, rhythm, and most importantly, the thinking and attitude of architects, have been an expression of beauty and in this way, while creating an identity for the visual elements, more and better architectural readability has been provided. These quality parameters have provided the basis for durability and added value to the architecture of historical houses in Kashan. In this way, architects have started to create diverse and novel combinations by using the geometry system.

This research tries to investigate the aspects of hidden geometry and its application methods in many historical monuments of Kashan city. In the meantime, the entrance, which is considered an important and strong detail for the architecture of the house, is the first place to enter the house and in interaction and harmony with the texture, establish a deep connection between the building and the surrounding environment. The entrance of Kashan houses as a residential territory is considered an important factor for collective protection (Danaeinia, 2021, p. 42). The entrance heads have beautiful and varied scales and compositions that are considered the identity card of the house. According to historical evidence and documents, due to the earthquake of 1778 AD in Kashan, unfortunately, many of these most valuable gates have been destroyed and over time replaced or have undergone many changes with unoriginal restorations. The lack of in-depth research about such entrance has caused their study to be neglected and their lack of application in contemporary architecture; in such a way that today's designed entrance in the historical context lacks the necessary quality and look like a senseless shell. This research aims to discover the hidden geometry in the entrances of the historical houses of Kashan City and to understand the correct application of geometry concepts in the architectural design of the entrances of the houses. In other words, getting to know the geometric methods used in the entrance gates of the past will make the architects of the present generation more familiar with the wide dimensions of this geometric language and as a result, the entrance design of the houses will benefit from the necessary quality and more geometric richness.

Kashan has many valuable houses that have been noticed in the last decade, and the beauty of the buildings is proof of the use of geometry knowledge and related principles in the design and construction of these houses (Farshchi & Majidi, 2021, p. 102). For this reason, we consider Iranian architecture to be in accordance with geometrical rules (Danaeinia & Erfan, 2023, p. 58).

Research in Iranian architecture without knowledge of the geometry is impossible. In Iranian architecture, geometrical and abstract patterns were emphasized as the best forming factor (of the whole structure) and decoration method (of components). Geometry has a fundamental role in designing of Iranian buildings (H. Mohammadi, 2017, p. 42). Architecture begins with geometry. Since the earliest times, architects have relied on mathematical principles (H. Mohammadi, 2017, p. 44).

The research conducted in the traditional architecture of Iran shows that the architects have sought to make the dimensions of the spaces as close as possible to some geometric proportions (Naseri & Amini Farsani, 2022, p. 66).

Traditional architects used natural geometry principles in the historical building. These rules have directly led to making buildings stable and sustainable besides their beauty and function. In regards to proportions and geometrical rules, it is possible to find a pattern type for rebuilding and reconstruction of lost parts of historical buildings (H. Mohammadi, 2017, p. 48).

2. Research background

Moblian and Kianmehr (2014) while investigating geometrical variables such as symmetry in the plan, proportions in the form (outer view of the building), and motifs of the main entrance of the Georgir Mosque in Isfahan (Iran) have discussed the similarities and influence of the architecture of this entrance main entrance of the Sassanid era architecture. Sarfaraz et al. (2012) have studied the combination of arches, the division of decorative frames, and the composition of geometric motifs at the entrance of Isfahan's Jurjir Mosque. Dousti Motlagh (2009) has investigated the importance of entrance design in achieving peace in the physical environment of human life and has evaluated the proportions and concepts of entrance in historical architecture. Shayestefar and Khaleghizadeh (2016) analyzed the geometric motifs used in the Ganj Ali Khan collection of Kerman (Iran) and showed that these motifs interact and coordinate with each other in square and rectangular frames to create a beautiful entrance. Afradi (2018) has studied how to classify the doors and gates of the historical context of the center of Rasht (Iran) to achieve the revival of the historical context. Tabassi and Fazelnasab (2012) compared and investigated the position of hierarchy in the entrance space system of Isfahan school mosques and architectural periods before and after it. Mohammadi (2019) while examining the inscriptions on the entrances of the houses from the Qajar period to the present has examined the geometric motifs and the existence of geometry, proportions, and symmetry in the entrances. PanjehBashi and Doulab (2018, p. 123) have investigated the type, number, composition, location, and theme of the tiles used in the facades of some prominent buildings in Shiraz. According to Christopher Alexander's theory of living structures, Shahiri Mehrabad (2013) describes some of the characteristics of traditional entries in contemporary architecture based on the concepts of scale and proportions, alternating repetition, definite space, local symmetry, coherence and deep ambiguity, contrast, gradation, heterogeneit, Pajhwok, space, simplicity inner peace, and the concepts of inseparability have been investigated. Karimi Khorasani (2017) focused on the frame above the arch of the entrance of the two Nimvard mosques and the Imamzadeh of the Imam Gate of Isfahan, explored the geometric proportions including the ratio of sizes, angles, golden shapes and the shape of the arches in the implementation of the structure of the arrangement of Iranian knots and how It has revealed the application of modern knowledge in ancient architecture.

Khamechian et al. (2018) have studied the geometry and proportions of the window structure in some samples of Kashan traditional houses. Ajami et al. (2023, p. 155), have investigated how the geometric shape and percentage of windows in the facade of residential houses affect the human brain when facing the buildings. The research of Abbasi et al. (2021) investigated the system of geometric proportions of the entrance space plan of the Abbasi and Seyed mosques of Isfahan using a comparative method. They also found that the system of geometric proportions organizing the entrance of the building is mostly visible in the structure of their entrance section (Abbasi et al., 2021, p. 39). Mozaffaripour (2014) has discussed the theoretical concepts of the sense of place at the entrance and its importance in creating invitingness. Danaeinia (2021) has identified three indicators of integration and integrity, the radius of vision, and visual depth as criteria for understanding the architecture of the entrance space of some historical houses in Kashan and form, visual quality, proportions in design, decorations, the possibility of presence and interaction, hierarchy and focused geometry have been introduced as architectural features of entrances. Cui and Huang (2012) pointed out the complex mechanism of the entrance of public buildings in cold regions and considered features such as proportions to be influential in their design. In the research related to an exhibition at the School of Interior Design in New York, Wolf investigates the history and evolution of the entrance in architecture by examining features such as the entrance proportions of some buildings and offers many design ideas and solutions for the entrance construction problem (Wolf, 2007, p. 6). According to excavations in two sacred Buddhist areas in Pakistan, Olivieri and Iori (2021) have interpreted the architectural language related to the entrance in the building of some of its shrines and stupas. Polpuech (1989), by studying features such as scale and proportions, has traced the role of the entrance through the classification of shape, form, size, color, and texture of the building type in different historical and cultural periods and has shown how the entrances can be considered as a combination of historical aspects. Taravati (2008, p. 12) has discussed the importance of the concepts of threshold, transition, and the in-between entrance in contemporary buildings and has investigated how the rapid pace of modernization and industrialization has gradually affected the interaction of a person with entrance doors as special architectural elements. Boettger (2014) has done a detailed analysis of the entrance space by presenting the parameters of delimitation, sequence, geometry, and materiality to discover what this space is, how to define it, and what functions it creates in the access and experience of a person in architecture. The researches show that despite the wide range of subjects related to the entrance head, the investigation, analysis, and implementation of the topics of theoretical geometry and proportions, especially in its hidden dimension, is a subject that has been less investigated.

3. Entrance space of the building

"Primitive tribes, to provide themselves and secure their sources of food, shelter, and warmth from wild animals, built shelters with only a small opening" (Taravati, 2008, p. 46).

Gates in prehistoric times were mostly considered fortified defensible entrances that were richly ornamented. Later on, they became free from these purposes and stood alone as an important symbolic architectural element, becoming bound to their ritual significance (Taravati, 2008, p. 57). Entrances as an architectural element have played a crucial role in design development throughout history (Polpuech 1989, p. 147).

Iranians used geometry to develop civilization in various fields such as architecture, and geometry has been their main tool in creating their works (Najafgholipour et al., 2017, p. 490).

From the distant past, the name of the entrance has been combined with the name of the threshold, and while marking the sanctity of two different spaces, it is considered an important factor in changing the view, weather, behavior, and rhythm (Daneshmand & Noghrekar, 2012, p. 80) and it is more like an arena than a border (Mozaffaripour, 2014, p. 20) and it is an indicator for identifying the value of the identity and architecture of each building (Mohammadi & Ali Rajabi, 2011, p. 20). Functionally, the entrance space acts as an interface with the surrounding environment and has an active connection and combination with other public spaces (Ganjavie, 2003, pp. 62–63).

The entrance, as a sign, has its characteristics and advantages, because in addition to its main function as a communication space in terms of visual and perception, it is also a space between the building and urban spaces (Danaeinia, 2021, p. 42) and interrupts spatial boundaries for a transition from one zone to another. The phenomenon of the threshold thrives on spatial ambivalence. Thresholds open up spaces and organize transitions (Boettger, 2014, p. 10). The entrance space is an important part of the exterior view, whose index is one of the functional and physical characteristics of the building, which is made and presented with an architectural design (Soltanzadeh, 1992, p. 11).

According to this, the entrance of Kashan houses shows the type and extent of people's interaction with each other and has an important role in the mental imagery of the inner space. "Audiences communicate with the home through the entrance space" (Danaeinia, 2021, p. 48).

4. Geometric symmetry

Geometry is the hidden essence of architecture in different periods (Silvayeh et al., 2013, p. 56), which is used as a tool to establish order and conscious relationships between architectural components (Najafgholipour et al., 2017, p. 478). Masters used geometrical methods and expanded and perfected them (El-Said & Parman, 2008, p. 21). Geometry is the language of architecture and has made it possible to expand various methods of facade construction (El-Said & Parman, 2008, p. 129). Geometric design in combination with proportions has led to the creation of stable buildings over the years (Feizollahbeig et al., 2021, p. 40). The shape of the entrances in the traditional architecture of Iran is among those all their components are arranged and designed based on geometry (Mohammadi, 2010, p. 100). Creating an opening in a building unit is an extremely delicate task that must be precisely proportioned and carefully placed in place, and the internal relationships reveal a meaningful composition (Krier, 2001, p. 119). Therefore, the beauty of Iranian Islamic architecture is the result of balance and balance obtained from observing geometric principles (Sharif, 2002, p. 108) and observing principles such as symmetry and the use of basic shapes with the golden ratio are geometric design principles that affect the stability of the building (Feizollahbeig et al., 2021, p. 39).

The entrances used symmetry to create balance, which indicates the movement from earth to sky or from matter to light (Mohammadi, 2010, p. 102). Symmetry means the ratio between the elements of the part in the whole (El-Said & Parman, 2008, p. 19). The use of symmetry in the facade, plan, and other entrance parts creates a feeling of being alive and intelligent in architectural spaces (Shahiri Mehrabad, 2013, p. 6). Placing a larger arch in the upper part of the entrance and two smaller concave arches on its sides; creates a long-standing combination (Moblian & Kianmehr, 2014, p. 47). In architecture, symmetry is mainly limited to two types of axial (shape rotation ⟦180⟧ ^° around a vertical or horizontal axis and the formation of two similar shapes at equal distances from that axis) and repetition (repetition of the shape in a row) (Grütter, 2009, p. 372). Figure 1 shows the geometrical analysis layers of axial symmetry in a historical Iranian house.

Geometrical analysis of many Persian historical buildings has proven that a complete of proportions, in particular the golden ratio was widely used in Persian architecture and it was the basis of Persian aesthetics. In many Persian buildings, the plan and elevation were set out in a framework of squares and equilateral triangles, whose intersections gave all the important fixed points, such as the width

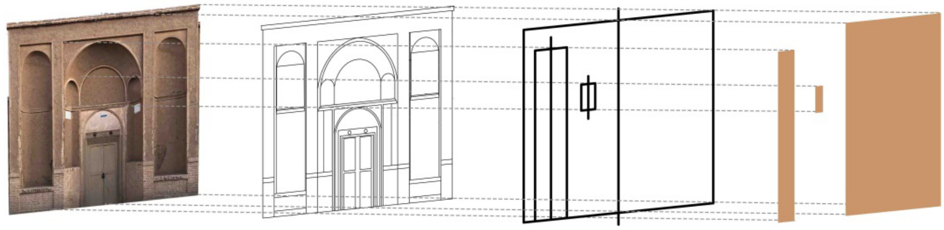


Figure 1. Geometrical analysis of axial symmetry in Bani Hosseini's house in Kashan

and height of doors; and the width, length, and height of galleries. A building was not, therefore a collection of odd components, but a harmonious configuration of proportionally related elements, which gave movement to space and satisfied the eye (Ghorbani, 2021, p. 166).

5. Golden Ratio

The concept of golden ratio division appeared more than 2400 years ago as evidenced in art and architecture. The magical golden ratio divisions of parts may be rather closely associated with the notion of beauty in pleasing, harmonious proportions expressed in different areas of knowledge (Sarwar Islam 2020, p. 37). True beauty lies in the right proportion of things, an internal order of the spirit, which acts on our senses, with harmony and clarity (Dewitte, 2015, p. 457). The golden ratio has been regarded as a useful ratio to be used in a design, where designers believe this proportioning tool can produce aesthetically pleasing designs (Page et al., 2010, p. 71). The Golden Proportion is a constant ratio derived from a geometric relationship which, like x and other constants of this type, is "irrational" in numerical terms (Lawlor, 1982, p. 46). The golden section is defined as a harmonic division of a line in extreme and mean ratio (Vico-Prieto et al., 2016, p. 187). "The proportion known as the Golden Mean has always existed in mathematics and the physical universe and it has been of interest to mathematicians, physicists, philosophers, architects, artists, and even musicians since antiquity" (Akhtaruzzaman & Shafie, 2011, p. 1).

"Golden ratio and aesthetic beauty are among the core principles that can be used effectively in design" (Fun et al., 2021, p. 486). The Golden Proportion can be considered as supra-rational or transcendent (Lawlor, 1982, p. 46). Historically this unique geometric proportion of two terms has been given the name golden proportion (Lawlor, 1982, p. 45). For years, architects have used the golden ratio, Φ, in their drawings (Posamentier & Ingmar, 2007, p. 64). The golden section endows a work of art with a particularly pleasing and "beautiful" shape (Posamentier & Ingmar, 2007, p. 244). It is probably fair to say that the Golden Ratio has inspired thinkers of all disciplines like no other number in the history of mathematics" (Alsina & Nelsen, 2010, p. 22). The Golden Ratio has considerable significance to geometry and has very unique and surprising algebraic properties (Bressler, 2020, p. 11).

In the design of traditional Iranian architecture, there are works that, in addition to using Iranian geometry and golden proportions, also follow the principles of Fibonacci's golden proportions (Kalantari et al., 2017, p. 490).

If the line is divided so that the ratio of the total length to the length of the longer segment is the same as the ratio of the length of the longer segment to the length of the shorter segment then this ratio is the golden ratio[1] (Dunlap, 1997, p. 2) (Figure 2). The golden ratio is an irrational number defined to be $\left(1+\sqrt{5}\right)/2$. It has been of interest to mathematicians, physicists, philosophers, architects, artists, and even musicians since antiquity and has a value of 1.61803 (Dunlap, 1997, p. 1). This root $\left(1+\sqrt{5}\right)/2$ is called the golden ratio or the divine proportion[2] (Alsina & Nelsen, 2010, p. 22) (Equation (1)). The Golden Proportion Though its concept is easy to understand its applications have been proven to be complex and its simplicity is very perplexing (Katyal et al., 2019, p. 3).

"Generally, the overall harmony of the building from floor to elevations can be achieved based on the use of the Golden Ratio" (Fun et al., 2021, p. 501).

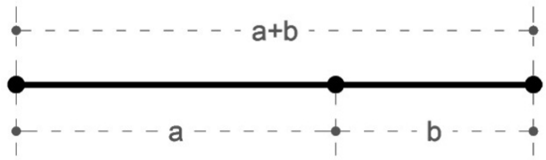


Figure 2. Representation of the golden ratio on a line segment

$$\frac{a}{b}=\frac{a+b}{a}=\frac{1+\sqrt{5}}{2}\cong 1.618=\Phi=1+\frac{1}{\Phi}=1+\varnothing\cong 1+0.618.$$

Equation 1: Calculation of the golden ratio on a line segment

6. Fibonacci

One of the best-known numerical sequences is the additive sequence (Dunlap, 1997, p. 7). The Fibonacci number series[3] is created by adding two numbers together to form the next in the series, 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, and so on. When you want to know the relative re-

1 It is a way to divide a line in such a manner as to create an ideal relationship between the parts (Borges, 2004, p. 402). The Golden Ratio is defined as the division of a line into two parts (Bressler, 2020, p. 3).

2 The whole is to its larger part, what the larger part is to the smaller (Borges, 2004, p. 402).

3 A numerical sequence is an ordered set of numbers which is generated by a well-defined algorithm (Dunlap, 1997, p. 7).

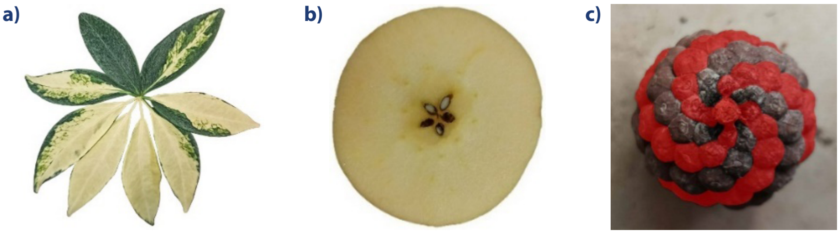


Figure 3. a – the dependence of the number of petals of Schefflera plant; b – the number of cut cores of an apple tree from Fibonacci numbers; c – representation of a logarithmic spiral on the scales of a pine tree fruit

lationship between two different numbers, you divide one by the other. If you divide 144 by 89, the result is a ratio of 1.6179. You can round this result to 1.618. So 1.618 (Φ) and the reciprocal 0.618 (1 / Φ) are ratios of significance when you compare two nearby numbers in the Fibonacci sequence (Brown, 2008, p. 22). The ratio of two consecutive Fibonacci numbers approaches the golden ratio, the greater the magnitude of the Fibonacci numbers, the closer the approximation of Φ (Posamentier & Ingmar, 2007, p. 232). Certain geometries describe growth patterns in nature, whereas other geometric shapes are employed in art and architecture to establish a sense of harmony and proportion in design (Borges, 2004, p. 401). Application of Fibonacci numbers in nature, the human body, and architecture is considered as the key to esthetics (Katyal et al., 2019, p. 8). Therefore, in traditional architecture, the use of proportions taken from nature has been an important principle in interaction with nature (Javadi Nodeh et al., 2022, p. 35)

The Fibonacci numbers form the best whole number approximations to the Golden Section (Akhtaruzzaman & Shafie, 2011, p. 10). We have seen the Fibonacci numbers as they appear in nature, as they manifest themselves among other numbers and sequences, and as they can be seen in geometry (Posamentier & Ingmar, 2007, p. 161). The golden ratio plays a role in the growth of many biological systems (Dunlap, 1997, p. 13). On the way up, nature uses additive and multiplicative relationships; on the way down, nature uses subtraction and division to diminish and create logarithmic growth and decay (Brown, 2008, p. 29). The Fibonacci series is closely related to the Golden Angle (Fun et al., 2021, p. 483). An interesting feature of living organisms on Earth is a seeming tendency for their morphology to be structured around the golden ratio (Larsen, 2021, p. 1). The beautiful spiral exhibits nature's exemplary display (Katyal et al., 2019, p. 4). Fibonacci numbers are also found in some spiral arrangements of leaves on the twigs of plants and trees (Koshy, 2018, p. 23) and most plants will have the number of such petals corresponding to a Fibonacci number (Posamentier & Ingmar, 2007, p. 71) (Figures 3a, 3b). The Fibonacci sequence can be considered more or less the beauty of nature (Akhtaruzzaman & Shafie, 2011, p. 12).

The clockwise spiral pattern is tighter than the counterclockwise spiral pattern. These numbers are almost always successive Fibonacci numbers. This property indicates that the growth angle of the scales in the pinecone is related to a rational approximation of the golden ratio (Dunlap, 1997, p. 130) (Figure 3c).

If the relative ratio is 1.618 for the components of any structure, the form will be convenient to the Golden Ratio, which is the perfect design (Akhtaruzzaman & Shafie, 2011, p. 15).

6.1. Fibonacci in Golden spiral

"A Golden spiral is one particular case of a logarithmic, or equiangular, spiral" (Bressler, 2020, p. 47). "The simple logarithmic growth spiral, or golden spiral, is derived from Fibonacci numbers" (Brown, 2008, p. 29). A Golden spiral is defined by its growth factor. For every quarter turn that a Golden spiral makes, it gets larger by a factor of φ (Bressler, 2020, p. 48). The number of spirals in each direction will be most often two successive Fibonacci numbers (Posamentier & Ingmar, 2007, p. 64). The golden spiral is a compositional overlay in the form of a logarithmic spiral made from the Golden Ratio (Gjonbalaj et al., 2021).

6.2. Fibonacci in the golden rectangle

Through this geometric algorithm, we can infinitely trace the smaller golden rectangles toward point P, which is the intersection of the two main diagonal lines in movement and creation (Figure 4a).

As we did with the golden rectangle, we can generate an approximation of a logarithmic spiral by drawing arcs to join the vertex angle vertices of consecutive golden triangles (Posamentier & Ingmar, 2007, p. 147) (Figure 4b). The logarithmic spiral is so rich in geometric and algebraic harmonies that traditional geometers named it Spira mirabilis, the miraculous spiral. While the radius of this spiral increases in a geometric progression, the radial angle increases in an arithmetic progression (Lawlor, 1982, p. 71).

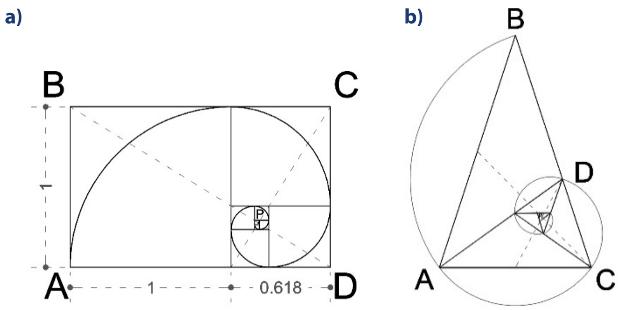


Figure 4. Logarithmic spiral representation on (a) golden rectangles and (b) golden triangles

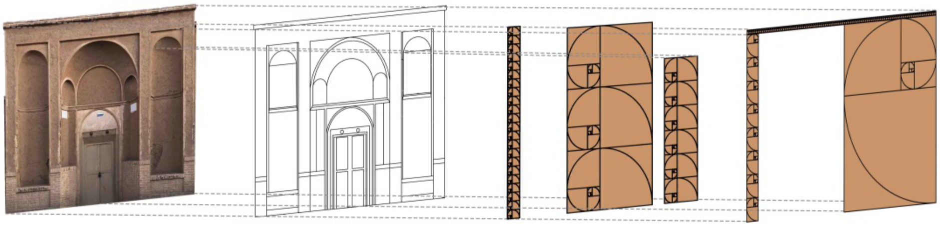


Figure 5. Geometrical analysis of the golden rectangle structure in Bani Hosseini's house in Kashan

In each growth phase characterized by a spiral, the new spiral is very close to the proportion of the golden ratio (a golden rectangle to a square) larger than the previous one (Borges, 2004, p. 404).

Figure 5 shows the layers of geometric analysis of the golden triangle structure in one of the historical houses of Kashan.

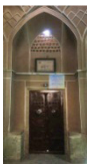
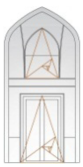
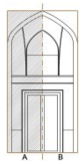
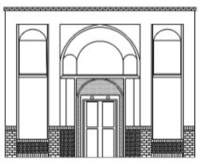
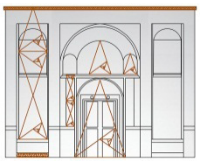
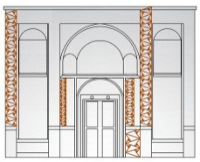
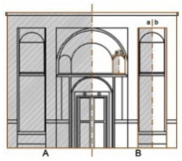
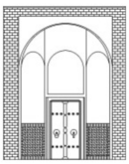
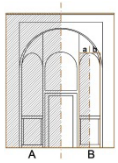
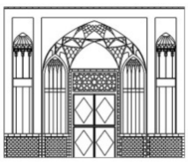
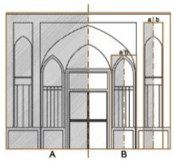
7. Research method

The scope of the research is in the city of Kashan, Iran, which includes a combination of library and field libraries. The research is descriptive-analytical. The research is descriptive-analytical. Library studies, focusing on theoretical geometry sciences, form the theoretical framework of the research. On the other hand, based on field studies and observation methods, eight types of remaining headstones from the Qajar period to the beginning of the first Pahlavi period of Kashan city, which have historical and architectural value, have been selected. To achieve a detailed understanding of the geometrical and architectural characteristics of the selected headstones, a regular and classified framework of samples has been obtained by drawing the data and modeling them in the Autocad 2014 software through image documents and taking the size of the required details. The obtained information has been separated and compared graphically in tables and charts, and finally, the main and secondary geometric rules and proportions of the inputs have been deduced and extracted.

8. Samples

Eight historical monuments in Kashan city have been selected and investigated. The examples are selected from among the most complete historical architectural patterns and include patterns that have minimal changes in the architectural space over time (Tables 1 and 2).

Table 1. Introduction of samples and examination of geometric proportions

Item	Case	Picture	Facade	Large unit	Medium unit	Geometric symmetry
1	Abbasian					
2	Banihosseini					
3	Ehsan					
4	Kheirieh					

End of Table 1

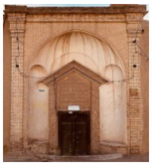
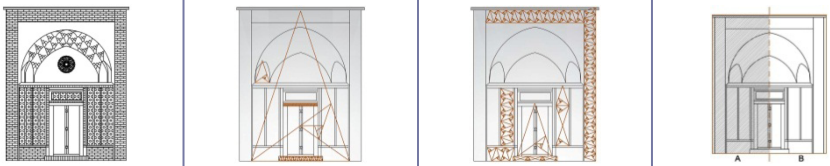
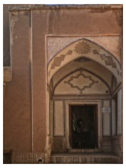
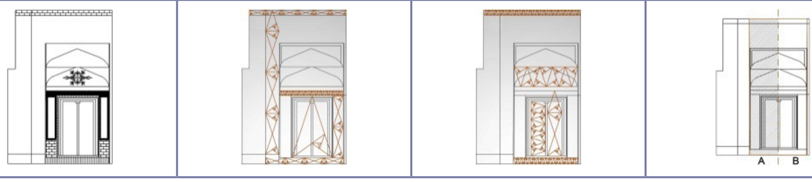
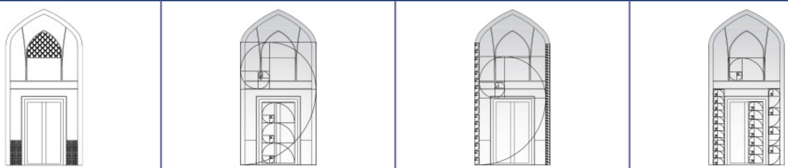
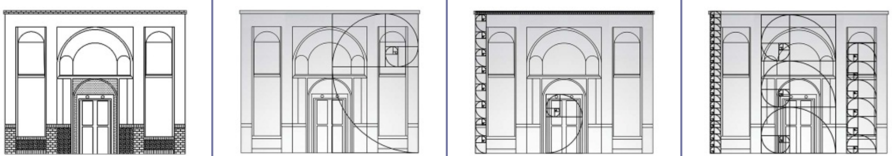
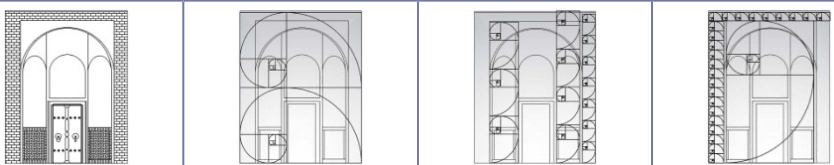
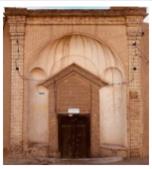
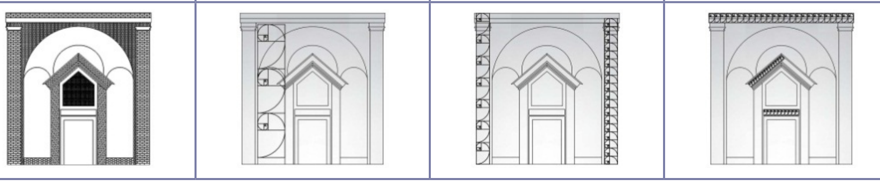
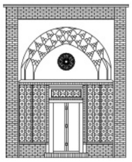
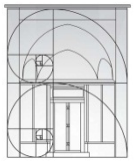
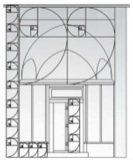
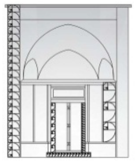
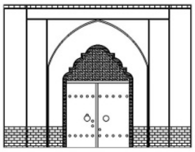
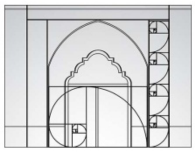
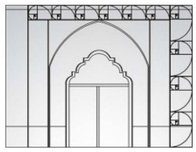
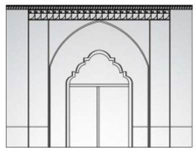
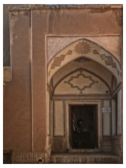
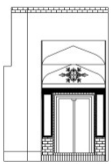
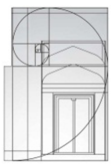
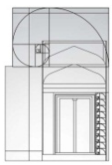
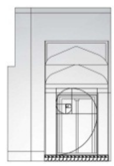
Item	Case	Picture	Facade	Large unit	Medium unit	Geometric symmetry
5	Mortazavi					
6	Roueintan					
7	Sharifian					
8	Taj					

Table 2. Investigating the golden rectangle in the logarithmic spiral in case studies

Item	Case	Picture	Facade	Large unit	Medium unit	Micro unit
1	Abbasian					
2	Banihosseini					
3	Ehsan					
4	Kheirieh					
5	Mortazavi					

End of Table 2

Item	Case	Picture	Facade	Large unit	Medium unit	Micro unit
6	Roueintan					
7	Sharifian					
8	Taj					

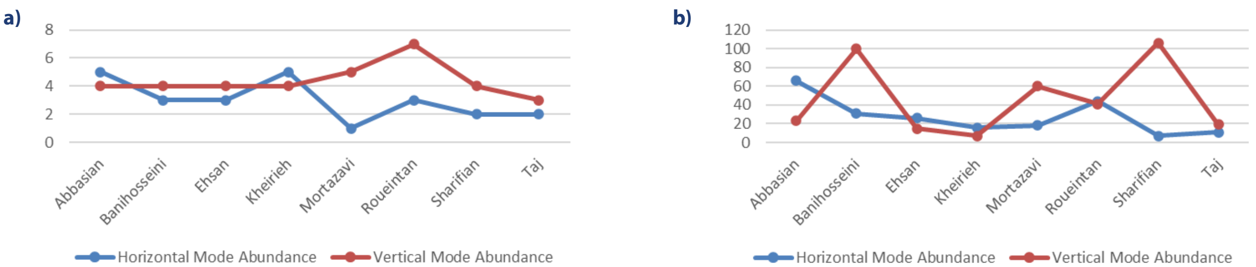


Figure 6. a – frequency diagram of the number of horizontal and vertical rows of golden rectangles without repetition in species b – and their constituent micro-units

Based on the non-repetition of the parts of the facade where golden rectangles are used, Figure 6 shows the frequency of the number of horizontal and vertical rows of golden rectangles (Figure 6a) and their constituent micro units (Figure 6b).

9. Dynamic Rectangle (A Root-Rectangle)

Based on the number of ratios, a rectangle can be categorized as a static rectangle or a dynamic rectangle (Fun et al., 2021, p. 481). In the creation of architectural buildings, the dynamic square is widely used, which shows the relationship of the square with the diameter and the square built on this diameter with its diameters, which is shown by the expansion of 1, √2, √3, 2, √5, etc. (Azar Khordad et al., 2018, p. 125).

A root rectangle is a rectangle in which the ratio of the longer side to the shorter is $\sqrt{n}$ for an integer n. We illustrate an iterative procedure for constructing $\sqrt{n}$ root rectangles beginning with a unit square. A circular arc is drawn clockwise with center (0, 0) and radius $\sqrt{n+1}$ from $\left(\sqrt{n},1\right)$. Such an arc then intersects the x-axis at $(\sqrt{n+1},0)$ (Alsina & Nelsen, 2010, p. 122) (Figure 7). Dynamic Rectangle produces a harmonious visual subdivision (Fun et al., 2021, p. 481). Static rectangles do not produce a series of visually pleasing ratios of surfaces while subdividing. On the other hand, dynamic rectangles produce an endless amount of visually pleasing harmonic subdivisions and surface ratios (Akhtaruzzaman & Shafie 2011, p. 5).

Figure 7 shows the layers of geometric analysis of dynamic rectangles in one of the historical houses of Kashan.

Table 3 shows the geometric proportions of the dynamic rectangles in both horizontal and vertical coordinates in the samples.

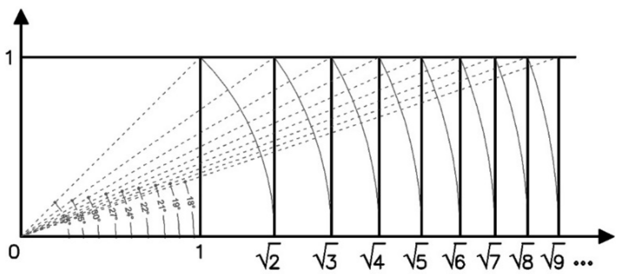


Figure 7. Geometrical analysis of the extensive structure of the network of radical rectangles of the nth root

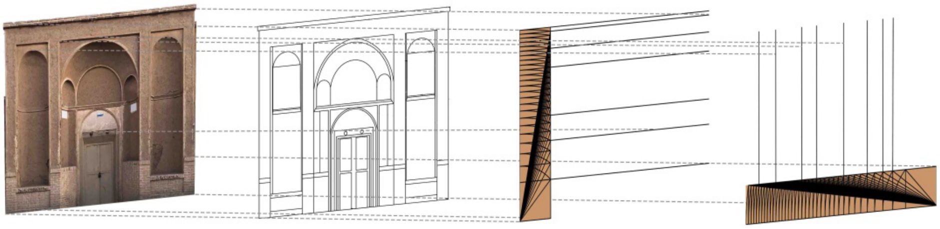
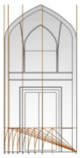
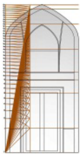
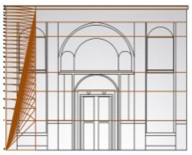
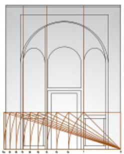
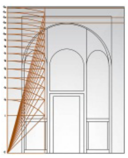
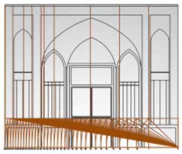
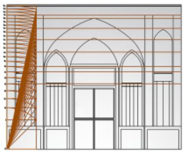
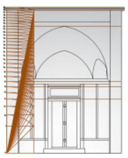
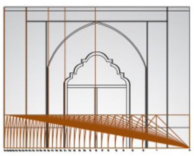
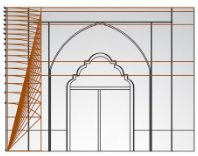


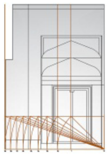
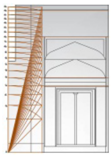
Table 3. Examining the geometric proportions of dynamic rectangle in species

Item	Case	Type	Analysis	Root	Intersection	Root	Intersection	Numbers
1	Abbasian	Horizontal		√2	Single hinge door	√8	The internal groove of the main frame	4
				√5	The inner frame of the door and the beginning of the central arch	√9	The main frame's outer groove	
		Vertical		1	Senior high level	√28	The top of the middle arch	9
				√6	Bottom of the door frame	√29	The top of the side arch	
				√9	Below the inscription	√36	The inner apex of the arch of the main frame	
				√11	Above the inscription	√37	The outer apex of the arch of the main frame	
				√18	Above the rectangle in the middle			
2	Banihosseini	Horizontal		√2	The edge of the side view frame	√15	The edge of the Pir Neshin	8
				√3	The edge of the middle frame	√19	The edge of the middle frame	
				√5	The edge of the middle frame	√27	In the middle of the middle frame edge	
				√9	The edge of Pir Neshin	√33	The edge of the side view frame	
		Vertical		1	Above the level of Pir neshin	√19	The end of the facade frames	7
				√4	Above the door frame	√23	The beginning of the roof edge	
				√7	Bottom of the middle eaves	√24	The end of the roof edge	
				√15	The starting point of the side arch			
3	Ehsan	Horizontal		1	The edge of the Pir Neshin	√7	The edge of the Pir Neshin	3
				√4	The external jamb of the door frame			
		Vertical		√12	The external vertex of the arc in the main frame	√15	Above the roof edge	3
				√13	The end of the main frame of the facade			

Continue of Table 3

Item	Case	Type	Analysis	Root	Intersection	Root	Intersection	Numbers
4	Kheirieh	Horizontal		1	The edge of the side frame	√22	The edge of the middle frame	10
				√4	The edge of Pirneshin	√25	The edge of the side frame	
				√9	Middle view	√28	The middle of the side frame	
				√18	The middle frame of Pirneshin	√32	Side frame	
				√21	The edge of Pirneshin	√35	The end of the facade	
		Vertical		√5	The end of the lower frame of Pirneshin	√17	The top of the arch of the side frame of the facade	9
				√7	The end of the inscription on the side frame of the facade	√20	The top of the arch of the middle frame	
				√8	The beginning of the arcs in the middle frame	√21	Bottom of the middle frame	
				√11	The top of the arch of Pirneshin	√24	Down the rokhbam	
				√14	The beginning of the arch of the side frame of the facade			
5	Mortazavi	Horizontal		√3	The middle frame of Pirneshin	√16	Middle view	10
				√4	The edge of the sloping door	√26	The outer edge of the door frame	
				√6	The outer edge of the door	√31	The outer edge of the door	
				√7	The arch above the sloping inscription	√41	The middle frame of Pirneshin	
				√9	The outer edge of the door frame	√65	The edge of the frame	
		Vertical		√8	The beginning of the door inscription	√46	The apex of the arch above the sloping inscription	10
				√10	The beginning of the door inscription	√62	The top of the main arch of the facade	
				√18	The beginning of the sloping inscription	√66	The beginning of the inscription under Rukhbam	
				√31	The top of the arch of Pirneshin	√70	Down the Rokhbam	
				√40	The top of the sloping inscription of the External door frame jamb	√75	The end of Rukhbam	
6	Roueintan	Horizontal		√2	External door frame jamb	√14	Vertical groove on the side of the facade	5
				√8	External door frame jamb	√18	The end of the facade	
				√11	The middle part of Pirneshin			
		Vertical		√4	The beginning of the door inscription	√22	The top of the main arch of the facade	6
				√6	End of the door inscription	√28	The beginning of Rukhbam	
				√7	Below the arch of the main facade	√29	The end of Rukhbam	
7	Sharifian	Horizontal		√8	Middle view	√24	The inner edge of the vertical groove on the side of the facade	5
				√14	The outer edge of the door frame	√31	The end of the facade	
				√18	The inner edge of the main facade frame			
		Vertical		√5	The broken edge of the arch above the door	√17	The beginning of Rukhbam	5
				√7	The broken edge of the arch above the door	√18	The end of the facade	
				√14	The broken edge of the arch above the door			

End of Table 3

Item	Case	Type	Analysis	Root	Intersection	Root	Intersection	Numbers
8	Taj	Horizontal		1	The middle of the main frame	√9	The end of the facade	3
				√2	The inner edge of the door frame			
		Vertical		√4	At the bottom of the door inscription	√12	The end of the main frame of the facade	4
				√5	Above the door inscription	√18	The end of the main frame of the facade	

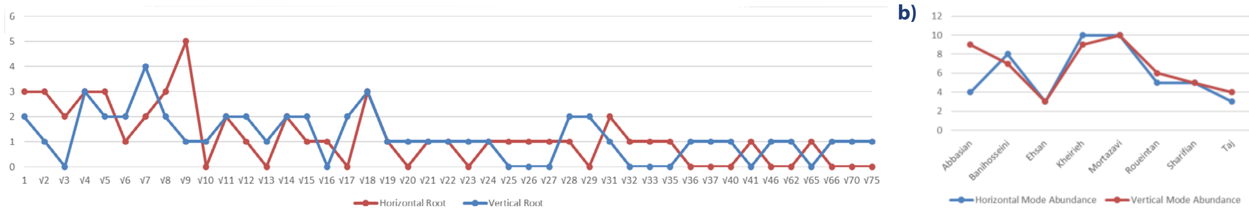


Figure 8. a – frequency of the number of each of the square derivatives in horizontal and vertical states; b – the total number of square derivatives in each of the horizontal and vertical states in species

Figure 8 shows the frequency of each of the square derivatives (Figure 8a) and their total number in each of the horizontal and vertical states of the species (Figure 8b).

10. Proportions of polygons

Geometry is the study of spatial order through the measure and relationships of forms (Lawlor, 1982, p. 7). In Iranian architecture, architects usually use the shapes of square, rectangular, hollow, pentagonal, hexagonal, and octagonal bases, which are formed by repeating, rotating, changing their size, and combining them (Feizollahbeigi et al., 2021, p. 39). Iranian architecture ensured the harmony of all parts of the building to achieve a unique nature through the connection of square, triangle, and pentagonal parts (Azar Khordad, et al., 2018, p. 125). Geometry attempts to recapture the orderly movement from infinite formlessness to an endless interconnected array of forms (Lawlor, 1982, p. 23). Among the most aesthetically appealing two-dimensional shapes are the regular polygons. These are Figures which have all edges equal and all interior angles equal and less than 180. The golden ratio plays an important role in the dimensions of many geometric figures, in both two and three dimensions. The simplest appearance of 1.618 in geometry occurs in two-dimensional figures (Dunlap, 1997, p. 15) and thus the golden ratio' appears in many constructions with regular polygons (Alsina & Nelsen, 2010, p. 121).

In general, in the proportions of regular polygons, we have four modes: square, five-pointed star, and six-pointed star, which are as follows.

11. Square quadrilateral proportions

Square is a subset of rectangle that is used in many buildings (Silvayeh et al., 2013, p. 57). The simple square, the first of the regular shapes gifted with Pythagorean mystical significance, conceals the means of commensurability's destruction (Havil, 2012, p. 25).

In the semicircle of the square ABCD of length x, if it is surrounded in such a way that the length of DF is equal to the unit number 1 (Figure 9a), Then the points G and F are connected to the vertex C of the mentioned square to obtain two similar triangles DCG and DCF, so from this similarity the relationship "x/1=(x+1)/x" is determined, from which the quadratic equation "x" ^"2" "=x+1" results that the positive root of x is equal to the golden number Φ and therefore the rectangle ABEF will be a golden rectangle (Figure 9b). In another case, if a square with side 1 is considered and a square of the same size is created by rotating 45 degrees, we have a star-shaped network in the form of eight-pointed stars. If the middles of each side of these squares are connected. Another square is obtained, which will have an area half of the area of the previous square, and the length of its side, according to the formula of the area of the square S = 1/2, will be equal to the square root of its area, i.e. "√2/1" (Figure 9c), which indicates is a geometric progression.

The combination of a square and the square is rotated by 45, which gives an octagon (Dewitte, 2015, p. 471) (Figure 9c). In this geometric demonstration of the relationship between proportion and progression, we are reminded of the alchemical axiom that everything in creation is formed from a fixed, immutable component (proportion) as well as a volatile, mutable component (progression) (Lawlor, 1982, p. 30).

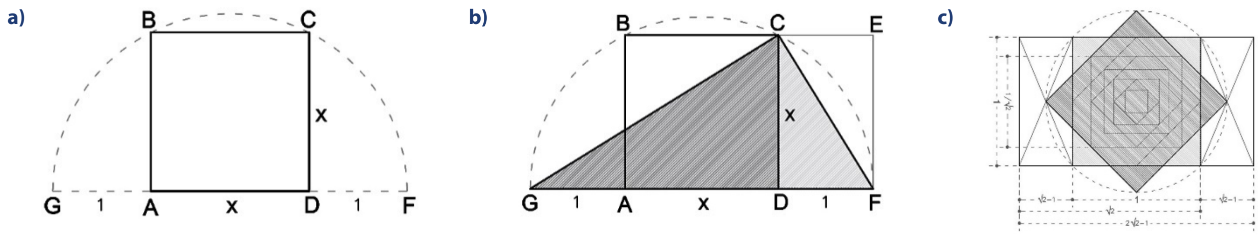


Figure 9. Geometrical analysis of the regular square quadrilateral structure

12. Pentagonal proportions

The pentagram, pent alpha, pentangle, or star pentagon is one of the most potent, powerful, and persistent symbols in the history of humankind (Havil, 2012, p. 25). The regular pentagon is a fascinating Figure with lots of useful properties (Posamentier & Ingmar, 2007, p. 151). The pentagram and the Golden Ratio are inexorably linked and the symbolism of the figure is directly related to the regenerative properties of the Golden Ratio which are perceptible in its construction (Bressler, 2020, p. 56). Regular pentagons provide us with many examples of the Golden Ratio in everyday life (Koshy, 2018, p. 362). The Golden Section can be found in all flowers having five petals or any multiple of five (Lawlor, 1982, p. 58). The golden triangle is embedded many times in the regular pentagram, which just happened to be the symbol of the Pythagoreans. According to Pythagoras, all geometric shapes could be described in terms of integers. The ratio of a side of the regular pentagram of the regular pentagon to the side length of the pentagon could not be expressed as a fraction of integers, in other words, this ratio is not rational (Posamentier & Ingmar, 2007, p. 149). The side of a pentagon is to its diagonal as the Golden Section (Lawlor, 1982, p. 51) (Figure 10a). To show that this length relationship is irrational, we use the relationship that in a regular pentagon every diagonal is parallel to the sides it does not intersect (Posamentier & Ingmar, 2007, p. 149).

The regular pentagon is widely used in Iranian architecture as a basic shape because this shape has a golden ratio inside (Feizollahbeigi et al., 2021, p. 46) Since the angle of a vertex in a regular pentagon is equal to 108 degrees, so the angle of the vertex of an equilateral triangle ABC is equal to 36 degrees, and based on the analysis made in the golden triangle section, every five-pointed star formed from a regular pentagon will be a golden triangle (Figure 10c). If we consider the pentagram with its defining regular pentagon, the eye is led to its many congruent or similar isosceles triangles and the nested regular pentagon, inverted and suggestive of an infinitely recursive extension (Havil, 2012, p. 25). A regular pentagram (the five-pointed star) is essentially composed of many golden triangles (Posamentier & Ingmar, 2007, p. 148). You can see the continued pattern in pentagrams and pentagons harboring the golden ratio and, consequently, the Fibonacci numbers[4] (Posamentier & Ingmar, 2007, p. 154) (Figure 10d). Two diagonals from a common vertex form an isosceles triangle, which can be partitioned by a segment of another diagonal into two smaller triangles (Alsina & Nelsen, 2010, p. 242) (Figure 10e).

13. Hexagonal proportions

It's associated double overlapping equilateral triangles, the hexagram, is one of the oldest and most widespread spiritual symbols (Havil, 2012, p. 27). The regular hexagon has great value in Iranian architecture, and the rectangle obtained from it is known as the Iranian golden rectangle, many architectural buildings have been formed based on the geometry hidden in it. Like what was observed in a regular pentagon, in any regular hexagon, the process of creating shapes through the connection of vertices can also be used. Inside every regular hexagon there are two parallel triangles (Figure 11a), one of the sides of each of these triangles is located along the length of the golden rectangle (Figure 11b). If we divide the hexagon into 24 equilateral triangles, a golden rectangle will be created with the ratio of one-half of the tribal golden rectangle (Figure 11c).

Figure 12 shows the layers of geometric analysis of golden angles in one of the historical houses.

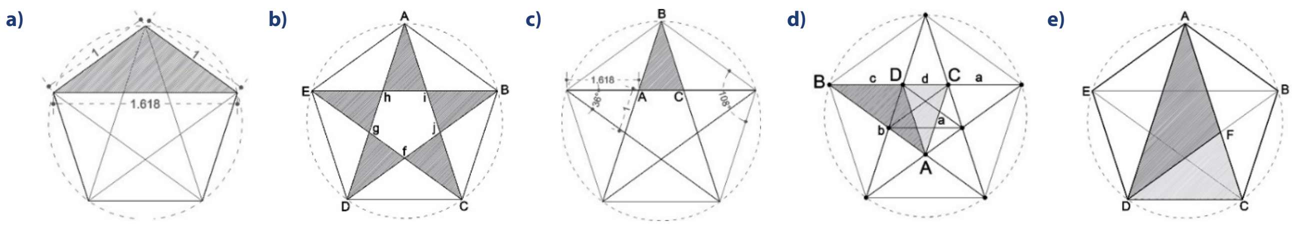


Figure 10. Analysis of the regular pentagonal geometric structure

4 For more information, refer to: (Alsina & Nelsen, 2010, p. 242).

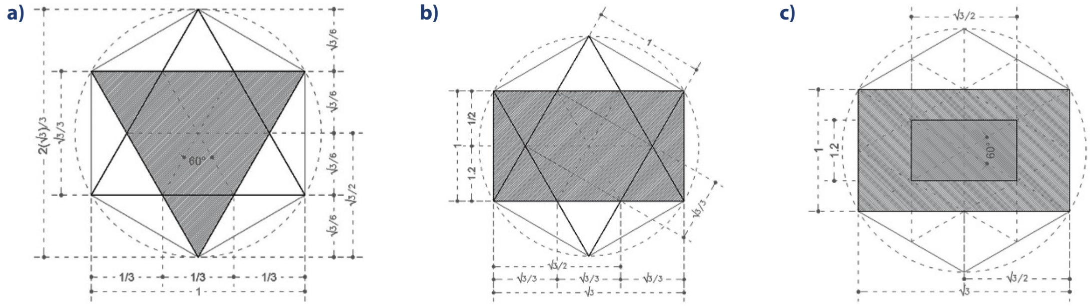


Figure 11. Geometrical analysis of regular hexagonal structure

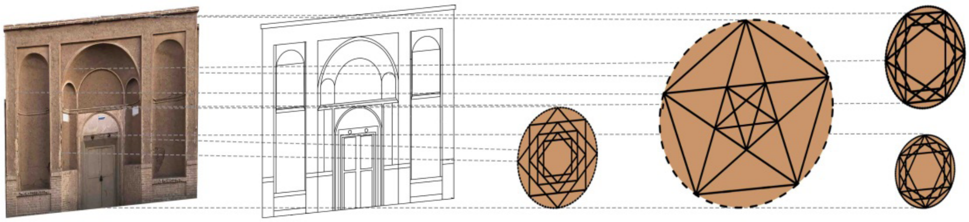
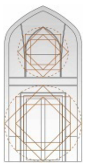
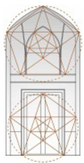
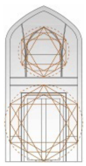
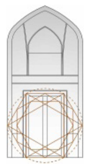
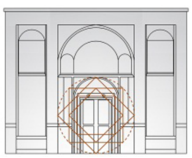
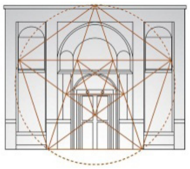
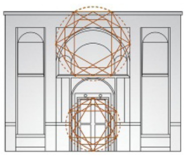
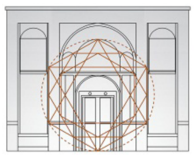
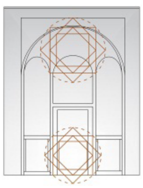
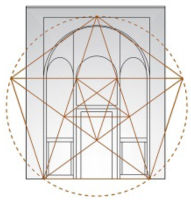
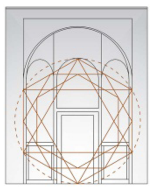
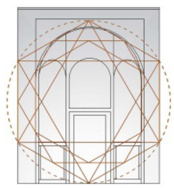
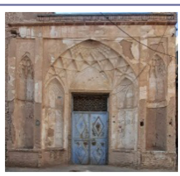
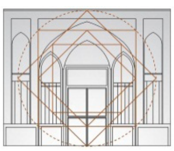
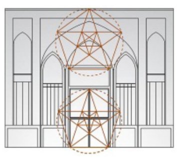
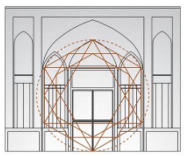
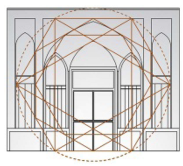
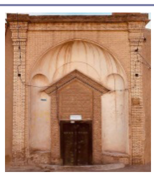
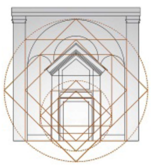
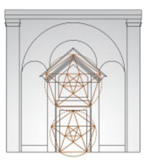
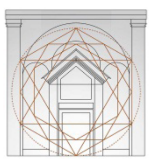
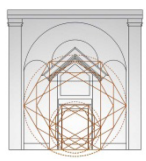
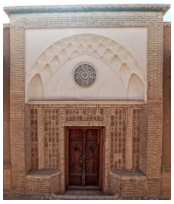
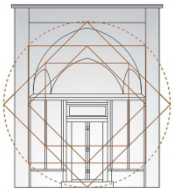
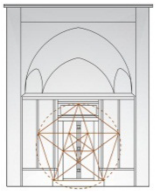
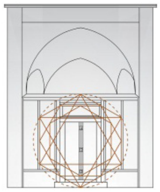
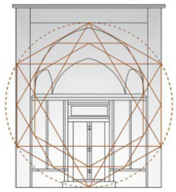
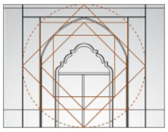
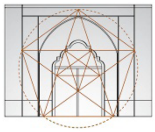
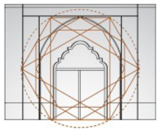
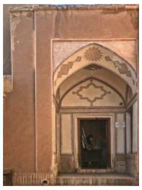
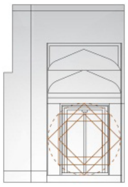
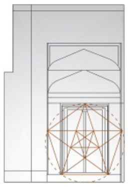
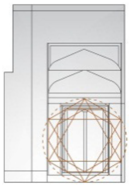


Figure 12. Geometrical analysis of square, pentagonal, and hexagonal proportions in Bani Hosseini house of Kashan

Table 4. Examining the geometric proportions of polygons in the samples

Item	Case	Picture	Square quadrilateral	Pentagonal proportions	Hexagonal proportions	
1	Abbasian					
2	Banihosseini					
3	Ehsan					
4	Kheirieh					
5	Mortazavi					

End of Table 4

Item	Case	Picture	Square quadrilateral	Pentagonal proportions	Hexagonal proportions	
6	Roueintan					
7	Sharifian					–
8	Taj					–

According to Table 4, tetragonal, pentagonal, and hexagonal proportions can be seen in the whole part of the studied buildings from the floor to the roof. In terms of quadrilateral proportions, the lowest frequency is related to Bani Hosseini, Khairai, Royintan, Sharifian, and Taj houses, and the highest frequency is related to Abbasian, Ehsan, and Mortazavi houses In pentagonal proportions, the lowest frequency is related to the house of Bani Hosseini, Ehsan, Ruyintan, Sharifian, and Taj, and the highest frequency is related to the house of Abbasian, Khairai and Mortazavi. In hexagonal proportions, the lowest frequency is related to the Sharifian and Taj houses, and the highest frequency is related to the Abbasid, Bani Hosseini, and Mortazavi houses.

14. Conclusions

The entrance of houses plays an important role in creating spatial proportions. In the examined samples, which are divided into two groups, simple and productive, geometric divisions have created a proportional framing in the entrance view and five types of geometry can be seen in them, including 1. Symmetry (axially, mainly in the middle of the facade of the houses). 2. Fibonacci in golden triangles (single or several sections next to each other in a row). 3. Fibonacci in the golden rectangle (single or several sections next to each other in a row). 4. Dynamic rectangles (in the horizontal and vertical mode, the square root of the unit, the second root (√4), and the third root (√9), in the horizontal mode, the fourth root (√16) and the fifth root (√25), and in the vertical mode, the root 6th (√36) square derivatives) and 5. Regular polygons (in quadrilateral, pentagonal, and hexagonal proportions).

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