Journal of Architecture and Urbanism
ISSN 2029-7955 / eISSN 2029-7947
2022 Volume 46 Issue 1: 58–67
https://doi.org/10.3846/jau.2022.15719

AN INVESTIGATION INTO GEOMETRIC RATIOS OF THE SUNKEN COURTYARDS IN TRADITIONAL IRANIAN HOUSES (A FIELD STUDY ON YAZD AND KASHAN)

Hosein NASERI [1*], Zahra AMINI FARSANI [2]

[1]*Department of Architecture, Faculty of Architecture and Urban Design, Art University of Isfahan, Isfahan, Iran*
[2]*School of Science, Statistics Department, Lorestan University, Khorramabad, Iran*

Received 08 October 2021; accepted 29 March 2022

Abstract. In Iranian architecture, the emphasis on the use of geometric ratios such as human scale and modularization, caused beauty and harmony. Unfortunately, in contemporary architecture, the use of these scales has been forgotten, and their presence has been diminished. Therefore, the main goal of the present study was to analyze and evaluate the geometry and proportions used in six remaining traditional Iranian sunken courtyards in Yazd and Kashan. For each house, the length, width, height and the ratio between these dimensions were measured for the sunken courtyard, courtyard and earth of the case studies. Then, to find out which kind of proportions were used in these sunken courtyards, we proposed some statistical tests to compare our measurements with the traditional proportions used. In the end, the results showed that the proportions used in the design of the sunken courtyard, courtyard, and earth of the case studies are related and the traditional Iranian sunken courtyards have been designed mostly based on the use of Gereh (a unit of measurement), which was the most appropriate and most widely used scale in housing architecture.

Keywords: sunken courtyard, geometric ratio, statistical analyze, Gereh.

Introduction

In the ancient world, two civilizations such as Babylon and Egypt made the foundation of geometric knowledge more than other civilizations. The first scientists of geometry never wanted to know geometry but turned to geometry for their daily needs. Probably the ancient Egyptians, Sumerians, and Babylonians were the first to discover the principles of geometry (Najafgholi Pour Kalantari et al., 2017, p. 481). In addition, in Islamic art, all universes can be understood by geometry, numbers and the alphabet. The development of Islamic geometric patterns began with the translation of Greek Sanskrit[1] texts (Akkach, 2012, p. 18). Of course, this happened about three centuries after the advent of Islam, and we can say that there is a gap in the growth and development of geometry of buildings from the beginning of the 7th century to the end of the 9th century. At that time, the first examples of the geometric layout of buildings in Islamic countries were created (Abdullahi & Embi, 2013, p. 244).

A quantitative approach to evaluating architectural design proposals for historic settings needs to supplement existing qualitative design guidance (Hu et al., 2017, p. 93). Using the geometry and proportions in design as an art to create shapes, patterns and proportions recalls the role of God as the architect of the world. In other words, geometry and proportions are considered as a key element in establishing the relationship between components of design which is expressed by its constructor and it is perceived by the audience and reference to his being (Nasrabadi et al., 2016, p. 42). Various built-environment attributes related to physical form are posited "to create a safe, inviting, and well-used public realm with visual interest" (U. S. Green Building Council, 2009, p. 17; Oreskovic et al., 2014, p. 220). In Iran, the main emphasis of architecture and urbanism has been based on a kind of aesthetic worldview. Over the centuries, the Iranian people have always had a great value for beauty and the science of geometry, and the proportions were considered as a powerful tool for creating balance, harmony, beauty, and order

[1] Sanskrit is an Old Indo-Aryan language. As one of the oldest documented members of the Indo-European family of languages, Sanskrit holds a prominent position in Indo-European studies.

*Corresponding author. E-mail: *hosei.n.aseri73@gmail.com*

in the world of creation (Nadimi, 1999, p. 32). The most popular proportions used both in Iranian architecture and in European architecture are golden ratio with a value of 1.618, 1.41 ratios (especially in palaces of Persepolis and Apadana), 1.73 ratios (Kasra palace in Ctesiphon), 1.118 ratio (Sarvestan and Ctesiphon palaces) (NoghrehKar, 2011, p. 189).

Geometry is a basic element for establishing unity in Iranian architecture and is the important subject for architects for its discipline and rule (Danaeinia & Azad, 2019, p. 21). In the past architecture of Iran, the design and implementation of building forms and sizes was done using the geometry knowledge. The use of the Hanjar triangle method was one of the methods in which the architect of the building performed all the large and small divisions of the building simply and accurately (Mehdizadeh et al., 2011, p. 15); Also from the Timurid period, the use of checkered plates based on geometry to draw a plan of the buildings has been common (Najiboghlo, 1999). Aboalghasemi emphasized that the design of the courtyard based on the Iranian golden ratio, provides a healthy and pleasant environment for residents throughout the year and provides the best lighting for the rooms and spaces around the courtyard (Aboalghasemi, 2005).

Paying attention to the geometry and proportions of buildings has always been of interest to architects and researches in this field, and the discovery of the elements of Iran's climatic architecture is of great importance (Mahdavi Nejad et al., 2013; Silvayeh et al., 2013; Nabavi & Ahmad, 2016; Sajjadzadeh et al., 2015; Abdi et al., 2016).

A house serves as a shelter and, ideally, matches its natural surroundings by being designed according to the exposure, views, daylight, sunlight, shade, greenery, and other conditions of the site (Bjerregaard, 2005; Forshed & Nylander, 2003; Wågø et al., 2016, p. 330). Considering the different-climate zones in Iran, the architects of building in each region have always sought solutions to create satisfactory conditions for life. To this end, architects in hot, dry and, desert climate of Iran, have been looking for a way to adaptation the burning heat of the desert and create climate-friendly space as well as access to flume water. The sunken courtyard is an architectural space that is an appropriate answer to the design of the building in this climate and to achieve the goals set by the architects of the buildings. In fact, the sunken courtyard has sunk into the deep soil to create a temperature balance and favorable climatic conditions (Guia et al., 2013, p. 91). There is little research on the proportions of this traditional architecture space in Iran.

More research on the sunken courtyard has been carried out into the history, definitions and the role of this architectural element. Hence, the lack of studies in the field of geometry and proportions of sunken courtyards in Iranian houses has led to this study. Furthermore, the answer to this question is very important, which kind of ratio has been used between the length and width of this architectural space?

The rest of this paper has been presented as following. The Section 1 named "materials and methods". Section 2 inclodes research literature. The results are presented in Section 3 and finally in the last section we show the conclusion.

1. Materials and methods

For such research, there is a need to use statistical analysis. For that, in this study, the measure of the length and the width of sunken courtyards have been calculated and analyzed via appropriate statistical methods and tools. Statistical tools are often used as a means of reporting results. When there is a discussion about which means to use for different metrics, the impact of the underlying assumptions involved in reporting results in the architecture community has been largely unexplored. In the current work, we plan to investigate the validity of assumptions such as the normality of the measured data and the use of significance tests in the 95% and 99% confidence level. The main assumptions for the statistical parametric test are to have: Normality: Data have a normal distribution (or at least is symmetric), Homogeneity of variances: Data from multiple groups have the same variance, Linearity: Data have a linear relationship, Independence: Data are independent. Several distribution tests have been suggested to check the normality of data and whether the data set is well-modeled by a normal distribution. We use the Shapiro-Wilk test because it is recommended in the NIST Handbook and the Kolmogorov-Smirnow test. The statistical null hypothesis is H0: Distribution is Normal and for p-values greater than 0.05 it works.

Finally, after confirmation of the normality of the data, by t-test and descriptive statistics, the superiority of each of the hypotheses is compared with others.

2. Research literature

2.1. Geometry, proportions and module

In Persian Dehkhoda Dictionary, geometry is referred to as a particular subfield of mathematics, in which the space, shapes, and imaginable objects are studied and discussed (Bemanian et al., 2010). Geometry is the fundamental science of forms and their order. Geometric Figures, forms, and transformations build the material of architectural design. In the history of architecture geometric rules, based on the ideas of proportions and symmetries formed, fixed tools for architectural design. Proportions were analyzed in nature and found as the general aesthetic categories across the nature and art (Leopold, 2006, p. 1). Geometry and geometrical patterns in Iranian architecture are such a critical issue that, without paying attention to their structural role, the spatial-structural analysis of this type of architecture will be essentially descriptive (Kiani & Amiriparyan, 2016).

The system of proportion exists everywhere in nature, such as the human body, shapes, artwork, music, painting, and even in the cosmic design of the universe. In architecture, the proportion is defined as a relationship between one part of the design to others or to the whole design that may create harmonious and aesthetical (Gangwar, 2017, p. 172). The proportional systems are mental, not visual constructs. Thus, you cannot see the usage of numerical ratios, which are expressed in terms of the local unit of measure, when a particular proportional system was designed (Cohen, 2014, p. 3). Proportion plays an important role in all three parts, as it provides guidelines for laying out useful spaces, for designing structural systems, and for creating an aesthetically pleasing environment. The proportions of a space can dramatically change how visitors feel in it, and the proportions of a facade design can affect whether a building appears welcoming, threatening, or impressive (Josephine, 2017, p. 266). An example of the proportions used in the design and construction of Iranian architectural buildings is the proportions of 1.118, 1.618, 1.41 and 1.73, each of these is briefly explained in the following.

2.1.1. Root rectangles

First, we draw the square whose side is 1 unit and then draw an arc as long as its diameter. The larger side of the rectangle obtained is equal to the square diameter (2) and the diameter of this rectangle is equal to 3. With a rectangle diameter of 3, we can draw a rectangle 4 and this process continues (see Figure 1). These rectangles are called dynamic rectangles (Jormouzi & Salehi, 2014).

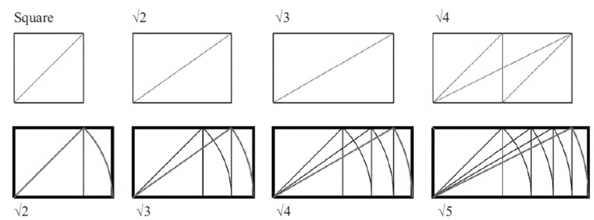


Figure 1. The root proportions based on the square (Dabbour, 2012, p. 383)

2.1.2. Golden ratio

The concept of the golden section is theorized by Greek Philosopher Plato[2]. He has established the relationship between the two-unequal segment of the line, as per his theory that if the ratio between the sum of two segments of the line to larger part is same as the ratio between large segment to small segment, the two-part is in the golden ration (Gangwar, 2017, p. 172) (see Figure 6). The Golden Ratio (also called the Golden Proportion, Golden Section, Golden Mean, Divine Ratio, and Divine Proportion) (Markowsky, 1992) is a transcendent ratio found in fundamental forms: plants, flowers, viruses, DNA, shells, planets, and galaxies. Although the Golden Ratio is primarily regarded as a proportion, not a number; as a numerical quantity, it is defined to be 1.618 (Hejazi, 2005). To draw a golden rectangle, divide the square with the side length of one unit into two parts, then draw the rectangle diameter and we draw the arc as much as the rectangle diameter. The resulting point represents the location of the golden rectangle formation (see Figure 2).

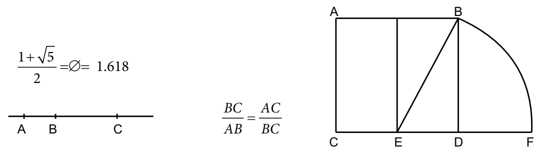


Figure 2. Golden ratio (authors)

2.1.3. 1.118 ratio

The ratio 1.118 multiplied by two equals 2.236, the square root of five. In another word, two $\sqrt{5}$ rectangles side-by-side has the ratio of 1.118 (Hedian, 1976, p. 412) (see Figure 3).

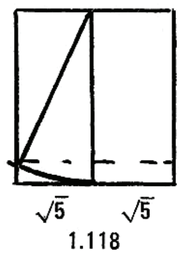


Figure 3. Golden ratio (Hedian, 1976, p. 413)

2.1.4. Gaz as an Iranian Module

Specific units were used for the majority of the parts in a traditional building. An example of this is the usage of the specific brick size. This is done for the purpose of harmonizing various buildings put together. The measurement unit in Iran is called Gaz[3] (see Table 4). All elements, especially openings used to be built based on this unit and its proportion (Nabavi & Ahmad, 2016, p. 146) (see Table 1).

Table 1. Unit of traditional measurement (Nabavi & Ahmad, 2016)

1	One Gereh = 1.16 Gaz = 6.66 cm
2	One Gaz = 16 Gereh = 106.66 cm

[2] Plato was a philosopher in Classical Greece and the founder of the Academy in Athens, the first institution of higher learning in the Western world.

[3] Gaz is from old units long. In the eleventh century AH, a gaz, according to Jean Shardin, was 94.745 cm, and according to Freire, it was 95 cm. Nowadays in Iran there is only a kind of Gaz and it is 106 cm.

3. Research area

People in the warm and dry regions of Iran have always been looking for ways to cope with the climate conditions of the region (Tavassoli, 2003, p. 649). In these areas, such as Yazd and Kshan, buildings are built in an introverted way, in the form of a central courtyard and providing a climate of comfort for the inhabitants of the building (Shateryan, 2008, p. 309). Using this method, the people of this climate were able to reduce the demand for energy compared to buildings on the ground using the soil insulation characteristics and land cover (Imani chat ghaye & Heidari, 2018, p. 90).

3.1. Central courtyard

The central courtyard is a kind of courtyard that has at least one side and a maximum four sides of it surrounded by a building and is the main space for providing lighting and ventilation and an internal connection to the spaces of the houses (Samarghandi, 1964, p. 59).

3.2. Sunken courtyard

The sunken courtyard is a kind of central courtyard whose height is lower than the level of the house and the passageway. In some cases, the sunken courtyard was part of the central courtyard which made the central courtyard have two levels (Soltan Zadeh, 2011, p. 82) (see Figure 4).

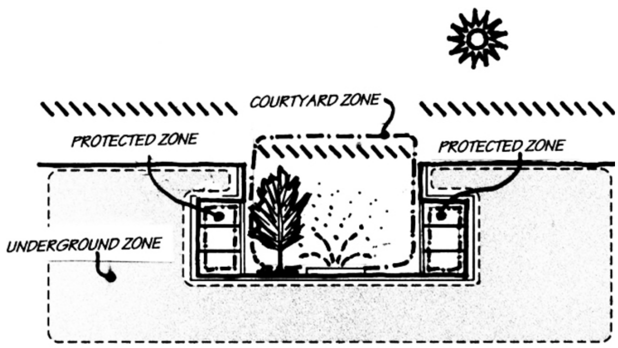


Figure 4. The sunken courtyard concept (Al-Mumin, 2001, p. 105)

In general, it can be said that the sunken courtyard is not related to a certain period, and during the time when the climate of the desert areas became hot and dry, the building was mostly dipped in a hole until they were formed in the sunken courtyard (Guia et al., 2013, p. 94). In Figure 5, the process of building a sunken courtyard has been shown schematically.

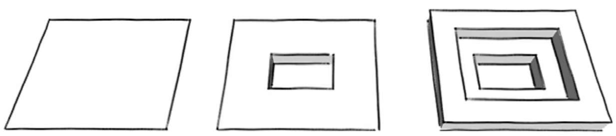


Figure 5. Process of building a sunken courtyard (authors)

4. Investigation and analysis of case studies

In this study, the case studies have been selected based on the following criteria: 1. The first criterion is that the building was built during the Qajar period. 2. Availability and access to architectural documents of the building and the possibility of field visits and measurements. 3. Having historical value and being located in the historical context of the city.

Therefore, based on the mentioned criteria, Among the residential buildings of the hot and dry climate of Iran, three houses from Yazd and three houses from Kashan have been selected which were made during the Qajar era (see Figures 6, 7, 8).

Figure 6. Situation of the case studies on Iran map (authors)

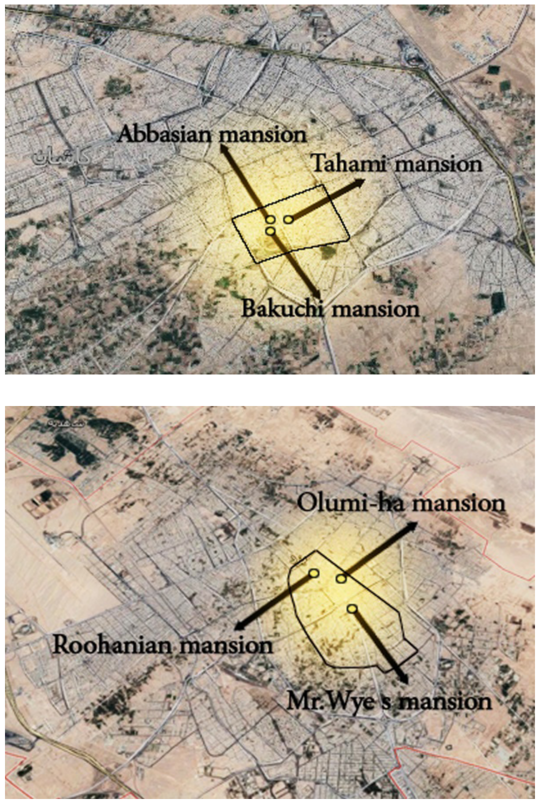


Figure 7. Situation of the case studies in Kashan and Yazd cities (authors)

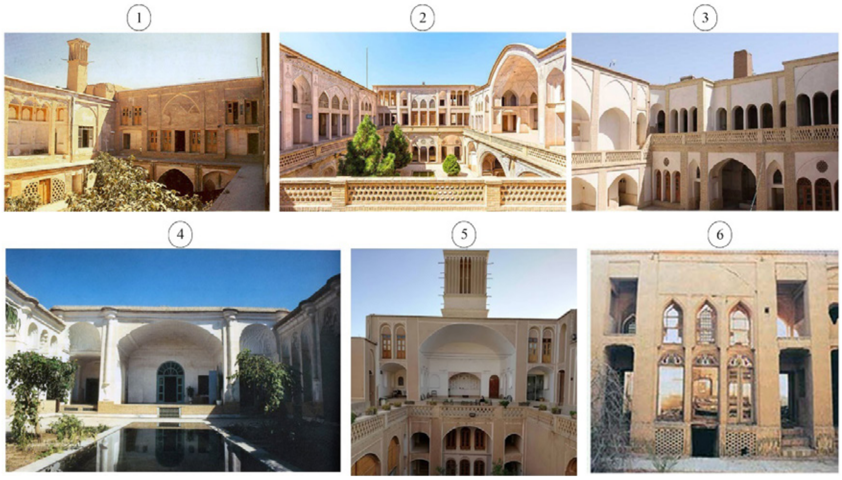
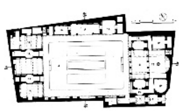
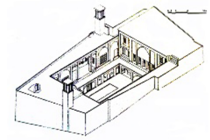
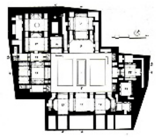
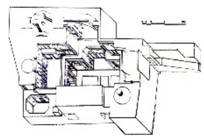
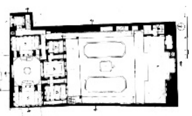
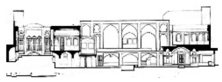
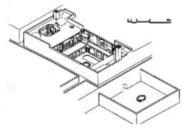
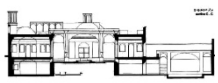
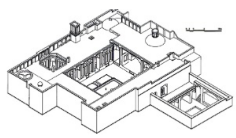
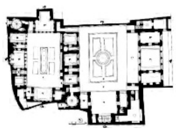
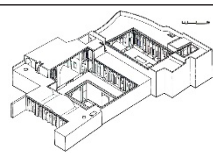
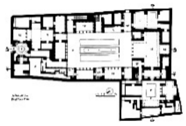
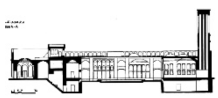
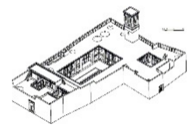


Figure 8. Case studies: 1 – Bakuchi mansion; 2 – Abbasian mansion; 3 – Tahami mansion; 4 – Roohanian mansion; 5 – Olumi-ha mansion; 6 – Mr. Wye mansion (authors)

Table 2. Introduction case studies (authors)

Row	Name	Era	Location	Plan	Section	Perspective
1	Tahami mansion	Qajar	Kashan			
2	Abbasian mansion	Qajar	Kashan			
3	Bakuchi mansion	Qajar	Kashan			
4	Roohanian mansion	Qajar	Yazd			
5	Olumi-ha mansion	Qajar	Yazd			
6	Mr. Wye mansion	Qajar	Yazd			

In Table 2 the brief introduction of these buildings and the position of the sunken courtyard on the plan and section of each house have been shown.

4.1. Analysis the proportion of case studies

To analyze the proportions of case studies, the length, width and height of the sunken courtyard, courtyard and earth of the case studies have been measured and documented. Since the mentioned cases have different length, width and height, the average of them has been considered. These dimensions and differences between their ratios and the other proportions introduced in the previous sections are shown in Table 3.

Table 3. Analysis of the sunken courtyard, courtyard and the earth of the case studies (authors)

The name of the mansion		Length (m)	Width (m)	Height (m)	$\frac{\text{length}}{\text{width}} = A$	$\frac{\text{Height(C)}}{\text{Height(S)}} = B$	Difference A, B with $\sqrt{2} = 1.41$		Difference A, B with $\sqrt{3} = 1.73$		Difference A, B with 1.118		Difference A, B with 1.618	
							A	B	A	B	A	B	A	B
Tahami	Sunken courtyard (S)	17.66	11.50	2.69	1.53	2.73	.12	2.32	.2	1	.412	1.612	.088	1.112
	Courtyard (C)	20.80	15.17	7.35	1.37		.04		.36		.252		.248	
	Earth (E)	42.02	23.48	–	1.78	–	.37	–	.05	–	.662	–	.162	–
Abbasian	Sunken courtyard (S)	18.06	11.54	4.67	1.57	1.52	.16	.11	.16	.21	.452	.402	.048	.098
	Courtyard (C)	18.06	11.54	7.11	1.57		.16		.16		.452		.048	
	Earth (E)	43.19	37.54	–	1.15	–	.26	–	.58	–	.032	–	.468	–
Bakuchi	Sunken courtyard (S)	14.27	11.09	3.24	1.09	2.84	.32	1.43	.64	1.11	.028	1.722	.528	1.222
	Courtyard (C)	23.30	15.60	9.23	1.49		.08		.24		.372		.128	
	Earth (E)	39.60	23.09	–	1.71	–	.3	–	.02	–	.592	–	.092	–
Roohanian	Sunken courtyard (S)	11.27	10.11	5.10	1.11	1.38	.30	.03	.62	.35	.008	.262	.508	.238
	Courtyard (C)	16.01	12.44	7.08	1.28		.13		.45		.162		.338	
	Earth (E)	40.12	37.47	–	1.07	–	.34	–	.66	–	.048	–	.548	–
Olumi-ha	Sunken courtyard (S)	14.39	8.61	6.80	1.67	1.25	.26	.16	.06	.48	.552	.132	.052	.368
	Courtyard (C)	19.73	13.49	8.52	1.46		.05		.27		.342		.158	
	Earth (E)	46.94	33.84	–	1.38	–	.03	–	.35	–	.262	–	.238	–
Mr. Wye	Sunken courtyard (S)	3.5	2.5	2.75	1.4	2.29	.01	.88	.33	.56	.282	1.172	.218	.672
	Courtyard (C)	7.68	6.89	6.30	1.11		.3		.62		.008		.508	
	Earth (E)	65.47	43.16	–	1.51	–	.1	–	.22	–	.392	–	.108	–

In order to analyze whether Iranian architects in designing and constructing sunken courtyard, courtyard and the earth of the mansions have used Gereh as a traditional measurement method, it is necessary to convert the length, width and height of the mentioned cases to Gereh. Therefore, if we convert the numbers obtained in Table 2 to centimeters and divided by 6.66, then the result will be in Gereh (see Table 4).

4.2. Statistical analysis

Table 5 shows the results of the tests, both *p*-values are greater than 0.05 which confirms that our measured data modeled by the Normal distribution. The decision is to fail to reject the null hypothesis. In consequence, the data in this study follow the normal distribution.

Table 4. Dimension of the sunken courtyard, courtyard and the earth of the case studies in Gereh (authors)

The name of the mansion		Length-Width-Height	In Gereh	Significant units of Gereh (.0–.25– .5– .75)	Difference
Tahami	Sunken courtyard	Length	265.16	.25	.09
		Width	172.67	.75	.08
		Height	40.39	.5	.11
	Courtyard	Length	312.31	.25	.06
		Width	227.77	.75	.02
		Height	110.36	.25	.11
	Earth	Length	630.93	.0	.07
		Width	352.55	.5	.05
Abbasian	Sunken courtyard	Length	271.17	.25	.08
		Width	173.27	.25	.02
		Height	70.12	.0	.12
	Courtyard	Length	271.17	.25	.08
		Width	173.27	.25	.02
		Height	106.75	.75	0
	Earth	Length	648.49	.5	.01
		Width	563.66	.75	.09
Bakuchi	Sunken courtyard	Length	214.26	.25	.01
		Width	196.54	.5	.04
		Height	48.64	.75	.11
	Courtyard	Length	349.84	.75	.09
		Width	234.23	.25	.02
		Height	138.58	.5	.08
	Earth	Length	594.59	.5	.09
		Width	346.69	.75	.06
Roohanian	Sunken courtyard	Length	169.21	.25	.04
		Width	151.80	.75	.05
		Height	76.57	.5	.07
	Courtyard	Length	240.39	.5	.11
		Width	186.78	.75	.03
		Height	106.30	.25	.05
	Earth	Length	602.40	.5	.1
		Width	562.61	.5	.11
Olumi-ha	Sunken courtyard	Length	216.06	.0	.06
		Width	129.27	.25	.02
		Height	102.10	.0	.1
	Courtyard	Length	296.24	.25	.01
		Width	202.55	.5	.05
		Height	127.92	.0	.08
	Earth	Length	704.80	.75	.05
		Width	508.10	.0	.1
Mr.Wye	Sunken courtyard	Length	52.55	.5	.05
		Width	37.53	.5	.03
		Height	41.29	.25	.04
	Courtyard	Length	115.31	.25	.06
		Width	103.45	.5	.05
		Height	94.59	.5	.09
	Earth	Length	983.03	.0	.03
		Width	648.04	.0	.04

Table 5. Tests of Normality (authors)

	Kolmogorov-Smirnov			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Measure	.147	18	.200*	.937	18	.260

In the next step, the significance test is used to determine if the mean of measured data significantly differs from the Iranian architecture standard measures in Table 3 (1.41, 1.73, 1.118, and 1.618) in 95% and 99% confidence level. For each case, we do the one-sample *t*-test. Based on the results in Table 6; the measured data is not significantly differing from the standard measure (*p*-values are greater than 0.01 or 0.05).

In Table 6, we have extended the confidence interval from 95% to 99% to have more chances to reject the inequality of the measurements. There is no significant difference between our data and the standard measure. According to data in Table 4, another way to analyze the significance of results is to use the independent student's *t*-test to check the difference in Gereh and its units. The null hypothesis can be stated as H0: the means of the measured Gereh is equal to the same means for the units with significance level α = 5%, So if the *p*-value is less than 5%, the null hypothesis is rejected, and the means have a 5% significant difference. The independent sample student's *t*-test is a good fit for this case. In the first part of the Table 7, there is a significant statistical test named Leven's test for equality of variances. The *p*-value of the test 0.719 is greater than 0.05 and it means the equality of variance in both groups. About the differences in the mean of group, with the *p*-value is 0.993, there is no significant differences between the two means.

Table 8 includes the descriptive statistic of the standard measures. The measure of the Coefficient of Variation (CV) is the main part of the table in which, we can easily inference that how much our measured data differed from the standard one. The smaller measurements of CV guarantee the smaller variation of our data from standard measures. In among all the CV's, Gereh has the smallest one and it could be chosen as a most appropriate measure in Iranian architecture.

Table 6. One-Sample *t*-test (authors)

Standard measure	*t*-test value	*p*-value	Mean	Test result
1.41	–.136	.893	1.4028	CI-95%-99% accepted
1.73	–6.161	.000	1.4028	CI-99% accepted
1.118	5.362	.000	1.4028	CI-99% accepted
1.618	–4.052	.001	1.4028	CI-95%-99% accepted

Table 7. Independent *t*-test for comparing the mean of the measurements to the ideal proportions (authors)

		Levene's test for equality of variances		*t*-test for equality of means					
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% CL Difference	
									Lower	Lower
Gereh	Equal variances assumed	.069	.793	1.243	94	.217	.06500	.05228	–.03881	.16881
	Equal variances not assumed			1.243	93.910	.217	.06500	.05228	–.03881	.16881

Table 8. Descriptive statistics (authors)

	N	Mean		Std. Deviation	Coefficient of Variation (Std. Deviation/Mean) ×100	Variance
	Statistic	Statistic	Std. Error	Statistic		Statistic
Gereh	48	.4672	.06350	.26942	57.6669	.073
1.41	18	.1850	.02847	.12079	65.2918	.015
1.73	18	.3328	.05104	.21655	65.0691	.047
1.118	18	.2950	.04972	.21095	71.5084	.044
1.618	18	.2492	.04350	.18454	74.0529	.034

Conclusions

The results obtained from the researches that have been done so far on the geometric ratios used in traditional Iranian buildings, indicate that the architects of this buildings sought to bring the dimensions of the spaces as close as possible to some geometric ratios, for this purpose, proportional adjustment systems such as (1.118, 1.618, 1.41, 1.73, Gereh) heve been used. This method, in addition to organizing and spatial arrangement of these buildings, provides maximum use of sunlight for the sunken courtyard and surrounding spaces for the comfort of the building's residents in different season. Therefore, it can be said in this research, based on the statistical analysis of all measurements and their coefficient of variation in Table 8, Gereh, has the better agreement to be an appropriate scale matching to the real scales of that architects. It means, the proportions used in the design of the sunken courtyard, courtyard, and earth of the case studies are related and the Gereh as a proportional adjustment system has been used in designing of sunken courtyards in Yazd and Kashan mansions.

References

Abdi, M., Abouei, R., & Beigzadeh Shahraki, H. R. (2016). *The study of the footprints of the garden pit in the architecture of Yazd during Atabakan and Al-Muzaffar era* [Conference presentation]. International Conference on Contemporary Islamic-Iranian Architectural Traditions, Ardebil.

Abdullahi, Y., & Embi, M. R. B. (2013). Evolution of Islamic geometric patterns. *Frontiers of Architectural Research*, *2*(2), 243–251. https://doi.org/10.1016/j.foar.2013.03.002

Aboalghasemi, L. (2005). The norm of formation in Islamic architecture of Iran. In *The endeavor of Mohammad Y. Kiani*. Samt Publication.

Akkach, S. (2012). *Cosmology and architecture in premodern Islam: An architectural reading of mystical ideas*. SUNY Press.

Al-Mumin, A. A. (2001). Suitability of sunken courtyards in the desert climate of Kuwait. *Energy and Buildings*, *33*(2), 103–111. https://doi.org/10.1016/S0378-7788(00)00072-4

Bemanian, M., Amirkhani, A., & Lilian, M. (2010). *Order and disorder in architecture*. Tahan and Hele Publications.

Bjerregaard, L. M. (2005). *Forsegling og symbiose: Naturvidenskab og naturromantik — en dialogi moderne arkitektur: Belyst via studier af grænsen mellem inde og ude (Danish)*. Arkitektskolens Forlag.

Cohen, M. (2014). Introduction: Two kinds of proportion. *Architectural Histories*, *2*(1), 1–14. https://doi.org/10.5334/ah.by

Dabbour, L. M. (2012). Geometric proportions: The underlying structure of design process for Islamic geometric patterns. *Frontiers of Architectural Research*, *1*(4), 380–391. https://doi.org/10.1016/j.foar.2012.08.005

Danaeinia, A., & Azad, M. (2019). The role of geometry of yard in the formation of the historical houses of Kashan. *Mathematics Interdisciplinary Research*, *4*(1), 21–35.

Forshed, K., & Nylander, O. (2003). *Bostadens omätbara värden (Swedish)*. HSB Riksförbund.

Gangwar, Ar. G. (2017). Principles and applications of geometric proportions in architectural design. *Journal of Civil Engineering and Environmental Technology*, *4*(3), 171–176.

Guia, A., Memarian, G. H., & Saffaran, E. (2013). The meaning of the sunken courtyard and its place in traditional Iranian architecture. *Athar Journal*, *34*(60), 90–104.

Hedian, H. (1976). The golden section and the artist. *The Fibonacci Quarterly*, *14*, 406–418.

Hejazi, M. (2005). Geometry in nature and Persian architecture. *Building and Environment*, *40*(10), 1413–1427. https://doi.org/10.1016/j.buildenv.2004.11.007

Hu, Y., Heath, T., Tang, Y., & Zhang, Q. (2017). Using quantitative analysis to assess the appropriateness of infill buildings in historic settings. *Journal of Architectural and Planning Research*, *34*(2), 91–113. https://www.jstor.org/stable/44987221

Imani chat ghaye, F., & Heidari, S. (2018). A comparison of energy consumption in an underground building with a similar on-ground model in the climates of Tehran, Yazd, and Tabriz. *JIAS*, *1*(13), 89–105.

Jormouzi, S., & Salehi, S. (2014). Beautiful proportions: Comparative comparisons of structural proportions of one of the marriages in the sacred treasury of Razavi with the conventional system of proportions in the West. *Ganjine-ye Asnad*, *23*(90), 120–137.

Josephine, E. E. C. (2017). The study of geometric forms, proportion and scale of heritage buildings due to architectural theory. *IPTEK Journal of Proceedings Series*, *3*(3), 263–272. https://doi.org/10.12962/j23546026.y2017i3.2455

Kiani, Z., & Amiriparyan, P. (2016). The structural and spatial analysing of fractal geometry in organizing of Iranian traditional architecture. *Procedia-Social and Behavioral Sciences*, *216*, 766–777. https://doi.org/10.1016/j.sbspro.2015.12.074

Leopold, C. (2006). *Geometry concepts in architectural design* [Conference presentation]. 12th International Conference on Geometry and Graphics, Salvador, Brazil.

Mahdavi Nejad, M. J., Yari, F., Silvayeh, S., & Kermani mazhari, A. R. (2013). *Explaining the position of geometry in traditional Iranian architecture in order to take advantage of ancient patterns to achieve sustainable architecture and urbanization* [Conference presentation]. The 1st Conference on Architecture and Sustainable Urban Spaces, Mashhad.

Markowsky, G. (1992). Misconceptions about the Golden Ratio. *The Collage Mathematics Journal*, *23*(1), 2–19. https://doi.org/10.1080/07468342.1992.11973428

Mehdizadeh, F., Tehrani, F., & Valibeig, N. (2011). Applying the 'Hanjar' triangles in the mathematical calculation, implementation and enforcement of traditional Iranian architecture. *Maremat-e Asar* & *Baft-haye Tarikhi-Farhangi*, *1*(1), 15–26.

Nabavi, F., & Ahmad, Y. (2016). Is there any geometrical golden ratio in traditional Iranian courtyard houses? *Archnet-IJAR: International Journal of Architectural Research*, *10*(1), 143–154. https://doi.org/10.26687/archnet-ijar.v10i1.744

Nadimi, H. (1999). The truth of the role. *Academy of Sciences*, *6*(14), 19–34.

Najafgholi Pour Kalantari, N., Etessam, I., & Habib, F. (2017). Manifestation of geometry and proportions in traditional architectural monuments of Iran in the Azeri style of the geographical range of Azerbaijan. *Geograpical Journal of Territory*, *14*(54), 115–131.

Najiboghlo, G. (1999). *Geometry, and decoration in Islamic architecture* (M. Ghaiomibidhendi, Trans.). Rozane Publication.

Nasrabadi, A., Pour Jafar, M. R., & Daghahi, A. A. (2016). An analysis of the aesthetic role of geometry in the formation of the urban space of four Abbasid gardens. *Urban Studies*, *5*(17), 41–54.

NoghrehKar, A. H. (2011). *Theoretical foundations of architecture*. Payame Noor University.

Oreskovic, N. M., Charles, P. R. S. L., Shepherd, D. T. K., Nelson, K. P., & Bar, M. (2014). Attributes of form in the built environment that influence perceived walkability. *Journal of Architectural and Planning Research*, *31*(3), 218.

Sajjadzadeh, H., Hemmati, S., & Farahanikia, B. (2015). *The effect of sunken courtyard on the climate comfort of the residents of Kashan district case study of Bakuchi House* [Conference presentation]. National Conference on Civil and Architecture with an Approach to Sustainable Development, Fouman.

Samarghandi, A. T. (1964). *Samarieh*. Farhang Iran Zamin.

Shateryan, R. (2008). *Climate and architecture of Iran*. Simaye Danesh.

Silvayeh, S., Daneshjo, K., & Farmihan Farahani, S. (2013). Geometry in pre-Islamic Iranian architecture and its manifestation in contemporary Iranian architecture. *Naqshejahan*: *Basic Studies and New Technologies of Architecture and Planning*, *3*(1), 55–66.

Soltan Zadeh, H. (2011). The role of geography on formation courtyards in traditional houses in Iran. *Human Geography Research*, *43*(74), 69–86.

Tavassoli, M. (2003). *Construction of city and architecture in warm and dry climate of Iran*. Payam Publisher.

U. S. Green Building Council. (2009). *LEED 2009 for neighborhood development rating system*. http://www.usgbc.org/resources/leed-neighborhood-development-2009-current-version

Wågø, S., Hauge, B., & Støa, E. (2016). Between indoor and outdoor: Norwegian perceptions of well-being in energy efficient housing. *Journal of Architectural and Planning Research*, *33*(4), 326–346. https://www.jstor.org/stable/44987209