PRICING MODELS OF COVERED BONDS – A NORDIC STUDY

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ABSTRACT. Covered bonds are an alternative way of investing indirectly in the debt side of real estate, which is beneficial for investors looking for alternatives to government or corporate bonds. Due to the dual nature of the protection offered by covered bonds, they have a justified place in investors' portfolios. This paper studies the pricing of covered bonds and tests it with data gathered from the Nordic countries. Using the tested reduced form model, it was possible to price covered bonds with satisfactory results. The estimated model was highly statistically significant and performed according to the economic reasoning behind it. The estimated model also worked well in comparison to research conducted earlier on competing models, such as the structural models.

KEYWORDS: Covered bonds; Reduced form models; Structural models; Pricing; Indirect real estate investment

1. INTRODUCTION

Indirect real estate investments can take many forms. The academic research focuses mainly on the equity side of indirect real estate investments, like REITs, funds and stocks, as the literature review by Worzala and Sirmans (2003) summarizes. There exists also the debt side of indirect real estate investments, where investors can invest indirectly in both consumer mortgages and commercial real estate debt. Banks being the main source for mortgage and commercial lending, it is possible to invest in the stocks of the banks to indirectly invest in the banks' lending business, but there are better solutions for this. Covered bonds and residential mortgage-backed securities are the answer if the investor wants to invest indirectly in consumer mortgage lending, while commercial mortgage-backed securities are the product for investing indirectly in commercial real estate lending.

Covered bonds are well-recognized products in the European fixed-income markets. According to the European Covered Bond Council (2009), covered bonds account for approximately one fifth of the European fixed-income markets and are also rising up in the US (Lucas et al., 2008), where the main source of funding for mortgage lending are mortgage-backed securities. With its characteristics, covered bonds offer investors a secure way of investing indirectly in mortgages and banks a cost-efficient way of funding the mortgage lending business.

Even though covered bonds and mortgage-backed securities are aimed at the same thing, funding mortgage lending, the structure of these products is very different. Covered bonds are on-balance sheet funding similar to corporate debt securities, but with supplementary protection for the investor due to the collateral of mortgages. This means that the security is backed by the collateral of mortgages and the financial solidity of the issuer. In contrast, mortgage-backed securities are off-balance sheet funding and they are only backed by the collateral of mortgages, which means that the financial performance of the security is solely dependent on the performance of the collateral. Typically, covered bonds issued in Europe have the highest credit ratings while the credit ratings of mortgage-backed securities vary more (Lucas et al., 2008).

The added protection of the covered bonds is reflected in the prices of covered bonds, as investors are willing to accept lower rates of return due to enhanced protection. For banks, issuing covered bonds is beneficial as the lower funding costs can increase their profitability, but the lower funding costs are also beneficial for consumers who are using mortgages to fund their housing investments. From the investors' perspective, the steady cash flows and low risks of covered bonds make them an attractive alternative to government debt securities and corporate debt securities (European Covered Bond Council, 2009). In the risk-return context, covered bonds lie between those two asset classes.

The high popularity of covered bonds among investors makes the covered bond markets quite efficient, which is always a positive feature if pricing applications are concerned. Especially if market based pricing models are used, efficient and liquid markets are a good foundation to build those pricing models on. The aim of this paper is to study pricing models that are suitable for covered bond applications and test the suitability of a chosen model

with data gathered from the Nordic countries. The results of previously published research with corresponding fixed-income securities are also reflected on in relation to the results of this study.

The structure of this paper is as follows. First, the possible pricing models for covered bonds are discussed, followed by the methodology of the study. Chapter four introduces the data used to test the pricing model and chapter five discusses the estimation of the model briefly. In chapter six, the results are presented and chapter seven concludes this paper.

2. PRICING MODELS FOR COVERED BONDS

Covered bonds differ from mortgage-backed securities in terms of cash flow patterns and risks. The main source of risk in covered bonds is the default of the issuer, i.e., credit risk. This is why the pricing models used for covered bond applications should focus on the credit risk and ignore the risk of prepayment, which is one of the main components behind the pricing of mortgage-backed securities. There are a few approaches to price the credit risk. These include the structural models originally by Black and Scholes in 1973 (Black and Scholes, 1973) and further developed by Merton (1974). The alternative approach are the so called intensity-based models, or reduced form models, created by Jarrow and Turnbull (1992) and further developed by Jarrow and Turnbull (1995) and Duffie and Singleton (1999).

The two approaches are regarded as somewhat competing models by academics and practitioners, but they approach the default mechanism from different viewpoints. Structural models assume that the default is triggered when the assets of the company are less than the liabilities of the company, whereas the reduced form models take into account the probability of the default and the potential loss in case of default. According to Jarrow and Protter

(2004), the key difference between the two models is in the informational assumptions. In structural models, the company's event of default is predictable, i.e., the company's managers have the information about the ratio of assets and liabilities. The reduced form models, on the other hand, rely on the information that is shared by the participants of the market in which the company's securities are traded and predict the probability of the default and the potential loss from the pricing decisions of the other market participants.

For market based pricing using financial market data, there is a very useful reduced form model developed by Jarrow in 2001 (Jarrow, 2001). The model prices both debt and equity securities by using observations of past data on security prices, stock index and interest rates. All the four components needed by the model are easily available from financial information providers like Thomson Reuters or Bloomberg. As this study focuses on the covered bonds, the equity pricing feature of the Jarrow 2001 model is ignored.

The existing literature concerning covered bonds and especially pricing them is minimal, but the reduced form models and structural models have been studied more. For example, the performance of structural models in investment grade and high-yield securities applications was studied by Turnbull (2005). Kau et al. (2006) presented reduced form models for mortgage-backed securities valuation as an alternative to option-pricing models. In commercial mortgage-backed securities setting reduced form models have been studied by Kau et al. (2009). Jobst (2005) studied collateralized debt obligations, mortgage-backed securities and covered bond spreads in a GARCH model application.

In addition to pricing applications, the reduced form models can be used in variety of other applications. Ambrose and Yildirim (2008) developed a reduced form credit risk model for leases. Crook and Bellotti (2010) introduced reduced form models as an alternative model to use in consumer loan credit risk applications. Edge et al. (2010) used reduced form models as benchmark models for economic forecasting in central bank application.

3. METHODOLOGY

As the data of covered bond prices for this study is gathered from the European financial markets and many covered bond issuers are not stock exchange listed companies, it is obvious that the structural models are not suitable for this study. The informational assumptions of having an information set similar to the company's management, like accounting data, is in the case of non-listed companies hard to meet, as the necessary information is not easily or publicly available. Furthermore, the perspective of the market participant making the pricing decisions supports the use of reduced form models in the market based pricing of covered bonds.

Jarrow's debt pricing model (Jarrow, 2001) uses three components for describing a price for a security. The components are price data for securities which are similar to that being priced, stock market index data which acts as a proxy of the state of the economy and interest rate data which is a risk-free benchmark investment. Using the above mentioned components, the model is used to estimate the hazard rate $\lambda(u)[1-\delta(u)]$, which is a function of the probability of default and the estimated loss in case of default. Using the hazard rate, the expected price at t of a security v can be written as presented in equation 1. The price of the security is dependent on two components of the discount factor, the spot rate of interest and the risk premium by the estimated hazard rate.

$$v(t,T:i) = E_t \left[e^{-\int_t^T \{r(u)+\lambda(u)[1-\delta(u)]\}du} \right] \qquad (1)$$

Jarrow's model uses Vasicek's (1977) extended term structure model to describe how the spot rate of interest evolves. The term structure model is a mean reverting process with the parameters a the constant mean reversion parameter, σ_r the constant of volatility of the spot rate, and $\bar{r}$ is a deterministic function of t chosen to match an initial bond price curve, with $W(t)$ being standard Brownian motion. The equation for the spot rate is presented in equation 2.

$$dr(t) = a_t\left[\bar{r}(t) - r(t)\right]dt + \sigma_r dW(t) \qquad (2)$$

In contrast to the spot rate model, the evolution of the stock market index is assumed to progress as presented in equation 3.

$$dM(t) = M(t)\left[r(t)dt + \sigma_m dZ(t)\right] \qquad (3)$$

where: σ_m is the constant of volatility of the market index and $Z(t)$ is standard Brownian motion; $Z(t)$ is correlated with $W(t)$ as $dZ(t)$ $dW(t) = \varphi_m dt \cdot \varphi_{rm}$ is the correlation between the market index and the spot rate. From equation 3, $Z(t)$ can be solved with observed dates *1, 2, 3, ..., t,*

$$Z(t) = Z(t-1) +$$

$$\left[\frac{\log\dfrac{M(t)}{M(t-1)} - \displaystyle\int_{t-1}^{t} r(u)du + \displaystyle\int_{t-1}^{t}\frac{1}{2}\sigma_m^2 du}{\sigma_m}\right] \qquad (4)$$

The market index follows a process that is based on geometric Brownian motion with drift $r(t)$ and volatility σ_m. The drift is the spot rate and the process $Z(t)$ measures the cumulative excess return per unit of risk on the market index and it is possible to estimate it using data containing time series observations.

Having obtained both of the needed parameters $r(t)$ and $Z(t)$, and assuming that they are normally distributed, the hazard rate parameters are:

$$\lambda(t) = \lambda_0 + \lambda_1 r(t) + \lambda_2 Z(t) \qquad (5)$$

and

$$\delta(t) = \delta \qquad (6)$$

With constants λ_0, λ_1, λ_2 and δ, the probability of default is a linear function of the variables $r(t)$ and $Z(t)$. It is possible that the function has values that imply negative default rates, i.e., $\lambda(t) < 0$, but it is reasonable to make the assumption that default rates cannot be negative, i.e., $\lambda(t) \geq 0$. Using the combined hazard rate of equations 5 and 6, the expected price of a security can be rewritten as presented in equation 7.

$$v(t, T : i) = E_t\left[e^{-\int_t^T \{r(u) + [\lambda_0 + \lambda_1 r(u) + \lambda_2 Z(u)](1-\delta)\}du}\right] \qquad (7)$$

The security prices can be estimated using equation 7 combined with nonlinear regression techniques, because the required parameters are easily obtained. Using both time series and cross sectional data containing various maturities of both risky and riskless debt prices, the initial forward curve $f(0, T)$, the term structure parameters a and σ_r, and the market index parameters φ_{rm} and σ_r can be solved. With the model parameters, the hazard rate function parameters λ_0, λ_1, λ_2, and δ can be estimated.

The pricing model parameter signs can be assumed to be the following, according to economic theories. The coefficient for the spot rate of interest, $r(t)$, should be positive, because an increasing spot rate should lead to rising interest expenses that lead to a higher probability of default. The coefficient for the excess market return, $Z(t)$, should be negative, because according to economic theory, a rising stock

market index should reflect an economic cycle of growth, which means that the default probabilities are decreasing.

4. DATA

The data used in this study is constructed from three different components. Interest rate data with various maturities is used to estimate the needed interest rate parameters used by the model as a benchmark of risk free investments. Stock market index data is used by the model to describe the economic conditions. In addition, data from Nordic covered bond prices is needed. Such data is gathered on a daily basis from the Thomson Reuters Datastream financial information system. The chosen interest rate data is the European Monetary Union Native Treasury Curve, a basket of Government bonds of European Monetary Union countries with maturities ranging from six months to fifty years. The stock market index used in this study is the OMX Nordic Price Index, which is a basket of 625 stocks from the Nordic exchanges. The covered bond data contains price information for 44 different covered bonds across all the Nordic countries except Iceland. The time window of the data ranges from July 2007 to the end of December 2008. Despite the short time horizon, the data set contains almost 27,000 observations of daily Euro nominated closing values of covered bond prices, stock index and interest rates, which should lead to reasonably accurate estimation results.

Descriptive statistics of the daily returns of the data are presented in exhibit 1. The daily returns of the stock market index are clearly more volatile than the daily returns of the interest rates and covered bonds, but the differences between interest rate returns and covered bond returns are very small. This illustrates the low risk involved in investing in

covered bonds, as the interest rates are considered as a proxy for the risk-free interest rates. In addition, one notable point is that the economic situation during the time window of the data was to some extent distressed.

5. ESTIMATION

The estimated model and the hazard rate function used in this study are presented as equations 8 and 9. We also need to estimate the interest rate model parameters a_t and σ_t and the stock market index model parameters σ_m and φ_{rm}. The interest rate model parameters, as well as the stock market index model parameters, were estimated using a rolling window of 130 days of observations. The estimated stock market index model is presented in exhibit 2.

$$v(t,T:i) = p(t,T)e^{-\lambda_0(T-t)}e^{-\lambda_1\mu_r-\lambda_2 Z(t)(T-t)+}$$

$$\frac{(2\lambda_1+\lambda_1^2)\sigma_r^2}{2}+\frac{\lambda_2^2(T-t)^3}{6}+(1+\lambda_1)\lambda_2\varphi_{rm}\eta(t,T) \tag{8}$$

$$\lambda(t)(1-\delta(t)) = \max\{\lambda_0+\lambda_1 r(t)+$$

$$\lambda_2 Z(t),0\} \tag{9}$$

Having obtained the needed parameters, the pricing model, i.e. the hazard rate parameters, were joint-estimated using the non-linear regression function of the Mathlab software package. Mathlab's function uses a standard Levenberg-Marquardt least squares estimation algorithm. The software also provides the residuals, the fitted values of dependent variables, and the estimated coefficients, the betas, so confidence intervals and different test statistics can be calculated to support the analysis of the results. The results of non-linear regression estimations can be treated and interpreted similarly to "normal" linear regression (McCabe and Leybourne, 1993).

6. RESULTS

The estimation results for the model in equations 8 and 9 are presented in exhibit 3. From the estimation results, it can be seen that the estimated model fits the data very well, as the r-squared is almost 0.8. The estimated model is also highly significant overall and in the estimated coefficients. The assumptions about the coefficients are met, as the coefficient for the spot rate is positive and for the excess market return negative. Thus, the model works according to the economic reasoning behind it and it is also performing well.

For the structural models, the existing study by Turnbull (2005) suggests r-squared values ranging from 0.3 to 0.8, depending on the type of the security. For investment grade securities, r-squared values were less than 0.3, but for high-yield securities, the values ranged from 0.6 to 0.8. This study, where covered bonds, which can be categorized as investment grade, were used, suggests r-squared values that are clearly higher than those for investment grade securities, which is a positive result and supports the use of reduced form models. In the study by Jobst (2005) concerning a GARCH model for collateralized debt obligations, mortgage-backed securities and covered bond spreads, the conclusion was drawn that covered bonds were the security class that fit the model better than others. This also supports the results obtained by this study.

7. CONCLUSIONS

Covered bonds are a well-recognized product in European fixed-income markets. Despite this, the existing research into them is minimal, which is unfortunate because covered bonds offer investors an alternative way of securely investing indirectly in mortgages and seeking risk-return profiles different from those offered by government or corporate debt securities. Being a liquid asset class, the use of covered bonds is also beneficial in well-diversified portfolios.

This study examined an empirical covered bond application of the reduced form model developed by Jarrow (2001). The results justified the use of reduced form models in market based pricing applications on covered bonds as the estimated model was statistically highly significant and outperformed its rival structural models when the results were compared to existing research. In addition to good statistical performance, the reduced form model of Jarrow (2001) followed the economic reasoning behind the model. Rising interest rates raise covered bond prices because of the rising interest expenses that increase the probability of default. Rising excess returns on the stock market, which is a sign of a rising economic cycle, should lead to decreasing prices of covered bonds due to the decreased probability of default.

The observed performance of the tested reduced form model proves its usability in both theoretical and practical applications when it is necessary to perform market based pricing of covered bonds. As this study was conducted using data from the Nordic covered bond markets, it would be interesting to do additional research to test whether the observed performance of the tested model would correspond to the performance of Europe-wide data. It would be also interesting to test the model by conducting out-if-sample testing.

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EXHIBITS

	Interest rates	Stock market index	Covered bonds
Mean	−0,004 %	−0,224 %	0,005 %
Standard Error	0,002 %	0,101 %	0,003 %
Median	−0,005 %	0,000 %	0,006 %
Standard Deviation	0,047 %	2,005 %	0,049 %
Kurtosis	3,626	6,019	5,488
Skewness	0,105	0,135	−0,248
Range	0,29 %	16,27 %	0,43 %
Minimum	−0,15 %	−8,42 %	−0,21 %
Maximum	0,15 %	7,85 %	0,22 %
Observations	9301	393	17292

Exhibit 1. Descriptive statistics on daily returns of the data
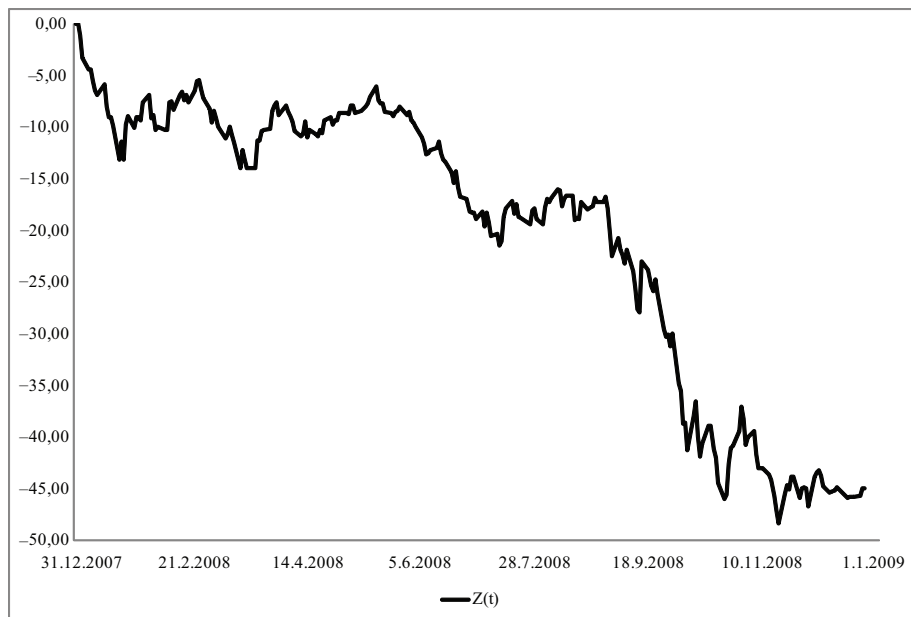
Exhibit 2. Estimated stock market index model

R2   0,795
F   21194,87 p<0,001

	Value	95% confidence intervals		t	p	Variable
λ0	−0,00473	−0,00489	−0,00458	−61,28	<0,001	Intercept
λ1	0,09588	0,08154	0,11022	13,11	<0,001	r(t)
λ2	−0,00018	−0,00018	−0,00018	−80,15	<0,001	Z(t)

Exhibit 3. Estimation results

SANTRAUKA

PADENGTŲ OBLIGACIJŲ KAINODAROS MODELIS: ŠIAURĖS ŠALIŲ TYRIMAS

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Padengtos obligacijos yra alternatyvus netiesioginio investavimo į nekilnojamąjį turtą būdas, naudingas investuotojams, ieškantiems vyriausybės ir įmonių obligacijų alternatyvų. Padengtoms obligacijoms siūlomos dvejopos apsaugos priemonės turi nustatytą vietą investuotojų vertybinių popierių portfeliuose. Straipsnyje tiriama padengtų obligacijų kainodara, kuri patikrinama naudojant duomenis, surinktus iš Šiaurės šalių. Taikant patikrintą modelį buvo galima nustatyti padengtų obligacijų kainą ir gauti patenkinamus rezultatus. Apytikris modelis buvo statistiškai labai reikšmingas ir parengtas remiantis jam priešingu ekonominiu pagrindimu. Apytikrį modelį sėkmingai taikyti, lyginant su anksčiau atliktu tyrimu, padėjo konkurencingi modeliai, pvz., struktūriniai modeliai.