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A DUAL HESITATION FUZZY VIKOR METHOD WITH INCOMPLETE ATTRIBUTE WEIGHTS FOR PROPERTY SERVICE QUALITY EVALUATION

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Abstract. The evaluation of property service quality is a typical multi-index evaluation problem, which has the characteristics of comprehensiveness and incomplete information in the modern service industry environment. To solve the above problem, we propose a dual hesitation fuzzy VIKOR comprehensive decision method based on incomplete attribute weights for evaluating property service quality. The method integrates innovative distance measurement, comprehensive weighting, and VIKOR decision technology to address challenges such as incomplete information, unknown weights, and evaluator hesitation. By incorporating the information value of dual hesitation fuzzy set hesitancy, we have designed a new distance measurement formula and developed a comprehensive weighting method that combines subjective and objective data. The dual hesitation fuzzy VIKOR model utilizes these tools to calculate utility values and regret values in order to make scientifically grounded decisions. This approach provides robust support for enhancing property service quality and demonstrates broad potential applications in fuzzy information processing and other domains.

Keywords: property service quality evaluation, dual hesitant fuzzy set, distance measure, incomplete attribute weights, VIKOR.

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1. Introduction

With the rapid progress of urbanization and the continuous improvement of residents' life quality requirements, property service, as an important part of community management, has a direct bearing on residents' life satisfaction and happiness (Guo et al., 2019). However, property service quality evaluation is a complex and multi-dimensional process, involving multiple evaluation indicators, such as service efficiency, service attitude, environmental maintenance and so on. Traditional evaluation methods are often based on clear attribute weights and complete information, but in practice, due to various subjective and objective reasons, attribute weights are often difficult to determine accurately, and the evaluation information is often incomplete and fuzzy (Kim & Ahn, 2019; Ali et al., 2022). In addition, evaluators often show some hesitation and uncertainty in the evaluation process, which is particularly prominent in the complex decision-making environment. The traditional evaluation method often ignores the psychological characteristics of the evaluator, resulting in the evaluation result may deviate from the actual situation (Zuo et al., 2023).

Despite efforts by some studies to address challenges in evaluating property service quality, shortcomings remain in handling incomplete attribute weights, fuzzy information, and evaluator hesitancy. For instance, earlier research employed relatively simplistic methods such as entropy weight, weighted average, and analytic hierarchy process to determine attribute weights (Zuo et al., 2019). When these methods are employed to determine attribute weights, they may yield inaccurate results or fail to fully capture the actual situation due to issues with data quality, subjective judgment, or inherent methodological limitations. Moreover, some studies overly relied on quantitative data, neglecting the inherent ambiguity in evaluation information and the psychological state of evaluators. Traditional approaches often overlook the complexity of the decision-making environment, which can compromise the scientific rigor and reliability of decisions owing to their strong subjectivity and limited applicability. In service quality assessment, customer perception and satisfaction are influenced by numerous factors that may be vague and challenging to quantify. Although certain studies (Shiu et al., 2016) have attempted to apply fuzzy multi-criteria

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decision making (MCDM) to select professional property management companies, they did not adequately account for evaluators' mental states, such as emotions, expectations, and biases, which can significantly impact evaluation outcomes. This oversight may lead to evaluation results that diverge from customers' true sentiments. Therefore, this study aims to bridge this research gap by proposing more flexible evaluation methods to effectively address issues like incomplete attribute weights, fuzzy information, and evaluator hesitancy, thereby enhancing the accuracy and reliability of property service quality evaluations. This will contribute to improving overall property service management levels and better meeting residents' needs for high-quality living.

The VlseKriterijumska Optimizacija Kompromisno Resenje (VIKOR) methodology, introduced by Opricovic (1998), is tailored for the multi-criteria optimization of intricate systems. This approach excels in balancing group utility maximization and individual regret minimization, while integrating subjective preferences of decision makers (DMs). Consequently, VIKOR has higher ranking stability and reliability, as highlighted in studies by Opricovic and Tzeng (2007), Yang et al. (2013), and Kim and Ahn (2019). The main reason the VIKOR decision method is suitable for property service quality assessment is that it can not only address various attributes such as service attitude and facility maintenance, but also incorporate the subjective preferences of the owner or assessor to obtain more realistic results. This method provides a compromise solution, which can effectively resolve the conflicts between attributes and make it more widely accepted.

The innovation point of this paper is that by combining the innovative distance measurement method, comprehensive weighting method and VIKOR under dual hesitation fuzzy (DHF) environment decision method, a comprehensive decision method to evaluate the quality of property service is proposed. This method effectively addresses complex scenarios involving incomplete information, unknown attribute weights, and hesitant evaluators in property service quality assessment. Through the integration of advanced measurement tools and algorithms, it enhances the accuracy, scientific rigor, and practicality of evaluation results while providing robust support for continual improvement in property service quality. Details are as follows.

(1) This paper proposes a novel method combining innovative distance measurement, DHF VIKOR, and comprehensive weighting to evaluate property service quality.

(2) The DHF VIKOR method manages incomplete information, unknown attribute weights, and evaluator hesitancy, overcoming traditional limitations in handling uncertainty and fuzziness.

(3) By integrating advanced tools and algorithms, the DHF VIKOR method improves accuracy, scientific rigor, and practicality, supporting continuous improvement and decision making in the property service industry.

The rest of this paper is organized as follows. In Section 2, the literature on property service quality evaluation, fuzzy set theory and VIKOR decision method are briefly reviewed. Section 3 provides a brief overview of the basic concepts of dual hesitation fuzzy set (DHFS). Section 4 introduces novel distance formulas for DHFS, analyzes their characteristics, proposes a comprehensive weight determination model, and develops an multi-criterion group decision making (MCGDM) approach combining incomplete attribute weights, DHF evaluation, and VIKOR. Section 5 presents a case study on property service quality assessment, discussing and comparing the proposed method. Section 6 summarizes the work and suggests future research directions.

2. Literature review

2.1. Property service quality evaluation

Service quality evaluation is the key link of service quality management, and the academic circle has formed a relatively mature theoretical system. In the practice of service quality evaluation, SCSB (Sweden, 1989), ACSI (USA, 1994), ECSI (Europe, 1999) and CCSI (China, 2002) customer satisfaction index models are constructed and applied in the macro field, and in-depth studies are carried out in the micro field around the evaluation dimensions and evaluation methods of customer perception service quality. The division of evaluation dimension is the basis of constructing evaluation model. Gronroos (2000) expanded the constituent elements of service quality into seven dimensions according to the characteristics of employees, customers and services. Parasuraman et al. (1985) summarized the general elements of service quality such as reliability, responsiveness and competence through empirical research, and then summarized them into the commonly adopted 5 dimensions of perceptivity, reliability, responsiveness, assurance and empathy (Zeithaml et al., 1993). Brady and Cronin (2001) believe that service quality is customers' perception of interaction quality, tangible environment and result quality. SERVQUAL model, rooted theory, BP neural network, IVWMM and comprehensive fuzzy evaluation are common methods of service quality evaluation (Zuo et al., 2019). Among them, econometric analysis based on SERVQUAL model is the most commonly used method in the evaluation of perceived service quality.

Service quality evaluation methods include SERVQUAL model (Huo, 2010), entropy method (Yang & Shen, 2012), structural equation model (Huang & Li, 2013), analytic hierarchy process (Lo et al., 2013), fuzzy evaluation method (Shiu et al., 2016), grade assessment (Yu & Zuo, 2024) and multidimensional preference analysis linear programming technique (Zuo et al., 2004). Among them, the method based on the SERVQUAL model is the most commonly used in service quality evaluation. At the same time, the research methods of service satisfaction, which began in the second half of the 20th century, possess good reference value. Cardozo (1965) conducted the first experimental study on customer satisfaction. Anderson (1973), Olshavsky and Miller (1972) discussed the expectation difference theory and its impact on product performance, and Mesarovic and Takahara (1972) conducted a systematic study on the satisfaction theory. In addition, traditional decision methods such as large group decision making (Wu et al., 2018) and interactive multi-attribute decision making can be used for service quality evaluation. D-S evidential reasoning is usually used for data fusion (Liu & Zhang, 2018), rough set theory can be used to deal with incomplete information (Liu & Wang, 2018), and operator theory uses some functions to transform data (Arora & Garg, 2019). However, these research methods cannot be directly and effectively applied to property service quality evaluation.

Property service quality refers to people's actual perceived level of property service provided by property service enterprise (Zuo et al., 2021). In contrast to the wide application and function of real estate service quality practice, the theoretical research of property service quality evaluation is relatively insufficient. Regarding the evaluation dimension of property service quality, Zuo et al. (2021) divided the evaluation dimension into customer service, cleaning, safety, greening and facilities. In practice, the most commonly used evaluation method of service quality is the econometric analysis method based on SERVQUAL, but this method lacks consideration for special real estate service scenarios. Therefore, Gomes and Luis (2009) proposed an improved TODIM method and applied it to property service quality evaluation. Zuo et al. improved the LINMAP model according to multi-source heterogeneous information fusion (Zuo et al., 2020) and the rational behavior of evaluators (Zuo et al., 2023) of property service quality evaluation. And these two studies have achieved good results in specific property service quality evaluation scenarios. Although these methods do enhance the accuracy and applicability of the evaluation system to a certain extent, they still insufficient in addressing the pervasive fuzziness and uncertainty in property service quality assessment, particularly in handling hesitation fuzzy information.

2.2. Dual hesitation fuzzy and distance measures

Zadeh (1965) introduced the concept of fuzzy set (FS), a groundbreaking mathematical tool for describing and managing ambiguous or uncertain entities. By introducing membership functions (or membership degrees), FSs adeptly translate the elusive concept of fuzziness into a precise mathematical format, enabling DMs to effectively handle fuzzy information using rigorous mathematical methods. This theory offers a fresh perspective and methodology for tackling complex and uncertain management issues within social systems. Building upon this foundation, Atanassov (1986) further extended the concept to intuitionistic fuzzy set (IFS) based on FS. IFS not only incorporates membership functions but also introduces non-membership functions, facilitating more detailed and comprehensive mathematical descriptions of intricate and ambiguous entities. The advent of IFS effectively addresses the challenges of incomplete information or information loss inherent in traditional FS, thereby enhancing the precision and efficiency of fuzzy information processing. Continuing the evolution of FS theory, Torra (2010) proposed hesitant fuzzy set (HFS) based on FS. HFS permits the membership degree of elements to be a set rather than a single value, rendering it more adept at describing and managing fuzzy information characterized by multiple possibilities and uncertainties, aligning closely with the diverse and uncertain nature of human understanding within social systems. Subsequently, Zhu et al. (2012) introduced the concept of DHFS based on HFS and IFS.

The key similarity between DHFS and IFS lies in their ability to depict individuals' attitudes towards fuzzy objects through three indicators the membership degree which represents approval, the non-membership degree which signifies opposition, and the hesitation degree which portrays abstention. The key distinction between DHFS and IFS lies in the utilization of sets to describe an individual's attitude towards fuzzy objects, whereas IFS employs discrete values encompassing membership, non-membership, and hesitancy. Sets offer numerous significant advantages over numerical values, enhancing not only the descriptive scope of fuzzy information but also facilitating the expression, processing, and analysis of relationships. In summary, DHFS, employing sets to depict individuals' attitudes towards fuzzy objects, possesses several advantages over IFS, making it a pivotal player in fuzzy data processing analysis and application.

Therefore, DHFS amalgamates the strengths of HFS and IFS, encompassing not only membership and non-membership degrees but also allowing both to be hesitant. This comprehensive framework enables DHFS to provide a more expansive description and processing capability for fuzzy information, particularly in addressing problems characterized by multiple uncertainties and complex correlations. In short, the evolution from FS to IFS to DHFS continues to improve our ability to understand and process ambiguous information. DHFS is regarded as a powerful tool to deal with complex fuzzy information, and has been developed and widely used in its theoretical framework. In recent years, scholars have widely discussed the theory and application of DHFS, including the application of DHFS in pattern recognition, cluster analysis, MCDM and other fields.

It's important to highlight that a fundamental concept underpinning numerous studies is distance measurement. The determination of a distance measure typically relies on axiomatic principles, and its formulation is not necessarily singular. Therefore, the selection of an appropriate or optimal distance measurement is a crucial matter that merits careful consideration. A significant number of scholars have delved into this issue and proposed diverse definitions for distance measurement in DHFSs. Singh (2015) introduced distance measures for DHFSs based on geometric distance. Su et al. (2015) presented a hybrid dual hesitant distance measure that integrates classical DHF distance and Hausdorff distance. Ren et al. (2017) proposed a novel distance

Table 1. Comparison of whether different distance measures satisfy axiomatic conditions

Sources	Processing asymmetric DHFS information (No complement)	(i) Non-negativity	(ii) Symmetric	(iii*) Degenerate triangle inequality	(iii) Triangle inequality
Li and Zhang (2016)	×	√	√	×	×
Singh (2015)	×	√	√	√	×
Su et al. (2015)	×	√	√	√	×
Ren et al. (2017)	√	√	√	×	×
Zeng et al. (2022)	√	√	√	×	×
Ali et al. (2022)	√	√	√	√	×
This paper	√	√	√	√	√

measure grounded in the DHFS scoring function and comparison method. Zeng et al. (2022) developed a distance measure tailored to specific DHFS features, such as mean function, variance function, and hesitation degree, and implemented it in practical medical diagnosis.

While the definition forms of the aforementioned distance measures vary, some are merely represented by the membership and non-membership degrees of DHFS. This suggests that certain valuable information encapsulated within the hesitation degree has not been fully considered, resulting in information loss. When dealing with complex DHFS containing inconsistent membership and non-membership information, some distance measures extend DHF information imprecisely by complementing (e.g., by means, modes, etc.). However, this operation not only alters the original decision information but also introduces subjective factors, compromising the independence of the distance measurement method and impeding scientific rigor in decision-making processes. Furthermore, existing distance measures solely satisfy the degenerate triangle inequality without fully meeting strict axiomatic conditions. Therefore, this paper proposes a distance measurement formula that not only considers the information value of DHFS hesitation degree but also effectively reduces the information distortion caused by complement operation, ensuring the accuracy and reliability of information processing. Simultaneously, the distance measure satisfies the axiomatic property condition, further enhancing its applicability in practical use. The details are outlined in Table 1.

2.3. MCDM methods

For the MCDM problem characterized by unknown attribute weights, Lei et al. (2024) developed a tripartite group decision framework grounded in regret theory to effectively address the complexities of MCGDM under DHF uncertainty. Song et al. (2024) incorporated psychological and behavioral factors into their research, proposing an interactive strategy based on prospect theory, combined with probabilistic DHFS distance measurement technology and the TODIM method, to enhance decision outcomes. Sha et al. (2021) constructed a TOPSIS emergency decision model in a probabilistic DHF environment, leveraging cumulative prospect theory and Lance distance measures to account for DMs' bounded rationality and risk preferences. Sun and Wang (2024) explored information processing strategies in Pythagorean fuzzy environments, defining hesitation factors and distance measures based on the centroid of the hesitation region, and subsequently developing an extended TOPSIS method. Aydoğoan et al. (2024) proposed a fuzzy TODIM method that integrates group utility and individual regret, introducing novel entropy and distance measures to specifically address engineering project management challenges in uncertain environments. Liu et al. (2024) combined the DHF information aggregation operator with the TODIM method to analyze the potential economic impacts of the Russia-Ukraine conflict on the global economy.

Tracing back to the work of Opricovic and Tzeng (2002), they introduced the VIKOR method, a MCDM approach designed to identify the closest ideal solution while considering group interests and minimizing individual regret. In subsequent, Opricovic and Tzeng (2004) compared the VIKOR and TOPSIS methods, revealing that although both approaches are grounded in proximity to the ideal solution, VIKOR offers a compromise solution, whereas TOPSIS emphasizes solutions furthest from the negative ideal and closest to the positive ideal, without accounting for the relative importance of these distances. Further comparisons by Opricovic and Tzeng (2007) between the extended VIKOR method and other techniques such as TOPSIS, ELECTRE, and PROMETHEE demonstrated VIKOR's superiority in managing conflicting and incomparable attributes. Wang and Tzeng (2012) combined DEMATEL, ANP and VIKOR methods to analyze the correlation between brand marketing strategies. Opricovic (2011) subsequently developed the fuzzy VIKOR method to address MCDM problems within a fuzzy environment. Riaz and Tehrim (2021) integrated VIKOR with bipolar FSs to propose a fuzzy MCDM strategy. Sarkar and Biswas (2022) constructed the Pythagorean fuzzy multi-objective optimal proportional analysis plus complete multiplicative form (PF-MULTIMOORA) method to tackle MCDM problems with unknown criterion weights. Lin et al. (2021) introduced the TOPSIS and VIKOR within the framework of probabilistic language term sets, utilizing score functions and a preprocessing algorithm. They conducted a comparative analysis of these two approaches, highlighting that VIKOR's ranking is more sensitive to variations in decision-making parameters.

In summary, the MCDM method plays a pivotal role in addressing complex decision-making problems, particularly when attribute weights are unknown or attribute values are fuzzy. The integration of psychological and behavioral economic theories, such as prospect theory, regret theory, and cumulative prospect theory, offers a novel perspective on managing uncertainty. Notably, the VIKOR method has garnered significant attention and widespread application due to its capability to balance group interests and individual regrets while effectively handling conflicts. Since its inception by Opricovic and Tzeng, the VIKOR approach has evolved from classical settings to accommodate various complex scenarios, including FS, IFS, and Pythagorean fuzzy environments, reflecting the growing complexity of decision-making processes. However, despite substantial advancements, the application of the VIKOR method in DHF environments remains challenging. DHFSs not only encompass membership degrees, non-membership degrees, and hesitation degrees but also introduce sets as eigenvalues, more closely mirroring the psychological states of DMs facing intricate problems. Consequently, effectively applying the VIKOR method and accurately determining unknown attribute weights in DHF environments is an urgent issue that requires further exploration.

In light of these considerations, this study innovatively proposes the VIKOR method within a DHF environment to enhance the effectiveness of property service quality evaluation. This approach not only effectively addresses conflicts and ambiguities among evaluation indicators but also identifies the optimal compromise by minimizing group utility loss and individual regret, thereby providing a more scientifically robust foundation for decision-making management. Building on this, the model has been further refined, introducing a more scientifically grounded strategy for solving unknown weights. Through this innovative methodology, the aim is to offer more detailed and comprehensive decision support for evaluating property service quality, ultimately ensuring enhanced property management efficiency and customer satisfaction.

3. DHF VIKOR method with incomplete attribute weights

3.1. DHFS concept

As an extension of FS and IFS, HFS and DHFS have emerged as a significant research tool in the field of decision analysis. DHFS, in particular, not only serves as a generalization of HFS but also offers a broader scope for addressing fuzzy information and decision problems.

Definition 1. (Zhu et al., 2012) Let the universe of discourse X be a nonempty set. Then a DHFS D on X is defined as $D = \{< x, h_D(x), g_D(x) > | x \in X\}$, where $h_D(x)$ and $g_D(x)$ are two sets of some values in $[0,1]$, representing the possible membership and non-membership degrees of the element $x \in X$ to the set D, respectively, and the conditions $0 \le \gamma, \eta \le 1$ and $\gamma^+ + \eta^+ \le 1$ are satisfied, where $\gamma \in h_D(x)$, $\eta \in g_D(x)$, $\gamma^+ = \max_{\gamma \in h_D(x)} \{\gamma\}$, $\eta^+ = \max_{\eta \in g_D(x)} \{\eta\}$ for all $x \in X$.

For a DHFS D on the universe of discourse X, let $E_D(x_0) =< x_0, h_D(x_0), g_D(x_0) >$ denote the fuzzy information corresponding to the element $x_0 \in X$. In Zhu et al. (2012), an ordered pair $< h_D(x_0), g_D(x_0) >$ is called a DHFE if the elements in sets $h_D(x_0)$ and $g_D(x_0)$ satisfy the following conditions $0 \le \gamma, \eta \le 1$ and $\gamma^+ + \eta^+ \le 1$, where $\gamma \in h_D(x_0)$, $\eta \in g_D(x_0)$, $\gamma^+ = \max_{\gamma \in h_D(x_0)} \{\gamma\}$, $\eta^+ = \max_{\eta \in g_D(x_0)} \{\eta\}$. For convenience, $E_D(x_0) =< x_0, h_D(x_0), g_D(x_0) >$ is often abbreviated as $E_D =< h_D, g_D >$. Thus, when h_D and g_D are two real numbers on the interval [0,1] and the condition $h_D + g_D \le 1$ is satisfied, $E_D =< h_D, g_D >$ is an IFN. When h_D and g_D are two sets, the values of the elements in sets are on the interval [0,1], and the condition $\max_{\gamma \in h_D}\{\gamma\} + \max_{\eta \in g_D}\{\eta\} \le 1$ is satisfied, $E_D =< h_D, g_D >$ is a DHFE.

We use $DHFE(X)$ to represent the set of all DHFEs, where X is the universe of discourse. That is,

$$DHFE(X) = \left\{ E_D =< h_D, g_D > \middle| \begin{array}{l} 0 \le \gamma, \eta \le 1;\ \gamma^+ + \eta^+ \le 1(\gamma \in h_D, \\ \eta \in g_D, \gamma^+ = \max_{\gamma \in h_D}\{\gamma\}), \eta^+ = \\ \max_{\eta \in g_D}\{\eta\} \end{array} \right\}.$$

We used $DHFS(X)$ to represent the set of all DHFSs, where X is the universe of discourse. That is

$$DHFS(X) = \left\{ E_D(x) =< h_D(x), g_D(x) > \middle| \begin{array}{l} 0 \le \gamma, \eta \le 1;\ \gamma^+ + \\ \eta^+ \le 1(\gamma \in h_D(x), \\ \eta \in g_D(x), \gamma^+ = \\ \max_{\gamma \in h_D(x)}\{\gamma\}), \eta^+ = \\ \max_{\eta \in g_D(x)}\{\eta\} \end{array} \right\}.$$

The numbers of elements in sets $h_D(x)$ and $g_D(x)$ are recorded as $|h_D(x)|$ and $|g_D(x)|$, respectively.

Remark 1. According to Definitions 2 and 3, for a DHFS $D_1 = \{< x, h_1(x), g_1(x) >, x \in X\}$, if the set $h_1(x)$ has only one element, meaning there is only one possible membership value for the element $x \in X$ to the set D_1. And the set $g_1(x)$ also contains only one element, implying there is only one possible non-membership value for the element $x \in X$ to the set D_1. Then the DHFS D_1 degenerates into an IFS. This demonstrates that DHFS is an extension of traditional IFS. Conversely, it can also be stated that traditional IFS is a special case of DHFS. Furthermore, according to Definition 1, if the sum of the values of the unique element in the set $h_1(x)$ and the unique element in the set $g_1(x)$ is equal to one, then IFS further degenerates to FS. Additionally, when the set $g_1(x)$ is empty, DHFS degenerates to HFS. In summary, DHFS is an extension of HFS, IFS, and FS, while the traditional HFS, IFS, and FS are all special cases of DHFS.

In the following section, we explore the remarkable potential of DHFS in vividly illustrating information through a

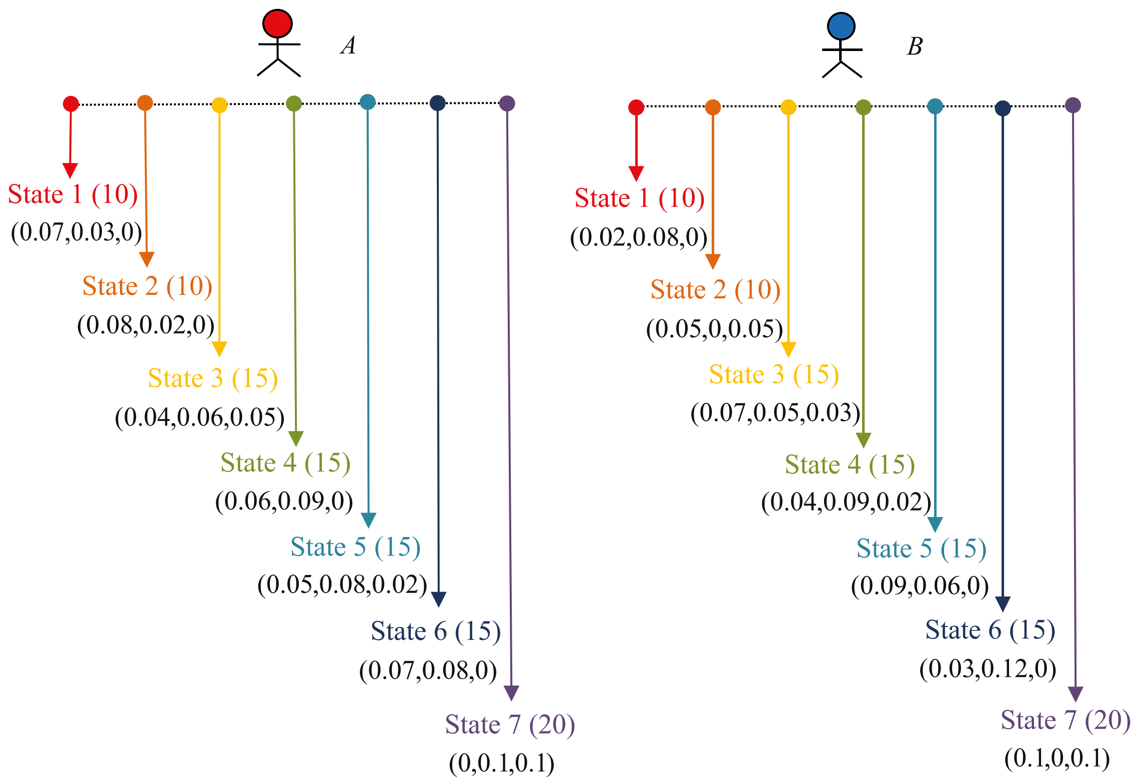

Figure 1. An infographic of voting results in DHFS

Table 2. Symbols and explanations

Symbols	Explanations
X	Universe of discourse
D	Dual hesitant fuzzy set (DHFS)
E	Dual hesitant fuzzy element (DHFE)
$h(x)$	Membership function
$g(x)$	Non-membership function
$\gamma^+ = \max_{\gamma \in h(x)} \{\gamma\}$	Maximum membership value
$\eta^+ = \max_{\eta \in g(x)} \{\eta\}$	Maximum non-membership value
$d(\cdot,\cdot)$	Distance measure
$A = \{A_1, A_2, \cdots, A_i, \cdots A_m\}$	Alternative set
$C = \{C_1, C_2, \cdots, C_j, \cdots, C_n\}$	Attribute set
$D = \{D_1, D_2, \cdots, D_k, \cdots, D_K\}$	Decision-maker set
$E = (E_{ij})_{m \times n}$	DHF decision matrix
$\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_k, \cdots, \lambda_K)$	Weight vector of DM
$\omega = (\omega_1, \omega_2, \cdots, \omega_t, \omega_{t+1}, \cdots, \omega_n)^T$	Weight vector of the attribute
$\bar{\omega} = (\omega_1, \omega_2, \cdots, \omega_t)^T$, $t = 1, 2, \cdots, n$	Incomplete attribute weight vector
$\bar{\omega}^k = (\omega_{k1}, \omega_{k2}, \cdots, \omega_{kt})^T$	Subjective attribute weight vector from DMs
α	Trade-off coefficient between subjective and objective factors
β	Compromise coefficient between group utility and individual regret

compelling electoral case study. This case not only aids in our comprehension of DHFS but also visually showcases its distinctive advantages in information presentation.

Let's delve into a scenario where two presidential candidates, denoted as Candidate *A* and Candidate *B*, are vying for one hundred electoral votes spread across seven pivotal states. Here, the set comprising these two candidates can be defined as a finite universe of discourse, denoted as $X=\{A,B\}$. For Candidate *A*, we meticulously gather and analyze the electoral votes received in each state. The ratio of the votes garnered by Candidate *A* in State 1 to the total one hundred electoral votes indicates a membership degree of 0.07, reflecting the preference of electors towards Candidate *A* for the presidency. Conversely, the ratio of votes against Candidate *A* in State 1 to the total electoral votes results in a non-membership degree of 0.03, indicating disapproval from electors towards their potential presidential choice. Additionally, considering the abstentions received by Candidate *A* in State 1 divided by the total electoral votes yields a hesitation degree of 0. Similarly, we conduct analogous calculations to assess the support for Candidate *B* across all seven states. The detailed information is visually represented in Figure 1.

From Figure 1, it is evident that the membership degree, non-membership degree, and hesitancy degree are all each represented as sets, with seven elements in each set corresponding to the seven states. As depicted in Figure 1, the voting information for Candidates *A* and *B* across these seven states can be obtained as DHFEs $E_D(A)=<\{0.07,0.08,0.04,0.06,0.05,0.07,0\}, \{0.03,0.02,0.06,0.09,0.08,0.08,0.1\}>$ and $E_D(B)=<\{0.02,0.05,0.07,0.04,0.09,0.03,0.1\}, \{0.08,0,0.05,0.09,0.06,0.12,0\}>$, respectively. By combining these two DHFEs into a DHFS, denoted as $D(A,B)=\{E_D(A),E_D(B)\}$, all the pertinent information regarding the election can be effectively conveyed.

The relevant symbols and explanations in this paper are summarized in Table 2.

3.2. Distance measures

Various factors, such as human error, technical failure, cleaning procedures, privacy concerns, data unavailability, limitations in sampling surveys, natural calamities, expired data, and deliberate concealment, often result in incomplete information marked by missing values. This incompleteness manifests in inconsistencies within the number of elements within both membership and non-membership sets in DHFS. Prior research commonly addresses this issue by resorting to mean, median, or mode imputation techniques before conducting relevant DHFS operations. However, such approaches compromise the authenticity and objectivity of the original data.

The Hamming distance, Euclidean distance, and Minkowski distance stand out as three commonly employed methods for measuring distances, each offering unique characteristics suited to specific application domains. The Hamming distance formula excels in its efficiency and ease of interpretation. Meanwhile, the Euclidean distance computes the straight-line distance between two points, unaffected by vector dimensionality, rendering it applicable to higher-dimensional vectors. However, it solely measures similarity in direction, neglecting differences in vector length. Hence, instances of substantial directional disparities between vectors may lead to inaccurate similarity calculations. The Minkowski distance, as a generalized version of the Euclidean distance, incorporates a parameter *p* into its formula to yield different measures based on varying values of *p*. While intuitive, the Minkowski distance formula suffers from limitations stemming from its independence from data distribution. Notably, if one dimension exhibits significantly larger amplitudes compared to others, it can unduly influence the overall calculated distances using this formula. In summary, each of these three methods boasts advantages and drawbacks, necessitating careful consideration of specific application scenarios and data characteristics when selecting an appropriate method. In practical settings, flexibility should guide the choice and utilization of these methods according to the prevailing circumstances.

To ensure scientific rigor in processing raw data for DHFE analysis, we propose three distance formulas for DHFE that circumvent the need for completing missing values. Inspired by the Hamming distance, Euclidean distance, and Minkowski distance measures, these formulas aim to enhance the reliability and robustness of DHFE analyses.

Definition 2. Let $E_1=<h_1,g_1>$ and $E_2=<h_2,g_2>$ be any two DHFEs. Then the hybrid DHF Hamming distance of E_1 and E_2 is defined as

$$d_{EH}(E_1,E_2)=\frac{1}{4}\times\left(\begin{array}{l}\max\limits_{\gamma_1\in h_1,\gamma_2\in h_2}\left\{\left|\gamma_1-\gamma_2\right|\right\}+\max\limits_{\eta_1\in g_1,\eta_2\in g_2}\left\{\left|\eta_1-\eta_2\right|\right\}+\\ \left|\frac{\sum_{\gamma_1\in h_1}\gamma_1}{|h_1|}-\frac{\sum_{\gamma_2\in h_2}\gamma_2}{|h_2|}\right|+\left|\frac{\sum_{\eta_1\in g_1}\eta_1}{|g_1|}-\frac{\sum_{\eta_2\in g_2}\eta_2}{|g_2|}\right|\end{array}\right). \quad (1)$$

The hybrid DHF Euclidean distance of E_1 and E_2 is

$$d_{EE}(E_1,E_2)=\sqrt{\frac{1}{4}\times\left(\begin{array}{l}\max\limits_{\gamma_1\in h_1,\gamma_2\in h_2}\left\{\left(\gamma_1-\gamma_2\right)^2\right\}+\max\limits_{\eta_1\in g_1,\eta_2\in g_2}\left\{(\eta_1-\eta_2)^2\right\}+\\ \left|\frac{\sum_{\gamma_1\in h_1}\gamma_1}{|h_1|}-\frac{\sum_{\gamma_2\in h_2}\gamma_2}{|h_2|}\right|^2+\left|\frac{\sum_{\eta_1\in g_1}\eta_1}{|g_1|}-\frac{\sum_{\eta_2\in g_2}\eta_2}{|g_2|}\right|^2\end{array}\right)}. \quad (2)$$

The hybrid DHF Minkovski distance of E_1 and E_2 is defined as follows:

$$d_{EG}(E_1,E_2)=\sqrt[p]{\frac{1}{4}\times\left(\begin{array}{l}\max\limits_{\gamma_1\in h_1,\gamma_2\in h_2}\left\{\left|\gamma_1-\gamma_2\right|^p\right\}+\max\limits_{\eta_1\in g_1,\eta_2\in g_2}\left\{\left|\eta_1-\eta_2\right|^p\right\}+\\ \left|\frac{\sum_{\gamma_1\in h_1}\gamma_1}{|h_1|}-\frac{\sum_{\gamma_2\in h_2}\gamma_2}{|h_2|}\right|^p+\left|\frac{\sum_{\eta_1\in g_1}\eta_1}{|g_1|}-\frac{\sum_{\eta_2\in g_2}\eta_2}{|g_2|}\right|^p\end{array}\right)}, \quad (3)$$

where $p(p\geq 0)$ is a distance parameter. The number of elements of a set h_1 is recorded as $|h_1|$.

Remark 2. When p = 1, the hybrid DHF Minkovski distance d_{EG} degenerates to the hybrid DHF Hamming distance d_{EH}. Similarly, when p = 2, it degenerates to the hybrid DHF Euclidean distance d_{EE}.

In general, distance measures are established through axiomatization with the axiom for the distance measure being precisely specified as follows.

Axiom 1. For any DHFEs $E_1 =< h_1, g_1 >$, $E_2 =< h_2, g_2 >$, $E_3 =< h_3, g_3 >$, a real function $d : DHFE(X) \times DHFE(X) \to R$ is called the distance measure of DHFEs, if d satisfies the following properties:

(i) (**Non-negativity**) $0 \le d(E_1, E_2) \le 1$ and $d(E_1, E_2) = 0$ if and only if $E_1 = E_2$.

(ii) (**Symmetric**) $d(E_1, E_2) = d(E_2, E_1)$.

(iii) (**Triangle inequality**) $d(E_1, E_2) \le d(E_1, E_3) + d(E_2, E_3)$.

(iii*) (**Degenerate triangle inequality**) If $E_1 \le E_2 \le E_3$, then $d(E_1, E_2) \le d(E_1, E_3)$ and $d(E_2, E_3) \le d(E_1, E_3)$.

Remark 3. Property (iii*) is a special condition for the triangle inequality property (iii). When the triangle inequality is holds, then the property (iii*) is obvious and is included in the triangle inequality (iii). Hence, property (iii*) can be viewed as a degenerate manifestation of the triangular inequality property (iii). Regrettably, the existing literature lacks a DHFE and DHFS distance formula that meets all the conditions stipulated in Axiom 1. Numerous frequently employed distance formulas only fulfill the degenerated triangular inequality property, i.e., conditions (iii*).

Subsequently, we demonstrate that the hybrid DHF Hamming d_{EH}, hybrid DHF Euclidean d_{EE}, and hybrid DHF Minkovski d_{EG} distance formulas for DHFE, as presented in Definition 2 adhere to all axiomatic conditions outlined in Axiom 1. Given that the hybrid DHF Hamming and hybrid DHF Euclidean distances are special cases of the hybrid DHF Minkowski distance, thus, by demonstrating the satisfaction of these properties by the hybrid DHF Minkowski distance, we inherently establish the satisfaction of the hybrid DHF Hamming and Euclidean distances as well.

Theorem 1. For any DHFEs $E_1 =< h_1, g_1 >$, $E_2 =< h_2, g_2 >$, $E_3 =< h_3, g_3 >$, the hybrid DHF Minkovski distance $d_{EG}(\cdot,\cdot)$ which is given by Equation (3) satisfies the following properties:

(i) (**Non-negativity**) $0 \le d_{EG}(E_1, E_2) \le 1$ and $d_{EG}(E_1, E_2) = 0$ if and only if $E_1 = E_2$.

(ii) (**Symmetric**) $d_{EG}(E_1, E_2) = d_{EG}(E_2, E_1)$.

(iii) (**Triangle inequality**) $d_{EG}(E_1, E_2) \le d_{EG}(E_1, E_3) + d_{EG}(E_2, E_3)$.

Proof: (i) (**Non-negativity**)

Firstly, due to $0 \le \gamma_1, \gamma_2, \eta_1, \eta_2 \le 1$, then

$$0 \le \max_{\gamma_1 \in h_1, \gamma_2 \in h_2} \left\{ \left| \gamma_1 - \gamma_2 \right| \right\} \le 1,\ 0 \le \max_{\eta_1 \in g_1, \eta_2 \in g_2} \left\{ \left| \eta_1 - \eta_2 \right| \right\} \le 1,$$

$$0 \le \left| \frac{\sum_{\gamma_1 \in h_1} \gamma_1}{|h_1|} - \frac{\sum_{\gamma_2 \in h_2} \gamma_2}{|h_2|} \right| \le 1,\ 0 \le \left| \frac{\sum_{\eta_1 \in g_1} \eta_1}{|g_1|} - \frac{\sum_{\eta_2 \in g_2} \eta_2}{|g_2|} \right| \le 1.$$

Further, $0 \le \max_{\gamma_1 \in h_1, \gamma_2 \in h_2} \left\{ \left| \gamma_1 - \gamma_2 \right|^p \right\} \le 1$,

$$0 \le \max_{\eta_1 \in g_1, \eta_2 \in g_2} \left\{ \left| \eta_1 - \eta_2 \right|^p \right\} \le 1,$$

$$0 \le \left| \frac{\sum_{\gamma_1 \in h_1} \gamma_1}{|h_1|} - \frac{\sum_{\gamma_2 \in h_2} \gamma_2}{|h_2|} \right|^p \le 1,\ 0 \le \left| \frac{\sum_{\eta_1 \in g_1} \eta_1}{|g_1|} - \frac{\sum_{\eta_2 \in g_2} \eta_2}{|g_2|} \right|^p \le 1.$$

Therefore, $0 \le d_{EG}(E_1, E_2) \le 1$.

Secondly, if $E_1 = E_2$, i.e., $\max_{\gamma_1 \in h_1, \gamma_2 \in h_2} \left\{ \left| \gamma_1 - \gamma_2 \right|^p \right\} = 0$,

$$\max_{\eta_1 \in g_1, \eta_2 \in g_2} \left\{ \left| \eta_1 - \eta_2 \right|^p \right\} = 0,\ \left| \frac{\sum_{\gamma_1 \in h_1} \gamma_1}{|h_1|} - \frac{\sum_{\gamma_2 \in h_2} \gamma_2}{|h_2|} \right|^p = 0,$$

$$\left| \frac{\sum_{\eta_1 \in g_1} \eta_1}{|g_1|} - \frac{\sum_{\eta_2 \in g_2} \eta_2}{|g_2|} \right|^p = 0, \text{ then } d_{EG}(E_1, E_2) = 0.$$

Conversely, if $d_{EG}(E_1, E_2) = 0$, that is, the sum of several non-negative numbers in the formula is zero, then these non-negative numbers are equal to zero.

Thence,

$$\max_{\gamma_1 \in h_1, \gamma_2 \in h_2} \left\{ \left| \gamma_1 - \gamma_2 \right|^p \right\} = \max_{\gamma_1 \in h_1, \gamma_2 \in h_2} \left\{ \left| \gamma_1^+ - \gamma_2^- \right|^p, \left| \gamma_2^+ - \gamma_1^- \right|^p \right\} = 0,$$

i.e., $\gamma_1^+ - \gamma_2^- = 0$, $\gamma_2^+ - \gamma_1^- = 0$. Similarly, $\eta_1^+ - \eta_2^- = 0$,

$\eta_2^+ - \eta_1^- = 0$. Therefore, we have $E_1 = E_2$.

(i) (**Symmetric**) We have

$$d_{EG}(E_1, E_2) = \sqrt[p]{\frac{1}{4} \times \left(\max_{\gamma_1 \in h_1, \gamma_2 \in h_2} \left\{ \left| \gamma_1 - \gamma_2 \right|^p \right\} + \max_{\eta_1 \in g_1, \eta_2 \in g_2} \left\{ \left| \eta_1 - \eta_2 \right|^p \right\} + \left| \frac{\sum_{\gamma_1 \in h_1} \gamma_1}{|h_1|} - \frac{\sum_{\gamma_2 \in h_2} \gamma_2}{|h_2|} \right|^p + \left| \frac{\sum_{\eta_1 \in g_1} \eta_1}{|g_1|} - \frac{\sum_{\eta_2 \in g_2} \eta_2}{|g_2|} \right|^p \right)} =$$

$$\sqrt[p]{\frac{1}{4} \times \left(\max_{\gamma_1 \in h_1, \gamma_2 \in h_2} \left\{ \left| \gamma_2 - \gamma_1 \right|^p \right\} + \max_{\eta_1 \in g_1, \eta_2 \in g_2} \left\{ \left| \eta_2 - \eta_1 \right|^p \right\} + \left| \frac{\sum_{\gamma_2 \in h_2} \gamma_2}{|h_2|} - \frac{\sum_{\gamma_1 \in h_1} \gamma_1}{|h_1|} \right|^p + \left| \frac{\sum_{\eta_2 \in g_2} \eta_2}{|g_2|} - \frac{\sum_{\eta_1 \in g_1} \eta_1}{|g_1|} \right|^p \right)} =$$

$d_{EG}(E_2, E_1)$.

(ii) (**Triangle inequality**)

Firstly, we prove that the d_{EH} (when p = 1, i.e., Equation (1)) satisfies the triangle inequality.

As we know

$$\max_{\gamma_1 \in h_1, \gamma_2 \in h_2} \left\{ \left| \gamma_1 - \gamma_2 \right| \right\} = \max_{\gamma_1 \in h_1, \gamma_2 \in h_2} \left\{ \left| \gamma_1^+ - \gamma_2^- \right|, \left| \gamma_2^+ - \gamma_1^- \right| \right\};$$

$$\max_{\gamma_1 \in h_1, \gamma_3 \in h_3} \left\{ \left| \gamma_1 - \gamma_3 \right| \right\} = \max_{\gamma_1 \in h_1, \gamma_3 \in h_3} \left\{ \left| \gamma_1^+ - \gamma_3^- \right|, \left| \gamma_3^+ - \gamma_1^- \right| \right\};$$

$$\max_{\gamma_3 \in h_3, \gamma_2 \in h_2} \left\{ \left| \gamma_3 - \gamma_2 \right| \right\} = \max_{\gamma_3 \in h_3, \gamma_2 \in h_2} \left\{ \left| \gamma_3^+ - \gamma_2^- \right|, \left| \gamma_2^+ - \gamma_3^- \right| \right\};$$

and

$$\left|\gamma_1^+ - \gamma_2^-\right| = \left|\gamma_1^+ - \gamma_3^- + \gamma_3^- - \gamma_2^-\right| \le \left|\gamma_1^+ - \gamma_3^- + \gamma_3^+ - \gamma_2^-\right| \le \left|\gamma_1^+ - \gamma_3^-\right| + \left|\gamma_3^+ - \gamma_2^-\right|;$$

$$\left|\gamma_2^+ - \gamma_1^-\right| = \left|\gamma_2^+ - \gamma_3^- + \gamma_3^- - \gamma_1^-\right| \le \left|\gamma_2^+ - \gamma_3^- + \gamma_3^+ - \gamma_1^-\right| \le \left|\gamma_2^+ - \gamma_3^-\right| + \left|\gamma_3^+ - \gamma_1^-\right|,$$

which infer that

$$\max_{\gamma_1\in h_1,\gamma_2\in h_2}\left\{\left|\gamma_1^+ - \gamma_2^-\right|,\left|\gamma_2^+ - \gamma_1^-\right|\right\} \le \max_{\gamma_1\in h_1,\gamma_3\in h_3}\left\{\left|\gamma_1^+ - \gamma_3^-\right|,\left|\gamma_3^+ - \gamma_1^-\right|\right\} + \max_{\gamma_3\in h_3,\gamma_2\in h_2}\left\{\left|\gamma_3^+ - \gamma_2^-\right|,\left|\gamma_2^+ - \gamma_3^-\right|\right\},$$

i.e.

$$\max_{\gamma_1\in h_1,\gamma_2\in h_2}\left\{\left|\gamma_1 - \gamma_2\right|\right\} \le \max_{\gamma_1\in h_1,\gamma_3\in h_3}\left\{\left|\gamma_1 - \gamma_3\right|\right\} + \max_{\gamma_3\in h_3,\gamma_2\in h_2}\left\{\left|\gamma_3 - \gamma_2\right|\right\}.$$

Similarly, we can prove

$$\max_{\eta_1\in g_1,\eta_2\in g_2}\left\{\left|\eta_1 - \eta_2\right|\right\} \le \max_{\eta_1\in g_1,\eta_3\in g_3}\left\{\left|\eta_1 - \eta_3\right|\right\} + \max_{\eta_3\in g_3,\eta_2\in g_2}\left\{\left|\eta_3 - \eta_2\right|\right\}.$$

In addition,

$$\left|\frac{\sum_{\gamma_1\in h_1}\gamma_1}{|h_1|} - \frac{\sum_{\gamma_2\in h_2}\gamma_2}{|h_2|}\right| \le \left|\frac{\sum_{\gamma_1\in h_1}\gamma_1}{|h_1|} - \frac{\sum_{\gamma_3\in h_3}\gamma_3}{|h_3|} + \frac{\sum_{\gamma_3\in h_3}\gamma_3}{|h_3|} - \frac{\sum_{\gamma_2\in h_2}\gamma_2}{|h_2|}\right| \le \left|\frac{\sum_{\gamma_1\in h_1}\gamma_1}{|h_1|} - \frac{\sum_{\gamma_3\in h_3}\gamma_3}{|h_3|}\right| + \left|\frac{\sum_{\gamma_3\in h_3}\gamma_3}{|h_3|} - \frac{\sum_{\gamma_2\in h_2}\gamma_2}{|h_2|}\right|;$$

$$\left|\frac{\sum_{\eta_1\in g_1}\eta_1}{|g_1|} - \frac{\sum_{\eta_2\in g_2}\eta_2}{|g_2|}\right| \le \left|\frac{\sum_{\eta_1\in g_1}\eta_1}{|g_1|} - \frac{\sum_{\eta_3\in g_3}\eta_3}{|g_3|} + \frac{\sum_{\eta_3\in g_3}\eta_3}{|g_3|} - \frac{\sum_{\eta_2\in g_2}\eta_2}{|g_2|}\right| \le \left|\frac{\sum_{\eta_1\in g_1}\eta_1}{|g_1|} - \frac{\sum_{\eta_3\in g_3}\eta_3}{|g_3|}\right| + \left|\frac{\sum_{\eta_3\in g_3}\eta_3}{|g_3|} - \frac{\sum_{\eta_2\in g_2}\eta_2}{|g_2|}\right|.$$

Hence, we get

$$d_{EH}(E_1,E_2) = \frac{1}{4}\times\left(\max_{\gamma_1\in h_1,\gamma_2\in h_2}\left\{\left|\gamma_1-\gamma_2\right|\right\} + \max_{\eta_1\in g_1,\eta_2\in g_2}\left\{\left|\eta_1-\eta_2\right|\right\} + \left|\frac{\sum_{\gamma_1\in h_1}\gamma_1}{|h_1|} - \frac{\sum_{\gamma_2\in h_2}\gamma_2}{|h_2|}\right| + \left|\frac{\sum_{\eta_1\in g_1}\eta_1}{|g_1|} - \frac{\sum_{\eta_2\in g_2}\eta_2}{|g_2|}\right|\right) \le$$

$$\frac{1}{4}\times\left(\max_{\gamma_1\in h_1,\gamma_3\in h_3}\left\{\left|\gamma_1-\gamma_3\right|\right\} + \max_{\eta_1\in g_1,\eta_3\in g_3}\left\{\left|\eta_1-\eta_3\right|\right\} + \left|\frac{\sum_{\gamma_1\in h_1}\gamma_1}{|h_1|} - \frac{\sum_{\gamma_3\in h_3}\gamma_3}{|h_3|}\right| + \left|\frac{\sum_{\eta_1\in g_1}\eta_1}{|g_1|} - \frac{\sum_{\eta_3\in g_3}\eta_3}{|g_3|}\right|\right) +$$

$$\frac{1}{4}\times\left(\max_{\gamma_3\in h_3,\gamma_2\in h_2}\left\{\left|\gamma_3-\gamma_2\right|\right\} + \max_{\eta_3\in g_3,\eta_2\in g_2}\left\{\left|\eta_3-\eta_2\right|\right\} + \left|\frac{\sum_{\gamma_3\in h_3}\gamma_3}{|h_3|} - \frac{\sum_{\gamma_2\in h_2}\gamma_2}{|h_2|}\right| + \left|\frac{\sum_{\eta_3\in g_3}\eta_3}{|g_3|} - \frac{\sum_{\eta_2\in g_2}\eta_2}{|g_2|}\right|\right) =$$

$$d_{EH}(E_1,E_3) + d_{EH}(E_3,E_2).$$

The proof of d_{EH} satisfying the triangle inequality has been successfully concluded at this juncture.

Further, we need to prove that d_{EH} satisfies the triangle inequality. Namely,

$$d_{EG}(E_1,E_2) = \sqrt[p]{\frac{1}{4}\times\left(\max_{\gamma_1\in h_1,\gamma_2\in h_2}\left\{\left|\gamma_1-\gamma_2\right|^p\right\} + \max_{\eta_1\in g_1,\eta_2\in g_2}\left\{\left|\eta_1-\eta_2\right|^p\right\} + \left|\frac{\sum_{\gamma_1\in h_1}\gamma_1}{|h_1|} - \frac{\sum_{\gamma_2\in h_2}\gamma_2}{|h_2|}\right|^p + \left|\frac{\sum_{\eta_1\in g_1}\eta_1}{|g_1|} - \frac{\sum_{\eta_2\in g_2}\eta_2}{|g_2|}\right|^p\right)} \le$$

$$\sqrt[p]{\frac{1}{4}\times\left(\max_{\gamma_1\in h_1,\gamma_3\in h_3}\left\{\left|\gamma_1-\gamma_3\right|^p\right\} + \max_{\eta_1\in g_1,\eta_3\in g_3}\left\{\left|\eta_1-\eta_3\right|^p\right\} + \left|\frac{\sum_{\gamma_1\in h_1}\gamma_1}{|h_1|} - \frac{\sum_{\gamma_3\in h_3}\gamma_3}{|h_3|}\right|^p + \left|\frac{\sum_{\eta_1\in g_1}\eta_1}{|g_1|} - \frac{\sum_{\eta_3\in g_3}\eta_3}{|g_3|}\right|^p\right)} +$$

$$\sqrt[p]{\frac{1}{4}\times\left(\max_{\gamma_3\in h_3,\gamma_2\in h_2}\left\{\left|\gamma_3-\gamma_2\right|^p\right\} + \max_{\eta_3\in g_3,\eta_2\in g_2}\left\{\left|\eta_3-\eta_2\right|^p\right\} + \left|\frac{\sum_{\gamma_3\in h_3}\gamma_3}{|h_3|} - \frac{\sum_{\gamma_2\in h_2}\gamma_2}{|h_2|}\right|^p + \left|\frac{\sum_{\eta_3\in g_3}\eta_3}{|g_3|} - \frac{\sum_{\eta_2\in g_2}\eta_2}{|g_2|}\right|^p\right)} =$$

$$d_{EG}(E_1,E_3) + d_{EG}(E_3,E_2).$$

Simplifying this inequality by substitution of variables is equivalent to proving the following theorem.

Let $a_1 = \max_{\gamma_1\in h_1,\gamma_2\in h_2}\left\{\left|\gamma_1-\gamma_2\right|\right\}$, $b_1 = \max_{\gamma_1\in h_1,\gamma_3\in h_3}\left\{\left|\gamma_1-\gamma_3\right|\right\}$,

$c_1 = \max_{\gamma_3\in h_3,\gamma_2\in h_2}\left\{\left|\gamma_3-\gamma_2\right|\right\}$, $a_2 = \max_{\eta_1\in g_1,\eta_2\in g_2}\left\{\left|\eta_1-\eta_2\right|\right\}$,

$b_2 = \max_{\eta_1\in g_1,\eta_3\in g_3}\left\{\left|\eta_1-\eta_3\right|\right\}$, $c_2 = \max_{\eta_3\in g_3,\eta_2\in g_2}\left\{\left|\eta_3-\eta_2\right|\right\}$,

$$a_3 = \left|\frac{\sum_{\gamma_1\in h_1}\gamma_1}{|h_1|} - \frac{\sum_{\gamma_2\in h_2}\gamma_2}{|h_2|}\right|,\quad b_3 = \left|\frac{\sum_{\gamma_1\in h_1}\gamma_1}{|h_1|} - \frac{\sum_{\gamma_3\in h_3}\gamma_3}{|h_3|}\right|,$$

$$c_3 = \left|\frac{\sum_{\gamma_3\in h_3}\gamma_3}{|h_3|} - \frac{\sum_{\gamma_2\in h_2}\gamma_2}{|h_2|}\right|,\quad a_4 = \left|\frac{\sum_{\eta_1\in g_1}\eta_1}{|g_1|} - \frac{\sum_{\eta_2\in g_2}\eta_2}{|g_2|}\right|,$$

$$b_4 = \left|\frac{\sum_{\eta_1\in g_1}\eta_1}{|g_1|} - \frac{\sum_{\eta_3\in g_3}\eta_3}{|g_3|}\right|,\quad c_4 = \left|\frac{\sum_{\eta_3\in g_3}\eta_3}{|g_3|} - \frac{\sum_{\eta_2\in g_2}\eta_2}{|g_2|}\right|.$$

If $a_1 \le b_1 + c_1$, $a_2 \le b_2 + c_2$, $a_3 \le b_3 + c_3$, $a_4 \le b_4 + c_4$, where $0 \le a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, c_1, c_2, c_3, c_4 \le 1$. Then $(a_1^p + a_2^p + a_3^p + a_4^p)^{1/p} \le (b_1^p + b_2^p + b_3^p + b_4^p)^{1/p} + (c_1^p + c_2^p + c_3^p + c_4^p)^{1/p}$. Theorem 1 is proved.

By amalgamating the Hausdorff metric with the mean value, we provide three distance formulas for DHFSs, all without the need for adding values into DHFSs. These formulations are detailed in Definition 3.

Definition 3. Let $D_1 = \{< x, h_1(x), g_1(x) >, x \in X\}$ and $D_2 = \{< x, h_2(x), g_2(x) >, x \in X\}$ be any two DHFSs, where $X = \{x_1, x_2, \cdots, x_n\}$. Then the hybrid DHF Hamming distance of D_1 and D_2 is defined as

$$d_{SH}(D_1, D_2) = \frac{1}{4n} \times \sum_{i=1}^{n} \left(\max_{\substack{\gamma_1 \in h_1(x_i),\\ \gamma_2 \in h_2(x_i)}} \left\{ \left| \gamma_1 - \gamma_2 \right| \right\} + \max_{\substack{\eta_1 \in g_1(x_i),\\ \eta_2 \in g_2(x_i)}} \left\{ \left| \eta_1 - \eta_2 \right| \right\} + \left| \frac{\sum_{\gamma_1 \in h_1(x_i)} \gamma_1}{|h_1(x_i)|} - \frac{\sum_{\gamma_2 \in h_2(x_i)} \gamma_2}{|h_2(x_i)|} \right| + \left| \frac{\sum_{\eta_1 \in g_1(x_i)} \eta_1}{|g_1(x_i)|} - \frac{\sum_{\eta_2 \in g_2(x_i)} \eta_2}{|g_2(x_i)|} \right| \right). \quad (4)$$

The hybrid DHF Euclidean distance of D_1 and D_2 is

$$d_{SE}(D_1, D_2) = \sqrt{ \frac{1}{4n} \times \sum_{i=1}^{n} \left(\max_{\substack{\gamma_1 \in h_1(x_i),\\ \gamma_2 \in h_2(x_i)}} \left\{ \left(\gamma_1 - \gamma_2 \right)^2 \right\} + \max_{\substack{\eta_1 \in g_1(x_i),\\ \eta_2 \in g_2(x_i)}} \left\{ (\eta_1 - \eta_2)^2 \right\} + \left| \frac{\sum_{\gamma_1 \in h_1(x_i)} \gamma_1}{|h_1(x_i)|} - \frac{\sum_{\gamma_2 \in h_2(x_i)} \gamma_2}{|h_2(x_i)|} \right|^2 + \left| \frac{\sum_{\eta_1 \in g_1(x_i)} \eta_1}{|g_1(x_i)|} - \frac{\sum_{\eta_2 \in g_2(x_i)} \eta_2}{|g_2(x_i)|} \right|^2 \right) }. \quad (5)$$

The hybrid DHF Minkovski distance of D_1 and D_2 is

$$d_{SG}(D_1, D_2) = \sqrt[p]{ \frac{1}{4n} \times \sum_{i=1}^{n} \left(\max_{\substack{\gamma_1 \in h_1(x_i),\\ \gamma_2 \in h_2(x_i)}} \left\{ \left| \gamma_1 - \gamma_2 \right|^p \right\} + \max_{\substack{\eta_1 \in g_1(x_i),\\ \eta_2 \in g_2(x_i)}} \left\{ \left| \eta_1 - \eta_2 \right|^p \right\} + \left| \frac{\sum_{\gamma_1 \in h_1(x_i)} \gamma_1}{|h_1(x_i)|} - \frac{\sum_{\gamma_2 \in h_2(x_i)} \gamma_2}{|h_2(x_i)|} \right|^p + \left| \frac{\sum_{\eta_1 \in g_1(x_i)} \eta_1}{|g_1(x_i)|} - \frac{\sum_{\eta_2 \in g_2(x_i)} \eta_2}{|g_2(x_i)|} \right|^p \right) }. \quad (6)$$

Remark 4. When p = 1, the hybrid DHF Minkovski distance d_{SG} degenerates to the hybrid DHF Hamming distance d_{SH}. When p = 2, it degenerates to the hybrid DHF Euclidean distance d_{SE}.

Furthermore, considering the significance of diverse attributes or indicators in economic and management decision-making problems, their corresponding attribute weights will inherently vary. Suppose that the weights of attributes $x_i (i = 1, 2, \cdots, n)$ are $\omega_i (i = 1, 2, \cdots, n)$ which satisfies $0 \le \omega_i \le 1$ and $\sum_{i=1}^{n} \omega_i = 1$. Then, the hybrid dual hesitant weighted Hamming distance of D_1 and D_2 is defined as

$$d_{SWH}(D_1, D_2) = \sum_{i=1}^{n} \frac{\omega_i}{4} \left(\max_{\substack{\gamma_1 \in h_1(x_i),\\ \gamma_2 \in h_2(x_i)}} \left\{ \left| \gamma_1 - \gamma_2 \right| \right\} + \max_{\substack{\eta_1 \in g_1(x_i),\\ \eta_2 \in g_2(x_i)}} \left\{ \left| \eta_1 - \eta_2 \right| \right\} + \left| \frac{\sum_{\gamma_1 \in h_1(x_i)} \gamma_1}{|h_1(x_i)|} - \frac{\sum_{\gamma_2 \in h_2(x_i)} \gamma_2}{|h_2(x_i)|} \right| + \left| \frac{\sum_{\eta_1 \in g_1(x_i)} \eta_1}{|g_1(x_i)|} - \frac{\sum_{\eta_2 \in g_2(x_i)} \eta_2}{|g_2(x_i)|} \right| \right). \quad (7)$$

The hybrid dual hesitant weighed Euclidean distance of D_1 and D_2 is defined as

$$d_{SWE}(D_1, D_2) = \sqrt{ \sum_{i=1}^{n} \frac{\omega_i}{4} \left(\max_{\substack{\gamma_1 \in h_1(x_i),\\ \gamma_2 \in h_2(x_i)}} \left\{ \left(\gamma_1 - \gamma_2 \right)^2 \right\} + \max_{\substack{\eta_1 \in g_1(x_i),\\ \eta_2 \in g_2(x_i)}} \left\{ (\eta_1 - \eta_2)^2 \right\} + \left| \frac{\sum_{\gamma_1 \in h_1(x_i)} \gamma_1}{|h_1(x_i)|} - \frac{\sum_{\gamma_2 \in h_2(x_i)} \gamma_2}{|h_2(x_i)|} \right|^2 + \left| \frac{\sum_{\eta_1 \in g_1(x_i)} \eta_1}{|g_1(x_i)|} - \frac{\sum_{\eta_2 \in g_2(x_i)} \eta_2}{|g_2(x_i)|} \right|^2 \right) }. \quad (8)$$

The hybrid dual hesitant weighed Minkovski distance of D_1 and D_2 is defined as

$$d_{SWG}(D_1, D_2) = \sqrt[p]{ \sum_{i=1}^{n} \frac{\omega_i}{4} \left(\max_{\substack{\gamma_1 \in h_1(x_i),\\ \gamma_2 \in h_2(x_i)}} \left\{ \left| \gamma_1 - \gamma_2 \right|^p \right\} + \max_{\substack{\eta_1 \in g_1(x_i),\\ \eta_2 \in g_2(x_i)}} \left\{ \left| \eta_1 - \eta_2 \right|^p \right\} + \left| \frac{\sum_{\gamma_1 \in h_1(x_i)} \gamma_1}{|h_1(x_i)|} - \frac{\sum_{\gamma_2 \in h_2(x_i)} \gamma_2}{|h_2(x_i)|} \right|^p + \left| \frac{\sum_{\eta_1 \in g_1(x_i)} \eta_1}{|g_1(x_i)|} - \frac{\sum_{\eta_2 \in g_2(x_i)} \eta_2}{|g_2(x_i)|} \right|^p \right) }. \quad (9)$$

Remark 5. When p = 1, the hybrid DHF Minkovski distance d_{SWG} degenerates to the hybrid DHF Hamming distance d_{SWH}. When p = 2, it degenerates to the hybrid DHF Euclidean distance d_{SWE}.

Axiom 2. For any DHFSs $D_1 = \{< x, h_1(x), g_1(x) >, x \in X\}$, $D_2 = \{< x, h_2(x), g_2(x) >, x \in X\}$, $D_3 = \{< x, h_3(x), g_3(x) >, x \in X\}$, where $X = \{x_1, x_2, \cdots, x_n\}$. Then a real function $d : DHFS(X) \times DHFS(X) \to R$ is called the dual hesitant fuzzy distance measure if d satisfies the three properties as follows:

(i) (**Non-negativity**) $0 \le d(D_1, D_2) \le 1$ and $d(D_1, D_2) = 0$ if and only if $D_1 = D_2$.

(ii) (**Symmetric**) $d(D_1, D_2) = d(D_2, D_1)$.

(iii) (**Triangle inequality**) $d(D_1, D_2) \le d(D_1, D_3) + d(D_3, D_2)$.

Theorem 2. For any DHFSs $D_1 = \{< x, h_1(x), g_1(x) >, x \in X\}$, $D_2 = \{< x, h_2(x), g_2(x) >, x \in X\}$, $D_3 = \{< x, h_3(x), g_3(x) >, x \in X\}$, where $X = \{x_1, x_2, \cdots, x_n\}$. The hybrid distance formulas d_{SH}, d_{SE}, d_{SG}, d_{SWH}, d_{SWE} and d_{SWG} which is given by Equations (4)–(9) satisfy the following properties:

(i) (**Non-negativity**) $0 \le d_{SH}, d_{SE}, d_{SG}, d_{SWH}, d_{SWE}, d_{SWG}(E_1, E_2) \le 1$, $d_{SH}, d_{SE}, d_{SG}, d_{SWH}, d_{SWE}, d_{SWG}(E_1, E_2) = 0$ if and only if $E_1 = E_2$.

(ii) (**Symmetric**) $d_{SH}, d_{SE}, d_{SG}, d_{SWH}, d_{SWE}, d_{SWG}(E_1, E_2) = d_{SH}, d_{SE}, d_{SG}, d_{SWH}, d_{SWE}, d_{SWG}(E_2, E_1)$.

(iii) (**Triangle inequality**)

$$d_{SH}, d_{SE}, d_{SG}, d_{SWH}, d_{SWE}, d_{SWG}(E_1, E_2) \le d_{SH}, d_{SE}, d_{SG}, d_{SWH}, d_{SWE}, d_{SWG}(E_1, E_3) + d_{SH}, d_{SE}, d_{SG}, d_{SWH}, d_{SWE}, d_{SWG}(E_2, E_3).$$

Proof: Analogous to the proof of Theorem 1, we can establish that Equations (4)–(9) satisfies the properties outlined in Axiom 2. A detailed proof is omitted for brevity.

3.3. Comprehensive weighting method

Suppose that $A = \{A_1, A_2, \cdots, A_i, \cdots A_m\}$ is a set of *m* feasible alternatives. The group of *K* DMs $D = \{D_1, D_2, \cdots, D_k, \cdots, D_K\}$ needs to rank these alternatives based on *n* attributes $C = \{C_1, C_2, \cdots, C_j, \cdots, C_n\}$ for an MCGDM problem. Suppose that the importance weight of the expert D_k be λ_k, where $\sum_{k=1}^{K} \lambda_k = 1,\ 0 \le \lambda_k \le 1$. Then DM D_k expresses his or her opinion about the performance (or evaluation values h_{ij}^k and g_{ij}^k) of the alternative A_i on the attribute C_j based on each DM's point from membership and non-membership degrees. Then the performance can be represented with DHFE $E_{ij} =< \{h_{ij}^1, h_{ij}^2, \cdots, h_{ij}^K\}, \{g_{ij}^1, g_{ij}^2, \cdots, g_{ij}^K\} >$. Thence, the DHF decision matrix is constructed as $E = (E_{ij})_{m\times n}$. The DHF decision matrix $E = (E_{ij})_{m\times n}$ can be shown in Table 3.

Without loss of generality, we assume that the weights of *t* attributes are unknown while the weights of the other (*n* − *t*) attributes have been given. Namely, the weight vector of the attributes can be recorded as $\omega = (\omega_1, \omega_2, \cdots, \omega_t, \omega_{t+1}, \cdots, \omega_n)^T$, where $\sum_{j=1}^{n} \omega_j = 1,\ 0 \le \omega_j \le 1$. Denote the incomplete attribute weight $\bar{\omega} = (\omega_1, \omega_2, \cdots, \omega_t)^T$. Due to each $\omega_j (j = t+1, \cdots, n)$ is constantly known, denote $\sum_{j=t+1}^{n} \omega_j = d$, then $\sum_{j=1}^{t} \omega_j = 1 - d$. When $t = 0$, it is the case where all the weights of attributes are known. When $t = n$, i.e., $d = 1$, it means that all weights of attributes are completely unknown. When $t = 1, 2, \cdots, n-1$, it means that the weights of attributes are partially unknown. Therefore, this study covers all cases, that is, the weights are fully known, completely unknown, and partially known.

When $C_j (j = 1, 2, \cdots, n)$ is the benefit, the DHF positive ideal solution (DHFPIS) and negative ideal solution (DHFNIS) can be defined as $E_j^+ =< \{1\}, \{0\} >$ and $E_j^- =< \{0\}, \{1\} >$, respectively. When the attribute C_j is cost, the DHFPIS and DHFNIS may be defined as $E_j^+ =< \{0\}, \{1\} >$ and $E_j^- =< \{1\}, \{0\} >$, respectively. Therefore, deviations of A_i from DHFPIS and DHFNIS can be taken as an optimality criterion to order all alternatives.

Then the sum of weighted deviations of all alternatives from DHFPIS and DHFNIS are as follows:

$$y_i^+(\bar{\omega}) = \sum_{j=1}^{t} \omega_j \cdot d(E_{ij}, E_j^+); \tag{10}$$

$$y_i^-(\bar{\omega}) = \sum_{j=1}^{t} \omega_j \cdot d(E_{ij}, E_j^-), \tag{11}$$

For weight vector $\bar{\omega}$, the smaller $y_i^+(\bar{\omega})$ and the bigger $y_i^-(\bar{\omega})$, the better A_i. Therefore, partially incomplete weight vector $\bar{\omega}$ should be obtained by each $y_i^+(\bar{\omega})$ reaches its minimum and each $y_i^-(\bar{\omega})$ reaches its maximum simultaneously. Therefore, we can solve the unknown weights by establishing the following multiple-objective programming model.

Table 3. DHF decision matrix

	C_1	C_2	$\cdots$	C_j	$\cdots$	C_n
A_1	$< \{h_{11}^1, h_{11}^2, \cdots, h_{11}^K\}, \{g_{11}^1, g_{11}^2, \cdots, g_{11}^K\} >$	$< \{h_{12}^1, h_{12}^2, \cdots, h_{12}^K\}, \{g_{12}^1, g_{12}^2, \cdots, g_{12}^K\} >$	$\cdots$	$< \{h_{1j}^1, h_{1j}^2, \cdots, h_{1j}^K\}, \{g_{1j}^1, g_{1j}^2, \cdots, g_{1j}^K\} >$	$\cdots$	$< \{h_{1n}^1, h_{1n}^2, \cdots, h_{1n}^K\}, \{g_{1n}^1, g_{1n}^2, \cdots, g_{1n}^K\} >$
A_2	$< \{h_{21}^1, h_{21}^2, \cdots, h_{21}^K\}, \{g_{21}^1, g_{21}^2, \cdots, g_{21}^K\} >$	$< \{h_{22}^1, h_{22}^2, \cdots, h_{22}^K\}, \{g_{22}^1, g_{22}^2, \cdots, g_{22}^K\} >$	$\cdots$	$< \{h_{2j}^1, h_{2j}^2, \cdots, h_{2j}^K\}, \{g_{2j}^1, g_{2j}^2, \cdots, g_{2j}^K\} >$	$\cdots$	$< \{h_{2n}^1, h_{2n}^2, \cdots, h_{2n}^K\}, \{g_{2n}^1, g_{2n}^2, \cdots, g_{2n}^K\} >$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
A_i	$< \{h_{i1}^1, h_{i1}^2, \cdots, h_{i1}^K\}, \{g_{i1}^1, g_{i1}^2, \cdots, g_{i1}^K\} >$	$< \{h_{i2}^1, h_{i2}^2, \cdots, h_{i2}^K\}, \{g_{i2}^1, g_{i2}^2, \cdots, g_{i2}^K\} >$	$\cdots$	$< \{h_{ij}^1, h_{ij}^2, \cdots, h_{ij}^K\}, \{g_{ij}^1, g_{ij}^2, \cdots, g_{ij}^K\} >$	$\cdots$	$< \{h_{in}^1, h_{in}^2, \cdots, h_{in}^K\}, \{g_{in}^1, g_{in}^2, \cdots, g_{in}^K\} >$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
A_m	$< \{h_{m1}^1, h_{m1}^2, \cdots, h_{m1}^K\}, \{g_{m1}^1, g_{m1}^2, \cdots, g_{m1}^K\} >$	$< \{h_{m2}^1, h_{m2}^2, \cdots, h_{m2}^K\}, \{g_{m2}^1, g_{m2}^2, \cdots, g_{m2}^K\} >$	$\cdots$	$< \{h_{mj}^1, h_{mj}^2, \cdots, h_{mj}^K\}, \{g_{mj}^1, g_{mj}^2, \cdots, g_{mj}^K\} >$	$\cdots$	$< \{h_{mn}^1, h_{mn}^2, \cdots, h_{mn}^K\}, \{g_{mn}^1, g_{mn}^2, \cdots, g_{mn}^K\} >$

$$\min\{y(\overline{\omega})=(y_1^+(\overline{\omega})-y_1^-(\overline{\omega}),y_2^+(\overline{\omega})-y_2^-(\overline{\omega}),\cdots,y_n^+(\overline{\omega})-y_n^-(\overline{\omega}))\}$$
$$s.t.\begin{cases}\sum_{j=1}^{t}\omega_j=1-d\\ \omega_j\ge 0,\ j=1,2,\cdots,t.\end{cases} \tag{12}$$

As A represents a set of non-inferior alternatives, there exists no overt preference among the alternatives. Consequently, each objective function is accorded equal weight. Thus, Equation (12) can be amalgamated into the subsequent single-objective programming Equation (13) as

$$\min\{\overline{y}(\overline{\omega})=\frac{1}{m}\sum_{j=1}^{t}(y_j^+(\overline{\omega})-y_j^-(\overline{\omega}))\}$$
$$s.t.\begin{cases}\sum_{j=1}^{t}\omega_j=1-d\\ \omega_j\ge 0,\ j=1,2,\cdots,t.\end{cases} \tag{13}$$

The attribute weight value can be derived by Equation (13), offering an objective method for weight determination. This scientifically grounded calculation process lays a robust quantitative foundation for this study. However, akin to two sides of the same coin, while the objective method presents irreplaceable advantages, it inevitably overlooks subjective factors in certain scenarios. These subjective factors often encompass profound professional backgrounds and extensive practical experience. Recognizing this, it becomes imperative to reassess and integrate both subjective and objective methods for weight determination. Hence, this paper proposes a hybrid approach that amalgamates subjective and objective factors to comprehensively incorporate and reflect the wisdom and value inherent in subjectivity while upholding objectivity.

From a subjective view, assume that DM $D_k(k=1,2,\cdots,K)$ gives subjective judgment of the importance of $C_j(j=1,2,\cdots,n)$. Let $\overline{\omega}^k=(\omega_{k1},\omega_{k2},\cdots,\omega_{kt})^T$ be the weight vector assigned by $D_k(k=1,2,\cdots,K)$, where $\sum_{j=1}^{t}\omega_{kj}=1-d$, $\omega_{kj}\ge 0(k=1,2,\cdots,K;j=1,2,\cdots,t)$.

In the decision-making process, each DM's opinion holds distinct value, stemming not only from their profound professional knowledge background but also from their extensive practical experience and unique insights. Consequently, allocating suitable weight to each expert during decision-making signifies both their authoritative status and influential role within a specific field, along with the contributions they offer to the decision-making process.

Assume the weight of the DM D_k be λ_k, where $\sum_{k=1}^{K}\lambda_k=1$, $0\le\lambda_k\le 1(k=1,2,\cdots,K)$.

After carefully evaluating the relative weight positions of each DM and their corresponding allocation of attribute importance, our objective is to minimize the discrepancy between attribute weights assigned by individual DMs and the subjective attribute weights consolidated through group decisions. This endeavor aims to foster widespread consensus in group decision-making processes. To achieve this goal, we can utilize a nonlinear mathematical programming Equation (14) for problem resolution.

$$\min\{\overline{z}(\overline{\omega})=\sum_{k=1}^{K}\sum_{j=1}^{t}\lambda_k(\omega_j-\omega_{kj})^2\}$$
$$s.t.\begin{cases}\sum_{j=1}^{t}\omega_j=1-d\\ \omega_j\ge 0,\ j=1,2,\cdots,t.\end{cases} \tag{14}$$

To comprehensively and thoroughly account for the comprehensive influence of both subjective and objective factors, we employ a multidimensional analysis approach that amalgamates subjective judgment with objective data, harnessing their respective strengths. Building upon this concept, we integrate the optimization Equation (13) (objective method) with the optimization Equation (14) (subjective method) to formulate a nonlinear bi-objective mathematical programming Equation (15). This Equation (15) aims to precisely ascertain unknown weights, ensuring that decision analysis adheres to objective principles while fully accommodating subjective needs. As a result, it furnishes a more scientific and rational foundation for final decision-making.

$$\min\{\overline{y}(\overline{\omega}),\overline{z}(\overline{\omega})\}$$
$$s.t.\begin{cases}\sum_{j=1}^{t}\omega_j=1-d\\ \omega_j\ge 0,\ j=1,2,\cdots,t.\end{cases} \tag{15}$$

In the process of constructing the model to solve subjective and objective weights, we have duly acknowledged the significant role played by both subjective and objective factors in decision analysis. To achieve a more refined balance and integration between them, it is imperative to introduce a pivotal element into our model: the trade-off factor for subjective and objective factors. This incorporation will further optimize our weight-solving process, ensuring that the model accurately reflects objective facts while also incorporating experts' subjective experiences and wisdom. By introducing this coefficient, we not only enhance the flexibility and adaptability of our model but also provide a more precise and scientifically grounded basis for decision-making. Considering these aspects, let α represent the trade-off coefficient between subjective and objective factors. Then, Equation (15) can be transformed into the following non-linear mathematical programming Equation (16).

$$\min\{\overline{g}(\overline{\omega})=\frac{\alpha}{m}\sum_{j=1}^{t}(y_j^+(\overline{\omega})-y_j^-(\overline{\omega}))+(1-\alpha)\sum_{k=1}^{K}\sum_{j=1}^{t}\lambda_k(\omega_j-\omega_{kj})^2\}$$
$$s.t.\begin{cases}\sum_{j=1}^{t}\omega_j=1-d\\ \omega_j\ge 0,\ j=1,2,\cdots,t.\end{cases} \tag{16}$$

Remark 6. When the trade-off coefficient $\alpha=0$, Equation (16) reduces to Equation (14), where the incomplete weight value $\overline{\omega}$ can be obtained by solving Equation (14), representing a subjective weighting method. Similarly, when the trade-off coefficient $\alpha = 1$, Equation (16) reduces to the Equation (13), the incomplete weight value $\bar{\omega}$ can be obtained by solving Equation (13), signifying an objective weighting method. When the trade-off coefficient $0 < \alpha < 1$, the unknown weight value $\bar{\omega}$ can be obtained by solving Equation (16), representing a comprehensive subjective and objective weighting method.

3.4. DHF VIKOR method and decision process

In the VIKOR, two key evaluation scheme values are calculated: group utility value and individual regret value. The group utility value primarily indicates the overall advantages and benefits of the scheme. It is derived by aggregating the utility value of each attribute and reflects the overall performance of the scheme within the group. On the other hand, the individual regret value pertains to the degree of dissatisfaction or regret of each DM regarding a specific plan. It showcases the disparity between the plan and the DM's ideal point. The core principle of the VIKOR lies in seeking the optimal compromise between maximizing group utility and minimizing individual regret. By amalgamating these two evaluation scenarios, the VIKOR assists DMs in selecting from multiple alternatives that best cater to both overall interests and individual needs. An essential aspect of this process is the precise calculation of the distance measure, a crucial indicator for assessing the gap between the alternative and the ideal solution. Through accurate distance measure calculation, the VIKOR can conduct a more scientific evaluation of the pros and cons of alternatives, providing dependable and effective decision support for DMs. In addition, in practical applications, the specific methodologies for computing group utility value and individual regret value may vary depending on the problem at hand, typically involving the determination of attribute weights. Therefore, we propose integrating the DHF distance measure and the subjective and objective comprehensive weighting method proposed in this paper with the VIKOR to devise a novel approach for solving MCGDM problems within the DHF evaluation system.

The DHF VIKOR for MCGDM with DHFSs can be illustrated in the following ten steps.

Step 1. Identify the alternative set $A = \{A_1, A_2, \cdots, A_i, \cdots A_m\}$ and attribute set $C = \{C_1, C_2, \cdots, C_j, \cdots, C_n\}$.

Step 2. Give the weight vector $\lambda = (\lambda_1, \lambda_2, ..., \lambda_k, ..., \lambda_K)^T$ of DMs. Determine the subjective attribute weight vector $\bar{\omega}^k = (\omega_{k1}, \omega_{k2}, \cdots, \omega_{kt})^T$ by DMs $D_k (k = 1, 2, ..., K)$. The weight vector of the attributes is recorded as $\omega = (\omega_1, \omega_2, \cdots, \omega_t, \omega_{t+1}, \cdots, \omega_n)^T$, where $\bar{\omega} = (\omega_1, \omega_2, \cdots, \omega_t)^T$, $\sum_{j=1}^{n} \omega_j = 1$, and $0 \le \omega_j \le 1$. Denote the incomplete attribute weight $\bar{\omega} = (\omega_1, \omega_2, \cdots, \omega_t)^T$. Where each $\omega_j (j = t+1, \cdots, n)$ is constantly known, $\sum_{j=t+1}^{n} \omega_j = d$. Then the sum of the unknown weights is $\sum_{j=1}^{t} \omega_j = 1 - d$.

Denote the incomplete attribute weight $\bar{\omega} = (\omega_1, \omega_2, \cdots, \omega_t)^T$. Due to each $\omega_j (j = t+1, \cdots, n)$ is constantly known, denote $\sum_{j=t+1}^{n} \omega_j = d$, then $\sum_{j=1}^{t} \omega_j = 1 - d$.

Step 3. Obtain DHF decision matrix $E = (E_{ij})_{m \times n}$.

Step 4. Calculate the weight DHF decision-making matrix $\bar{E} = (\bar{E}_{ij})_{m \times n}$, where

$$\bar{E}_{ij} = \lambda \otimes E_{ij} = <\{\lambda_1 h_{ij}^1, \lambda_2 h_{ij}^2, \cdots, \lambda_K h_{ij}^K\}, \{\lambda_1 g_{ij}^1, \lambda_2 g_{ij}^2, \cdots, \lambda_K g_{ij}^K\}>. \quad (17)$$

Step 5. Construct the DHF positive ideal solution (PIS) $E_j^+ = <\{1\}, \{0\}>$ and negative ideal solution (NIS) $E_j^- = <\{0\}, \{1\}>$ when the attribute $C_j (j = 1, 2, \cdots, n)$ is benefit, or the PIS $E_j^+ = <\{0\}, \{1\}>$ and NIS $E_j^- = <\{1\}, \{0\}>$ when the attribute C_j is cost.

Step 6. Calculate the sum of weighted deviations of all alternatives from PIS and NIS $y_i^+(\bar{\omega})$ and $y_i^-(\bar{\omega})$ by Equations (10) and (11), respectively.

Step 7. Calculate the comprehensive attribute weight vector $\bar{\omega} = (\omega_1, \omega_2, \cdots, \omega_t)^T$ by the Equation (16).

Step 8. Calculate S_i (group utility) and R_i (individual regret) for each alternative.

$$S_i = \sum_{j=1}^{n} \omega_j d(\bar{E}_{ij}, E_j^+) / d(E_j^+, E_j^-), \; i = 1, 2, \cdots, m; \quad (18)$$

$$R_i = \max_j [\omega_j d(\bar{E}_{ij}, E_j^+) / d(E_j^+, E_j^-)], \; i = 1, 2, \cdots, m, \quad (19)$$

where ω_j is the computed comprehensive weighting attribute weight by step 7.

Step 9. Calculate the aggregate score Q_i for each alternative.

$$Q_i = \beta \frac{S_i - S^-}{S^+ - S^-} + (1 - \beta) \frac{R_i - R^-}{R^+ - R^-}, \; i = 1, 2, \cdots, m, \quad (20)$$

where $S^+ = \max_i S_i$, $S^- = \min_i S_i$, $R^+ = \max_i R_i$, $R^- = \min_i R_i$. The compromise coefficient $\beta (0 \le \beta \le 1)$ is a weight introduced to support the strategy of maximum group utility, while $(1 - \beta)$ is used to weigh the individual regret.

Step 10. Rank the alternatives in descending order based on their respective aggregate score Q_i.

4. Case analysis

With the rapid development of urbanization in China, urban residents' requirements for living environment are increasing day by day. As an important part of community management, the quality of property service is directly related to the quality of life and happiness of residents. However, the current property service market is mixed with different service standards. How to evaluate and select high-quality property companies scientifically and objectively has become the focus of common attention of community managers, owners and industry experts. In this context, in order to further improve the level of property management and enhance the satisfaction of residents, a community decided to adopt a more scientific and comprehensive evaluation method to screen the property service companies in the market. This section will expand on four aspects: Alternatives, DMs, Attributes, and Decision-making process.

4.1. Alternatives

After a thorough qualification review and extensive investigation, three property services companies were identified as part of an alternative set $A=\{A_1,A_2,A_3\}$.

Alternative A_1 is Poly Property Services Co., LTD, a property management enterprise under Poly Development Holding Group Co., LTD. It holds a national property management qualification and provides services for community residential properties as well as urban landmark office buildings, government public buildings, urban scenic spots, characteristic industries, colleges, hospitals and other property types. The company prides itself on its professional team of industry elites with rich experience and knowledge in property management, offering owners comprehensive and high-quality services. Embracing technological advancements, Poly Property has implemented an advanced intelligent service system to enhance the intelligence and information of property management. Additionally, the company prioritizes equipment maintenance to ensure the normal operation of various facilities and create a safe and comfortable living environment for owners. These efforts have solidified Poly Property's leading position in the industry.

Alternative A_2 refers to Country Garden Life Service Group Co., LTD, an integrated property management enterprise affiliated with the Country Garden Group and listed on the main Board of the Hong Kong Stock Exchange. The company is distinguished by its emphasis on talent as the primary driving force, attracting a substantial number of high-quality property management professionals. It actively leverages cutting-edge technologies such as AI and big data to advance the development of intelligent communities and deliver convenient and efficient service experiences for owners. Furthermore, Country Garden services prioritize the intelligent upgrading of equipment and facilities, elevating the overall level of intelligence within communities through the introduction of advanced smart equipment and systems. These initiatives not only enhance service efficiency but also contribute to heightened resident satisfaction.

Alternative A_3 is Shenzhen Qianhai Field Intelligence Service Holding Group Co., LTD. He is a comprehensive high-end property service brand, ranked No. 25 on the list of "*2024 China Property Service Top 100 Enterprises with Comprehensive Strength*", and won a number of honors. The company is distinguished by its core value of "wisdom" and is dedicated to forging a new community ecology in the future. Through its self-developed E-control center and other intelligent management systems, the company achieves efficient management and swift response to community needs. Furthermore, it prioritizes service innovation and continuously explores value-added services closely linked to homeowners' lives in order to meet their diverse needs. Its top-notch property services, intelligent living experiences, and ongoing efforts to enhance community satisfaction have garnered high praise and trust from homeowners.

4.2. DMs

In making decisions regarding property services it is crucial to ensure scientific rigor. To achieve this goal effectively within our community setting we engaged three distinct types of DMs. Community managers (D_1) who oversee overall operations. Owner (D_2) representatives with first-hand resident experiences. As well as Property industry experts (D_3) with specialized knowledge. Community managers are responsible for overseeing all aspects related to managing properties within our community including personnel quality assessment team collaboration efforts along with equipment maintenance status reviews. Owner representatives offer unique perspectives based on their direct interactions with residents focusing on factors such as resident satisfaction personnel attitudes towards services provided along with equipment utilization efficiencies. Property industry experts bring invaluable insight through their deep understanding gained from years working within this field allowing them to assess technology applications innovation capabilities while also evaluating equipment renewal strategies offering guidance based upon established standards.

Considering the expertise of each individual, it is acknowledged that DMs evaluating potential property management companies may hold divergent viewpoints stemming from their distinct backgrounds, professional experiences, and personal inclinations. To ensure equitable consideration of all perspectives, it is imperative to assign appropriate weights reflecting their respective significance throughout the evaluation process, thereby ensuring comprehensive deliberation and optimal outcomes. Consequently, a judicious weight is assigned based on each evaluator's proficiency in the selection criteria for property management companies. The weight vector representing these DMs' contributions can be expressed as $\lambda=(0.35,0.35,0.3)^T$.

4.3. Attributes

As the determination of evaluation indicators is an important and complex issue, it is not the focus of this paper, so this paper does not carry out special research. With reference to the existing theoretical research and property management expert opinions, the following evaluation indicators are selected, as shown in Table 4.

These three primary indicators form an attribute set $C=\{C_1,C_2,C_3\}$. As a crucial determinant in the decision-making process, the weights of attributes are still unknown. Hence, we will employ the subjective and objective comprehensive attribute weighting method proposed in

Table 4. Property management company evaluation index system

Primary indicators	Secondary indicators	Index meaning
Personnel (C_1)	Service attitude and response speed	To assess how friendly and patient property staff are when dealing with owners or tenants, and how quickly they respond to requests for repairs, inquiries, etc.
	Professional skills and training	To examine whether the property team members have the necessary professional skills, such as facility maintenance, safety management, customer service, etc., and whether they receive relevant training regularly to improve the quality of service
	Teamwork and communication efficiency	To assess the ability to collaborate among members of the property team, the efficiency of information sharing, and the smoothness of communication across departments
Technology (C_2)	Intelligent management system	Assess whether the property adopts intelligent management systems (such as access control systems, parking management systems, repair platforms, etc.) to improve management efficiency and service quality
	Data analysis and decision support	Focus on whether the property uses big data, cloud computing and other technical means to collect and analyze service data to provide a scientific basis for management decisions
	Technological innovation and application	Assess whether the property is actively introducing new technologies and methods, such as Internet of Things, artificial intelligence, etc., to improve service quality and user experience
Equipment (C_3)	Facility maintenance and maintenance	Pay attention to the regular maintenance and maintenance of public facilities (such as elevators, water and electricity systems, fire fighting facilities, etc.) in the property area to ensure their normal operation and safety
	Equipment update and upgrade	To assess whether the property is based on actual needs and technological developments, and timely update or upgrade old equipment to improve service efficiency and living experience
	The environment is clean and green	To investigate the environmental health status and green maintenance level in the property area, including the cleanliness of public areas, garbage disposal, pruning and maintenance of green plants, etc.

this paper to systematically and scientifically address the weighting of each attribute. Nevertheless, within an enterprise decision-making system, ultimate decision-making authority lies with the top leader whose subjective judgment on attribute weight cannot be disregarded. In this context, it can be assumed that the top leader, based on his or her expertise and discernment, assigns a weight of 0.3 to the attribute C_3 while possessing an ambiguous understanding of the weights associated with attributes C_1 and C_2. In other words, the top leader's subjective judgment of attribute weight is a vector $\omega = (\omega_1, \omega_2, 0.3)$. This information will be fully incorporated into our weight-determination process to ensure comprehensiveness and rationality in decision-making.

4.4. Decision-making process and results

Each DM independently evaluates the performance of the three property management companies on the three attributes, based on their expertise and experience. In the evaluation problem of property management company, the traditional decision-making method usually uses real number as the evaluation value to quantify the performance of each option in different attributes. However, in complex business environments, DMs are frequently confronted with incomplete information, high uncertainty, and personal preference differences. These factors make it challenging to accurately reflect the true intentions and judgments of DMs.

To overcome this limitation, we propose using DHFS as evaluation values. The DHFS not only encompasses the quantitative properties of real values but also allows DMs to express hesitations and uncertainties. This enables a better simulation of the complexity and diversity present in the real world. Specifically, the DHFS can accommodate multiple possible values to form a set that reflects various possibilities regarding a DM's assessment of alternative under a specific attribute. This mode of expression is not only closer to reality but also more accurately captures DMs' true intentions and judgments. By adopting this approach, potential risks and opportunities can be revealed while enhancing flexibility and adaptability in decision-making processes. The evaluation value provided by each DM is summarized to form a decision matrix.

$$E = (E_{ij})_{3\times 3} = \begin{pmatrix} <\{0.5,/,0.3\},\{0.4,/,0.3\}> & <\{0.5,0.6,0.4\},\{0.3,0.4,0.2\}> & <\{0.3,0.2,0.1\},\{0.6,/,0.5\}> \\ <\{/,0.4,/\},\{/,0.5,0.4\}> & <\{0.7,0.6,0.4\},\{0.3,0.2,0.2\}> & <\{0.6,0.5,0.4\},\{0.1,0.3,0.2\}> \\ <\{0.5,0.5,0.4\},\{0.4,0.2,0.2\}> & <\{/,0.6,0.4\},\{/,0.2,0.2\}> & <\{0.4,0.3,0.2\},\{0.6,0.5,0.5\}> \end{pmatrix},$$

where the DHFE $<\{0.5,/,0.3\},\{0.4,/,0.3\}>$ in the matrix E means that the three DMs D_1, D_2, D_3 think that the membership and non-membership degrees of the alternative

A_1 on the attribute C_1 are $(0.5, 0.4)$, $(/, /)$, and $(0.3, 0.3)$, respectively. There are three indicators below the primary indicators, so there are three pairs of arrays. "/" means that the membership and non-membership evaluation value of the expert D_2 is not obtained since the expert D_2 abstains or data loss. Other elements in the decision matrix E can be similarly explained. Let the weight vector of DMs is $\lambda = (0.35, 0.35, 0.3)^T$ and the attribute weight vector is $\omega = (\omega_1, \omega_2, 0.3)$.

Then, the DHF VIKOR proposed in this paper is used to solve the MCGDM problem of evaluating and selecting a property management company in this community. The specific steps involved in making these decisions are outlined below.

Step 1. Identify the alternative set $A = \{A_1, A_2, A_3\}$ and attribute set $C = \{C_1, C_2, C_3\}$.

Step 2. Determine DMs' weight vector $\lambda = (\lambda_1, \lambda_2, \lambda_3)^T = (0.35,\ 0.35,\ 0.3)^T$. The top leader's subjective judgment of attribute weight is a vector $\omega = (\omega_1, \omega_2, 0.3)$. These three DMs' subjective weight vectors for attributes are $\overline{\omega}^1 = (\omega_{11}, \omega_{12}, \omega_{13})^T = (0.5, 0.2, 0.3)^T$, $\overline{\omega}^2 = (\omega_{21}, \omega_{22}, \omega_{23})^T = (0.4, 0.3, 0.3)^T$, and $\overline{\omega}^3 = (\omega_{31}, \omega_{32}, \omega_{33})^T = (0.3, 0.4, 0.3)^T$, respectively.

Step 3. Obtain DHF decision matrix $E = (E_{ij})_{3\times 3}$.

Step 4. Considering the difference in the degree of importance between these three DMs, i.e., DMs' weight vector $\lambda = (\lambda_1, \lambda_2, \lambda_3)^T = (0.35, 0.35, 0.3)^T$, we calculate the weight DHF decision-making matrix by Equation (17) as

$$E = (E_{ij})_{3\times 3} = \begin{pmatrix} \langle \{0.175, /, 0.09\}, \{0.14, /, 0.09\} \rangle & \langle \{0.175, 0.21, 0.12\}, \{0.105, 0.14, 0.06\} \rangle & \langle \{0.105, 0.07, 0.03\}, \{0.21, /, 0.15\} \rangle \\ \langle \{/, 0.14, /\}, \{/, 0.175, 0.12\} \rangle & \langle \{0.245, 0.21, 0.12\}, \{0.105, 0.07, 0.06\} \rangle & \langle \{0.21, 0.175, 0.12\}, \{0.035, 0.105, 0.06\} \rangle \\ \langle \{0.175, 0.175, 0.12\}, \{0.14, 0.07, 0.06\} \rangle & \langle \{/, 0.21, 0.12\}, \{/, 0.07, 0.06\} \rangle & \langle \{0.14, 0.105, 0.06\}, \{0.21, 0.175, 0.15\} \rangle \end{pmatrix}.$$

Step 5. Identify the DHF PIS and NIS. Since all attributes are beneficial in this problem, therefore DHF PIS and NIS are $E_j^+ = \langle \{1\}, \{0\} \rangle$ and $E_j^- = \langle \{0\}, \{1\} \rangle$, respectively.

Step 6. Compute the sum of weighted deviations of all alternatives from DHF PIS and NIS $y_i^+(\overline{\omega})$ and $y_i^-(\overline{\omega})$ by Equations (10) and (11), respectively. As shown in Table 5.

Step 7. According to the comprehensive weighting method of subjective and objective attribute weights which is proposed in Section 3, the incomplete weight vector of the comprehensive weighted attribute is calculated by Equation (16).

$$\min\{\overline{g}(\overline{\omega}) = 0.35\alpha[(\omega_1 - 0.5)^2 + (\omega_2 - 0.2)^2 + (\omega_1 - 0.4)^2 + (\omega_2 - 0.3)^2] + (1-\alpha)(-0.01\omega_1 - 0.08\omega_2 + 0.01)\}$$
$$s.t. \begin{cases} \omega_1 + \omega_2 = 0.7 \\ \omega_1, \omega_2 \geq 0. \end{cases} \quad (21)$$

By the method of Lagrange multipliers, we can get that the solution of Equation (21) is $\overline{\omega} = (\omega_1, \omega_2)^T = (0.48 - \frac{0.03}{\alpha}, 0.22 + \frac{0.03}{\alpha})^T$. Since α is a trade-off coefficient between subjective and objective factors, we might as well assume that $\alpha = 0.5$. Then the comprehensive weighting attribute weight vector that can be solved is $\omega = (0.465, 0.235, 0.3)^T$.

Step 8. According to Equations (18) and (19), the group utility and individual regret values can be calculated as $S_1 = 0.52$, $S_2 = 0.49$, $S_3 = 0.5$, $R_1 = 0.24$, $R_2 = 0.24$ and $R_3 = 0.23$, respectively.

Step 9. According to Equation (20), the aggregate score values of the three alternatives are $Q_1 = 1$, $Q_2 = 1 - \beta$, and $Q_3 = \frac{\beta}{3}$, respectively.

Step 10. Determine the best alternative.

The aggregate scores of the three alternatives obtained in step 9 reveal that they are three functions dependent on the compromise coefficient β, which serves as a trade-off between group utility and individual regret. The graphical representation of these functions can be observed in Figure 2.

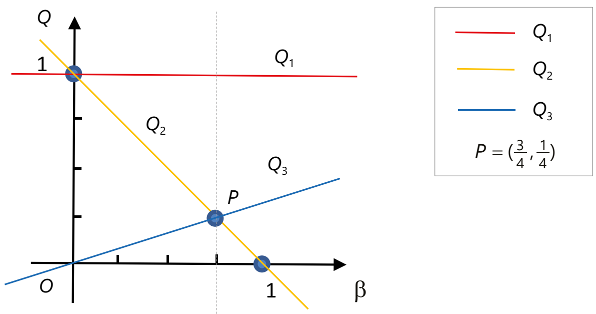

Figure 2. The relationship between the aggregate score function and the compromise coefficient β

Table 5. The sum of weighted deviations of all alternatives from DHF PIS and NIS

	$y_i^+(\overline{\omega})$	$y_i^-(\overline{\omega})$
A_1	$y_1^+(\overline{\omega}) = 0.51\omega_1 + 0.48\omega_2 + 0.17$	$y_1^-(\overline{\omega}) = 0.52\omega_1 + 0.55\omega_2 + 0.14$
A_2	$y_2^+(\overline{\omega}) = 0.52\omega_1 + 0.47\omega_2 + 0.14$	$y_2^-(\overline{\omega}) = 0.49\omega_1 + 0.57\omega_2 + 0.17$
A_3	$y_3^+(\overline{\omega}) = 0.49\omega_1 + 0.47\omega_2 + 0.17$	$y_3^-(\overline{\omega}) = 0.55\omega_1 + 0.55\omega_2 + 0.14$

The best alternative A_1 remains unchanged regardless of the variation in compromise coefficient β, as depicted in Figure 2. However, the relative positioning of alternatives A_2 and A_3 leads to qualitative changes on either side of the dotted line. On the left side, the function Q_2 intersects with Q_3 at point $P=(\frac{3}{4},\frac{1}{4})$. Notably, the alternative A_2 outperforms the alternative A_3 on this side. Conversely, on the right side of the dotted line, the alternative A_2 is inferior to the alternative A_3. In other words, when the value of the compromise coefficient β is greater than 0.75, that is, when the group utility is sufficiently valued, the alternative A_3 outperforms the alternative A_2. The ranking results of the three alternatives can be summarized as shown in Table 6.

Table 6. The ranking results of the three alternatives

β	Alternative ranking results
$\beta = 0$	$A_1 = A_2 \succ A_3$
$\beta \in (0, 0.75)$	$A_1 \succ A_2 \succ A_3$
$\beta = 0.75$	$A_1 \succ A_2 = A_3$
$\beta = (0.75, 1]$	$A_1 \succ A_3 \succ A_2$

Therefore, through an in-depth analysis of the above four scenarios, we can infer that the reason they present different alternative ranking results is the significant differences in the importance of group utility maximization and individual regret in each scenario.

In the first scenario, the DM may be entirely regret-oriented, prioritizing personal emotions and avoiding regret over maximizing group utility. Consequently, less consideration is given to maximizing group utility, resulting in decision-making biased towards meeting individual needs and wishes.

Conversely, in the fourth scenario, DMs may prioritize maximizing group utility, focusing on overall interests and maximizing overall benefits. Here, individual regret may be viewed as a secondary factor or even ignored or sacrificed in the pursuit of maximizing group utility.

The second scenario may aim to strike a balance between group utility and individual regret, considering overall good while minimizing the likelihood of individual regret. Adjustments to find this balance point may be necessary based on specific circumstances and DMs' preferences to achieve relatively optimized decision outcomes.

The third scenario represents a more complex mix where DMs are not solely driven by group utility maximization or individual regret avoidance. They consider multiple factors, including group utility, individual regret, and other potential benefits and risks, to make more comprehensive decisions.

5. Analysis and discussion

This paper proposes a MAGDM method applicable in a DHF environment where evaluation values are incomplete and attribute weights are not fully known. To verify the proposed models and method, a comparative analysis of decision-making methods is provided in this section. In this section, first, the necessity of using DHFS to evaluate the quality of property services is analyzed. Secondly, the impact of incomplete attribute weights on MAGDM is discussed, and in the DHF environment, combined with the VIKOR decision-making idea, the necessity and significance of the proposed new method for evaluating the quality of property services are presented. Finally, the effectiveness and superiority of the method proposed in this paper are verified through a comparative analysis with existing methods.

5.1. Necessity analysis of using DHFS to evaluate property service quality

It is imperative to employ DHFS for the evaluation of property service quality, which primarily encompasses the following three aspects.

(1) DHFS possess inherent capabilities in handling uncertainty and fuzziness. When assessing property service quality, numerous indicators such as service attitude and maintenance efficiency are challenging to measure with precise values, relying more on subjective perceptions and judgments from owners. This subjectivity introduces fuzziness into the evaluation information. By introducing concepts like membership degree and non-membership degree, DHFS can accurately express this fuzziness, resulting in evaluation outcomes that closely align with reality.

(2) Compared to traditional binary logic (yes/no, good/bad), FS, IFNs, gray numbers, etc., DHFS can better represent fuzziness and group decision attributes during property service quality evaluations through their membership degree (set) and non-membership degree (set). They provide more comprehensive and accurate data for evaluation information while enhancing decision accuracy.

(3) DHFS can accommodate missing values. During actual property service quality evaluations, various factors such as owners not completing certain assessment indicators or errors in the data collection process may lead to a loss of evaluation information. Traditional evaluation methods often struggle with addressing this situation by either directly ignoring missing values or adopting simple interpolation methods that could compromise the accuracy of results. Both membership and non-membership degrees of DHFS are a set, which is more inclusive of missing values. This set form improves accuracy and reliability while retaining more evaluation information.

To conclude, the utilization of DHFS is imperative for describing the evaluation information pertaining to property service quality. This approach not only effectively addresses the ambiguity and uncertainty associated with such evaluations but also accommodates missing values, thereby enhancing the accuracy and reliability of the assessment.

5.2. The necessity and significance of the new method

The evaluation of property service quality typically involves assigning weights to each attribute or criterion, reflecting their relative importance in the decision-making process. However, due to factors such as inconsistent expert opinions and challenges in expressing preferences clearly, determining these attribute weights completely can be difficult, resulting in incomplete weight information. This incompleteness poses challenges for the decision-making process. Traditional decision methods rely on deterministic values for accurate calculation and ranking, making them less applicable when dealing with unknown weights or potentially leading to inaccurate results. Therefore, a decision method capable of handling incomplete weight information is necessary.

The basic principle of the VIKOR decision-making method is to first determine the PIS and NIS, and then determine the degree of closeness of the attributes of each alternative to the ideal solutions based on their evaluation values. It seeks a balance between maximizing group utility and minimizing individual regret, which is a method of optimal compromise solution. The solution obtained is a compromise solution, which is the result of mutual concession between the best and worst attributes.

The decision-making methods currently proposed include the VIKOR method, TOPSIS method, ELECTRE method, AHP method, LINMAP method, DEA method, and so on. Each method has its own advantages, disadvantages, and applicable scenarios, and needs to be carefully selected based on specific problems and needs. The unique feature of the VIKOR method is the aggregation function and the decision mechanism coefficient. The VIKOR method proposed a compromise solution with an advantage rate based on the TOPSIS method. In the TOPSIS method, the distance is simply the sum, without considering their relative importance, while in VIKOR, the DM will determine their importance based on their own needs. The VIKOR method has one more compromise coefficient $\beta(\beta \in [0,1])$, representing the decision mechanism coefficient, which can be set according to the personal preferences of the decision-maker, which can enable the decision-maker to make more radical or conservative decisions.

The VIKOR method is suitable for DMs who cannot or do not know how to accurately express their preferences, where evaluation criteria are in conflict and incommensurable (different measurement units), and where DMs are willing to accept compromise solutions. There are three reasons why the VIKOR decision-making idea is applicable to the problem of evaluating property service quality.

(1) Property service quality evaluation involves multiple aspects, such as service attitude, cleanliness, and facility maintenance, which can be regarded as different attributes or criteria. The VIKOR method can handle MCGDM problems, so it is suitable for evaluating property service quality.

(2) In evaluating property service quality, different owners or evaluators may have different preferences for various aspects of service quality. The VIKOR method can incorporate the subjective preferences of DMs (such as owners or evaluators) so that the evaluation results are more in line with actual needs.

(3) Since property service quality evaluation often involves multiple aspects, and different aspects may have conflicting or incommensurable problems. The VIKOR method obtains a compromise solution, which is the result of mutual concessions among different attributes, and is therefore more easily accepted by DMs.

In summary, the VIKOR method, with its unique compromise ranking method, MCGDM ability, and ability to incorporate DMs' subjective preferences, has broad application prospects in property service quality evaluation problems. Therefore, It is necessary and meaningful to put forward a new decision making method which is suitable for the evaluation of property service quality. This method can not only deal with the case where the attribute weights are not completely known, but also make the decision-making process more flexible and practical. It can also adapt to the fuzziness and uncertainty of evaluation information and reflect the actual situation more accurately. The proposed method not only effectively addresses the scenario where attribute weights are partially unknown, enhancing flexibility and practicality in decision-making processes, but also accommodates fuzziness and uncertainty in evaluation information, leading to a more accurate reflection of the actual situation.

5.3. Comparative analysis

In this subsection, we will employ several existing MCGDM approaches to address the issue of property service quality evaluation presented in Section 4. The methods for comparison include the TOPSIS method based on the cumulative prospect theory in the probabilistic HF environment (Sha et al., 2021), the TOPSIS method based on the centroid distance measure in the Pythagorean FS environment (Sun & Wang, 2024), the fuzzy TODIM method (Aydoğan et al., 2024), and the TODIM method in the DHF environment (Liu et al., 2024).

In order to conduct quantitative analysis more effectively, the total discrimination degree (TDD), a commonly used standard for evaluating the quality of MAGDM method rankings, was introduced (Li & Wan, 2014). The validity and superiority of the MAGDM method proposed in this paper were verified by comparing the analysis results. The higher the degree of discrimination between alternative solutions, the more accurate and reliable the ranking results will be. Variance is an effective indicator for measuring sample differences, and can be used as a measure of the TDD between solutions. The TDD based on variance (TDDV) is defined as

$$TDDV = [\frac{1}{m}\sum_{i=1}^{m}(\nu(A_i) - \overline{\nu})^2] \times 100\%, \tag{22}$$

where $\nu(A_i)$ denotes the ranking value of A_i. In this paper, $\nu(A_i)$ is Q_i value. $\overline{\nu}$ is the average value of $Q_i (i = 1, 2, \cdots, m)$.

Table 7. Decision making indicator values and decision making results of different methods

Methods	Values of decision making indicators	Alternative ranking	TDDV
In Sha et al. (2021)	$R(A_1)=0,\ R(A_2)=-0.207,\ R(A_3)=-0.104$	$A_1 \succ A_3 \succ A_2$	1.07%
In Sun and Wang (2024)	$c(A_1)=0.758,\ c(A_2)=0.503,\ c(A_3)=0.427$	$A_1 \succ A_2 \succ A_3$	3.01%
In Aydoğan et al. (2024)	$Q_1=0.938,\ Q_2=0.837,\ Q_3=0.826$	$A_1 \succ A_2 \succ A_3$	0.38%
In Liu et al. (2024)	$PI(A_1)=-0.104,\ PI(A_2)=0.075,\ PI(A_3)=-0.002$	$A_2 \succ A_3 \succ A_1$	0.81%
In this paper	$Q_1=1,\ Q_2=1-\beta,\ Q_2=\frac{\beta}{3},\ (\beta\in[0,1])$	$A_1 = A_2 \succ A_3$ $(\beta=0)$ $A_1 \succ A_2 \succ A_3$ $(\beta\in(0,0.75))$ $A_1 \succ A_3 \succ A_2$ $(\beta\in(0.75,1])$	[2.38%, 22.2%]

The larger the TDDV value, the better the discrimination ability between alternatives and the more accurate and reliable the ranking results.

To ensure an objective and equitable comparison of decision results obtained by both methods, we employ identical PIS and NIS, denoted as $E_j^+ =<\{1\},\{0\}>$ and $E_j^- =<\{0\},\{1\}>$ respectively. The weight vector for DM is represented as $\lambda=(0.35,0.35,0.3)^T$, while the attribute weight vector is $\omega=(0.465,0.235,0.3)^T$.

The comparison methods and ranking results are shown in Table 7.

From the exhaustive data in Table 7, we can extract the following several key analytical outcomes.

(1) The ranking results of some traditional methods differ from those derived by the innovative DHF VIKOR method proposed in this paper. This contrast starkly reveals the profound influence of different distance measurement selections and comprehensive weight determination strategies on the final ranking results, highlighting the significance of method choice in guaranteeing the fairness and accuracy of the assessment.

(2) The method proposed in this paper has a distinctive advantage: it is capable of deducing an explicit scheme utility value function, which is closely associated with the compromise coefficient. This feature enables us to precisely calculate the corresponding ranking results for any given compromise coefficient. Further observation indicates that as the compromise coefficient changes, the ranking results adjust accordingly, demonstrating a dynamic characteristic. Nevertheless, although the ranking order may vary, overall, the optimal solution remains consistent, highlighting the advantages of this method in terms of stability and consistency.

(3) From the methodological perspective, the proposed method generates a scheme utility value function, rather than the single utility value real number provided by traditional methods. Hence, in the crucial indicator of TDDV (that is, the decision diversity metric value, specifically the interval value of [2.38%, 22.2%] in this paper, which is calculated by a Python program), the proposed method exhibits a unique interval characteristic. Among all the methods involved in the comparison, the TDDV value obtained by the proposed method is the highest. This data outcome not only strongly proves the superiority of the proposed method in the evaluation effect from a quantitative perspective but also further validates its extensive applicability and potential value in practical applications.

Compared with other methods, the superiority of the method presented in this paper lies in the following aspects:

(1) DHF VIKOR employs the DHF metric to accurately characterize the interactions among criteria, thereby enhancing the rationality of decision-making. By fully considering hesitant information and satisfying strict inequality property conditions, the distance measure we proposed not only accurately depicts the interactions among criteria but also improves expressiveness and precision. Consequently, we adopt this new distance metric to determine the distances between alternative points as well as the distances between PIS and NIS in DHF VIKOR.

(2) DHF VIKOR based on the comprehensive determination of subjective and objective weights achieves a balance between subjective judgments and objective data, thus making the decision-making more comprehensive and accurate. This approach takes into account the experience and preferences of data managers while integrating the objectivity of data sources. Therefore, it helps alleviate subjective biases and blind spots in decision-making and simultaneously enhances the scientific rigor and reliability of decisions.

(3) Provide support for high-precision management decision-making. The proposed method is a practical and universal non-additive preference information fusion approach. It can not only aggregate non-additive preference

information more reasonably, making the ranking results more objective and accurate, but also enhance the differences among alternative ranking values, thereby improving the robustness of alternative rankings. Therefore, it can offer strong support for the high-precision decision-making requirements in real scenarios.

6. Conclusions

With the rapid advancement of social sciences, technology, and the economy, practical challenges have become increasingly complex and multifaceted. To address these challenges, DHFS theory offers a robust framework. This paper, through an in-depth analysis of DHFS information characteristics, addresses the issue of information distortion inherent in traditional complementary methods when processing DHF information. We propose several novel distance measurement formulas that comprehensively considers the informational value of DHFS hesitation, thereby ensuring the accuracy and reliability of information processing. The proposed formulas satisfy axiomatic property conditions and enhances the applicability of DHFS theory.

Furthermore, this paper innovatively introduces a comprehensive weighting methodology that integrates subjective evaluation with objective data analysis to enhance the precision of decision-making processes. This approach not only addresses the ambiguity in DMs' preferences and the incompleteness of attribute weights but also successfully applies to the assessment and selection of property service quality. The proposed DHF VIKOR decision-making method calculates group utility values and individual regret values based on the DHFS distance measure, and employs a combined subjective-objective weighting technique to determine incomplete attribute weights. This method offers the advantages of a standardized process, reliable outcomes, and flexible application.

While this study has achieved significant advancements in the DHF evaluation system, future research should strive for breakthroughs in both technological and managerial innovation. Blockchain technology, with its distinctive advantages, holds substantial potential for enhancing economic management evaluation and decision-making processes. Future studies could investigate the integration of blockchain technology into DHF evaluation systems to bolster data authenticity and credibility.

In conclusion, this paper has made notable contributions to the theoretical and practical aspects of DHFS. However, further exploration and refinement are necessary to address the increasingly intricate real-world challenges.

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Author contributions

Jingjing AN was responsible for the model, method and the conclusion. Xingxian ZHANG was responsible for project supervision and research design. Lijun LIU was responsible for the manuscript revision. Wenjin ZUO was responsible for method application and scenario design.

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