



## SETTING RENTS IN RESIDENTIAL REAL ESTATE: A METHODOLOGICAL PROPOSAL USING MULTIPLE CRITERIA DECISION ANALYSIS

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**ABSTRACT.** The real estate sector has been negatively affected by the recent economic recession, which has forced structural changes that impact property value and price. Recent pressures have also motivated reduced liquidity and access to credit, causing a drop in property sales and, thus, boosting the rental housing market. It is worth noting, however, that the rental housing segment is not without difficulties and complexity, namely in terms of legislation and rental value revaluation. In light of this reasoning, this study aims to develop a multiple criteria decision support system for calculation of residential rents. By integrating cognitive maps and the measuring attractiveness by a categorical based evaluation technique (MACBETH), we also aim to introduce simplicity and transparency in the decision making framework. The practical implications, advantages and shortfalls of our proposal are also analyzed.

**KEYWORDS:** Multiple criteria analysis; Decision support; Residential rental; Rental definition; Real estate

### 1. INTRODUCTION

The real estate sector is of great importance for the economic and social development of a country, and any change within it has repercussion in other sectors of economic activity. Indeed, major changes in the housing market have been taking place in recent years, which have been revealing new trends, such as the fall in house prices and declining property values (Campbell 2009; Eichholtz *et al.* 2012; Ferreira *et al.* 2013). Additionally, fostered by the economic crisis, a visible lack of purchasing power of the population has been taking place, coupled with liquidity shortages and lending retraction. These factors led to a decline in real estate sales and, in parallel, to a strong growth of the rental housing market (cf. Gomes, Rangel 2009a, 2009b; Lee, Chung 2010). In this sense, it seems likely that the rental segment will be, in conjunction

with urban regeneration, one of the pillars of the housing market in the coming years. Due to the fact that most of the leases currently in place are outdated (for discussion, see Titman, Twite 2013), several studies and analyses have been made to update these contracts. Nevertheless, the process of rental revaluation has been difficult to resolve.

With this background in mind, there is a clear need for the rental housing market to be consistent with the landlords' requirements, the tenants' expectations and, simultaneously, with the economic and social interests of the State. It is from this perspective that the use of structuring techniques and multiple criteria evaluation methods seems to make sense, as these approaches have been reported in the literature as able to clarify complex decision problems guided by conflicting positions (Belton, Stewart 2002). Because valuation models are based on certain variables that determine property value, the combined use of cognitive mapping tech-

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niques and the multiple criteria decision analysis (MCDA) approach holds great potential for rental estimates in residential real estate. Thus, it seems to be of practical interest to propose a methodological framework, which would allow residential rents to be estimated. Simply put, this study aims to develop a decision support model for rental estimates in the housing market by combining cognitive mapping techniques and the MCDA approach.

In order to accomplish this aim, and following the strategic options development and analysis (SODA) approach (Ackermann, Eden 2001; Eden, Ackermann 2001), cognitive mapping techniques will be used to select the evaluation criteria. Next, measuring attractiveness by a categorical based evaluation technique (MACBETH) (Bana e Costa, Vansnick 1994) will be used to calculate the trade-offs among them. The entire process will be grounded on group meetings with real estate professionals.

The remainder of the paper is structured as follows. The next section addresses the importance of setting rents in the rental housing market, and presents some of the main limitations of the current valuation models. The ensuing section presents the methodological background of the evaluation system developed, discussing how the combination of cognitive maps with the MACBETH technique can surpass some of the limitations identified. The next section explains the procedures followed to develop and test our framework. The final section presents the conclusions of the study.

## 2. RELATED WORK

Lack of liquidity, low wages, household indebtedness, unemployment and limited access to credit negatively affect the real estate market and society at large. Indeed, real estate sales have been declining due to households' lack of liquidity, which has additionally originated an increase in default rates on mortgages (see Campbell 2009; Ferreira *et al.* 2014a). Families' incapacity to respect payments, namely mortgage loans, led banks to impose severe restrictions on access to credit and, in sequence, an exponential growth in the property rental market has been taking place in opposition to the sharp drop in the buying and selling market.

Due to its increasing influence on the financial, service and labor markets, the rental housing market is of great importance for the economy of a country. However, it is worth noting that this strong growth of the rental segment is not without difficulties, making it necessary to reconcile the

landlords' priorities with the tenants' expectations, as well as to contemplate the social and economic interests of the State itself. Taking into account that decision making in this context is very important but extremely complex, further contributions enhancing the consistency and transparency of the decisions are paramount. As pointed out by Gomes and Rangel (2009a: 204), *"the rental evaluation of a property is one of the most important tasks for those who work in the [...] real estate market. This evaluation is generally based on quantitative and qualitative criteria employing various simple methods"*. It is not surprising therefore that remarkable progress has been taking place over the years to address this issue.

Despite the variety and dynamism revealed by the studies presented to date (which use hedonic modeling and other mass appraisal techniques, non-parametric or semi-parametric regressions, comparative and income approaches, and spatial models that capture correlations within submarkets allowing for temporal asymmetry) (e.g. Ginsburgh, Waelbroeck 1998; Bourassa *et al.* 2003; Bin 2004; Hongyu, Yue 2005; Diappi, Bolchi 2006; Čeh *et al.* 2012; Goswami, Tan 2012; Leung, Tsang 2012; Aysoy *et al.* 2014), it is worth noting that, like all research, they are not without their shortcomings. Regarding hedonic modeling in particular, despite its undeniable merit, Eichholtz *et al.* (2012: 272) defend that *"an important drawback of the hedonic regression method constitutes the requirement to select a set of appropriate property quality characteristics. However, identifying a complete set of historic property characteristics is practically impossible and one has to make assumptions about which factors to include and which not. This bears the risk of omitted variables and functional misspecification"* (see also Quigley 1995; Gouriéroux, Laferrère 2009). In addition, it is worth noting that the current approaches generally fail to provide insight into the relationships between the determinants that influence rent definition, such as: individual factors; conjuncture and market maturity. Indeed, as noted by Bin (2004: 69), *"there is little guidance from economic theory about the proper functional relationship between housing price and its attributes"*. In light of this reasoning, there are limitations common to most extant studies, which can broadly be grouped into two major categories: (1) the way the determinants/criteria are selected and incorporated into the appraisal systems; and (2) the way the trade-offs between criteria are presented. Indeed, as pointed out by Bourassa *et al.* (2010: 139), *"caution [...] should be*

exercised [...]. *Appropriate variables must be selected carefully and measured accurately*". Following this, the development of a system that can support the definition of rents in residential real estate, while overcoming these two general categories of methodological limitations, is a very relevant aim.

Because real estate decision processes are complex, inherently subjective and involve multiple stakeholders with different and often conflicting perspectives, there is considerable scope for methodological proposals that are both comprehensive and flexible enough to deal with such issues. In this regard, and as pointed out by Belton and Stewart (2002) and Eden (2004), the MCDA approach is particularly suited to such decision problems, namely because its constructivist stance acknowledges and incorporates the underlying subjectivity, allowing for a better structuring of the decision making process. That said, the application of MCDA methodologies in real estate in general, and in rent definition of residential housing in particular, is not new (see, for instance, Zavadskas et al. 2005; Gomes, Rangel 2009a; Pereira et al. 2013). While applying different MCDA techniques (e.g. analytic hierarchy/network process (AHP/ANP), complex proportional assessment (COPRAS) and interactive and multicriteria decision making (TODIM)), none of the previous studies use cognitive mapping to obtain the evaluation criteria, allowing our study to provide relevant progress in this regard. In fact, the combined use of cognitive maps and MACBETH for setting rents in the residential real estate market, as proposed and applied in this study, is to the best of our knowledge a novel application of these decision support tools, and provides relevant progress in terms of criteria selection and trade-offs calculation, creating a conceptually coherent framework that can complement and add to the extant literature. This methodological approach is presented in the next section.

### 3. METHODOLOGICAL BACKGROUND

This study is grounded on the foundations of the MCDA approach (see Roy 1985; Belton, Stewart 2002). As stated by Bana e Costa et al. (1997: 30), *"the one basic conviction underlying every MCDA approach is that the explicit introduction of several criteria, each representing a particular dimension of the problem to be taken into account, is a better path for robust decision-making when facing multidimensional and ill-defined problems, than optimizing a single-dimensional objective function"*. From this premise, it is known that the MCDA approach is an

open field of research, which recognizes the limits of the mathematical optimum and aims to build something that does not pre-exist (i.e. something grounded on a constructivist basis, which allows working with the decision makers' own value systems). Indeed, as already pointed out by Santos et al. (2002: 1252), *"MCDA methodologies can help decision makers to learn about the problems they face, about their own priorities and those of other stakeholders, and consequently to make better informed and justifiable choices"*. By integrating cognitive maps and MACBETH, this study is grounded on the underlying convictions of the MCDA approach.

#### 3.1. Cognitive maps

This study follows a methodological approach known as SODA, which has been (re)named *Journey Making* (Ackermann, Eden 2001). This approach was originally developed by Colin Eden, with the goal of supporting decision makers in structuring complex decision problems, which are usually characterized by the existence of different and conflicting perspectives and value systems, which in turn should be explored in order to achieve a compromise-solution to the problem. Based on the development of cognitive maps, this approach promotes dialogue and exchange of ideas among decision makers, allowing decision situations to be clarified, structured and potentially more informed (cf. Ferreira et al. 2014b).

Despite its subjective nature, cognitive mapping holds great potential for structuring complex problems, namely because it allows to reduce the rate of omitted criteria in the decision making framework and leverages knowledge synergies through discussion and reflexive analysis (Nutt 2002; Tegarden, Sheetz 2003; Mansingh et al. 2009). According to Eden (2004: 673), *"a cognitive map is the representation of thinking about a problem that follows from the process of mapping"*. Additionally, Shaw (2004: 366) defends that *"negotiation between group members ensures rigor in the outcomes which are agreed upon [...]"*. As such, the application of the SODA approach in this study will allow a collective cognitive map to be constructed, which should result from the negotiation process established among the panel members during workgroup sessions.

#### 3.2. The MACBETH approach

The MACBETH approach was developed in the early 1990s, by Carlos Bana e Costa and Jean-Claude Vansnick (cf. Bana e Costa, Vansnick 1995;

Bana e Costa *et al.* 2005). This approach allows cardinal scales to be constructed and differences of attractiveness between choice alternatives to be measured based on the decision makers' value judgments. It follows the MCDA constructivist conviction and holds great potential in the definition of trade-offs between evaluation criteria.

According to Bana e Costa and Vansnick (1994), the MACBETH approach is based on numerical representations of semi-orders for multiple thresholds, and was initially inspired by the mathematical principles of Doignon (1984). Mathematically, this means that in a structure of  $m$  binary relations  $[P^{(1)}, \dots, P^{(k)}, \dots, P^{(m)}]$ , where  $P^{(k)}$  stands for a value preference that is stronger the greater  $k$ , it is possible to convert these preference relations into numbers. The codification procedure consists in associating each choice alternative of  $A$ , with  $A = \{a, b, \dots, n\}$  being a finite set of  $n$  choice alternatives, to a value  $x$  (resulting from a value function  $v(\cdot): A \rightarrow \mathbb{R}$ ) such that differences as  $v(a) - v(b)$  (with  $a$  strictly more attractive than  $b$  (*i.e.*  $a P b$ )), are as compatible as possible with the experts' value preferences. Thus, for every pair of alternatives  $(a, b)$  allocated to a particular category of difference of attractiveness  $C$ , the differences  $v(a) - v(b)$  belong to the same interval, without overlaps (*cf.* Bana e Costa, Vansnick 1995). As shown in Figure 1, to define the thresholds between successive categories of difference of attractiveness, it is essential to calculate the limits  $s_k$  and  $s_{k+1}$ , which are positive real constants.

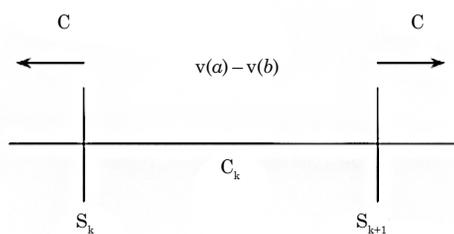


Fig. 1. Allocation of  $v(a) - v(b)$  to a category  $C_k$

Following this, semi-multiple orders can be easily introduced as long as value preferences are represented by a value function  $v$  and function thresholds  $s_k$ , as presented in formulation (1):

$$a P^{(k)} b : s_k < v(a) - v(b) < s_{k+1} . \tag{1}$$

As discussed by Bana e Costa *et al.* (2005: 412), "the basic idea underlying the initial development of MACBETH was that limits of these intervals should not be arbitrarily fixed a priori, but determined simultaneously with numerical value scores for the elements of  $X$  [or "A" following the nomenclature of this paper]". Broadly, the procedure consists in the elicitation of a qualitative value judgment representative of the difference of attractiveness between

$a$  and  $b \in A$ , using the semantic categories of difference of attractiveness presented in Table 1.

Table 1. Semantic categories of difference of attractiveness

Category	Difference of attractiveness
$C_0$	Null
$C_1$	Very weak
$C_2$	Weak
$C_3$	Moderate
$C_4$	Strong
$C_5$	Very strong
$C_6$	Extreme

For instance, if  $a$  is more attractive than  $b$ , and the difference between both alternatives is considered *extreme*, then  $(a, b) \in C_6$ . In this sense, and according to Bana e Costa and Vansnick (1994), during the elicitation process of value judgments, it helps to construct an upper triangular matrix for each criterion (also known as *Fundamental Point of View* (FPV)), namely because this matrix allows to control the projection of value judgments and to identify inconsistencies. Bana e Costa *et al.* (2005) further defend that the construction of this matrix can be done in two ways: (1) through direct projection of the differences of attractiveness between the alternatives with reference to a given FPV; or (2) through projection of indirect value judgments, *i.e.* through comparisons against fictitious alternatives associated to a given impact level. In either case, it is necessary to ensure the consistency of the value judgments through formulations (2) and (3):

$$\forall a, b \in A : v(a) > v(b) \Leftrightarrow a P b, \tag{2}$$

$$\forall k, k^* \in \{1, 2, 3, 4, 5, 6\}, \forall a, b, c, d \in A \text{ with } (a, b) \in C_k \text{ and } (c, d) \in C_{k^*} : k \geq k^* + 1 \Rightarrow v(a) - v(b) \geq v(c) - v(d). \tag{3}$$

Once confirmed the consistency of the value judgments, linear programming is applied in accordance with formulation (4) (*cf.* Junior 2008; Ferreira *et al.* 2012), which generates an initial scale that should be presented to the decision makers for discussion:

$$\text{Min } v(n)$$

$$\text{S.T.} : \forall a, b \in A : a P b \Rightarrow v(a) \geq v(b) + 1$$

$$\forall a, b \in A : a I b \Rightarrow v(a) = v(b)$$

$\forall (a, b), (c, d) \in A$ , if the difference of attractiveness between  $a$  and  $b$  is bigger than between  $c$  and  $d$ , then:

$$v(a) - v(b) \geq v(c) - v(d) + 1 + \delta(a, b, c, d)$$

$$v(a^-) = 0$$

where:

$n$  is an element of  $A$  so that  $\forall a, b, c, \dots \in A : n(P \cup I)a, b, c, \dots$

$A : n(P \cup I)a, b, c, \dots$

$a^-$  is an element of  $A$  so that  $\forall a, b, c, \dots \in A : a, b, c, \dots (P \cup I)a^-$

$A : a, b, c, \dots (P \cup I)a^-$

$\delta(a, b, c, d)$  is the minimal number of categories of difference of attractiveness

between the difference of attractiveness between  $a$  and  $b$  and the difference of attractiveness between  $c$  and  $d$ .

(4)

Formulation (4) explains the mathematical procedure used to achieve the value function, where  $n$  represents the most attractive (or at least as attractive as the others) element of  $A$  (i.e.  $n(P \cup I)a, b, c, \dots$ ), and its value minimization guarantees the minimal length of the initial scale. Also,  $a^-$  represents the least attractive (or at least as attractive as the others) element of  $A$  (i.e.  $a, b, c, \dots (P \cup I)a^-$ ), and its value is anchored to the “zero” of the scale (cf. Bana e Costa et al. 2005). This process is repeatedly executed until a preference scale is defined for each descriptor (i.e. a set of impact levels for each FPV).

Following this, it is worth underlying that the MACBETH methodology is based on a direct question-answer procedure, where the panelists compare pairs of alternatives and project qualitative judgments about the difference in attractiveness between these alternatives. As such, several ar-

rays of value judgments are defined until a range of local preference for each descriptor included in the process is obtained. This methodology helps to systematize complex problems, allowing decision makers to manage information, to exchange knowledge and experience, and to generate consensus among stakeholders. It seems clear, therefore, that the combined use of cognitive maps and the MCABETH approach holds great potential to support the definition of rents in the housing market.

### 4. MODEL DEVELOPMENT

#### 4.1. The structuring phase

Considering that “many major decisions in organizations are made by groups, collectively” (Turban in Ferreira 2011: 222), our study involved a group of seven residential real estate experts (i.e. 2 appraisers, 2 realtors and 3 landlords), who operate in the Lisbon metropolitan area, and expressed interest in collaborating in the definition and analysis of our decision problem. Two group sessions with an average duration of 4.5 hours were coordinated by two facilitators (i.e. researchers), assisted by an ICT technician, who was responsible for registering the sessions’ outcomes.

The first group session began with a short presentation of the research objectives and methodological procedures. This allowed misunderstandings to be avoided and increased the confidence of the group members. After this introduction, and in order to focus the decision makers’ attention on the

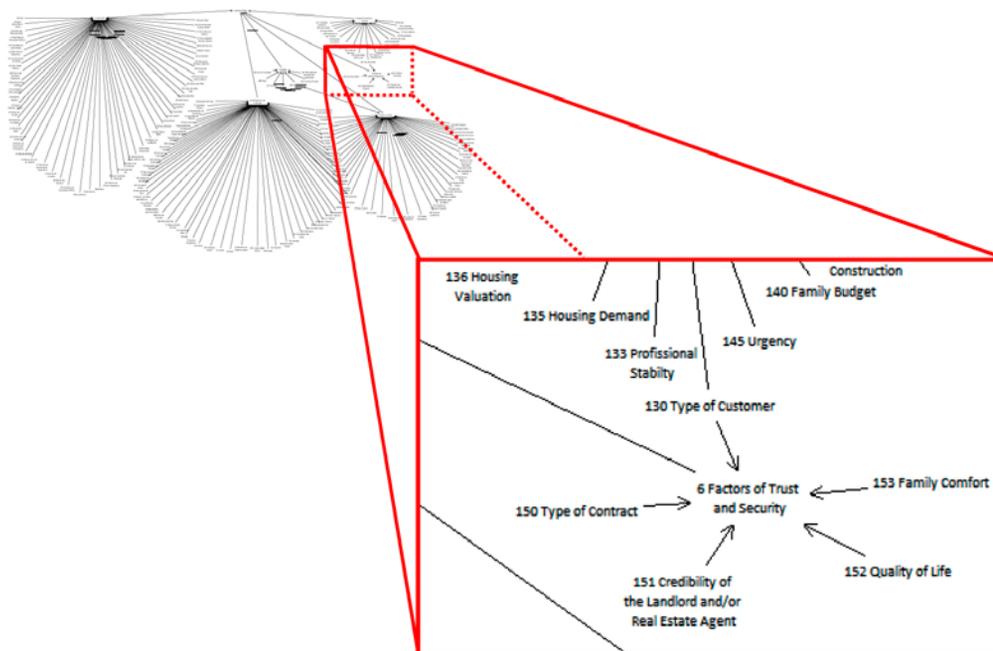


Fig. 2. Final version of the collective cognitive map

problem at hand (i.e. setting rents in residential real estate), they were asked the following trigger question: “Based on your own values and professional experience, what are the main factors and characteristics of a house that influence its rental value?”. It is worth noting that the concept of house was associated to *single family apartment*, because this is the most common type of residential real estate in the region where this study took place. Given these clarifications, the next step consisted in the application of the “post-its technique” (Ackermann, Eden 2001), which, according to Bana e Costa *et al.* (2002: 229), “helps to identify clusters of linked aspects”. During the application of the technique, the group members were invited to share experiences and values and, grounded on permanent discussion, to identify relevant criteria and write them on post-its. This procedure allowed a wide range of evaluation criteria to be made explicit.

In the second phase of the process, the decision makers were asked to group the criteria by clusters (also known as “areas of concern”), and six areas of concern were identified. The next step consisted in the analysis of the cause-and-effect relationships between the criteria in each cluster. This analysis was performed using the *Decision Explorer* software (<http://www.banxia.com>), which allowed a collective cognitive map to be obtained. Figure 2 illustrates the final version of the map, which was discussed and approved by the group.

The analysis of the cognitive structure of the map allowed the FPVs to be identified, and a value tree to be obtained (for technical details, see Keeney 1992, 1996). Figure 3 illustrates the tree of FPVs, which was created using the M-MACBETH software (<http://www.m-macbeth.com>). It should be highlighted that this value tree was discussed and approved by the panel members, and the FPVs were carefully tested to guarantee mutual preferential independence among them (cf. Belton, Stewart 2002).

According to the panel members’ interpretation of the value tree, FPV<sub>1</sub> – *Factors of Trust and Security* – concerns issues related to trust and credibility

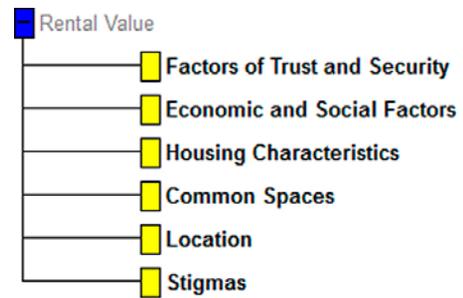


Fig. 3. Tree of points of view

of the landlord or of the real estate agent. FPV<sub>2</sub> – *Economic and Social Factors* – addresses issues such as economic and social conjuncture, taxes, financing conditions and housing supply. FPV<sub>3</sub> – *Housing Characteristics* – highlights the internal characteristics of the apartment, such as state of conservation and plant, amenities, finishings and lighting. FPV<sub>4</sub> – *Common Spaces* – addresses issues related to the physical characteristics of the common spaces, such as state of conservation, cleanliness and building plant. FPV<sub>5</sub> – *Location* – stands for the location of the apartment and surrounding environment (e.g. neighborhood safety and reputation, proximity to economic agents, accessibilities). Finally, FVP<sub>6</sub> – *Stigmas* – underlines the importance of social stigmas, namely: proximity to cemeteries, haunting and ghosts and past deaths in the house.

In the second group session, the decision makers were asked to focus their attention on the value tree and to define a descriptor and respective impact levels for each FPV. In practice, they were asked to identify the most important criteria in each cluster and, using an adaptation of Fiedler’s scale (1965, 1967), to define levels of partial performance, including *Good* and *Neutral* levels to facilitate cognitive comparisons. Given that the descriptors could be quantitative, qualitative or mixed (see Ferreira *et al.* 2013), this exercise proved easily manageable. As an example of this procedural step, Figure 4 presents the descriptor obtained for FPV<sub>1</sub>, which resulted from the negotiation process established among the group members.

Descriptor FPV01 - Factors of Trust and Security			Level	Description
Contract without guarantees	1 2 3 4 5 6 7 8	Contract with excellent guarantees	Good	Index FTS =40
Low Credibility of the landlord or RE agent	1 2 3 4 5 6 7 8	High Credibility of the landlord or RE agent	Neutral	Index FTS € [36-39]
Low quality of life	1 2 3 4 5 6 7 8	Excellent quality of life	L3	Index FTS € [5-35]
No comfort	1 2 3 4 5 6 7 8	Excellent comfort		
Low credibility of the tenant	1 2 3 4 5 6 7 8	High Credibility of the tenant		

Fig. 4. Descriptor and impact levels for FPV<sub>1</sub>

As shown in Figure 4,  $FPV_1$  was operationalized by a FTS index, which brought together the five most important factors of trust and security, from the panel members' point of view. These were rental contract with guarantees, credibility of the landlord or of the real estate agent, quality of life, comfort and credibility of the tenant. Impact level  $L_1$ , also identified by the panel members as the *Good* reference level, represents the best possible performance, comprising an estate where the index (i.e. the sum of values assigned to each criterion) belongs to the maximum practicable value. In contrast, impact level  $L_3$  represents a clearly negative performance, indicating an estate classified by the minimum range of values. It is worth noting that this procedure was repeated for the remaining five FPVs.

The structuring phase is usually considered complete once a descriptor for each FPV has been defined (cf. Ferreira et al. 2014b). Although very enlightening, it is worth noting that the structuring process followed in this study, which is based on the use of cognitive maps, is subjective in nature and strongly dependent on the participants' willingness and availability. Still, it is possible to identify several advantages that resulted from its application,

namely: (1) it assisted the systematization of the complex decision problem at hand; (2) it enabled information management by the group; (3) it allowed the exchange of knowledge and experiences; (4) it allowed the generation of consensus among the participants; (5) it is easily adaptable to change; and (6) it allowed for the inclusion of criteria that are frequently omitted in the current appraisal models.

**4.2. The evaluation phase**

The evaluation phase took place in the second stage of the second group workshop. With the goal of creating scales of local preference, the panel members were asked to fill in an array of value judgments for each of the descriptors defined. The definition of these scales is extremely important, as they allow alternatives to be partially evaluated. To achieve this aim, the panel members were asked to pairwise compare the impact levels in each descriptor and, based on the MACBETH methodology, to collectively express value judgments regarding their difference of attractiveness. Figure 5 illustrates this conceptual procedure, which resulted in a value function for each FPV included in the model.

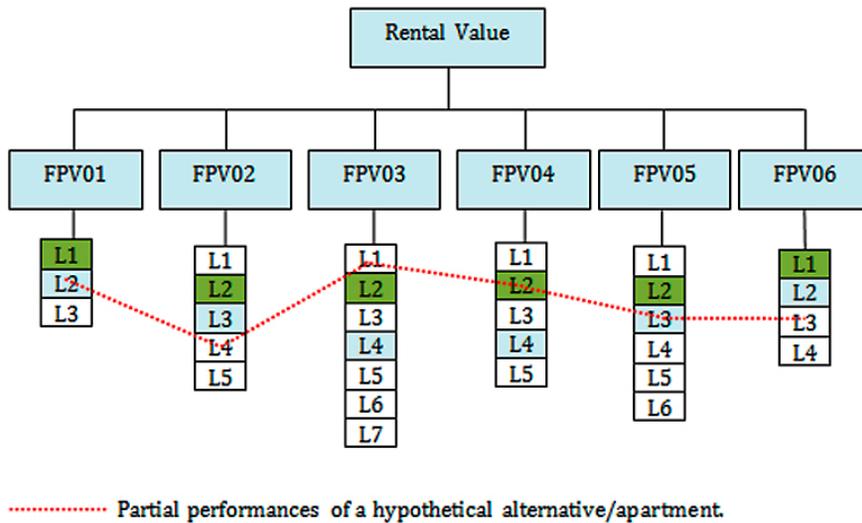


Fig. 5. Descriptors and value functions for partial evaluation

	PVF01	PVF02	PVF03	PVF04	PVF05	PVF06	Total	R
Factors of Trust and Security	PVF01	0	0	0	0	1	1	5
Economic and Social Factors	PVF02	1	0	0	0	1	2	4
Housing Characteristics	PVF03	1	1	1	0	1	4	2
Common Spaces	PVF04	1	1	0	0	1	3	3
Location	PVF05	1	1	1	0	1	5	1
Stigmas	PVF06	0	0	0	0	0	0	6

Fig. 6. Raking of global attractiveness

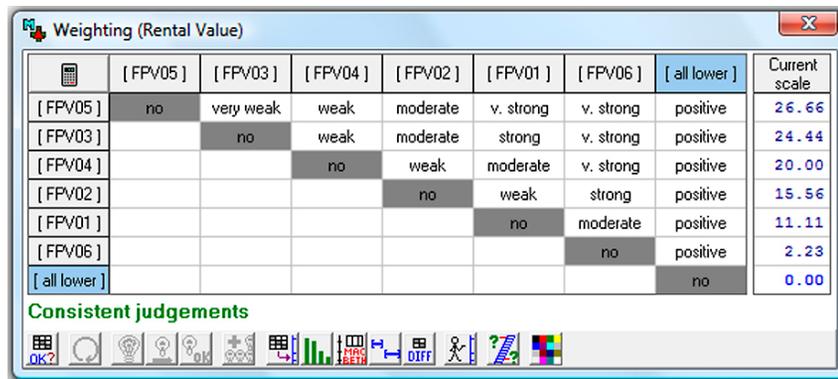


Fig. 7. Value judgments and trade-offs calculation

Once a value function for each FPV had been obtained, the decision makers were asked to concentrate on the FPVs and tabulate them according to their level of global attractiveness, attributing a value of “1” to a FPV that was deemed globally attractive to another; and a value of “0” when the opposite occurred. This step was important to obtain the trade-offs among the FPVs. The ranking was obtained from the sum of the values attributed in each peer-to-peer comparison, and the final hierarchy was validated by the group (Fig. 6).

After sorting the FPVs and approved their hierarchy, a new matrix of value judgments was built as shown in Figure 7. Grounded on the semantic categories of the MACBETH approach, this matrix was important to obtain the differences of overall attractiveness between FPVs, which allowed the trade-offs to be calculated. As in previous stages, the final scale was discussed and approved by the group.

As illustrated in Figure 7, FPV<sub>5</sub> was considered the most relevant criterion when setting rents in residential real estate, presenting the highest weight (26.66%). Contrarily, FPV<sub>6</sub> presents relative little importance (2.23%). At this stage, overall assessment of different apartments was possible, using the additive model presented in formulation (5):

$$V(a) = \sum_{i=1}^n x_i v_i(a) \text{ with } \sum_{i=1}^n x_i = 1$$

$$\text{and } x_i > 0 \text{ and } \begin{cases} v_i(\text{good}_i) = 100 \\ v_i(\text{neutral}_i) = 0 \end{cases} \quad (5)$$

This additive model aggregates the partial scores  $v_i(a)$ , considering the respective weight  $x_i$  and allowing an overall score  $V(a)$  for each apartment to be calculated. This means that  $V(a)$  represents a holistic appraisal measure. It is worth noting that  $v_i(\text{good}_i)$  and  $v_i(\text{neutral}_i)$  represent the partial scores of two specific impact levels (i.e. good and neutral), which were identified by the panel members in each descriptor and included in the system to facilitate cognitive comparisons. One should bear in mind, in addition, that the trade-offs obtained should be interpreted with some caution, namely because the information gathered is based on semantic judgments of a particular group of decision makers.

In order to test the model, the next step consisted in the calculation of global values (i.e. overall scores) for four “artificially” created apartments (called “Deltas”). Because this procedure allowed reference levels to serve as “calculation anchors”, it served as a starting point for estimating the value of the rents (for technical details about this procedure, see Ferreira *et al.* 2013). Table 2 presents the values obtained for each Delta.

As shown in Table 2, the fictitious apartment Delta 1 (hereafter “Great”) presents the highest scores on all FPVs. Delta 2 (hereafter “Good”) gathers the “good” levels of all the FPVs. Delta 3 is considered “Neutral”, because it gathers the neutral levels of all the FPVs. Finally, Delta 4 (hereafter referred to as “Terrible”) is an apartment with

Table 2. Partial and overall scores for the Deltas

	Overall score	FPV1	FPV2	FPV3	FPV4	FPV5	FPV6
Delta 1	163.61	100.0	150.0	137.5	200.0	200.0	100.0
Delta 2	100.00	100.0	100.0	100.0	100.0	100.0	100.0
Delta 3	0.00	0.0	0.0	0.0	0.0	0.0	0.0
Delta 4	-423.57	-120.0	-150.0	-187.5	-550.0	-850.0	-200.0
Weights		0.1111	0.1556	0.2444	0.2000	0.2666	0.0223

the worst performance on all the FPVs. The panel members were at this point asked to associate a rent to each of the four Deltas, as a means of defining monetary/rent anchors. This was the step that sparked more interest among decision makers, having been considered essential in the estimation of residential rents. Table 3 shows the values of the rents (in Euros), which were established by the experts after negotiation.

Table 3. Estimated anchors

Delta	Overall score	Estimated anchors [monthly rent in Euros]
Great	163.61	12.000.00
Good	100.00	1.000.00
Neutral	0.00	500.00
Terrible	-423.57	150.00

Because the estimated anchors were based on semantic perceptions directly projected by the panel members after group negotiation and experience sharing, we are aware that they are non-linear and inherently subjective, thus likely to be imprecise. Still, it should be noted that an important feature of the software used (i.e. M-MABETH) is that it allows for interactive explorations of changes in the inputs. This means that the impact of a variation in any particular variable on the model as a whole can immediately be seen, offering opportunities for further discussion and supporting the estimated anchors. In addition, one should bear in mind that the nature of this proposal is such that it is updated as new data is introduced. This means that, after a certain amount of time, the estimated anchors will be replaced with new ones based on actual/observed data, introducing realism into the calculi.

Once the anchors have been obtained, it becomes possible to use linear interpolation and estimate the rental value of any apartment. In order to demonstrate this process, the following section is devoted to the practical application of the model, including sensitivity and robustness analyses.

### 4.3. Testing the new model and presenting recommendations

In order to test the model, the group proceeded with its implementation on a set of ten real apartments (called “Alphas”), whose physical characteristics allowed for the identification of the respective levels of partial performance. Figure 8 shows the values obtained for each Alpha (*Good* and *Neutral* included).

Options	Overall	FPV01	FPV02	FPV03	FPV04	FPV05	FPV06
Alpha 01	-23,05	100,00	0,00	137,50	-150,00	-150,00	100,00
Alpha 02	90,27	100,00	-75,00	62,50	100,00	200,00	100,00
Alpha 03	-12,50	0,00	0,00	62,50	-150,00	0,00	100,00
Alpha 04	23,33	0,00	-150,00	100,00	100,00	0,00	100,00
Alpha 05	73,34	100,00	100,00	100,00	100,00	0,00	100,00
Alpha 06	55,28	100,00	0,00	62,50	0,00	100,00	100,00
Alpha 07	80,00	100,00	100,00	100,00	0,00	100,00	100,00
Alpha 08	1,67	-120,00	-75,00	100,00	0,00	0,00	100,00
Alpha 09	-57,23	100,00	-75,00	100,00	-550,00	100,00	100,00
Alpha 10	22,50	-120,00	0,00	137,50	0,00	0,00	100,00
Good	100,00	100,00	100,00	100,00	100,00	100,00	100,00
Neutral	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Weights:		0,1111	0,1556	0,2444	0,2000	0,2666	0,0223

Fig. 8. Partial and overall performances of the Alphas

Given the overall scores for the assessed properties, it was possible to sort a final ranking of Alphas and Deltas, as shown in Figure 9. The ranking was then presented to the panel members for discussion and subsequent validation.

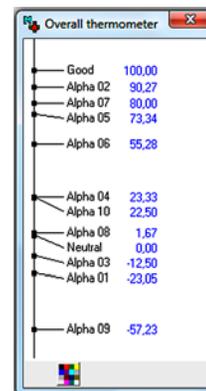


Fig. 9. Ranking of Alphas and Deltas

Based on the overall scores and anchoring references presented in Table 3, it was possible to use linear interpolation and estimate the monthly rents for the Alphas (Table 4).

Table 4. Setting rents for the Alphas

Rents for residential real estate [Values in Euros]			
Alpha/ Delta	Overall score	Estimated an- chors [monthly rents]	Estimated rents [per month]
<b>Great</b>	<b>163.61</b>	12.000.00	12.000.00
<b>Good</b>	<b>100.00</b>	1.000.00	1.000.00
Alpha 02	90.27	--	951.35
Alpha 07	80.00	--	900.00
Alpha 05	73.34	--	866.70
Alpha 06	55.28	--	776.40
Alpha 04	23.33	--	616.65
Alpha 10	22.50	--	612.50
Alpha 08	1.67	--	508.35
<b>Neutral</b>	<b>0,00</b>	500.00	500.00
Alpha 03	-12.50	--	489.67
Alpha 01	-23.05	--	480.95
Alpha 09	-57.23	--	452.70
<b>Terrible</b>	<b>-423,57</b>	150.00	150.00

It should be noted that the accuracy of our system tends to increase whenever further Alphas are introduced, because the introduction of new data allows the intervals between overall scores, and in sequence the estimation error, to be reduced. In other words, the introduction of new data (i.e. actual values) will allow us to replace the estimated anchors, introducing realism into the model and strengthening the results obtained.

In practice, when comparing the estimated rents against the effective/real ones (i.e. the Alphas stand for real apartments), it was found that the average oscillation was around 50 Euros. In addition, it is worth noting that the panel members revealed their overall preference for the values projected by the model developed in this study. Nevertheless, to test the sensitivity and robustness of our evaluation system, some additional analyses were performed. Figure 10 shows the sensitivity analysis carried out on FPV<sub>5</sub>, which was considered the most important criterion in the evaluation system developed.

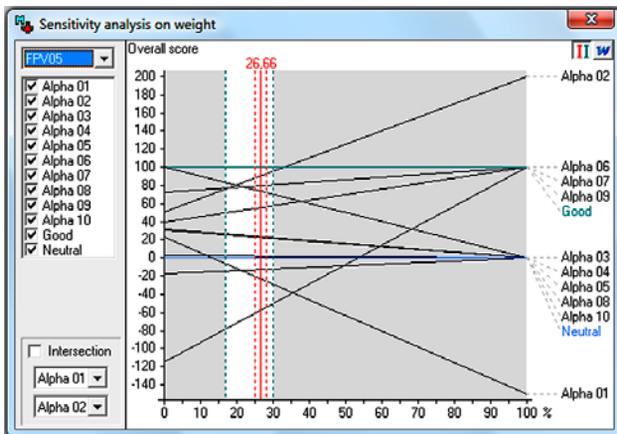


Fig. 10. Sensitivity analysis on FPV<sub>5</sub>

According to Figure 10, the oscillation interval for the weight of the FPV<sub>5</sub> is quite significant, resulting in a considerable safety margin where potential weight fluctuations will not affect the Alphas' ranking or the panel members' value judgments. In light of these results, and because sensitivity analyses were performed for all FPVs, one may assume that our evaluation system is strong enough to guarantee that the discrimination of Alphas (i.e. apartments) is correctly done.

Because sensitivity analysis deals with isolated variations with regard to the FPVs' weights, robustness analyses were also performed. According to Bana e Costa *et al.* (2002), the robustness analysis considers, implicitly, an analysis of dominance, which may be classified in two categories: (1) *absolute or classic* (▲) where *a* is overall better

than *b*, and partially better than (or equal to) *b* in all FPVs; and (2) *additive* (+), where *a* is overall better than *b*, but is not partially better than *b* in at least one FPV. In this regard, several weight simulations were performed in order to draw conclusions about the robustness of the model. Figure 11 exemplifies this exercise.

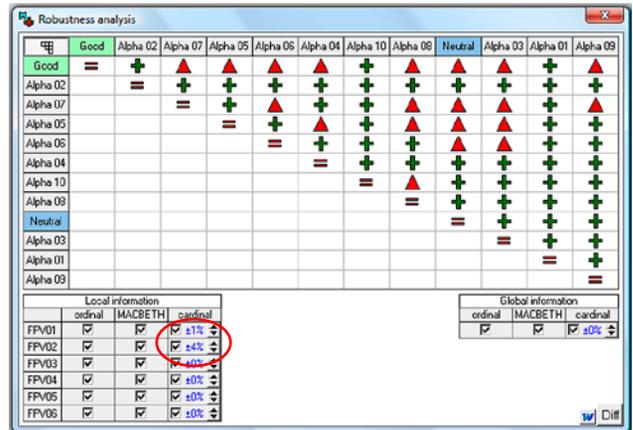


Fig. 11. Robustness analysis

Through the robustness analysis carried out, it is possible to conclude that our evaluation system is robust. This is due to the stability observed during the simultaneous variations in the weights of different FPVs. For example, only when changing the weight of FPV<sub>1</sub> in ±1% and the weight of FPV<sub>2</sub> in ±4% simultaneously, the ranking position of the Alphas changes. Naturally, the sensitivity and robustness analyses carried out in this study are group-contextualized. Thus, generalizations are discouraged without the necessary adjustments.

### 5. CONCLUSIONS

This study confirms that the combined use of cognitive maps and MCDA methodologies, MACBETH in particular, allows residential rents to be defined in a transparent and robust manner. Previous contributions in this field present certain common methodological limitations, as far as the selection and weighting of the criteria involved in the rent definition process is concerned. While having limitations of its own, the combined use of these two approaches was able to overcome previously encountered shortcomings. Indeed, the collective map developed in this study allowed different individuals' opinions to be aggregated in a shared framework within which cause-and-effect relationships between criteria could be detected and understood. MACBETH, in turn, allowed the resulting criteria to be compared and weighted according to the panel members' own values and professional experience.

It is worth highlighting that other papers have been published on the combined use of cognitive mapping and the MACBETH approach. In fact, the authors of this study have practical experience in the integrated use of these two techniques to assist decision making across several contexts. Observation and reflection on this experience led the authors to the proposal, initially as an abstract concept, that there is synergy to be obtained from the integrated use of cognitive mapping and MCDA tools in the definition of rents in residential real estate. The development of this concept has led us to the framework proposed in this paper, which has been successfully tested in other contexts (e.g. Ferreira 2011; Ferreira *et al.* 2015; Filipe *et al.* 2015), but which requires further applications involving different decision makers and different decision situations to ensure the potential and the generalization of the integrated use of these two approaches. It is in this testing stage that we have inserted this paper.

This study had the active collaboration of seven real estate experts (i.e. 2 appraisers, 2 realtors and 3 landlords), who have been working in this field over the past 2-3 decades in the Lisbon metropolitan area. It should be noted, however, that the process can work well with different groups of participants, including tenants with differing preferences or landlords with distinct priorities. That said, our proposal is not a substitute for statistical approaches; its practical application can provide real estate decision makers with insights on the role of key cause-and-effect relationships in the system, which might otherwise go undetected by statistical methods alone. Indeed, the construction of a group cognitive map in this study allowed ideas and experiences to be shared and cause-and-effect relationships between criteria to be explored, answering to questions such as “why does this happen?”. When combined with the MACBETH approach, which also relies on a constructivist stance, our framework allowed the number of omitted criteria to be decreased and the calculation of trade-offs to be facilitated, while adding simplicity and transparency to the decision process.

From a practical perspective, this study can be of great use to all those operating in the residential real estate market, be they agencies, tenants or landlords. Although caution is required in generalizing the results, namely because our framework is context-dependent, much of the value created is in the learning process associated with the application of techniques, which reward the opportunity to reflect on the assessments and

make suggestions for improvement. At the theoretical level, the combined use of cognitive maps and MACBETH for setting rents in the residential real estate market is, to the best of our knowledge, a novel application of these decision support tools. As such, our study provides an advance in their use and understanding.

As far as future research is concerned, a comprehensive comparison of our framework to other approaches with different epistemological stances is part of our longer term aims. In addition, to explore its integration within established frameworks (e.g. hedonic modeling and other mass appraisal techniques), to replicate the process in other countries and/or to carry out comparative analyses would also be of great interest. Last but not least, it would be of particular relevance to replicate the process with different expert panels. This would increase the generalizability of the process and greatly enrich the discussion, contributing to our understanding of the processes whereby tenants and landlords reach an agreement.

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