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CONSUMPTION AND INVESTMENT VALUES IN HOUSING PRICE: A REAL OPTIONS APPROACH

Chih-Hsing HUNG[1], Shyh-Weir TZANG[2,*]

[1] *Department of Money and Banking, National Kaohsiung University of Science and Technology, Kaohsiung, Taiwan, R.O.C.*
[2] *Department of Finance, Asia University, Taichung, Taiwan, R.O.C.*

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Abstract. Homeowners can be viewed as the put option holders who can sell housing to lenders when the housing price is lower than its mortgage value and sell houses when the housing price rises above a certain threshold. On the basis of the theory of investment under uncertainty, we model the housing value from the perspective of houseowners who can choose to either live in their houses or switch houses for comfort improvement and price appreciation. We can decompose the housing value into consumption and investment values by exploring parameters affecting housing value and decision making of houseowners. We find that the proportion of investment value to housing value increases with the volatility of the housing market, indicating the possible formation of housing bubbles. In addition, the comfort and utility provided by housing are critical for homeowners to decide whether to sell their houses. The analysis provides policymakers and market participants in the real estate market with insights into the price formation of real estate.

Keywords: consumption value, housing value, investment value, real options, utility rental benefit.

Introduction

Housing can be treated not only as a consumer good that generates a stream of services to satisfy household demands but also as an investment asset that may appreciate in price in the real estate market. The dual role of housing results in its heterogeneity and thus raises the volatility of price and possibility of an overheating housing market. For example, the Chinese Central Government has set up the housing policy of encouraging the consumptive demand for housing and curbing the speculative one. To differentiate between the consumption and investment demands for housing, one main stream of literature analyzes the decision of buy-or-rent (i.e., tenure choice), which is based on households' preferences for the tenure choice of either owning or renting (Rosen, 1979; King, 1980; Henderson & Ioannides, 1983; Goodman, 1988; Ioannides & Rosenthal, 1994). Henderson and Ioannides (1983) further proposed that under the constraint of investment, optimal consumption for housing can be obtained from the intertemporal utility maximization model. They also found that investment and consumption motives are separable when the investment constraint is not binding. Their findings showed that the cost of owning houses and income stream from rental and tax laws are critical factors that affect household decisions. The portfolio approach was also introduced by Brueckner (1997) and further expanded by Yao and Zhang (2005) to analyze the effect of investment constraint on homeowners' consumption and choice of housing tenure.

However, the traditional approach in evaluating housing by the rental income stream or household utility cannot fully account for uncertainty in the real estate market. As a result, the real options approach, an alternative approach that can measure the value of uncertainty in decision making, is applied to model the renters' decision to buy and landlords' decision to sell as the exercise of real options. Similar to financial options, the real option is the option held by renters and homeowners to buy or sell their houses at some time in the future. The value of real option can reflect the value of waiting that can help investors to make optimal decisions in their asset al.ocation (Dixit & Pindyck, 1994; Bloom, 2009). In other words, the uncertainty of housing value in the market will affect the option value of waiting, which may affect homeowners' decision making in switching houses or hold renters back from purchasing houses. The application of real options in real estates has been widely documented in the literature (Titman, 1985; Childs et al., 1998; Qian, 2013). By applying this approach

*Corresponding author. E-mail: *swtzang@gmail.com*

to the real estate and rental market of Hong Kong and cities in Mainland China, Wang et al. (2020) found that different from the theory of housing price being determined by future rents, housing price uncertainty and volatility have significant causal effects on rental changes, thereby offering insights into the housing price of real estate.

The purchasing prices paid by homeowners when they decide to switch houses can be decomposed into consumption and investment values. By adopting real options models for evaluation by including an income approach and assume that the variables are randomly determined, we find that the proportion of investment value to housing value increases with the volatility of the housing market, indicating the possible formation of housing bubbles. In addition, the comfort and utility provided by housing are critical for homeowners to decide whether to sell their houses.

One of the most important factors for owning houses is utility rental benefit, which is defined as the satisfaction from owning houses and the saving of rental expenses from owning houses. By the results of numerical simulation, we find that for houseowners with higher-quality houses (i.e., not necessarily luxurious houses but ones that can deliver higher utility rental benefit), the consumption value of the housing may not become extremely important compared with investment value when they decide to switch houses in a highly volatile housing market. The fact that the consumption value is less important for high-quality houses also results in a lower probability to sell houses. In other words, houses with a lower utility rental benefit rate will be more likely to be sold in exchange for higher-quality houses, especially in a highly volatile real estate market. In addition, for a given utility rental benefit, we also find that houseowners will care more about consumption value delivered by the housing when the interest rate is low, but they do less when the interest rate becomes higher, especially for highly leveraged mortgage loans.

1. Literature review

The utility function approach has been widely adopted in the literature to explain the duality of housing demands. Piazzesi et al. (2007) and Lustig and Van Nieuwerburgh (2005, 2006, 2010) used utility function approach to analyze the value difference between residences that served as consumption and nondurable goods. They found that residences are consumption and investment assets, and although homebuyers can be categorized as consumers or investors, consumers will consider dual values when purchasing houses. Therefore, prospective homebuyers have consumption and investment motivations; they tend to consider future sales value in addition to comfort. In comparison with investors, prospective homebuyers exhibit a lower level of investment motivation. Homebuyers appraise housing value differently according to their purchasing motivations.

Consumption and investment demands for housing vary across markets. After examining the consumption and investment demands for the French housing market, Arrondel and Lefebvre (2001) reached a different conclusion than Ioannides and Rosenthal (1994) in their study of houses in the United States. The two studies applied the same research methodology. For French homebuyers, no substantial difference was found between consumption and investment demands. However, in the United States, homebuyers primarily evaluated housing consumption value, whereas consumers with investment motivation tended to assess housing value according to investment value. Regarding investment demand in the Taiwan market, Chen et al. (2012) showed that real estate investment demand increases when individuals have more capital and that this speculation causes structural changes in real estate prices. They also found that real estate prices are substantially influenced by stock prices and inflation during periods with high growth in the money supply. The results confirmed that real estate prices are influenced by investment demand and that investment decision largely determines real estate prices.

By contrast, Lin and Lin (1994) divided housing demand into investment, self-owned housing, and rental housing demand. They concluded that regardless of lease duration, tenants tend to view real estate as a necessity, whereas homebuyers tend to view real estate as a luxury. They proposed that housing value can generally be categorized as consumption and investment values. Moreover, they stated that the value of luxurious housing can be divided into consumption, investment, and luxury values. Typically, consumers who rent housing exhibit strong consumption demand, and consumers who buy ordinary housing show consumption and investment demands. Generally, decomposing the housing price into distinguishable components by the heterogeneous demands of households is rather difficult. The housing price paid by housing demanders includes consumption and investment values, whereas the housing price paid by housing investors includes rental and investment values. Teng et al. (2017), who defined the bubble prices as the difference between fundamental housing value and market prices using the data of Taiwan housing market, attempted to decompose the housing value. The fundamental housing value is estimated from real permanent income, and the bubble prices, mainly driven by investment motivation, can thus be regarded as a proxy for the investment value of housing.

Our model adds to the literature that, to our knowledge, has not covered the decision making of homeowners on switching houses. From the perspective of homeowners, we assume that with the budget constraint that does not allow homeowners to afford a second house, homeowners have the option to sell their houses if housing prices increase above a certain threshold and switch to new houses for better comfort and utility. Homeowners can also choose not to exercise the option if housing price decreases, such that they decide to stay with their houses and save rental costs. The present rental value can be treated as a consumption value for homebuyers, and the profit generated by selling houses represents the housing investment value. We apply the real options model to housing valuation by measuring the consumption value per se from rental cash flow and additional investment value. Through a numerical simulation by including parameters that affect homebuyer decision making, such as utility benefit, initial housing cost, depreciation rate, housing price volatility, interest rate, and property tax, we can measure their influences on housing value and assess the extent to which housing prices are affected.

2. Housing evaluation model

The model adopted in this study is based on Merton's (1973b) option theoretic approach to default by borrowers. The housing value, similar to the firm value in the context of corporate finance, follows a geometric Brownian motion, and the default can be regarded as the put option, held by housing owners, to sell housing (asset) to lenders with a strike price to mortgage (debt) value. Leland (1994) applied this approach to analyze debt value and estimate the optimal capital structure. On the basis of the framework of the arbitrage-free price of a contingent claim, Hung (2012) used real options to estimate housing prices and optimal rental. However, distinct from the model of Hung, who focused on general investors of housing, the model in this study stresses the other demands of homebuyers for their living satisfaction, not merely for investment purposes. By owning houses, homebuyers can benefit from the utilities generated from the services provided by the housing. However, when housing price increases, homebuyers may seek to sell their current houses and purchase new ones.

2.1. Residential real estate price index model

A specific average housing price index can be calculated for each region according to its population and urbanization. We treat the house price index (i.e., the Federal Housing Finance Agency House Price Index) as an exogenous variable to represent the mean housing price of a city or region. We assume that the housing price index (H) follows a diffusion-type stochastic process, as shown as follows:

$$\frac{dH}{H} = (\mu - \delta)dt + \sigma dW_t , \tag{1.1}$$

where: μ is the housing price index return; δ is the housing price depreciate rate; σ is the residential property price index return volatility; W_t is a standard Brownian motion.

Similar to Merton (1973a) and Black and Cox (1976), we assume that there exists a contingent claim Y whose value can be expressed as a function of housing value and time, that is, $Y = F(H, t)$. By taking the partial differential equation (PDE) of F with respect to H and t, we obtain the following:

$$0.5H^2\sigma^2 F_{hh} + (r - \delta)HF_h + F_t - rF + b = 0 , \tag{1.2}$$

where: F_h and F_{hh} are the first- and second-order partial derivatives with respect to H, respectively; r is the risk-free rate; b is the payout to the holders of the F per unit time.

2.2. Rental value

Allen et al. (2009) proposed that the real estate value can be affected by the asking rent and turnover of renters. Landlords can also enhance real estate value by asking a lower rent in return for a shorter marketing time for their properties. Therefore, homebuyers can save the rent, and the indispensable housing rent for renters can accordingly be regarded as the derived value from housing demand. We define w as the *utility rental benefit* with two implications:

1. Satisfaction of housing buyers: There exist many types of housing in the housing market, and every home has unique characteristics. Regardless of the rental cost, the degree of comfort and/or utility gained from owning houses is proxied by w.
2. Rental expenses: Before homebuyers purchase their own houses, they will live in a rental house and pay rental expenses w, which can be regarded as the cost that homebuyers can save if they buy their own houses rather than renting.

According to the aforementioned description, w can be defined as the utility rental benefit rate for housing. In our study, w is assumed to be an exogenous variable[1].

More formal rent–price ratio setups can be found in the literature, such as Gallin (2006, 2008) and Kishor and Morley (2015). By not complicating the model in the study, we assume the rent to be exogenously determined by inflation rate, which is consistent with the condition in Taiwan real estate market[2]. Therefore, the housing value per se (F^H) is determined by its rental value and residential property price index. The PDE of F^H is calculated as follows:

$$0.5H^2\sigma^2 F^H_{hh} + (r - \delta)HF_h + F^H_t - rF^H + H(w - c) = 0 , \tag{2}$$

where: w is the utility rental benefit rate; r is the risk-free rate; c is the ratio of the maintenance and management expense to the housing value.

According to Leland (1994), Dixit and Pindyck (1994), Sarkar and Zapatero (2003), and Lambrecht and Myers (2008), we assume that the waiting time for homeowners' decision to sell their houses is indefinite and theoretically unlimited; thus, time will not affect the real housing value. Therefore, F^H is also regarded as a project investment with a real option to be sold in the future at any time without an expiration date. In this model, $F^H_t = 0$; thus, the general solution to Equation (2) is as follows[3]:

$$F^H = X_0 + X_1 H^{\lambda_1} + X_2 H^{\lambda_2} , \tag{3}$$

[1] Rental utility can be estimated according to the model used by Campbell and Cocco (2015). As this is not the concern of this study, we assume w to be exogenously given.

[2] Different from the Housing index of Taiwan (Shinyi Real Estate and Research), rental index of Taiwan has shown a smooth increasing trend from 96.01 (2010) to 102.71 (2019).

[3] As F^H_t approaches 0 when the expiry date is infinite, we suppress the subscript t in the general solution to the PDE.

where:

$$\lambda_1 = 0.5 - \frac{r-\delta}{\sigma^2} + \sqrt{\left(\frac{r-\delta}{\sigma^2} - 0.5\right)^2 + \frac{2r}{\sigma^2}}\,, \tag{4.1}$$

$$\lambda_2 = 0.5 - \frac{r-\delta}{\sigma^2} - \sqrt{\left(\frac{r-\delta}{\sigma^2} - 0.5\right)^2 + \frac{2r}{\sigma^2}}\,. \tag{4.2}$$

The constants X_0, X_1, and X_2 can be determined by the boundary conditions of F^H.

2.3. When to sell the house?

This study assumes that housing demanders are homebuyers; owning houses enables them to save rental expenses. However, when the residential real estate price index rises to a certain satiation level (H_L), homeowners will sacrifice rental income savings by selling houses for profit, the capital gains of which are liable to government taxes t_c. By contrast, when the residential real estate price index falls, homebuyers will hold real estate to avoid paying rent and gain the present value of net rental $\frac{(w-c)H_0}{\delta}$. We must determine the constants X_0, X_1, and X_2, and the value and boundary conditions of F^H is given by

$$H = H_L\,,\ F^H = H_L - (H_L - H_0)\times t_c\,; \tag{5.1}$$

$$H \to 0,\ F^H = \frac{(w-c)H_0}{\delta}, \tag{5.2}$$

where: H_0 is the initial housing price index; H_L is the selling price based on the housing index.

From Equation (3), using Equation (5.2) yields $X_2 = 0$. Given Equation (5.2) and $H^{\lambda_1} \to 0$ when $\to 0$, $X_0 = \frac{(w-c)H_t}{\delta}$. Finally, by Equation (5.1), $X_2 = \left[H_L - (H_L - H_0)\times t_c - \frac{(w-c)H_0}{\delta}\right] H_L^{-\lambda_1}$. Thus,

$$F^H = \left[\frac{(w-c)H_0}{\delta}\right]\times\left[1 - \left(\frac{H}{H_L}\right)^{\lambda_1}\right] + \left[H_L - (H_L - H_0)\times t_c\right]\times\left(\frac{H}{H_L}\right)^{\lambda_1}, \tag{6}$$

where $\left(\frac{H}{H_L}\right)^{\lambda_1}$ is the probability for homeowners to sell their houses. We generate the simulation results based on Equation (6), in which we combine the income approach to residential real estate valuation with the real options model. The simulated results can be obtained by assuming reasonable parameterized values of the residential real estate.

Given that housing demanders are homebuyers, they will buy other houses to live in if they sell their original houses. Therefore, related benefit and cost will be generated. We assume that housing demanders purchase houses by using financial leverage with fixed-rate mortgages. When housing demanders sell their houses and prepay their loan balance, the present value of interest on ordinary loans can be saved. However, using financial leverage to buy the new house is likely to produce a new stream of the present value of loan interest. In addition, a house-switching cost is incurred.

1. Interest cost

Interest cost refers to the present value of the loan interest that buyers pay when purchasing a house with a loan. Given that houses are highly expensive goods, most homebuyers will use financial leverage when buying houses. We consider the condition of when homeowners will switch homes. When the house price increases to H_L, they will sell the house and need not pay the interest expense thereafter. However, if the house price has not reached H_L, then they will continue to pay the interest expense $I\times(1-t_I)$, which is the net interest expense after tax, and t_I is the tax rate. We denote F^I as the after-tax interest cost of a mortgage, and the marginal conditions can be expressed as Equations (7.1) and (7.2).

$$H = H_L, F^I = 0\,; \tag{7.1}$$

$$H \to 0, F^I = I\times(1-t_I). \tag{7.2}$$

Homebuyers buy a house with a loan, and I is the total interest expense of the loans, which can be expressed as follows:

$$I = H_0 \times LTV \times \left[\frac{(1+r_b)^n \times r_b \times n}{(1+r_b)^n - 1} - 1\right], \tag{7.3}$$

where: r_b is the interest rate of housing loans; LTV is the ratio of loan to housing value; n is the term of housing loans.

According to the aforementioned condition, the interest cost F^I is as follows:

$$F^I = I\times(1-t_I)\times\left[1 - \left(\frac{H}{H_L}\right)^{\lambda_1}\right]. \tag{8}$$

The interest cost is the after-tax interest expense multiplied by the probability that the homeowners decide to hold the house. Equation (8) can also be written as $F^I = I\times(1-t_I)\times\left[1 - P_L^H\right]$, where $P_L^H \equiv \left(\frac{H}{H_L}\right)^{\lambda_1}$ represents the probability of homeowners deciding to sell the house (i.e., H increases to H_L).

2. House-switching cost

We assume that when homebuyers sell their original house and purchase new houses, the value of which is assumed to be π more than that of the old house, where π can be positive or negative. If π is negative, then homebuyers are considering downsizing their housing demand. However, we assume in this study that they only purchase houses that are superior to their old houses. In addition, selling houses will incur trading cost θ such as agency fees; therefore, we denote it as the house-switching cost F^C. When homeowners determine whether to sell their houses, they must also consider the house-switching cost, which is restricted by the following conditions:

$$H = H_L, F^c = H_L \times (\theta + \pi), \quad (9.1)$$

$$H \to 0,\ F^c = 0. \quad (9.2)$$

According to the aforementioned restriction equation, the house-switching cost is expressed as follows:

$$F^c = \left[H_L \times (\theta + \pi)\right] \times \left(\frac{H}{H_L}\right)^{\lambda_1}. \quad (10)$$

3. Debt value

By following Chen et al. (2009), we assume that continuous payment is an approximation of the actual value of fixed-rate mortgage security. We denote the initial mortgage loan amount as M, the scheduled mortgage maturity as T, the mortgage time into term as t, the annualized effective mortgage contract rate as R_0, and the loan to value as LTV. $M(R_0, t)$ is the mortgage balance at time t, which is the present value of the remaining payment stream discounted at R_0, as shown as follows:

$$M(R_0, t) = M\frac{1 - e^{-R_0(T-t)}}{1 - e^{-R_0 T}} = H_0 \times LTV \times \frac{1 - e^{-R_0(T-t)}}{1 - e^{-R_0 T}}.$$

4. Decision to sell

When the housing price index increases, homeowners may consider selling their old houses and purchasing new ones. Consequently, we assume that when the residential real estate price index reaches the price index at H_L, investors will be highly likely to sell old houses and purchase new ones. However, selling decisions will be made by maximizing the homeowners' benefit, that is, the homeowners' equity E, which can be obtained by deducting the interest and house-switching cost from the housing price, as shown as follows:

$$E = F^H - F^I - F^c - M(R_0, t) = \left[\frac{(w-c)H_0}{\delta}\right] \times \left[-\left(\frac{H}{H_L}\right)^{\lambda_1}\right] + \left[H_L - (H_L - H_0) \times t_c\right] \times \left(\frac{H}{H_L}\right)^{\lambda_1} - I \times (1 - t_I) \times \left[1 - \left(\frac{H}{H_L}\right)^{\lambda_1}\right] - \left[H_L \times (\theta + \pi)\right] \times \left(\frac{H}{H_L}\right)^{\lambda_1} - H_0 \times LTV \times \frac{1 - e^{-R_0(T-t)}}{1 - e^{-R_0 T}}. \quad (11)$$

To find the optimal value of H_L^*, we have

$$\left.\frac{\partial E(H)}{\partial H_L}\right|_{H=H_L^*} = \left.\frac{\partial\left(F^H - F^I - F^c\right)}{\partial H_L}\right|_{H=H_L^*} = 0. \quad (12.1)$$

To maximize the real estate equity value, the optimal selling price H_L^* can be obtained as follows by the smooth-pasting condition (Dixit & Pindyck, 1994; Leland, 1994; Merton, 1973b):

$$H_L^* = \frac{\lambda_1\left[\frac{(w-c)H_0}{\delta}\right] - \lambda_1 H_0 t_c - \lambda_1 \times I \times (1 - t_I)}{(\lambda_1 - 1)(1 - t_c - \theta - \pi)}, \quad (12.2)$$

where:

$$\lambda_1 = 0.5 - \frac{r - \delta}{\sigma^2} + \sqrt{\left(\frac{r - \delta}{\sigma^2} - 0.5\right)^2 + \frac{2r}{\sigma^2}}.$$

(See proof in Appendix)

Based on Equation (12.2), we find that the housing price H_L^* will increase with higher initial housing price H_0 when $\frac{(w-c)}{\delta} > t_c$. In addition, H_L^* also decreases with the higher ratio of maintenance and management expense to housing value c and increases with higher w. H_L^* is also positively related to θ and π. By plugging H_L^* into Equation (6), the housing price can be reformulated as follows:

$$F^H = \left[\frac{(w-c)H_0}{\delta}\right] \times \left[1 - \left(\frac{H}{H_L^*}\right)^{\lambda_1}\right] + \left[H_L^* - (H_L^* - H_0) \times t_c\right] \times \left(\frac{H}{H_L^*}\right)^{\lambda_1}. \quad (13)$$

In Equation (13), housing price is affected by utility rental benefit rate (w), cost of housing management (c), capital gain taxes (t_c), loan conditions (r), selling cost, and house-switching cost (θ and π). H_L^* also affects the probability of selling a house in the future and the housing value.

2.4. Investment value of housing

The investment income of housing can be divided into two types: rental income and capital gains from future sales. Excluding rental income, the remaining future capital gains F^{IG} are the investment value of houses, namely, the real options value of homeownership. The PDE of the investment value is as follows:

$$0.5H^2\sigma^2 F_{HH}^{IG} + (r - \delta)HF_H^{IG} + F_t^{IG} - rF^{IG} = 0. \quad (14)$$

When homeowners sell their houses, they acquire after-tax capital gains. If they do not sell their houses, no investment value will be realized. The restriction is as follows:

$$H = H_L,\ F^{IG} = (H_L - H_0) \times (1 - t_c), \quad (15.1)$$

$$H \to 0,\quad F^{IG} = 0. \quad (15.2)$$

According to the smooth-pasting condition $\left(\left.\frac{\partial F^{IG}}{\partial H}\right|_{H=H_L} = 1 - t_c\right)$, the optimal selling price $H_L^{\#}$ with the maximum residential real estate investment value can be obtained as follows:

$$H_L^{\#} = \frac{\lambda_1 H_0}{\lambda_1 - 1}. \quad (16)$$

Therefore, the housing investment value F^{IG} is as follows:

$$F^{IG} = (H_L^{\#} - H_0) \times (1 - t_c) \times \left(\frac{H}{H_L^{\#}}\right)^{\lambda_1}. \quad (17)$$

2.5. Structural analysis of housing price

Housing value can be divided into consumption value HC and investment value F^{IG}. Therefore, the housing demand value is obtained by subtracting the investment value from the real value of the house. In this study, we assume that the per ping[4] house price equals the sum of the per ping consumption and investment values. The value equation is as follows:

$$HC = F^{H} - F^{IG}. \tag{18}$$

When real estate is in a bull market, the housing price will increase, as well as HC and F^{IG}. Similarly, when the real estate is in a bear market, investment and consumption values will decline. However, in response to different economic situations of the real estate market, the changes in investment and consumption values of housing must be examined more precisely to measure the degree to which their values are affected by the economic cycles. We therefore conduct our numerical analysis in Section 3.

3. Numerical analysis

According to the research group Demographia in its 2020 survey, Hong Kong still ranks first for ten years in a row with a housing price-to-income ratio of 20.8. However, Taipei City, the capital city of Taiwan and not ranked in the survey, posted 13.9 by the end of 2019 (Ministry of Interior of Taiwan), followed by Vancouver (11.9), Sydney (11.0), and Melbourne (9.5). The housing market of Taiwan has seen a sharp rise since 2009 when the government lowered the inheritance tax rate from 50% to 10% and the home price index of Taiwan almost doubled, from 150 in 2009 to 300 in the middle of 2014 (Shinyi Real Estate and Research). Even after the repercussion from COVID-19, the home price index of Taiwan rose to 302.6 as of the end of July 2020, from 287.2 in January 2020. According to the UBS Global Real Estate Bubble Index 2019, "anyone who acquired residential property in the last 40 years, even at the height of a local price bubble, has nevertheless enjoyed long-term capital gains in most centers." This notion applies to the Taipei housing market, in which an average home price of newly built houses as of June 2020 has posted a record high of TWD 870,000 (US $29,000) per ping.

The main objective of this study is to deconstruct housing value and specifically measure the influence of economic factors on housing value. Table 1 presents the description of model parameters and their values. The model parameters are specified with initial values[5] subject to variations to measure how the investment and consumption values change accordingly, thereby providing more insight into homebuyers' decision making and the implications of housing policy. We assume the initial housing price index H_0 to be 100, following a stochastic process. Housing value F^H is then evaluated, and the consumption value HC and investment value F^{IG} can be computed separately. To curb the speculation in Taiwan housing marking, Taiwan has passed the amendment of Income Tax Act (also called Integrated House and Land Income Tax Act) in 2016 to impose the capital gains tax rate of 20% if homeowners sell houses holding less than 10 years. Consistent with housing mortgage rate in Taiwan, it is assumed to be 3% in annual rate, and we convert it into monthly rate of 0.25% for simulation by month.

3.1. Effect of utility rental benefit rate on housing price

Table 2 shows the value per ping of HC, F^{IG}, and F^H across the volatility of property price index σ and utility rental benefit rate w. We find that F^{IG} is positively related to σ, increasing from 4.12 to 15.37 as σ increases from 9% to 15%, whether w is low (3%) or high (5%). The huge

Table 1. Parameter assumptions in the housing model

	Symbols	Numerical values	Description
Initial price of housing index	H_0	100	Housing index is initialized to be 100 as a proxy for market price
Residential property price index volatility	σ	12%	
Capital gains tax	t_c	20%	Capital gains tax for selling houses
Interest tax	t_I	15%	Tax shield for interest expense
Depreciation rate of real estate	δ	2.5%	
Mortgage rate	r_b	0.125%	1.5%/12 = 0.125% (monthly rate)
Term of loan	n	240	20 years with 240 months
Loan-to-value (LTV) ratio	LTV	0.7	
Utility rental benefit rate	w	3%	Without considering other costs
Expense ratio of maintenance and management	c	1%	
Risk-free rate	r	1.5%	Annual rate
Related costs of sale	θ	10%	
Cost of switching house	π	20%	

[4] *Ping* is a traditional unit for measuring the area of the housing in Taiwan. One ping is approximately equal to 3.3 m^2.

[5] We adopt the parameter setup similar to Geltner et al. (2014) as the baseline value. The parameter setup for numerical analysis here is partially adapted from Chapter 14 "After-tax investment analysis and corporate real estate".

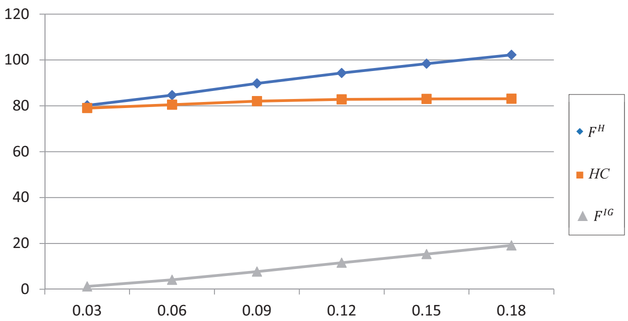
increase in investment value implies that homebuyers are expecting potential profits in real estate investment in an ever-increasingly volatile housing market, which makes homebuyers more willing to pay higher prices for houses. The higher volatility of housing price may also be caused by lower housing prices due to economic recession, which is subsumed by the assumption of a negative inflation rate. The consumption and investment values will decrease when the inflation rate is negative.

Table 2 also shows that HC is negatively related to σ when w is higher than 4%. When w is 3%, HC increases with σ, from 82.07 (σ = 9%) to 83.05 (σ = 15%). However, when w rises to 4% or even higher (5%), HC decreases slightly with higher σ. For example, when w is 5%, HC shows an obvious downtrend from 155.89 to 150.05 as σ increases from 6% to 15%. This finding implies that for houseowners with high-end houses (i.e., luxurious houses that can deliver higher utility rental benefit), the consumption value of the housing may not become highly important compared with the investment value when the houseowners decide to switch houses in a highly volatile housing market. Nevertheless, the houses that can deliver high w to residents are still highly valued by homeowners or homebuyers in terms of their share in the housing value. For the case of σ = 15%, as w increases from 3% to 5%, the proportion of HC to F^H still increases from 84.37% to 90.70%, compared with 95.13% and 97.41% for the case of σ = 6%. Consequently, homeowners will continue to hold real estate not only because they expect to profit from the increase in asset value but because they can also gain comfort and utility from living in the houses.

As the volatility of housing price increases, homebuyers seem to weigh the future investment value of the real estate more than the consumption value when deciding to purchase houses. In comparison with HC, F^{IG} has a large percentage of increase as σ rises. Therefore, the price volatility of residential real estate is found to have a strong effect on housing demanders' investment decision.

On the basis of Table 2, Figure 1 shows the positive relationship between σ and the value of F^H per ping for the case of w = 3%. When σ < 0.09, F^{IG} and HC increase, causing F^H to increase. When σ > 0.09, F^H continues to increase, but HC only barely increases. Given that the increase in F^{IG} is larger than the decrease in HC, F^H still increases. In addition, as σ increases, the proportion of F^H, represented by F^{IG}, increases; whereas the proportion of F^H, represented by HC, decreases. Therefore, when the housing market becomes more volatile, the housing investment motivation increases, and the proportion of expected investment value also increases.

On the basis of Table 2, Figure 2 shows the relationship between w and price per ping for the case of σ = 15%; as w increases, F^H and HC also increase, whereas F^{IG} remains constant. Therefore, when w increases to 5%, the proportion of HC to F^H increases as high as 91%. w also affects the housing demander consumption motivation. When w increases, a housing demander has higher motivation to buy houses, which will make housing demanders more willing to pay higher prices to purchase houses. Figure 3 shows that given σ = 9%, the proportion of HC in F^H increases with w, whereas the proportion of F^{IG} decreases. Therefore, for a given volatility of housing price, the increasing w will induce houseowners to weigh HC more than F^{IG} and encourage houseowners to hold houses that can deliver higher utility benefit.



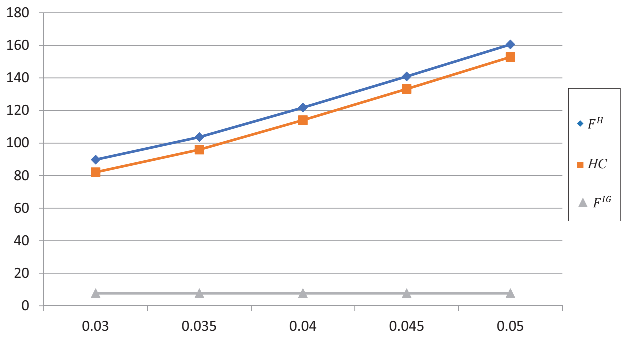
Note: Vertical axis represents the price per ping. Horizontal axis represents the housing price volatility σ. HC is the consumption value. F^{IG} is the investment value. F^H is the housing value.

Figure 1. Volatility of housing price index and housing value

Table 2. Consumption, investment, and housing values across the volatility of housing price index and utility rental benefit rate*

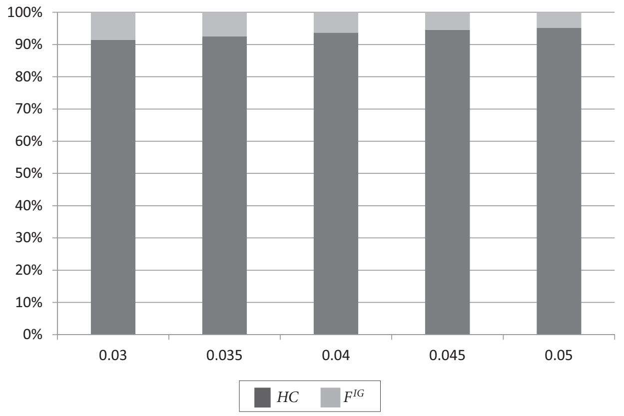
	Volatility of housing price index					
Utility rental benefit rate	w	Value	σ = 6%	σ = 9%	σ = 12%	σ = 15%
	3%	Consumption value HC	80.51	82.07	82.78	83.05
		Investment value F^{IG}	4.12	7.72	11.53	15.37
		Housing value F^H	84.63	89.78	94.32	98.43
	4%	Consumption value HC	116.03	114.05	113.41	113.43
		Investment value F^{IG}	4.12	7.72	11.53	15.37
		Housing value F^H	120.15	121.77	124.94	128.80
	5%	Consumption value HC	155.89	152.86	150.92	150.05
		Investment value F^{IG}	4.12	7.72	11.53	15.37
		Housing value F^H	160.02	160.57	162.46	165.42

Note: * Simulated values in the table are the value per ping, which is a traditional unit for measuring the area of the housing in Taiwan. One ping is approximately equal to 3.3 m^2.



Note: Vertical axis represents the price per ping. Horizontal axis represents the utility rental benefit rate (w). *HC* is the consumption value. F^{IG} is the investment value. F^H is the housing value.

Figure 2. Utility rental benefit rate and housing value



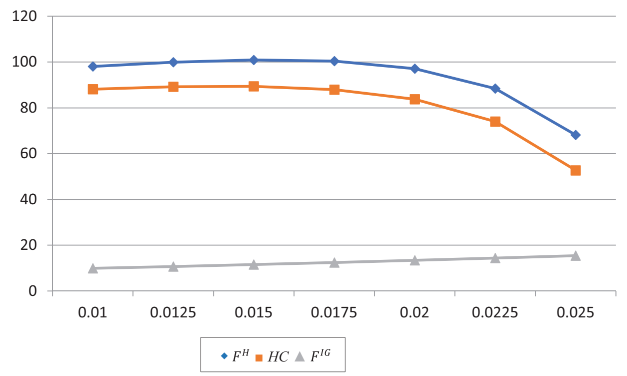
Note: Vertical axis represents the housing value composition. Horizontal axis represents the utility rental benefit rate *w*. *HC* is the consumption value. F^{IG} is the investment value.

Figure 3. Housing value composition and utility rental benefit rate

3.2. Effects of LTV ratio and interest rate on housing value

Table 3 presents the value per ping of *HC*, F^{IG}, and F^H across the LTV ratio and interest rate. Given that houses are expensive assets, we assume that all housing demanders purchase houses through mortgage loans. Given w = 3%, Table 3 shows that *HC* increases with the LTV ratio when the interest rate is as low as 1.25%. When the interest rate rises above 1.5%, *HC* starts to decrease when the LTV ratio is higher than 0.8. When the LTV ratio reaches 1, F^H decreases with the rising interest rate because of the decrease in *HC*. In this situation, the increase in F^{IG} is not sufficiently large to offset the decrease in the consumption value, and F^H thus decreases. For example, *HC* substantially decreases to 77.17 from 87.98 when the LTV ratio rises from 0.8 to 1.0 for the interest rate of 1.75%.

Figure 4 describes the relationship between risk-free rate and housing value. It shows that F^H initially increases and then decreases with the increasing interest rate.



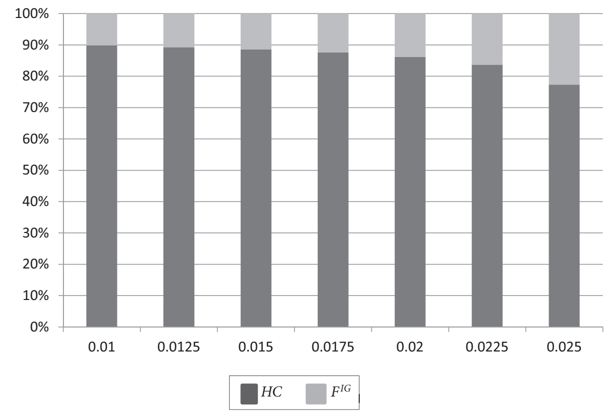
Note: Vertical axis represents the price per ping. Horizontal axis represents risk-free rates *r*. *HC* is the consumption value. F^{IG} is the investment value. F^H is the housing value.

Figure 4. Risk-free rate and housing value

Table 3. Consumption, investment, and housing values across LTV ratio and interest rate*

		LTV			
r		0	0.4	0.8	1
1.25%	Consumption value *HC*	82.39	85.76	89.18	89.82
	Investment value F^{IG}	10.69	10.69	10.69	10.69
	Housing value F^H	93.09	96.45	99.88	100.51
1.5%	Consumption value *HC*	82.51	86.42	89.38	87.04
	Investment value F^{IG}	11.53	11.53	11.53	11.53
	Housing value F^H	94.05	97.96	100.91	98.57
1.75%	Consumption value *HC*	82.62	86.97	87.98	77.17
	Investment value F^{IG}	12.44	12.44	12.44	12.44
	Housing value F^H	95.05	99.40	100.42	89.60

Note: * Simulated values in the table are the value per ping, which is a traditional unit for measuring the area of the housing in Taiwan. One ping is approximately equal to 3.3 m^2.



Note: Vertical axis represents the housing value composition. Horizontal axis represents the risk-free rate r. HC is the consumption value. F^{IG} is the investment value.

Figure 5. Housing value composition and risk-free rate

When the interest rate exceeds a threshold, such as 1.5%, HC starts to decrease substantially and results in a lower F^H. Before that threshold, HC has a positive relationship with the interest rate on the basis of numerical results. Therefore, F^H shows a positive relationship with interest rate when the interest rate is low. F^{IG} also increases when the interest rate starts to rise, indicating enhanced investment motivation while the economy is expanding. We also observe that the consumption and investment values are negatively related when the interest rate exceeds the threshold of 1.5%. HC and F^H will decline more when the interest rate exceeds 2%.

Figure 5 shows the share of HC and F^{IG} in different risk-free rates. When the interest rate rises, the proportion of F^{IG} to F^H also rises. This finding implies that facing higher interest rate, housing demanders are willing to pay more to buy houses from the perspective of investment because they believe that they can gain more wealth from owning housing assets as the economy is expanding. Furthermore, a higher interest rate implies that loan conditions tend to be less favourable to mortgage borrowers, and they will lower their motivation to switch houses. Therefore, they tend to value less for the housing consumption value. However, when the interest rate is low, housing demanders will care more about the consumption value delivered by the housing and less about the investment value. As shown in Figure 5, when the interest rate decreases, the larger share of consumption value in the total value can be accounted for by the housing demanders' physiological search of either safety or comfort from owning new houses. The simulated results show that when the interest rate starts to decrease from 0.025 to 0.0175 (Figure 4), a substantial increase is observed in the housing value, which is mainly attributed to the consumption incentive instead of investment incentive. The increase in housing value or housing "prices" observed in the market may be claimed to be the result of strong demand from investors in the market. However, homeowners who have to decide whether to switch to new houses may weigh investment and consumption demands differently from the market traders. In summary, interest rate is positively related to investment value and negatively related to consumption value when the interest rate rises above a certain threshold.

3.3. Probability to sell housing

Table 4 shows the probability for homeowners to sell houses across the volatility of the real estate price index and utility rental benefit rate. It reveals that the probability to sell house is positively related with σ and negatively related with w. This finding implies that when the housing location is unfavorable or the utility delivered by houses is lower than other houses, the homeowners are more likely to sell houses as σ increases. By contrast, when σ is low (6%) and w is high (5%), the probability to sell houses is only 0.05%, suggesting that homeowners prefer to live in their current houses rather than switching to new houses when the housing prices are stable. The high probability to sell house is commanded by the housing with lower w and higher σ. In summary, houses with a lower utility rental benefit rate will be more likely to be sold in exchange for higher-quality houses, especially when real estate market becomes volatile.

Table 5 shows the effects of LTV ratio and interest rate on the probability for homeowners to sell houses. As almost all homeowners use financial leverage to purchase houses, their decision to sell houses is affected by the changes in mortgage interest rate. Specifically, when the LTV ratio is as low as 0.2, the probability to sell a house increases slightly from 20.28% to 24.31% as the interest increases from 1% to 1.75%. However, when the LTV ratio is 0.8, the probability to sell houses sharply increases from

Table 4. Probability for homeowners to sell houses across the volatility of the real estate price index and utility rental benefit rate*

		Volatility of real estate price index			
Utility rental benefit rate		σ = 6%	σ = 9%	σ = 12%	σ = 15%
w	3%	11.78%	17.54%	19.43%	19.64%
	4%	1.20%	4.82%	7.77%	9.49%
	5%	0.05%	0.80%	2.17%	3.44%

Note: * Probability to sell housing is $\left(\frac{H}{H_L}\right)^{\lambda_1}$, which is defined in Equation (6).

42.92% to 94.19%. In summary, homeowners with high LTV ratios are more sensitive to changes in interest rate when deciding whether to sell houses.

w is the utility rental benefit and represents the homeowners' subjective satisfaction with houses. A higher w indicates a higher satisfaction with the current houses, and homeowners will thus be less probable to sell or switch houses. Table 6 compares the effects of LTV ratio and w on the probability for homeowners to sell houses. It shows that the utility delivered by houses is more important for homeowners with a high LTV ratio in deciding whether to sell or hold houses. For example, when w is 3.5%, homeowners with an LTV ratio of 0.8 have a probability to sell houses 22 times higher than they do when w is 6%. By contrast, homeowners with an LTV ratio of 0.2 have a probability to sell houses only 13 times higher than they do when w is 6%.

Table 7 presents the summary of the relationship among consumption value, investment value, housing value, probability to sell, and parameters in the model. When the interest rate is low, the housing value will increase as the interest rates rise. When the interest rate rises above the threshold value, the housing value starts to decrease. This study is conducted from the perspective of homeowners and thus analyzes whether houses should be sold in exchange for real options value. Therefore, when the interest rates increase, the investment motivation of housing demanders increases, causing a higher probability for them to sell houses for profit-taking purposes. By contrast, when the LTV ratio is higher than 0.8 and the interest rate rises above the threshold, the decrease in housing consumption value will become greater than the increase in housing investment value. Therefore, the interest rate and housing value are inversely correlated when the interest rate is above the threshold. This finding implies that a reasonable increase in interest rate does not necessarily result in a decrease in housing value. The result that the housing value starts to decrease will occur only when the interest rate exceeds the threshold level.

This study has divided housing value into consumption and investment values and analyzed how consumption and investment values are affected by housing conditions and economic factors. F^{IG} is not affected by utility rental benefit rate w, LTV, house selling cost θ, or cost of switching houses π. Furthermore, F^{IG} is positively related to the interest rate and volatility of the housing price index σ; whereas F^{IG} is negatively affected by maintenance and management fees and real estate depreciation rate. By contrast, w and LTV ratio affect HC; whereas interest rate r, maintenance and management fees c, real estate depreciation rate δ, and θ negatively influence HC. However, σ and π initially positively and then negatively affect HC.

Table 5. Probability for homeowners to sell houses across LTV ratio and interest rate*

		Interest rate			
Loan to value		r = 1%	r = 1.25%	r = 1.5%	r = 1.75%
LTV	0.2	20.28%	21.70%	23.05%	24.31%
	0.5	28.91%	33.59%	38.68%	44.22%
	0.8	42.92%	55.63%	72.13%	94.19%

Note: * Probability to sell housing is $\left(\frac{H}{H_L}\right)^{\lambda_1}$, which is defined in Equation (6).

Table 6. Probability for homeowners to sell houses across LTV ratio and utility rental benefit rate

		Utility rental benefit rate			
Loan to value		w = 3%	w = 4%	w = 5%	w = 6%
LTV	0.2	8.82%	4.25%	1.44%	0.65%
	0.5	12.76%	5.66%	1.75%	0.75%
	0.8	19.43%	7.77%	2.17%	0.88%

Table 7. Relationship among housing value, probability to sell, and parameters in the model (r is the risk-free rate; σ is the volatility of housing price index; w is the utility rental benefit rate; LTV is the LTV ratio; δ is the depreciation rate of real estate; c is the expense ratio of maintenance and management; θ is the related costs of sale; π is the cost of switching house; t_I is the interest tax; and t_c is the capital gain tax

	r	σ	w	LTV	δ	c	θ	π	t_I	t_c
Consumption value (HC)	–	+(–)	+	+	–	–	–	+(–)	+	–
Investment value (F^{IG})	+	+	x	x	–	–	x	x	x	–
Housing value (F^H)	+(–)	+	+	+	–	–	–	+(–)	+	–
Probability to sell house	+	+	–	+	+	+	–	–	+	–

Note: "+" represents positive relationship. "–" represents negative relationship. "*x*" represents unrelated. "+(–)" represents an initially positive and then a negative relationship).

Therefore, the effect of housing conditions and economic factors on housing value differs. Specifically, F^H is negatively affected by δ, c, and θ, but it is positively affected by σ, w, and LTV ratio. In addition, F^H is initially positively related to r and π and then becomes negatively related to these factors. In addition, w, θ, t_c, and π are negatively related to the probability for homeowners to sell their house. r, σ, LTV, δ, and c are positively related to the probability for homeowners to sell their house.

Conclusions

Consumption and investment values are two primary components of housing value. The ratio of consumption value to housing value should always account for the most share of housing value. However, bubbles in the real estate market may be forming when the share of investment value to housing value is increasing. Policymakers should take heed of the changes in the share of investment and consumption values in housing value. According to the model analysis in this study, homebuyers consider investment value when purchasing houses and are willing to pay different housing prices based on several economic and financial factors. Factors affecting housing value include interest rate, LTV ratio, utility rental benefit rate, initial housing cost, housing price volatility, current real estate prices, maintenance and management cost, house-switching cost, and real estate taxes. When the economy is booming, the housing market will become volatile. We find that as the volatility in the housing market becomes higher, the share of housing consumption value to housing value will be lower for houses with a high utility rental benefit rate. However, the share of investment value increases. This finding implies that housing demanders tend to provide higher weight to investment value than consumption value delivered by houses, which may promote the formation of a housing bubble.

When the housing prices are low, governments can enhance the quality and comfort of housing and thus increase the utility benefit delivered by housing consumption for house demanders. By contrast, during the booming of an economy when housing prices start to rise, policies can be implemented to reduce the volatility of housing price, for example, by lowering the LTV ratio and raising the utility rental benefit rate, which can reduce the homeowners' motivation to sell or switch houses. For example, subsidies to houseowners in revamping their houses or policies to improve public facilities for residents will help houseowners to lower their motivation to switch houses. By providing a structural analysis of housing value, this study can serve as a start for housing regulators to differentiate consumption and investment values in formulating housing regulations.

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Author contributions

Chih-Hsing Hung conceived the study and was responsible for the theoretical work and the simulations based on the assumptions of the model. Shyh-Weir Tzang was responsible for the data analysis and interpretation.

Disclosure statement

Authors do not have any competing financial, professional, or personal interests from other parties.

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Appendix

Proof of optimal selling price for H_L^* where:

$$H_L^* = \frac{\lambda_1\left[\frac{(w-c)H_0}{\delta}\right] - \lambda_1 H_0 t_c + \lambda_1 \times dI \times (1-t_I)}{(\lambda_1-1)(1-t_c-\theta-\pi)}.$$

Home equity is as follows (a.1):

$$E = F^H - F^I - F^c. \tag{a.1}$$

Substituting (6), (8), and (11) into (a.1) leads to

$$\left[\frac{(w-c)H_0}{\delta}\right]\times\left[-\left(\frac{H}{H_L}\right)^{\lambda_1}\right]+\left[H_L-(H_L-H_0)\times t_c\right]\times \left(\frac{H}{H_L}\right)^{\lambda_1} - I\times(1-t_I)\times\left[1-\left(\frac{H}{H_L}\right)^{\lambda_1}\right]-\left[H_L\times(\theta+\pi)\right]\times\left(\frac{H}{H_L}\right)^{\lambda_1}. \tag{a.2}$$

Differentiating Equation (a.2) with respect to H_L, a "smooth-pasting" condition (Leland, 1994) at $H = H_L$, and solving the optimal sell price for H_L^*:

$$\left.\frac{\partial E(H)}{\partial H_L}\right|_{H=H_L} = \left.\frac{\partial\left(F^H - F^I - F^c\right)}{\partial H_L}\right|_{H=H_L^*} = 0$$

$$\left.\frac{\partial E(H)}{\partial H_L}\right|_{H=H_L} = \lambda_1\left[\frac{(w-c)H_0}{\delta}\right]\left(\frac{1}{H_L}\right)+$$
$$(1-t_c)-\lambda_1\left[H_L-(H_L-H_0)\times t_c\right]\times\left(\frac{1}{H_L}\right)+$$
$$\lambda_1\times dI\times(1-t_I)\times\left(\frac{1}{H_L}\right)-(\theta+\pi)+\lambda_1\left[H_L\times(\theta+\pi)\right]\times\left(\frac{1}{H_L}\right)=0. \quad \text{(a.3)}$$

Both multiply H_L

$$\lambda_1\left[\frac{(w-c)H_0}{\delta}\right]-\lambda_1 H_0 t_c+\lambda_1\times dI\times(1-t_I)=$$
$$\lambda_1 H_L(1-t_c)+(\theta+\pi)H_L(1-\lambda_1)-(1-t_c)H_L.$$
$$(\lambda_1-1)H_L(1-t_c)+(\theta+\pi)H_L(1-\lambda_1)=$$
$$\lambda_1\left[\frac{(w-c)H_0}{\delta}\right]-\lambda_1 H_0 t_c+\lambda_1\times dI\times(1-t_I). \quad \text{(a.4)}$$

We can arrange (a1-a.4) and thus obtain:

$$H_L^* = \frac{\lambda_1\left[\frac{(w-c)H_0}{\delta}\right]-\lambda_1 H_0 t_c+\lambda_1\times dI\times(1-t_I)}{(\lambda_1-1)(1-t_c-\theta-\pi)}.$$