



## ANALYSIS OF ANGLE MEASUREMENTS AND A NEW APPROACH TO VERTICAL ANGLE CALIBRATION

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Received 16 November 2011; accepted 21 November 2011

**Abstract.** Testing and calibrating geodetic instruments is an important metrological procedure for precise instrumentation used in industry and geodesy. Electronic geodetic measuring instruments consist, among other elements, of circular scales and angular encoders for angle determination in two perpendicular horizontal and vertical planes. The accuracy of the instrument mostly depends on the accuracy of angle standards used for angle calibration in the horizontal plane. Although many methods are developed for angle calibration, the majority of those are not available for calibrating angles in the vertical plane. The method described in the International Standards of vertical angle calibration validates field testing geodetic instruments by taking precise angle readings between the calibrated points placed at long distance and height. Difficulties in creating such calibration basis and inconveniences in proceeding with this field at long distances may appear. Also, the traceability of length and angle measurements is not ensured. The new method of calibrating the angles in the vertical plane for geodetic instruments was developed. The article describes the principle of indoor testing and the calibration of the vertical angle measuring system.

**Keywords:** circular scales, calibration, reference measure, angle.

### 1. Introduction

The accuracy of measurement is based on a proper performance of geodetic tasks using appropriate measuring instruments that must undergo metrological verification or calibration processes valid for all kinds of measurements, such as a flat and vertical angle or length, distance measurements or monitoring construction deformation. Measurement accuracy depends on the instruments, devices and methods used for performing a task. This article describes methods of angle calibration, improvement in angle calibration and circular scales used for angle measuring systems and geodetic instruments. The paper is prepared on the basis of the presentation made at the 8th International Conference “Environmental Engineering”, Natal, Brazil (Šiaudinyte, Giniotis 2011; Rybokas *et al.* 2011).

A number of methods are employed in the process of angle calibration. Classical angle measurement methods originally implemented in geodesy and astronomy and applied by Wild, H. Bruns, Heuvelink, S. Jelisejev

are frequently used for taking control over measuring the angle accuracy of geodetic instruments. Most of the methods are used for calibrating flat angles; however, the conducted experiments show that the accuracy of vertical and horizontal angle measuring systems differs. Due to this reason, the raster scales of vertical angle measuring systems can be measured using the methods of flat angle calibration. Moreover, in case no possibility of measuring a separate raster scale exists, there are some ways to determine the accuracy of the whole angle measuring system used in the instrument. The article presents a case study on some angle measurement systems used in geodetic instruments and describes a new method created for vertical angle calibration of the instrument.

### 2. Circular Scales

Circular scales together with angle encoders are mounted in geodetic total stations (tacheometers). The modulation of the light beam crossing the raster scale and indication scale is the main principle of operating the rotary

encoder. A circular scale is an efficient and reliable mean of getting and passing information about the angular position of an object or axis of the measuring instrument. Scales are made of various materials depending on operational conditions, accuracy and price.

A rotary encoder is an electromechanical device used for encoding the angular movement of the shaft or axis of the turntable to a certain analogue or digital signal. There are two widely used types of angle encoders: absolute and incremental. The main difference between an absolute and incremental encoder is that the absolute encoder always has a fixed zero position; however, this position varies in the incremental encoder showing the difference between the previous and present position of the encoder.

A digital encoder generates a unique digital binary code for each turn of an axle. The rotary encoder has a circular raster scale mounted on the shaft and divided into many parts that define angular degrees, minutes, seconds and the decimal parts of seconds of arc (depending on the discretion of the required data). The coded scale consists of the parts of the circle covered by the layer of a black or white colour and has a preset binary value (1 or 0, Fig. 1). During operation, the angle encoder is turned to the needed angle position and stops at a certain combination of the black and white parts of the scale. Each part has a unique binary code that is recognized as angle reading transmitted to further data processing devices. An optical disk of an absolute angle encoder is intended to assign digital codes to a certain position of a shaft (i.e. if a circular scale consists of 8 tracks with engraved marks, this scale is able to generate 256 different positions or angular movements with the accuracy of 1.406 degrees equal to  $360/256$ ). Most common numerical encoding for this type of encoders is based on the position of the black and white parts and binary or Gray coding systems. Upon rotation of the circular scale and shaft on the axle, photodetectors read the pattern and generate a digital code. During the revolution of a digital code generating the angle encoder, all elements change their position. All track readings of the code encoder have outputs separately from each other.

The way the glass circle is divided into black and white blocks is not completely standardised and depends on a manufacturer. Using the red light from LED (Light Emitting Diode) and a mirror, the marks are projected onto the CCD (Charge – Coupled Device) linear array.

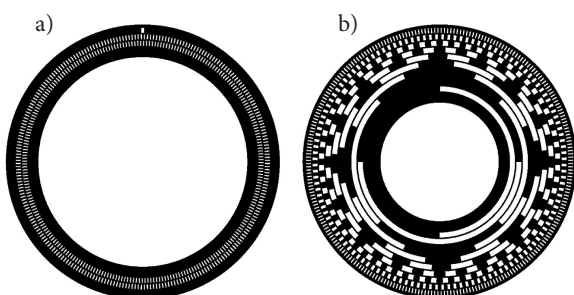


Fig. 1. Types of circular scales used in  
a) incremental encoders; b) absolute encoder

Most of geodetic instruments have two angle measuring systems, including horizontal and vertical where the difference between them consists of the main standard of angle measurement – the raster or coded scale is placed in the horizontal or vertical position (Cooper 1987; Giniotis 2005). The instrument operates by pointing its optical system to the object to be measured and taking the readings from angle measuring systems about the angular position of the object in two perpendicular planes. Neither in the vertical nor horizontal measuring system is the electronic parts moved relatively to the alidade, so that all electronics are fixed to it. Vertical and horizontal array boards have the same circuitry but different mounting and packaging. The calibration of angle standards used for readings in the horizontal plane is widely applied when the same problem of vertical readings in the instrument is more complicated and needs special methods and means usually developed by manufacturers for their intra-company needs (Cooper 1987; Surveying... 1991).

### 3. Methods of Angle Calibration

The errors of the angular position of the strokes of circular scales are often determined by comparing the angle between the strokes with the angle “created” by other angle measuring devices or standards.

The method of constant angle repetition in a circular scale was improved by many famous scientists such as H. Bruns, G. Schreiber, A. Perard, H. Wild, Heuvelink, S. Jelisejev and others (Giniotis *et al.* 2004). The errors of circular scales are determined by the methods approved in written standards such as the method of approximation, the method of an opposite matrix, the method of Heuvelink and the method of Wild. The methods of calibrating angular scales used for machine engineering and instrumentation are a comparison of the angular values of scale strokes with the values of the reference scale or other reference measure of the angle (Fig. 2).

A comparison of the angular position of the strokes of the scale with reference to the angle created by the strokes of the same scale is also called calibration with setting a constant angle in full circumference. The disadvantage of these methods is that the accuracy of error determination differs depending on the position of the strokes on the circular scale.

According to the classical method of placing the constant angle in the circular scale, the error of the position of circular scale strokes can be defined (Key 2005; Rybokas *et al.* 2011; Siaudinyte, Giniotis 2011):

$$x = \frac{1}{3} \sum (x^I + x^{II} + x^{III}), \quad (1)$$

where  $x^I = -\bar{x}_m$ ,  $x^{II} = \bar{x}_{+60^\circ} - \bar{x}_m$ ,  $x^{III} = \bar{x}_{+120^\circ} - \bar{x}_m$  are the readings of the stroke position in the scale at different angles.

A standard deviation of determining the error of the stroke position can be defined as

$$m_x = \sqrt{\frac{\sum \Delta x^2}{6n}}, \quad (2)$$

where  $\Delta x^I = x - x^I$ ,  $\Delta x^{II} = x - x^{II}$ ,  $\Delta x^{III} = x - x^{III}$ .

Random and systematic errors of the angular position of the circular scale can be determined using the method of Heuvelink. The measurement process and data processing of the angle standards used in geodesy are widely performed employing the method of Wild.

Some of the methods and means for circular scale measurement are shown in Fig. 2. The main parts of the measurement systems are: 1 – a circular scale to be measured, 2 – an instrument for reading a stroke position, 3 – a multi-angular prism, 4 – autocollimator, 5 – pulse sequences coming from the scale and reference measure, 6 – Moore Index Table, 7 – measurement using a moiré pattern. The etalon of the angle used in the diagram is (a) a constant angle placed in the circle, (b) – a multi-angle prism – polygon, (c) – pulses from the reference scale or rotary encoder and standard time intervals in conjunction with the constant rotation of the scale for measurement ( $\delta\varphi_i$  is angle deviation between reference and measuring pulses), (d) – the values of the standard angle fixed by the Index Table, (e) – a constant angle set by two microscopes placed at etalon  $\pi$  rad and the angle between them (half circle method) and (f) – using the geometry of moiré fringe pattern created by the reference raster scale and the scale to be measured (Giniotis *et al.* 2004).

**4. Angle References Used for the New Proposed Methods**

Some existing differences in applying recently proposed angle measuring methods are shown in Fig. 2 (e) and (f) using the etalon of the angle equal to  $\pi$  rad and applying moiré fringe features for angle measurements.

An angle of  $180^\circ$  can be set up with high precision using the circular raster scale to be measured, two opposite microscopes and the axis for scale rotation, Fig. 2 (e), (Siaudinyte, Giniotis 2011; Giniotis 2005). By adjusting the microscopes in the tangential direction, a position

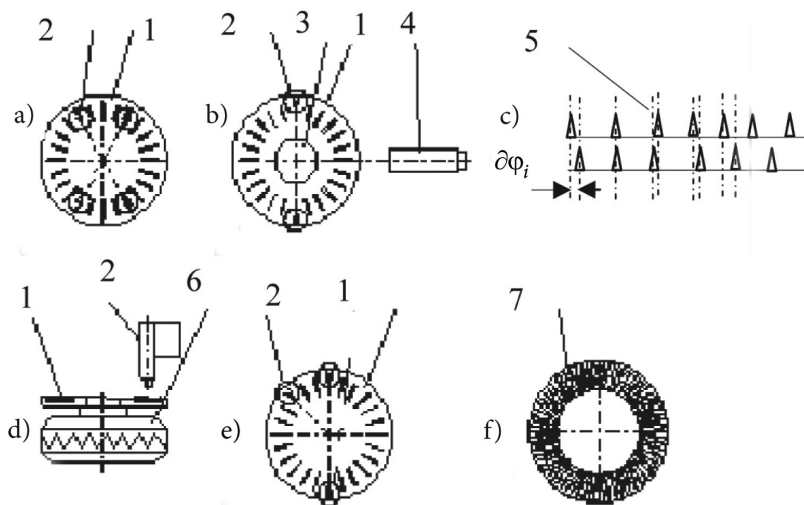
can be reached when readings from both microscopes are equal in an absolute value after the rotation of the scale at  $0^\circ$  and  $180^\circ$ . The position of starting the process of measurement can be freely chosen.

The “half circle” method (p rad method) for circular scale calibration is based on the application of  $180^\circ$  angle created by the scale to be measured and the instruments for reading strokes during the same calibration process (Giniotis, Rybokas 2010). Such standard of measure can be set with the accuracy not less than  $(0.1-0.3)^2$  of standard deviation in case of using a high-accuracy rotation axis and high-accuracy photoelectric microscopes. The task is to create an angle standard of the measure – a circular scale having a great number of calibrated discrete angular values against which the output signals of the rotary encoder, geodetic or other optical instruments could be compared. The suggested method of angle calibration allows measuring and calculating practically all systematic errors of the angular position of the strokes on the scale.

The moiré fringe pattern is used applying the high accuracy raster scale previously calibrated as reference measure and joining it with the raster scale to be measured. The bias of regular moiré fringes from periodic structure shows errors persistent in this area of the scale (Giniotis, Grattan 2002).

**5. New Method for Vertical Angle Calibration**

The methods of measuring circular scales described above can be used for calibrating the scales used in all positions of the measuring instrument, including the vertical measuring plane. Difference consists of calibrating the instrument when the instrument itself is positioned in the vertical direction from which horizontal and vertical angles must be measured. Then, the calibration of the measuring system must also be performed in the vertical plane (Key 2005; Giniotis *et al.* 2009).



**Fig. 2.** Methods for circular scale measurement: a) constant angle placement and using 4 microscopes in the circle; b) calibration using a prism, autocollimator and 2 microscopes; c) rotating and comparing transmitted signals from the measured scale and reference measure; d) calibration while comparing strokes with precise Moore Indexing Table standard; e) calibration using three microscopes and half circle etalon; f) the method of a moiré fringe pattern

The proposed arrangement for vertical angle calibration is based on determining the trigonometric angle using the reference scale of length for vertical readings employing the tacheometer and another reference measure of length for the distance from the tacheometer axis to vertical scale determination.

After placing the instrument under control at the initial position, an auxiliary instrument position is achieved by moving the instrument along the slide ways of the test bench for testing geodetic instruments (Giniotis et al. 2009; Ingensand 2008). At distance  $l$  from the axis of the instrument, the linear scale is fixed in the vertical position to the horizontal axis of the instrument. The distance from both positions of the instrument is fixed by using the reference measure of length, for example, end length gauge (length standard) applied for determining the distance from the axis of the instrument to the surface of the scale, which is a quite complicated task to be done initially. At both positions of the instrument, reading  $h'$  from the scale is taken at the angle  $\varphi$  of the axis of the telescope of the instrument and horizontal line. Readings from the scale are taken and the angle of interest is expressed

$$\varphi_t = \arctg \frac{h}{l_m}. \quad (3)$$

A case study would be a simplified scheme for measurement consisting of a distance measuring instrument (DMI) fixed to the geodetic instrument for angle calibration in the vertical plane (Fig. 3).

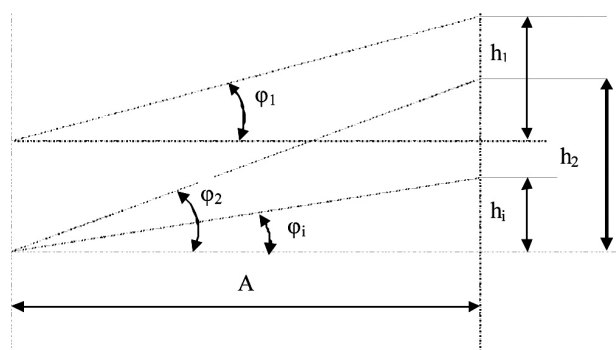


Fig. 3. A simplified diagram of angle measurement in the vertical plane

In our case, distance  $A$  is measured using a distance measuring instrument or any of the standards for length measuring. When putting the instrument in different vertical positions parallel to the linear scale, readings  $h_1$ ,  $h_2$  and  $h_i$  are taken. The value of the angle in the vertical plane is determined using the same expression (3) inserting the known value of distance  $A$  from the instrument to the surface of the linear scale.

## 6. Conclusions

Most of circular scales used in geodetic measuring instruments are calibrated in the horizontal plane using classical and newly developed methods of measurement.

The new method for vertical angle calibration has advantages over the methods used in geodesy due to higher accuracy of vertical angle determination and possibilities of performing it in the laboratory environment. Simplified setup of vertical angle calibration is proposed permitting the performance of calibration in the laboratory environment.

## Acknowledgment

The research is funded by the European Social Fund under the Global Grant measure.

## References

- Cooper, M. A. R. 1987. *Modern Theodolites and Levels*. Second ed. BSP Professional Books.
- Giniotis, V. 2005. *Padėties ir poslinkių matavimas*. Vilnius: Technika. 216 p. (in Lithuanian).
- Giniotis, V.; Grattan, K. T. V. 2002. Optical method for the calibration of raster scales, *Journal of the International Measurement Confederation (IMEKO)* 32(1): 23–29. doi:10.1016/S0263-2241(01)00057-4
- Giniotis, V.; Rybokas, M. 2010. Creation of the PI angle standard for the flat angle measurements, *Journal of Physics: Conference Series*. 13th IMEKO TC1-TC7 Joint Symposium London, UK, 1–3 September 2010 IOP Publishing Ltd, UK, 238: 1742–6596.
- Giniotis, V.; Rybokas, M.; Petroškevičius, P. 2004. Kampų kalibravimo tikslumo tyrimas, *Geodezija ir kartografija* [Geodesy and Cartography] 30(3): 65–70. doi:10.1080/13921541.2004.9636644
- Giniotis, V.; Bručas, D.; Šiaudinytė, L. 2009. Arrangement for vertical angle calibration of geodetic instruments, *Mechanika* 5(79): 59–62.
- Ingensand, H. 2008. *Einführung in die geodätische Messtechnik*. Zurich: ETH.
- Key, H. 2005. Angular measurements. *GIM International*. The Netherlands, *GITC* 9(10): 13.
- Rybokas, M.; Šiaudinytė, L.; Giniotis, V. 2011. New approach to precision angle calibration means and methods, in *Metrologia 2011. Metrology as Factor of Quality, Innovation and Competitiveness*, National Institute of Metrology, Standardization and Industrial Quality (Inmetro) Brazilian Society of Metrology (SBM) International Measurement Confederation (IMEKO)TC 4 IMEKO Committee - Measurement of Electrical Quantities, 27–30 September 2011, Natal, Brasil.
- Siaudinytė, L.; Giniotis, V. 2011. New approach to vertical angle calibration, in *Environmental Engineering, Proceedings of the 8<sup>th</sup> International Conference*. May 19–20, 2011, Vilnius, Lithuania. Vilnius: Technika, 1466–1469.
- Surveying Instruments and their Operational Principles*. 1991. Ed.-in-Chief: Lajos Fialovszky, Elsevier.

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