GEODESY AND CARTOGRAPHY
ISSN 2029-6991 print / ISSN 2029-7009 online
2011 Volume 37(1): 5–14
doi: 10.3846/13921541.2011.558326

UDK 528.21

THE EVALUATION OF THE NEW ZEALAND'S GEOID MODEL USING THE KTH METHOD

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Received 13 Oct. 2010; accepted 20 Dec. 2010

Abstract. We compile a new geoid model at the computation area of New Zealand and its continental shelf using the method developed at the Royal Institute of Technology (KTH) in Stockholm. This method utilizes the least-squares modification of the Stokes integral for the biased, unbiased, and optimum stochastic solutions. The modified Bruns-Stokes integral combines the regional terrestrial gravity data with a global geopotential model (GGM). Four additive corrections are calculated and applied to the approximate geoid heights in order to obtain the gravimetric geoid. These four additive corrections account for the combined direct and indirect effects of topography and atmosphere, the contribution of the downward continuation reduction, and the formulation of the Stokes problem in the spherical approximation. The gravimetric geoid model is computed using two heterogonous gravity data sets: the altimetry-derived gravity anomalies from the DNSC08 marine gravity database (offshore) and the ground gravity measurements from the GNS Science gravity database (onshore). The GGM coefficients are taken from EIGEN-GRACE02S complete to degree 65 of spherical harmonics. The topographic heights are generated from the 1×1 arc-sec detailed digital terrain model (DTM) of New Zealand and from the 30×30 arc-sec global elevation data of SRTM30_PLUS V5.0. The least-squares analysis is applied to combine the gravity and GPS-levelling data using a 7-parameter model. The fit of the KTH geoid model with GPS-levelling data in New Zealand is 7 cm in terms of the standard deviation (STD) of differences. This STD fit is the same as the STD fit of the NZGeoid2009, which is the currently adopted official quasigeoid model for New Zealand.

Keywords: corrections, geoid, gravity, least-squares analysis, Stokes integral.

1. Introduction

The geodetic vertical reference system in the South and North Islands of New Zealand was realized by 12 major local vertical datums (LVDs) relative to the mean sea level (MSL) observed at 11 different tide-gauge stations (cf., Amos and Featherstone 2003; Amos and Featherstone 2009). The LVD Dunedin-Bluff 1960 was defined by fixing the heights of two levelling benchmarks from the LVDs Dunedin 1958 and BLUFF 1955 instead of using the tide-gauge station as the origin. Moreover, additional vertical datums were established for surveying purposes throughout the country. Since gravity was not observed along the precise levelling lines, LVDs are defined in the system of the approximate normal-orthometric heights. The cumulative normal-orthometric correction to the levelled height differences was defined based on the GRS67 normal gravity formula and computed approximately using a truncated form of Rapp's equation (Rapp 1961) for the mean normal gravity along the normal plumbline. Amos and Featherstone (2009) applied the iterative gravimetric approach to unify the LVDs in New Zealand using a regional gravimetric quasigeoid model and GPS-levelling data on each LVD. The principle of this method is based on an iterative quasigeoid modelling where the LVD offsets computed from earlier model are used to apply additional gravity reductions from each LVD to that model. The result of this procedure was the first detailed regional gravimetric quasigeoid model of New Zealand NZGeoid05. NZGeoid05 was computed jointly by the Land Information New Zealand (LINZ) and the Western Australian Centre for Geodesy – Curtin University of Technology (Amos 2007). NZGeoid05 was calculated from different heterogeneous ground, seaborne and altimetry-derived gravity data sets using a deterministic modification of the Stokes kernel (Featherstone *et al.* 1998). NZGeoid05 was complied on a 2×2 arc-min geographical grid at the computational area of New Zealand and its continental shelf (bounded by the parallels of 25 and 60 arc-deg southern geodetic latitude and the meridians of 160 and 190 arc-deg western

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longitude). The estimated LVD offsets relative to the regional quasigeoid model NZGeoid05 are from 26 cm (One Tree Point 1964; Nelson 1955, and Dunedin-Bluff 1960 LVDs) up to 59 cm (Gisborne 1926 LVD). The quasigeoid model NZGeoid2009 is the currently adopted official height reference surface for New Zealand. NZGeoid2009 was computed using a similar approach as NZGeoid05 (cf. Claessens *et al.* 2009). The substantial improvement of NZGeoid2009 comes from using a more recent global geopotential model (GGM); NZGeoid05 was computed using EGM96 (Lemoine *et al.* 1998), while EGM2008 (Pavlis *et al.* 2008) was used for the computation of NZGeoid2009. NZGeoid2009 model is provided to users on a 1×1 arc-min geographical grid over the same area as NZGeoid05. GPS-levelling data were used to determine LVD offsets. The estimated LVD offsets relative to NZGeoid09 are within 6 cm (One Tree Point 1964 LVD) and 49 cm (Dunedin 1958 LVD).

In this study we apply the method developed at the Royal Institute of Technology (KTH) in Stockholm to compute the geoid model at the computation area of New Zealand and its continental shelf. The KTH method utilizes the least-squares modification of the Stokes integral for the biased, unbiased and optimum stochastic solutions. The principle of this modification is to match the errors within terrestrial gravity data and the GGM omission and commission errors by means of the least squares. The GGM contribution is estimated using the satellite-only GGM. The reasons of using the satellite-only GGM in geoid modelling are discussed, for instance, by Vaníček and Sjöberg (1991). Various least-squares stochastic solutions are applied to estimate the maximum spherical distance of the near-zone surface integration area and the maximum degree of the GGM coefficients based on empirical models for the harmonic and terrestrial gravity anomaly degree variances. The modified Bruns-Stokes formula combines the observed gravity anomalies and GGM. The gravimetric geoid heights are obtained after applying four additive corrections to the approximate geoid heights. These additive corrections account for the gravitational effects of topography and atmosphere, the downward continuation reduction, and the ellipsoidal approximation of the Earth's shape. The least-squares analysis is finally applied to combine the gravity and GPS-levelling data using a 7-parameter model formed for the observation equations of differences between the geometric and gravimetric geoid heights (cf. Kotsakis and Sideris 1999).

The principal difference between the KTH method and conventionally used approaches for the gravimetric geoid determination comes from a different treatment of the gravity corrections and consequently different types of gravity anomaly data used in the Stokes integral convolution. In conventional Stokesian approaches, the observed gravity anomalies are first corrected for the topographic and atmospheric gravitational effects and subsequently reduced to the geoid surface. The integral convolution of the (modified) Bruns-Stokes kernel with the corrected and reduced gravity anomalies provides the final gravimetric geoid after subtracting the primary indirect topographic effect on the geoid. In the KTH method, the Stokes integration is applied directly to the observed gravity anomaly data at the Earth's surface. The integral convolution of the (modified) Bruns-Stokes kernel with the observed gravity anomalies provides the approximate geoid heights. The complete contribution of the direct and secondary indirect effects of topography and atmosphere on the gravity anomalies and consequently the primary indirect effects of topography and atmosphere on the geoid heights are treated as the combined topographic and atmospheric corrections applied to the approximate geoid heights (Sjöberg 2003c). Similarly, the contribution of the downward continuation of the gravity anomalies from the Earth's surface onto the geoid surface is treated as the downward continuation correction to the approximate geoid heights. The formulation of the modified Bruns-Stokes formula in the spherical approximation yields the correction for the ellipsoidal approximation of the Earth's shape. The theoretical and numerical aspects of the KTH method are described in Sjöberg (1984, 1991, 2003b, 2003c, 2003d). The practical numerical aspects of the KTH method are explained in Ågren *et al.* (2009). This method was successfully applied for a gravimetric geoid determination in several countries. For results of the regional geoid modelling using the KTH method we refer readers to Nahavandchi (1998), Ågren (2004), Ellmann (2001, 2004), Nsombo (1996), Hunegnaw (2001), Kiamehr (2006b), Daras (2000), Abdalla (2009), Ulotu (2009), and Ågren *et al.* (2009).

The KTH method is briefly reviewed in Section 2. This method is applied to determine the first experimental geoid model at the computation area of New Zealand and its continental shelf. The input data description and the numerical results are provided in Section 3. The combination of the gravimetric solution with the GPS-levelling data is done in Section 4. The validation of the KTH-geoid model using GPS-levelling data in New Zealand and its comparison with the regional and global quasigeoid models NZGeoid2009 and EGM2008 is done in Section 5. The summary and conclusions are given in Section 6.

2. The geoid determination using the KTH method

According to the KTH method, the gravimetric geoid height N is computed as a sum of the following components (Sjöberg 2003b):

$$N = \tilde{N} + \delta N^{T} + \delta N^{A} + \delta N^{DWC} + \delta N^{ell}, \qquad (1)$$

where $\tilde{N}$ is the approximate geoid height, δN^{T} the combined topographic correction, δN^{A} the combined atmospheric correction, δN^{DWC} the downward continuation correction, and δN^{ell} the ellipsoidal correction for the formulation of the Bruns-Stokes formula in the spherical approximation to the problem. The approximate geoid height $\tilde{N}$ in Eq (1) is computed using the modified Bruns-Stokes formula in the following form (Sjöberg 2003d)

$$\tilde{N} = \frac{\mathrm{R}}{4\pi\gamma_0} \int_{\alpha=0}^{2\pi} \int_{\psi=0}^{\psi_0} S^{\bar{n}}(\psi)\, \Delta g \sin\psi \, d\alpha \, d\psi + \frac{\mathrm{R}}{2\gamma_0} \sum_{n=2}^{\bar{n}} b_n \, \Delta \mathrm{g}_{\mathrm{n}}^{\mathrm{GGM}}. \qquad (2)$$

The first constituent on the right-hand side of Eq (2) represents the terrestrial gravity anomaly contribution to the approximate geoid heights. This contribution is computed by the integral convolution of the observed gravity anomalies Δg at the Earth's surface with the modified Stokes kernel $S^{\bar{n}}(\psi)$ which is defined as (ibid.)

$$S^{\bar{n}}(\psi) = S(\psi) - \sum_{n=2}^{\bar{n}} \frac{2n+1}{2} b_n \, \mathrm{P_n}(\cos\psi), \tag{3}$$

where $S(\psi)$ is the (original) Stokes kernel, $\mathrm{P_n}(\cos\psi)$ are the Legendre polynomials of degree n for the argument of cosine of the spherical distance ψ. The surface integration element $d\sigma = \sin\psi \, d\alpha \, d\psi$ is defined in the polar spherical coordinates (α, ψ) with the spherical azimuth α and the spherical distance ψ. The near-zone surface integration domain $\int_{\alpha=0}^{2\pi} \int_{\psi=0}^{\psi_0} \sin\psi \, d\alpha \, d\psi$ is limited by the spherical distance ψ_0. The Earth's mean radius in Eq (2) is denoted as R, and γ_0 is the normal gravity evaluated at the surface of the reference ellipsoid GRS80 (Moritz 1980). The second constituent on the right-hand side of Eq (2) represents the GGM contribution to the approximate geoid heights. This contribution is computed from the GGM coefficients up to a maximum degree $\bar{n}$ of spherical harmonics and from a set of the least-squares modification parameters $\{b_n : n = 2, 3, ..., \bar{n}\}$. The Laplace spherical harmonics $\Delta \mathrm{g}_\mathrm{n}^\mathrm{GGM}$ for the gravity anomalies of degree n in Eq (2) are defined as (e.g., Heiskanen and Moritz 1967: 89)

$$\Delta \mathrm{g}_\mathrm{n}^\mathrm{GGM} = \frac{\mathrm{GM}}{\mathrm{a}^2}\left(\frac{\mathrm{a}}{r}\right)^{n+2} (n-1) \sum_{m=-n}^{n} \mathrm{c}_\mathrm{n,m}^T \, \mathrm{Y_{n,m}}, \tag{4}$$

where GM is the geocentric gravitational constant, a the major semi-axis of the reference ellipsoid, r the geocentric radius of the computation point, $\mathrm{c}_\mathrm{n,m}^T$ are the GGM coefficients of the disturbing potential T, and $\mathrm{Y_{n,m}}$ the surface spherical harmonics (e.g., Heiskanen and Moritz 1967).

The least-squares modification parameters b_n are defined by the following linear system of observation equations (cf. Sjöberg 2003d)

$$\mathrm{c}\,(k = 2, 3, ..., \bar{n}). \tag{5}$$

The coefficients $\{a_{k,r} : k, r = 2, 3, ..., \bar{n}\}$ of the design matrix read

$$a_{k,r} = (\sigma_r^2 + \mathrm{dc_n^{GGM}}) \cdot {}_{k,r} - \mathrm{E_{k,r}}\,\sigma_r^2 - \mathrm{E_{r,k}}\,\sigma_k^2 + \sum_{n=2}^{\infty} \mathrm{E_{n,k}}\,\mathrm{E_{n,r}}\,(\sigma_n^2 + C_n), \tag{6}$$

The coefficients $\{h_k : k = 2, 3, ..., \bar{n}\}$ of the observation vector are given by

$$h_k = \Omega_k - \mathrm{Q_k}\,\sigma_k^2 + \sum_{n=2}^{\infty} \mathrm{E_{n,k}}\,[\mathrm{Q_n}(\sigma_k^2 + C_n) - \Omega_k]. \tag{7}$$

The parameters Ω_k, $\mathrm{E_{n,k}}$, $\delta_{k,r}$ and C_k in Eqs (6) and (7) read

$$\Omega_k = \frac{2\sigma_k^2}{k-1}, \tag{8}$$

$$\mathrm{E_{n,k}} = \frac{2k+1}{2}\mathrm{e_{n,k}}, \tag{9}$$

$$\delta_{k,r} = \begin{cases} 1 & \text{if } \; k = r \\ 0 & \text{otherwise} \end{cases}, \tag{10}$$

$$C_n = \sigma_n^2 + \begin{cases} \mathrm{c_n^{GGM}}\,\mathrm{dc_n^{GGM}} / (\mathrm{c_n^{GGM}} + \mathrm{dc_n^{GGM}}) & \text{if } 2 \le n \le \bar{n}, \\ \mathrm{c_n^{GGM}} & \text{if } n > \bar{n}, \end{cases} \tag{11}$$

where σ_n^2 are the terrestrial gravity anomaly error degree variances, and $\mathrm{c_k^{GGM}}$ and $\mathrm{dc_k^{GGM}}$ are the GGM gravity anomaly degree variances and their error degree variances. Molodensky's truncation coefficients $\mathrm{Q_n}$ and the functions $\mathrm{e_{n,k}}$ are defined in Eqs (19) and (20). The GGM gravity anomaly degree variances $\mathrm{c_n^{GGM}}$ are computed from the GGM coefficients $C_{n,m}^T$ and $S_{n,m}^T$ of the disturbing potential as follows

$$\mathrm{c_n^{GGM}} = \frac{\mathrm{GM}^2}{\mathrm{a}^4}(n-1)^2 \sum_{m=0}^{n} (C_{n,m}^2 + S_{n,m}^2). \tag{12}$$

In practice, the infinite series in Eqs (6) and (7) are truncated at a chosen upper limit of the expansion. In this study, we used the maximum degree of $n_{\max} = 2000$. The GGM gravity anomaly degree variances $\mathrm{c_n^{GGM}}$ of degree $\bar{n} < n \le n_{\max}$ are generated synthetically using the analytical model developed by Tscherning and Rapp (1974), see also Ågren (2004) and Ellmann (2005a). It reads

$$\mathrm{c_n^{GGM}} = \alpha \frac{(n-1)}{(n-2)(n+24)}\left(\frac{\mathrm{R_B}}{\mathrm{R}}\right)^{n+4}, \tag{13}$$

where $\alpha = 425.28$ mGal^2, $\mathrm{R} = 6371 \times 10^3$ m, and $\mathrm{R_B} = \mathrm{R} - 1225$ m. The GGM gravity anomaly error degree variances $\mathrm{dc_n^{GGM}}$ are calculated from the standard errors $dC_{n,m}$ and $dS_{n,m}$ of the GGM coefficients as follows (cf. Rapp and Pavlis 1990)

$$\mathrm{dc_n^{GGM}} = \frac{\mathrm{GM}^2}{\mathrm{a}^4}(n-1)^2 \sum_{m=0}^{n} (dC_{n,m}^2 + dS_{n,m}^2). \tag{14}$$

The GGM gravity anomaly error degree variances $\mathrm{dc_n^{GGM}}$ of degree $\bar{n} < n$ are usually neglected. The terrestrial gravity anomaly error degree variances σ_n^2 are calculated according to the procedure described in Ågren (2004) and Ågren *et al.* (2009).

The system of observation equations in Eq (5) is formed for the biased least-squares solution. The corresponding system of normal equations is then solved directly, for instance, by applying the Gauss elimination method. Alternative methods of solving the system of normal equations for finding the modification parameters b_n are discussed in Sjöberg (1984). When forming the system of observation equations for either the optimum or unbiased least-squares solutions, the system of normal equations becomes ill-conditioned (cf. Sjöberg 1991, 2003d). The regularization techniques are applied. The determination of the unbiased and optimum least-squares modification parameters and the regularization techniques are discussed in Ågren (2004) and Ellmann (2005a).

The combined topographic correction δN^T in Eq (1) is computed approximately using the following simple expression (cf. Sjöberg 2001)

$$\delta N^T \approx -\frac{2\pi}{\gamma_0} \mathrm{G} \rho^{\mathrm{T}} H^2, \tag{15}$$

where $\mathrm{G} = 6.673 \times 10^{-11}$ m^3 s^{-2} kg^{-1} is Newton's gravitational constant, $\rho^{\mathrm{T}} = 2670$ kg m^{-3} the adopted value of the average topographic density (cf. Hinze 2003), and H the height of the computation point above sea level.

The combined atmospheric correction δN^A in Eq (1) is defined as (cf. Sjöberg and Nahavandchi 2000)

$$\delta N^A = -\frac{2\pi \mathrm{R}}{\gamma_0} \rho_0^A \sum_{n=2}^{\bar{n}} \left(\frac{2}{n-1} - b_n - \mathrm{Q}_{\mathrm{n}}^{\bar{\mathrm{n}}} \right) \mathrm{H}_{\mathrm{n}} - \frac{2\pi \mathrm{R}}{\gamma_0} \rho_0^A \sum_{n=\bar{n}+1}^{\infty} \left(\frac{2}{n-1} - \frac{n+2}{2n+1} \mathrm{Q}_{\mathrm{n}}^{\bar{\mathrm{n}}} \right) \mathrm{H}_{\mathrm{n}}, \tag{16}$$

where ρ_0^A is the adopted nominal value of the atmospheric density at sea level, i.e., $\rho_0^A = 1.230$ kg m^{-3} (cf. Sjöberg 2001). The surface (topographic) height functions H_{n} of degree n in Eq (16) read

$$\mathrm{H}_{\mathrm{n}} = \sum_{\mathrm{m}=-\mathrm{n}}^{\mathrm{n}} \mathrm{H}_{\mathrm{n,m}} \mathrm{Y}_{\mathrm{n,m}}, \tag{17}$$

where $\mathrm{H}_{\mathrm{n,m}}$ are the numerical coefficients of the global elevation model (GEM) of degree n and order m. The modified Molodensky's truncation coefficients $\mathrm{Q}_{\mathrm{n}}^{\bar{\mathrm{n}}}$ are given by (cf. Sjöberg and Nahavandchi 2000)

$$\mathrm{Q}_{\mathrm{n}}^{\bar{\mathrm{n}}} = \mathrm{Q}_{\mathrm{n}} - \sum_{k=2}^{\bar{n}} \frac{2k+1}{2} b_k \, \mathrm{e}_{\mathrm{n,k}}, \tag{18}$$

where the Molodensky's truncation coefficients Q_{n} read (Molodensky *et al.* 1960)

$$\mathrm{Q}_{\mathrm{n}} = \int_{\psi_0}^{\pi} S(\psi) \, \mathrm{P}_{\mathrm{n}}(\cos\psi) \sin\psi \, d\psi. \tag{19}$$

Molodensky's truncation coefficients Q_{n} are computed recurrently according to formulae derived by Hagiwara (1975). Alternatively, they can be computed using Paul's (1973) algorithm. The functions $\mathrm{e}_{\mathrm{n,k}}$ of the spherical distance ψ_0 are defined in the following integral form (cf. Sjöberg and Nahavandchi 2000)

$$\mathrm{e}_{\mathrm{n,k}} = \int_{\psi_0}^{\pi} \mathrm{P}_{\mathrm{n}}(\cos\psi) \, \mathrm{P}_{\mathrm{k}}(\cos\psi) \sin\psi \, d\psi. \tag{20}$$

The downward continuation correction δN^{DWC} in Eq (1) consists of three terms which are computed individually (cf. Ågren 2004)

$$\delta N^{DWC} = \delta N^{DWC,1} + \delta N^{L1,Far} + \delta N^{DWC,L2}. \tag{21}$$

The first term $\delta N^{DWC,1}$ in Eq (21) is defined as (Sjöberg 2003a)

$$\delta N^{DWC,1} = \frac{\Delta g}{\gamma_0} H + 3 \frac{\bar{\varsigma}}{r} H - \frac{1}{2‡_0} \left. \frac{\partial \Delta g}{\partial r} \right|_{r=\mathrm{R}+H} H^2, \tag{22}$$

where $\bar{\varsigma}$ denotes the approximate value of the height anomaly at the computation point. Due to the diminutive value of $\delta N^{DWC,1} = 1$ mm that corresponds to an error of about 1 m for the height of the computation point of $H = 2000$ m, it is convenient to compute $\bar{\varsigma}$ in Eq (22) using the following simplified formula (Sjöberg 2003a)

$$\bar{\varsigma} \cong \frac{\mathrm{R}}{4\pi\gamma_0} \int_{\alpha=0}^{2\pi} \int_{¨=0}^{\psi_0} S^{\bar{n}}(\psi) \, \Delta g \sin\psi \, d\alpha \, d\psi + \frac{\mathrm{R}}{2\gamma_0} \sum_{n=2}^{\bar{n}} \left(b_n + \mathrm{Q}_{\mathrm{n}}^{\bar{\mathrm{n}}} \right) \Delta \mathrm{g}_{\mathrm{n}}^{\mathrm{GGM}}. \tag{23}$$

The linear vertical gravity anomaly gradient $\partial \Delta g / \partial r$ at the computation point is calculated according to the expression for the analytical continuation given in Heiskanen and Moritz (1967: 115). It reads

$$\left. \frac{\partial \Delta g}{\partial r} \right|_{r=\mathrm{R}+H} = \frac{\mathrm{R}^2}{2\pi} \int_{\alpha=0}^{2\pi} \int_{\psi=0}^{\psi_0} \frac{\Delta g(\alpha,\psi) - \Delta g}{\ell_0^3(\psi)} \sin\psi \, d\alpha \, d\psi - \frac{2}{\mathrm{R}} \Delta g, \tag{24}$$

where Δg and $\Delta g(\alpha,\psi)$ are the values of the surface gravity anomaly at the positions of the computation and running integration points, respectively. The Euclidean spatial distance $\ell_0(\psi)$ in Eq (24) reads

$$\ell_0(\psi) = 2\mathrm{R} \sin\frac{\psi}{2}. \tag{25}$$

The downward continuation correction terms $\delta N^{L1,Far}$ and $\delta N^{DWC,L2}$ in Eq (21) are computed using the following expressions

$$\delta N^{L1,Far} = \frac{\mathrm{R}}{2\gamma_0} \sum_{n=2}^{\bar{n}} (b_n + \mathrm{Q}_{\mathrm{n}}^{\bar{\mathrm{n}}}) \left[\left(\frac{\mathrm{R}}{r} \right)^{n+2} - 1 \right] \Delta \mathrm{g}_{\mathrm{n}}^{\mathrm{GGM}}, \quad (26)$$

and

$$\delta N^{DWC,L2} = \frac{\mathrm{R}}{4\pi\gamma_0} \int_{\alpha=0}^{2\pi} \int_{=0}^{\psi_0} S^{\bar{n}}(\psi) \left. \frac{\partial \Delta g}{\partial r} \right|_{r=\mathrm{R}+H} \times$$
$$\left[H - H(\alpha,\psi) \right] \sin\psi \, d\alpha \, d\psi, \quad (27)$$

where H and $H(\alpha,\psi)$ are the topographical heights at the positions of the computation and running integration points, respectively.

The ellipsoidal correction δN^{ell} in Eq (1) is computed approximately as (cf. Sjöberg 2004a)

$$\delta N^{ell} \approx \psi_0 [(0.12 - 0.38 \sin^2 \mathrm{j}) \, \Delta g + 0.17 \tilde{N} \cos^2 \mathrm{j}], (28)$$

where δN^{ell} is given in millimeters, Δg in mGal, and $\tilde{N}$ in meters. j is the geocentric spherical latitude of the computation point.

3. Numerical realization

The 2×2 arc-min gravity anomalies at the Earth's surface over the data area bounded by the parallels of 25 and 60 arc-deg southern geodetic latitude and the meridians of 160 and 190 arc-deg western longitude were used to determine the gravimetric geoid heights. The 2×2 arc-min gravity anomalies were reconstructed from the gravity measurements provided by the GNS Science gravity database (onshore) according to the procedure described in Janák and Vaníček (2005) and extracted from the DNSC08 marine gravity database (offshore). The DNSC08 marine gravity database is provided by the Danish National Space Centre (Andersen *et al.* 2008). The 2×2 arc-min gravity anomalies over the data area of New Zealand and its continental shelf are shown in Figure 1. The values of gravity anomalies vary from 252.6 to 310.7 mGal with the mean of 2.0 mGal, and the standard deviation (STD) is 35.1 mGal. The EIGEN-GRACE02S (cf. Reigber *et al.* 2004) was used to model the GGM contribution. The topographic heights were generated from the 1×1 arc-sec detailed digital terrain model (DTM) of New Zealand and from the 30×30 arc-sec global elevation data of SRTM30_PLUS V5.0 (Becker *et al.* 2009).

Various least-squares stochastic solutions are applied in the KTH method to estimate the maximum spherical distance ψ_0 of the near-zone surface integration area and the maximum degree $\bar{n}$ of the GGM coefficients based on empirical models for the harmonic and terrestrial gravity anomaly degree variances.

The GGM gravity anomaly degree variances $\mathrm{c}_{\mathrm{k}}^{\mathrm{GGM}}$ and their error degree variances $\mathrm{dc}_{\mathrm{k}}^{\mathrm{GGM}}$ were computed from the EIGEN-GRACE02S coefficients according to Eqs (12) and (14). The inaccuracy of modelling the GGM contribution increases proportionally with increasing degree of the GGM coefficients. Abdalla (2009) shown that the GGM error degree variances of EIGEN-GRACE02S significantly increases above the degree 77 of spherical harmonics.

Since the accuracy of marine and terrestrial gravity data used in this study is unknown, we assessed the accuracy of input gravity data according to the approach described in detail in Tenzer (2008). This approach utilizes the variance component estimation (VCE) technique (see Förstner 1979; Koch and Kusche 2002; Kusche 2003) for observation groups weighting. The gravity data were separated into two data sets consisting of the terrestrial and marine gravity data from the GNS Science and DNSC08 databases. The parameterization of gravity field was done in terms of the spherical radial basis functions. The representative value of the variance $C(0) = 2.1$ mGal^2 of the entire input gravity data was obtained as the weighted mean of the corresponding values estimated for these two observation groups. We note here, that this value is more likely unrealistic, especially in mountainous regions where the accuracy of the gravity anomalies is much lower due to the errors in determined heights of the observation points.

The selection of the parameters $\psi_0 = 3$ arc-deg and $\bar{n} = 65$ was done empirically. As demonstrated in Figure 2, the modified Stokes kernel $S^{\bar{n}}(\psi)$ converges to zero for $\psi \to 3$ arc-deg and thus minimize the truncation bias for the chosen parameter $\psi_0 = 3$ arc-deg. For more details we refer readers to study by Ellmann (2005a). The example of truncation bias of the original Stokes function $S(\psi)$ is also illustrated in Figure 2. For comparison, the parameters $\psi_0 = 1.5$ arc-deg and $\bar{n} = 40$ were adopted in computing the regional quasigeoid model NZGeoid05 (cf. Amos 2007). Claessens *et al.* (2009) used $\psi_0 = 2.5$ arc-deg and $\bar{n} = 40$ in computing NZGeoid2009. We note that Amos (2007) and Claessens *et al.* (2009) used the deterministic modification of the Bruns-Stokes formula.

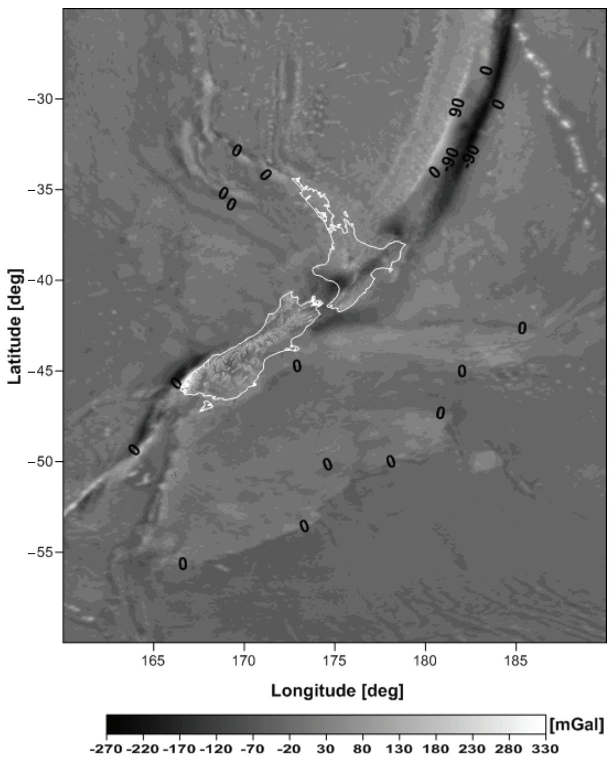


Fig. 1. The gravity anomalies compiled on a 2×2 arc-min grid at the Earth's surface

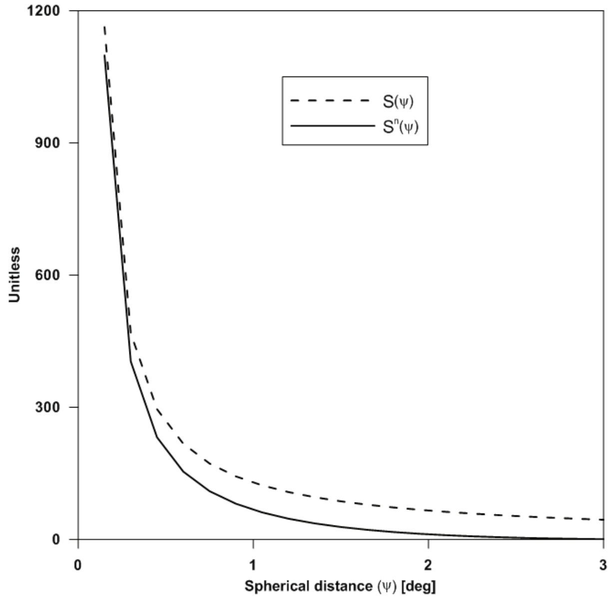
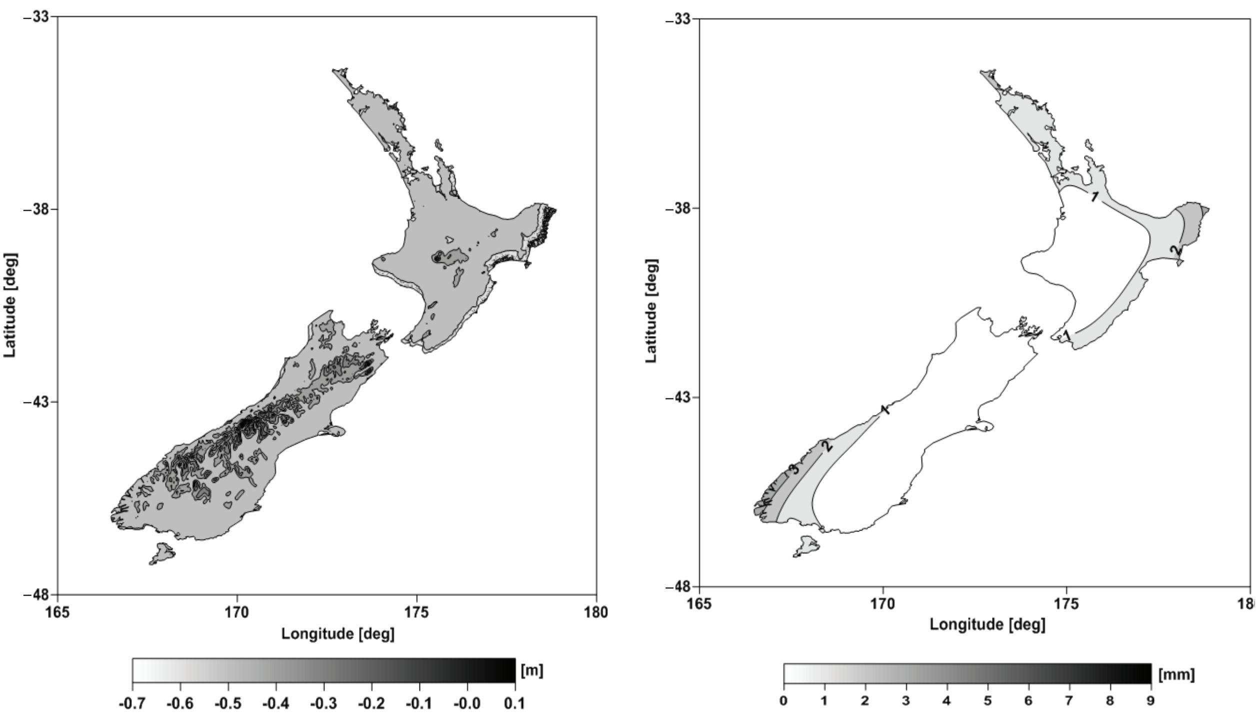


Fig. 2. The comparison of the modified Stokes function $S^{\overline{n}}(\psi)$ computed for the parameters $\psi_0 = 3 \neq$ arc-deg and $\overline{n} = 65$ and the original Stokes function $S(\psi)$ at the interval of $0 \leq \psi \leq 3$ arc-deg

The 2×2 arc-min surface gravity anomaly data up to $\psi_0 = 3$ arc-deg of the spherical distance around the computation point and the EIGEN-GRACE02S coefficients up to degree $\overline{n} = 65$ of spherical harmonics were used to calculate the approximate geoid heights $\tilde{N}$ according to Eq (2). The spherical harmonics of the normal gravity field were computed for the parameters of the GRS80 reference ellipsoid. The discrete values of the combined topographic correction δN^T were computed according to Eq (15) on a 1×1 arc-sec geographical grid using the 1×1 arc-sec detailed DTM of New Zealand and adopting the average topographical density of kg m^{-3} (cf. Hinze 2003). The 2×2 arc-min mean values of the combined topographic correction were then computed by a spatial averaging of the corresponding 1×1 arc-sec discrete values. The 2×2 arc-min mean values of the combined topographical correction vary from –69.0 to 0.0 cm with the mean of –0.2 cm, and the standard deviation is 2.1 cm (see Fig. 3a). The 30×30 arc-sec global elevation data of SRTM30_PLUS V5.0 were used to generate the GEM coefficients $\mathrm{H}_{\mathrm{n,m}}$. These coefficients complete to degree and order 2160 were used to compute the combined atmospheric correction δN^A at the 2×2 arc-min geographical grid according to Eq (16). The combined atmospheric correction is shown in Fig. 3b. It varies from 0.0 to 1.2 cm with the mean of 0.6 cm, and the standard deviation is 0.3 cm. The 2×2 gravity anomalies and the mean topographical heights averaged for 2×2 arc-min geographical grid cells were used to compute the near-zone contribution to the downward continuation correction δN^{DWC} according to Eqs (21–27). The corresponding long-wavelength contribution was computed using the EIGEN-GRACE02S coefficients complete do degree 65 of spherical harmonics. The downward continuation correction is shown in Fig. 3c. It varies from –3.7 to 58.7 cm with the mean of 2.5 cm, and the standard deviation is 3.2 cm. The ellipsoidal correction δN^{ell} was computed using Eq (28). Over the study area of New Zealand this correction is negligible; the maxima of this correction are less than 1 mm.

The gravimetric geoid model compiled on a 2×2 arc-min geographical grid at the computation area of New Zealand and its continental shelf bounded by the parallels of 28 and 57 arc-deg southern geodetic latitude and

a) b)



c)

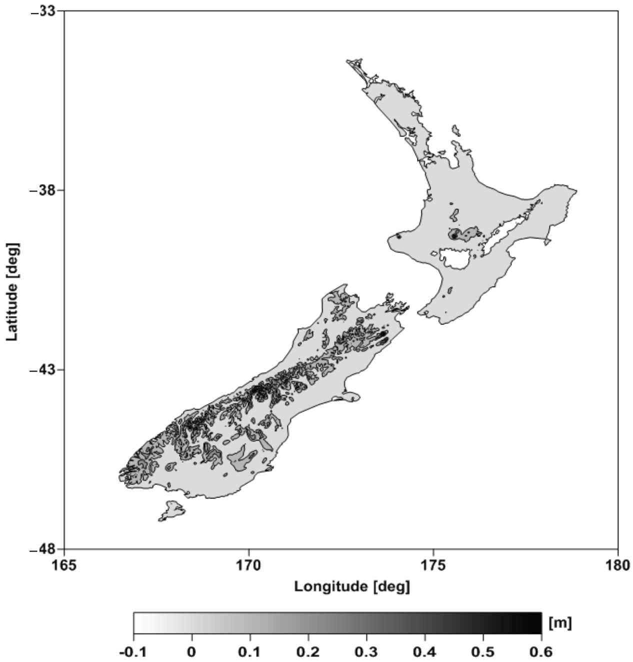


Fig. 3. The additive corrections to the approximate geoid heights computed on a 2×2 arc-min geographical grid: a) the combined topographic correction, b) the combined atmospheric correction, and c) the downward continuation correction

the meridians of 163 and 187 arc-deg eastern longitude is shown in Fig. 4. The geoid heights vary from –39.69 to 49.58 m with the mean of 8.03 m, and the standard deviation is 24.57 m.

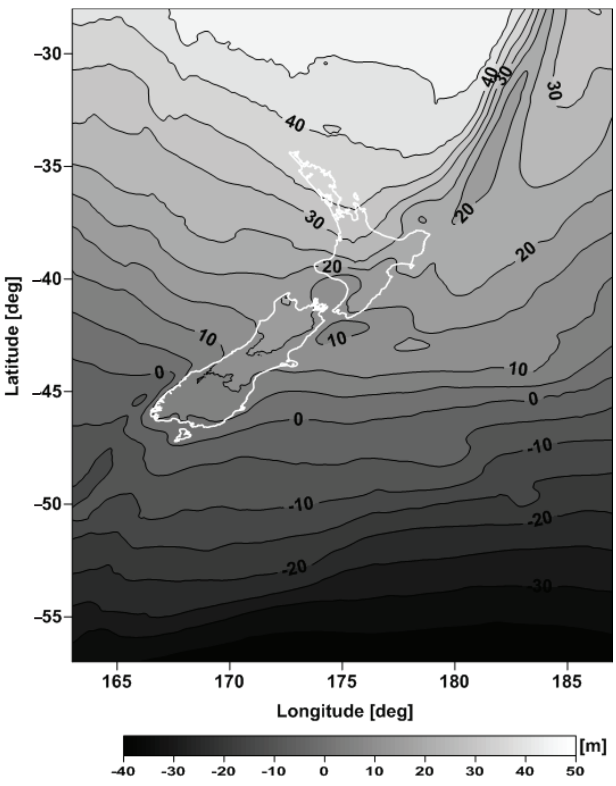


Fig. 4. The KTH gravimetric geoid model compiled on a 2×2 arc-min geographical grid at the computation area of New Zealand and its continental shelf

4. Combination of the gravimetric solution with GPS-levelling data

The GPS-levelling testing network in New Zealand consists of 2320 points from the LINZ geodetic database. The ellipsoidal heights above the GRS80 geocentric reference ellipsoid are defined in the New Zealand Geodetic Datum 2000 (NZGD2000). The NZGD2000 is aligned to the International Terrestrial Reference Frame 1996 (ITRF1996) at the reference epoch of January 1st, 2000 (Blick *et al.* 2005). Since the normal-orthometric heights at the points of GPS-levelling testing network in New Zealand are aligned to 18 different LVDs, we utilized the geopotential value approach (cf. Burša *et al.* 1999, 2001, 2002) to estimate the average offsets of LVDs relative to the World Height System (WHS). WHS is defined by the adopted value of the geoidal geopotential W_0 = 62636856 m^2s^{-2}. The estimated average offsets of 18 LVDs in New Zealand relative to WHS are summarized in Table 1. The LVDs within the South and North Islands of New Zealand are positive and range from 1 cm (Wellington 1953 LVD) to 37 cm (One Tree Point 1964 LVD).

The gravimetric geoid solution was further combined with the GPS-levelling data corrected for the average LVD offsets in order to reduce additional systematic distortions between the geometric and gravimetric geoid heights. The systematic distortions were modelled by a 7-parameter model (see *Kotsakis* and *Sideris* 1999) formed for the observation equations of differences between the geometric and gravimetric geoid heights at GPS-levelling points and solved applying the least-squares analysis.

Table 1. The offsets of 18 LVDs in New Zealand relative to WHS

LVD	LVD offset [m]
One Tree Point 1964	0.37
Auckland 1946	0.12
Moturiki 1953	0.19
Gisborne 1926	0.10
Napier 1962	0.24
Taranaki 1970	0.12
Wellington 1953	0.01
Nelson 1955	0.20
Lyttelton 1937	0.13
Dunedin 1958	0.07
Dunedin-Bluff 1960	0.23
Bluff 1955	0.17
MSL	0.15
Deep Cove 1960	0.30
Port 1954	0.32
Tarakohe 1982	0.23
Tararu	0.21
Unahi	0.19

5. Geoid validation

The KTH-geoid was validated at the GPS-levelling testing network in New Zealand. The geometric geoid heights were calculated from the NZGD2000 ellipsoidal heights by subtracting the normal-orthometric heights corrected for the average LVD offsets relative to WHS (see Table 1). The same validation was done for the NZGeoid2009 and EGM2008 quasigeoid models. The average LVD offsets relative to WHS (see Table 1) were applied for a validation of EGM2008. The average offsets of 12 major LVDs relative to NZGeoid2009 (adopted from Claessens *et al.* 2009) were applied to the geometric geoid heights for a validation of NZGeoid2009. Statistics of the differences between the geometric and gravimetric geoid/quasigeoid heights at the GPS-levelling testing network are given in Table 2. The differences between normal, normal-orthometric and orthometric heights were not taken into consideration. We note that the accuracy estimation of the geometric geoid heights at GPS-levelling points in New Zealand is problematic due to several reasons, mainly due to unknown errors in leveling data and large vertical tectonic deformations throughout the country.

Table 2. Statistics of the differences between the geometric and gravimetric geoid/quasigeoid heights calculated for the NZGeoid2009 and EGM2008 quasigeoid models and the KTH-geoid model

Geoid/Quasigeoid Model	Differences at GPS-levelling points			
	Min. [m]	Max [m]	Mean [m]	STD [m]
NZGeoid2009	−0.42	0.38	−0.01	0.07
EGM2008	−0.42	0.40	0.00	0.08
KTH-geoid	−0.49	0.42	0.00	0.07

6.Summary and conclusions

We have applied the KTH method to determine the geoid model at the computation area of New Zealand and its continental shelf bounded by the parallels of 28 and 57 arc-deg southern geodetic latitude and the meridians of 163 and 187 arc-deg eastern longitude. The KTH-geoid model was compiled in two principal numerical steps. First, the approximate geoid heights were computed using the modified Bruns-Stokes integral. It combines the regional terrestrial gravity data with the GGM coefficients. The gravimetric geoid heights were obtained after applying four additive corrections. These additive corrections to the approximate geoid heights account for the effects of the topography, atmosphere, downward continuation reduction, and spherical approximation. The final KTH-geoid model was obtained after combining the gravimetric geoid with GPS-levelling data using a 7-parameter model. The KTH-geoid model was validated at the GPS-levelling testing network in New Zealand and compared with the regional and global quasigeoid models NZGeoid2009 and EGM2008. The analysis of the accuracy revealed that the STD fit of the KTH-geoid model with GPS-levelling data is 7 cm (cf. Table 2). The same STD fit was found for NZGeoid2009. The STD fit of EGM2008 in New Zealand is 8 cm.

The KTH-geoid model has the same accuracy (by means of the STD fit with GPS-levelling data) as the NZGeoid2009 quasigeoid model computed using the iterative gravimetric approach (Amos and Featherstone 2009, Claessens *et al.* 2009).

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