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THE UNITED GEODETIC VERTICAL NETWORK OF LATVIA AND LITHUANIA

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Abstract. A unified and continuous national vertical network is the back-bone for geodesy, cartography, civil engineering and global positioning. International institutions are working to reach homogenous and unified vertical datum all around the globe. Levelling evaluation on the border between Latvia and Lithuania is of particular interest. Connection between vertical networks is made in three places, so connecting lines construct the two first order levelling loops. A joined loop adjustment produces a good basis for analysis and evaluation of height connection between Latvia and Lithuania not only as neighbouring countries but also as parties to the EVRS.

Keywords: geodetic vertical network, vertical reference system, levelling.

1. Introduction

The existing levelling networks of Latvia and Lithuania are a part of the United Precise Levelling Network (UPLN). They were developed after the Second World War and no longer fit the nowadays requirements of geodetic control of the countries. Consequently, the project of the Fundamental Vertical Network of Lithuania was initiated (Parseliunas *et al.* 1998; Buga *et al.* 1999; Krikstaponis *et al.* 2007; Zakarevičius *et al.* 2008). The network was observed in 1998–2007. Connections to Polish and Latvian networks were observed in 2007–2010. Projection and construction works of the Latvian National First Order Levelling Network (NFOLN) commenced in 2000 and finished in 2010 (Celms, Kaminskis 2005; I, II and II 2000). Currently, computing of the networks continues, and discussions on the vertical reference system adoption in each country are taking place as well.

Following the Resolution of the European Reference Frame (EUREF) Symposium adopted in Bad Neuenahr-Ahrweiler in 1998 (Augath *et al.* 2000) – requesting to extend and improve the Vertical Network around the Baltic Sea – both Latvian and Lithuanian geodesists included the new state levelling networks to the United European Levelling Network (UELN) (Ernsperger, Kok 1986; Lang, Sacher 1995; Sacher *et al.* 1998, 1999; Paršeliūnas *et al.* 2000).

In order to unify the geodetic datums of Latvia and Lithuania and have a reliable basis for geodynamic studies, it was decided to connect both levelling networks (Paršeliūnas *et al.* 2000; Krikštaponis *et al.* 2011). Thus, the new first order levelling lines Būtingė–Rucava, Joniškis–Eleja and Turmantas–Demene were observed by Latvian and Lithuanian geodesists in 2007–2010. An adjustment of the joint united vertical network was carried out. The main results achieved are presented in this paper.

2. An overview of the Latvian vertical network

Development of the Latvian National First Order Levelling Network (NFOLN) commenced in 2000, and field measurements were finished in 2010 (Celms, Kaminskis 2005). Geodetic measurements were taken by specialists of the Latvian Land Service in 2000–2005 and the Latvian Geospatial Information Agency in 2006–2010. The development of the network and geodetic measurements were undertaken on the basis of technical requirements "I, II and III classes of levelling instruction" (I, II and II 2000). The NFOLN consists of 15 loops of precise levelling lines (Fig. 1).

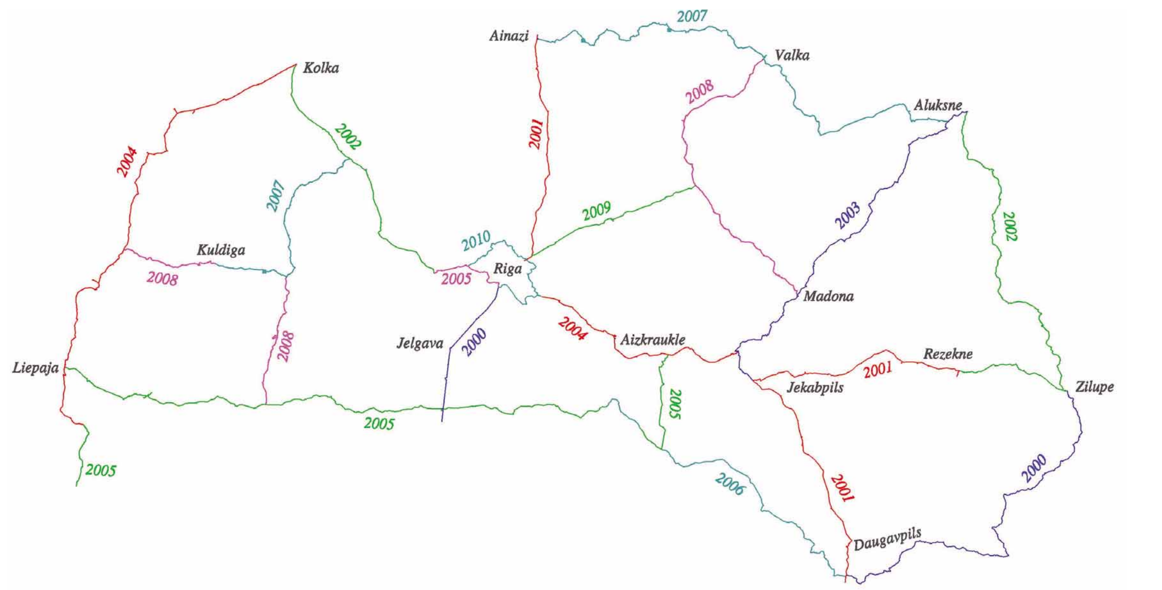

Fig. 1. The first order levelling lines of the Latvian NFOLN and years of field observations

To develop the network, geodetic and gravimetric observations were performed. The general requirement to not exceed the RMS error of 0.5 mm/km of the measured height differences was followed in the course of development of the National First Order Levelling Network.

Digital levels Leica NA3003, Zeiss DiNi12, Zeiss Ni002 and Trimble DiNi0.3 and invar rods with bar code scales Wild GPCL3, Zeiss LD13, Zeiss LD11 and gravimeters *Scintrex CG-3* and *CG-5* were used for measurements. All levelling lines were divided into sections. Section length amounted to approx. 0.5 km in an urban area and approx. 2 km in a rural area. Every section was levelled forwards and backwards.

Differences in section height were corrected adjusting the calibration of levelling rods and temperature. Staff readings were reduced to the staff calibration temperature or +20°C. The temperature correction was computed using the following formula:

$$\delta_c = \Delta h \cdot \left(k_{rod} + k_{termal} \cdot \left(t_m^o - t_{cal}^o \right) \right), \qquad (1)$$

where Δh – measured height difference; k_{rod} – rod length thermal dependency; k_{termal} – thermal expansion coefficient; t_m^o – mean temperature during measurements; t_{cal}^o – calibration temperature.

The refraction effect was minimized by keeping an equal sight distance from a level to a rod (the maximum sight distance was 40 m, but usually – 36 m) and by carrying out measurements at mornings and evenings.

Corrections due to non-homogeneity of the gravitational field were computed on the basis of parameters of real and normal gravitational fields. The gravimetric data and normal gravitational field GRS 80 were used for this purpose (Kaminskis, Forsberg 1996). The gravity acceleration was measured along the precise levelling lines. The distance between gravimetrically measured benchmarks was approximately 1 km in urban areas and 2 km in rural areas. All gravimetric measurements were connected to the Second Order Gravimetric Network, which is constraint by the First (Absolute) Order Network and is realized in IGSN71. No special correction for the tidal effect due to the Moon and the Sun was applied so far.

Geopotential numbers are determined in GRS 80 normal field, applying the new European gravity system and evaluating non-linearity of GRS 80 normal field equipotential surfaces (Moritz 1988). Normal correction computed from formula:

$$f_{ik}^{80} = \frac{1}{\gamma_{80}} (g_{71} - \gamma_{80})_v h_{ik} - \frac{1}{\gamma_{80v}} (\gamma_{80k} - \gamma_{80i}) H_v, \quad (2)$$

where g_{71} – gravity value measured in the IGSN71 system; h_{ik} – measured height difference; H_v – mean normal height between points; γ_{80} – GRS 80 normal field gravity value on rotation telluroid surface:

$$\gamma_{80} = \gamma_0 - 0.3086 \cdot H, \qquad (3)$$

where H – point height rounded till meter. GRS 80 normal field gravity value γ_0 computed using the following formula:

$$\gamma_0 = 97802.7 \times$$
$$(1 + 0.0053024 \cdot \sin^2 B - 0.0000058 \cdot \sin^2 2B), \qquad (4)$$

where B – geodetic latitude in the LKS-92 system.

The observed height differences were corrected by temperature and calibration, so the corrected height differences for forward and backward levelling were computed. Mean height difference values were computed and corrected by normal correction f_{ik}^{80} and the final height differences were computed for all lines of the Latvian National First Order Levelling Network lines.

All levelling lines were reduced to common epoch 2000.0. The empirical Latvian land uplift model was taken into account.

The empirical land uplift model was calculated comparing height differences of common points in two levelling campaigns. The first was measured in 1929–1939 and the second in 2000–2010. The empirical model for Latvia showed the land uplift rate of 2 mm per year if to compare the majority of north-western and south-eastern areas.

For each nodal point, the speed of the land uplift was calculated and a correction for the measured height difference of a line to the common epoch was computed using a formula (5). Every section was corrected proportionally to the length of levelling.

$$h_{ep} = h_m + (t_{ep} - t_m)\cdot(v_{end} - v_{start}), \quad (5)$$

where h_m – measured height difference of line corrected by temperature and staff calibration; t_{ep} – common reduction epoch 2000.0; t_m – epoch of measurements taken in the middle of a year; v_{end} – speed of the land uplift at the end point; v_{start} – speed of the land uplift at the start point.

The Latvian NFOLN was adjusted using the program HOENA (Schoch 1995).

The allowed misclosures of closed loops (Table 1) were computed with the help of the following formula:

$$f_h = 2\cdot m_0\sqrt{L}, \quad (6)$$

where m_0 – a priori standard deviation of point heights in mm; L – the loop perimeter in km.

Table 1. Misclosures of loops of the Latvian NFOLN

Loop No.	Actual misclosure, kgal·mm	Loop perimeter, km	Allowable misclosure, m_0 = 1.0 mm
1	−2.44	36.800	11.89
2	−7.66	35.500	11.68
3	−18.86	429.800	40.63
4	−15.62	353.300	36.84
5	−16.69	97.000	19.30
6	−11.76	363.200	37.35
7	+1.82	42.700	12.81
8	−7.72	36.400	11.83
9	−15.33	342.400	36.27
10	+4.10	366.900	37.54
11	−7.49	362.800	37.33
12	+33.50	501.600	43.90
13	+12.25	548.500	45.90
14	+11.66	423.800	40.35
15	−9.10	420.600	40.20

Misclosures closest to the allowable value were obtained in loops 5 and 12. Exact reasons are unknown, but we could guess, that misclosure in the loop No 5 emerge because lines of the polygon go through the centre of the capital city Riga, cross river Daugava in two places and river Lielupe in two places as well.

One cross of Daugava is over the Rock Bridge in the centre of Riga but the second one is between moles in the harbour. River Lielupe is crossed over the Lielupes Bridge at the entrance to the city of Jurmala and near the mouth of the Riga Gulf.

The misclosure in the loop No 12 resulted from the lack of experience, because the loop consists of levelling lines, which were established from the very start of all levelling works and depend on local geodynamic individuality. In Soviet times, this polygon did not offer the best results either, thus geodesists made some re-measurements.

Here are the parameters of the adjustment of the Latvian NFOLN with one fiducial point (point code 014----A):

- Number of fixed points: 1,
- Number of unknowns: 1889,
- Number of measurements: 1904,
- Degrees of freedom: 15,
- Standard deviation (from results of the adjustment): 0.870 kgal·mm/km,
- A-posteriori standard deviation referred to a levelling distance of 1 km (based on actual misclosures): 0.889 kgal·mm,
- The mean value of the standard deviation of the adjusted geopotential differences: 1.11 kgal·mm,
- The mean value of the standard deviation of the adjusted geopotential heights: 8.18 kgal·mm,
- The greatest value of the standard deviation of the adjusted geopotential heights: 12.47 kgal·mm,
- The average redundancy: 0.008.

The adjustment of geopotential height differences of enlarged UELN including new Latvian and Lithuanian levelling networks was performed as an unconstrained adjustment linked to the reference point No 13600 in Amsterdam, the geopotential height of which was set to 0.70259 kgal×m, with normal height at 0.71599 m. The data for initial points of Latvian (Table 2) and Lithuanian (Table 4) vertical networks was received from this adjustment. These datum points were used in the adjustment of the united vertical network of Latvia and Lithuania.

Table 2. Datum points of the Latvian NFOLN

No.	National code	UELN code	LKS-92 coordinates	Geopotential number, $m^2\cdot s^{-2}\cdot 10^{-1}$	Accuracy of geopotential number in UELN network, $m^2\cdot s^{-2}\cdot 10^{-1}$	Normal height, m	IGSN71 gravity acceleration, $m\cdot s^{-2}$
1	012--002	2110009	56° 56′ 44.064′ 23° 36′ 46.210′	2.239	0.0138	2.281	9.816420
2	007----4	2110262	56° 26′ 16.597′ 21° 33′ 57.398′	49.783	0.0139	50.715	9.816343
3	031---50	2110070	57° 46′ 38.465′ 26° 01′ 18.304′	48.937	0.0146	49.847	9.817236

End of Table 2

No.	National code	UELN code	LKS-92 coordinates	Geopotential number, $m^2 \cdot s^{-2} \cdot 10^{-1}$	Accuracy of geopotential number in UELN network, $m^2 \cdot s^{-2} \cdot 10^{-1}$	Normal height, m	IGSN71 gravity acceleration, $m \cdot s^{-2}$
4	001--766	2110238	57° 45′ 7.924′ 22° 35′ 48.308′	4.232	0.0147	4.310	9.817144
5	014----A	2110281	56° 57′ 16.737′ 24° 06′ 43.893′	5.156	0.0135	5.252	9.816479
6	038-3939	2110087	56° 50′ 18.746′ 26° 12′ 7.821′	138.716	0.0140	141.310	9.816187
7	040-1484	2110340	57° 29′ 40.496′ 27° 20′ 18.772′	153.926	0.0146	156.797	9.816766
8	049-1174	2110366	56° 25′ 20.766′ 28° 03′ 20.514′	128.130	0.0146	130.531	9.815665
9	034-2913	2110323	57° 51′ 45.518′ 24° 21′ 31.032′	6.093	0.0148	6.206	9.817452
10	033-2083	2110322	58° 04′ 32.081′ 25° 11′ 30.075′	71.244	0.0154	72.568	9.817204
11	042--538	2110347	57° 33′ 08.763′ 26° 39′ 59.427′	77.341	0.0149	78.782	9.817141
12	006-1684	2110259	56° 04′ 50.446′ 21° 07′ 22.582′	10.980	0.0132	11.185	9.815799
13	026-0718	2110301	56° 21′ 55,157′ 23° 40′ 26,333′	38.670	0.0129	39.394	9.816106
14	046-2285	2110354	55° 42′ 13,666′ 26° 28′ 03,768′	136.431	0.0130	138.997	9.815243
15	035-3433	2110326	57° 52′ 27.015′ 24° 21′ 39.366′	3.336	0.0148	3.398	9.817488
16	044-2128	2110349	57° 46′ 35.152′ 26° 01′ 23.200′	50.146	0.0146	51.079	9.817232

3. An overview of the Lithuanian vertical network

The development of the Lithuanian National Geodetic Vertical First Order Network (NGVN) extended from 1998 till 2007 (Parseliunas *et al.* 1998; Buga *et al.* 1999; Krikstaponis *et al.* 2007). The contracting authority for the network establishment was the National Land Service under the Ministry of Agriculture. The Lithuanian National Geodetic Vertical Network was established following the technical regulation on Requirements for the Lithuanian National Geodetic Vertical Network. The latest requirements on development of vertical networks were considered (European… 2000; Ihde, Augath 2000). The NGVN consists of 5 loops of precise levelling lines (Fig. 2).

To develop the network, data of the geodetic and gravimetric observations were used. Geopotential heights of points were determined from results of the precise levelling and gravimetric data. The ellipsoidal heights of the network points were obtained by means of the GNSS positioning.

The general requirement to not exceed the RMS error of 0.5 mm/km of measured height differences was followed in the course of development of the National Geodetic Vertical First Order Network.

Digital levels *Leica NA3003,* invar precise staffs bar coded staffs *Wild GPCL-3*, GPS receivers *Ashtech Z12, Z-Surveyor, Trimble 5700* and gravimeters *La Coste* & *Romberg* were used for measurements. All levelling lines were divided into sections. Every section was levelled forwards and backwards. The field measurements of height differences were corrected adjusting the calibration of staffs and temperature. The refraction effect was also taken into account. Corrections due to non-homogeneity of gravitational field were computed on the basis of parameters of real and normal gravitational fields. Sufficiently accurate gravimetric data and normal gravitational fields of Helmert and GRS 80 were used for this purpose. Gravitational acceleration measurements in control gravimetric first order network were performed with *La Coste & Romberg* gravimeters. Gravimetric observations were tied to the Lithuanian National Zero Order Gravimetric Network, at stations, the absolute gravitational acceleration of which was measured (Paršeliūnas *et al.* 2010b).

Tides are caused by the tidal effect of the Moon and the Sun (Petroškevičius 2000, 2004; Torge 1989; Petroškevičius *et al.* 2008). They result in periodic fluctuation of height difference between the Earth surface points. On the Lithuanian territory, for the points separated by 2.5 km, the change in height difference caused by the Moon may vary from –0.18 mm to 0.18 mm; and that caused by the Sun – from –0.07 mm to 0.07 mm.

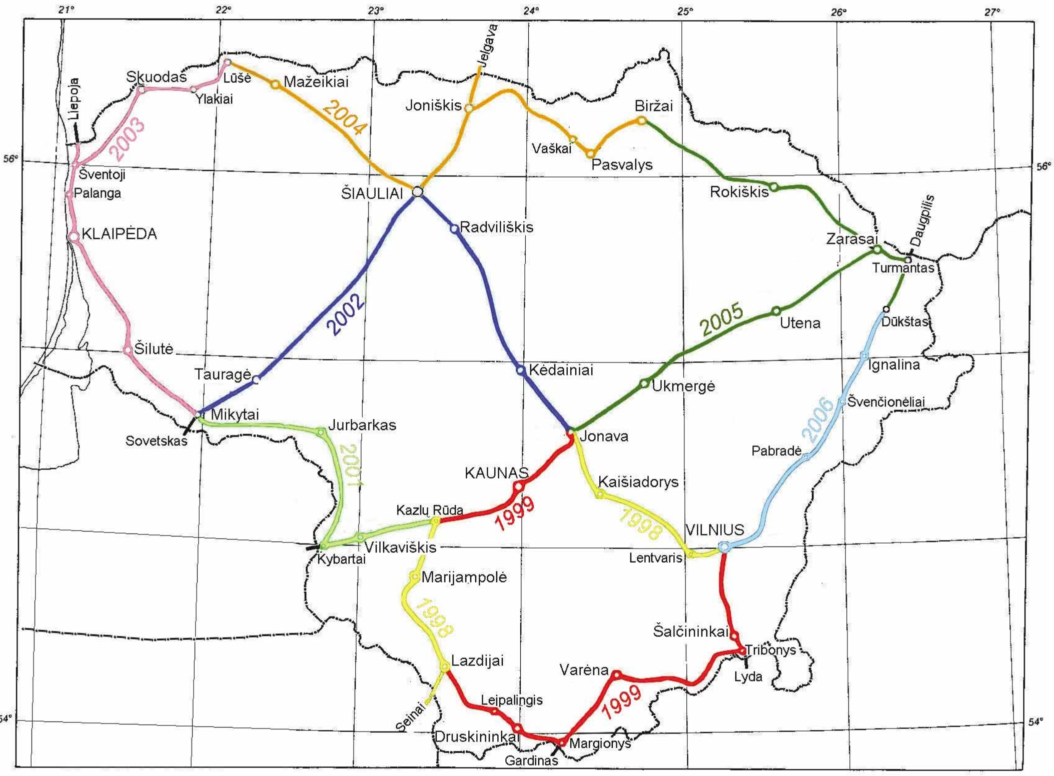

Fig. 2. The first order levelling lines of the Lithuanian NGVN and years of field observations

There are two maximums and minimums during the day time. The largest effect of both celestial bodies is during the full and young Moon phases. Tidal corrections δ_{MS} for the height differences were computed using the following formulas:

$$\delta_{MS} = \delta_M + \delta_S, \tag{7}$$

where δ_M – correction due to the Moon and

$$\delta_M = \nu_M S \cos(A_M - A), \tag{8}$$

δ_S – correction due to the Sun:

$$\delta_S = \nu_S S \cos(A_S - A), \tag{9}$$

S – line between points of the vertical network in km; ν_M and ν_S – deflection of the vertical due to the Moon and the Sun; A_M and A_S – azimuths of the Moon and the Sun, A – azimuth between the points.

Staff readings were reduced to the staff calibration temperature of +20°C (Putrimas 1999; Skeivalas 2000; Skeivalas *et al.* 2009; Zakarevičius, Puzienė 2010; Krikštaponis 2001, 2002; Paršeliūnas *et al.* 2010a). The temperature correction was computed with the help of the following formula:

$$\delta_t = a_m \times \alpha / 2.93, \tag{10}$$

where a_m – staff reading; α – equation of temperature dependency of staff invar strip of 2.93 m, which common expression is

$$\alpha = k_1 (t_m - 20°) + k_2, \tag{11}$$

where k_1 and k_2 – coefficients of equation of staff length thermal dependency, determined at the Finnish Geodetic Institute; t_m – temperature of invar strip during the levelling.

Staff readings were corrected by staff calibration corrections:

$$\delta_k = k_3 + k_4\, a_m + k_5\, a_m^{\,2} + \ldots + k_N\, a_m^{N-3}. \tag{12}$$

Height difference at the station:

$$h'_s = (a_m + \delta_t^a + \delta_k^a) - (p_m + \delta_t^p + \delta_k^p), \tag{13}$$

where a_m and p_m – backsight and foresight staff readings; δ_t^a and δ_t^p – temperature corrections for backsight and foresight readings; δ_k^a and δ_k^p – calibration corrections for backsight and foresight readings.

Height differences were corrected for refraction

$$\delta_r = A\, \Delta t\, S^2\, h'_s, \tag{14}$$

where A – coefficient; Δt – temperature difference between heights Z_2 and Z_1 above the ground; S – length of collimation line; h'_s – height difference at the station.

Coefficient A was computed as follows:

$$A = \frac{4{,}76 \cdot 10^{-4}}{Z_2^c - Z_1^c} \left\{ \frac{p^{c+1} - a^{c+1}}{c+1} - Z_0^c (p - a) \right\}, \tag{15}$$

where c – coefficients; Z_0 – levelling instrument height. Values used: Z_0 = 1.5 m, Z_1 = 1.0 m, Z_2 = 2.0 m. The coefficient c was taken from intermediate values derived from the second order conformal transformation.

To determine normal height differences of points, it is necessary to evaluate non-paralellity of normal field equipotential surfaces as well as real and normal field non-coincidence. For this purpose, normal corrections for height differences determined by levelling in real gravity field were computed (Petroškevičius 2004; Petroškevičius *et al.* 2008). The gravity value g_{71r} of the European system at the marks height of the first order network points were computed on the basis of Bouguer anomalies $(g_p - \gamma_H)_{2,3}$, taken from the gravity map, scale 1:200 000. The gravity value g_{71z} at the surface was derived from the formula:

$$g_{71z} = (g_p - \gamma_H)_\delta + \gamma_H^0 - 0{,}3086 H_z + 0{,}0419 \delta H_z - 14, \qquad (16)$$

where g_P – free fall acceleration in the Potsdam system; γ_H – normal gravity value of the Helmert's field at the telluroid; H_z – approximate normal height of the earth's surface; $\delta = 2.3$ g/cm^3 – the density of the Earth's crust; γ_H^0 – normal gravity value at the ellipsoid surface, from the Helmert's formula:

$$\gamma_H^0 = 978030 \times (1 + 0{,}005302 \sin^2 B_{42} - 0{,}000007 \sin^2 2B_{42}), \qquad (17)$$

where B_{42} – geodetic latitude in the coordinate system of 1942.

Gravity value g_{71r} at the mark height H was computed as follows:

$$g_{71r} = g_{71z} + dg. \qquad (18)$$

If $H_z > H$, then

$$dg = 0{,}3086 dh - 2 \cdot 0{,}0419 \delta dh, \qquad (19)$$

where $dh = H_z - H$.

If $H_z < H$, then

$$dg = -0{,}3086 dh, \qquad (20)$$

where $dh = H - H_z$.

Normal height difference in LKS 94 (the Lithuanian Coordinate System of 1994) was determined in the GRS 80 normal field, applying the new European gravity system and evaluating the non-linearity of GRS 80 normal field equipotential surfaces (Moritz 1988). Normal correction was computed using the formula (2), therefore the GRS 80 normal field gravity value γ_{80}^0 was computed as follows:

$$\gamma_{80}^0 = \gamma_{80e}^0 \frac{1 + k_{80} \sin^2 B_{94}}{\sqrt{1 - e_{80}^2 \sin^2 B_{94}}}, \qquad (21)$$

where B_{94} – geodetic latitude in the LKS 94 system; normal gravity value at the equator on equipotential ellipsoid surface γ_{80e}^0 = 978032.67715 mGal; e_{80} – the first eccentricity of ellipsoid; e_{80}^2 = 0.00669438002290; coefficient k_{80} = 0,001931851353.

The mean normal gravity value between ellipsoid and telluroid for the territory of Lithuania: γ_{80v} = 981 500 mGal.

Free air gravity anomaly of vertical network points:

$$(g_{71} - \gamma_{80}) = g_{71} + \delta g_a - \gamma_{80}^0 - \Delta\gamma_{80}, \qquad (22)$$

where atmospheric gravity correction (Petroškevičius 2004)

$$\delta g_a = 0{,}874 - 0{,}99 \cdot 10^{-4} H + 0{,}356 \cdot 10^{-8} H^2, \qquad (23)$$

and height correction (H in metres) (Torge 1989)

$$\Delta\gamma_{80} = -0{,}30877(1 - 0{,}00142 \sin^2 B_{94}) H + 0{,}75 \cdot 10^{-7} H^2. \qquad (24)$$

Observed height differences were corrected by temperature, calibration, refraction and tidal corrections, so the corrected height differences for forward and backward levelling were computed. Mean height difference values were computed and corrected by the normal correction f_{ik}^{80} and the final height differences were computed.

In summer of 2007, the NGVN was integrated into the UELN (Fig. 3) (Krikštaponis *et al.* 2011).

Fig. 3. The Lithuanian NGVN in UELN (red ellipses show the datum points of the NGVN)

Data preparation was divided into a number of steps:
- Computing the gravity values of benchmarks and heights differences,
- Computing the geopotential height differences and geopotential heights of benchmarks,
- Preliminary control and adjustment of the single Lithuania levelling network,
- Detecting the connections with levelling networks of neighbouring countries,
- Encoding the nodal benchmarks according to the coding system of the UELN,
- Adjustment of the total UELN,
- Calculation of normal heights of benchmarks,
- Comparison of the received normal heights with normal heights of the national height system.

Later, the NGVN of Lithuania was also adjusted using the program HOENA (Schoch 1995).

The misclosures of the closed loops are presented in Table 3.

All actual misclosures are below the allowable values. This proves that the right method was used to determine the normal height difference and that fieldworks were of the highest quality. Therefore, the misclosure in the loop No 3 is close to the allowable misclosure. Exact reasons are unknown, but we could guess, that big misclosure results from geodynamic processes, because the levelling lines were established in different epochs: 1998, 2005 and 2006.

Table 3. Misclosures of loops of the Lithuanian NGVN

Loop No.	Loop perimeter, km	Actual misclosure, mm	Allowable misclosure, m_0 = 1.0 mm
1	491.5	+4.29	43.4
2	525.4	–14.80	44.9
3	576.3	-32.83	47.0
4	452.0	+11.66	41.6
5	510.0	+3.04	44.2

Below are the parameters of the adjustment of the new NGVN of Lithuania with one fiducial point (point code 73S-0271):
- Number of fixed points: 1,
- Number of unknowns: 1381,
- Number of measurements: 1454,
- Degrees of freedom: 73,
- Standard deviation (from the results of adjustment): 0.187 kgal·mm/km,
- A-posteriori standard deviation referred to a levelling distance of 1km (based on actual misclosures): 0.722 kgal·mm,
- The mean value of the standard deviation of adjusted geopotential differences: 0.22 kgal·mm,
- The mean value of the standard deviation of adjusted geopotential heights: 2.02 kgal·mm,
- The greatest value of the standard deviation of adjusted geopotential heights: 2.73 kgal·mm,
- Average redundancy: 0.050.

All datum points of the NGVN are presented in Table 4. These datum points were used in the adjustment of the United Network of Latvia and Lithuania.

Table 4. Datum points of the Lithuanian NGVN

No.	Name	National code	UELN code	LKS94 coordinates	Geopotential number, $m^2 \cdot s^{-2} \cdot 10^{-1}$	Accuracy of geopotential number in UELN network, $m^2 \cdot s^{-2} \cdot 10^{-1}$	Normal height, m	IGSN71 gravity acceleration, $m \cdot s^{-2}$
1	ŠIAULIAI	55S-0128	2412001	55°54′48.78202″ 23°22′17.18605″	138.795	0.0127	141.402	9.815339
2	VILNIUS	73S-0271	2412002	54°39′11.30417″ 25°17′55.19158″	211.797	0.0128	215.801	9.814334
3	MOLAS	25S-1522	2412004	55°43′47.23801″ 21°04′58.88606″	4.590	0.0136	4.676	9.815498
4	ŽELVIAI	26V10300	2412015	56°00′41.96954″ 21°06′51.86654″	9.126	0.0138	9.297	9.815762
5	MIKYTAI	34V10201	2412020	55°07′54.06812″ 21°57′34.81749″	16.370	0.0116	16.678	9.814947
6	JONAVA	64V--217	2412023	55°05′55.95392″ 24°16′20.64503″	67.575	0.0122	68.848	9.814745
7	KAZLAI	53V12421	2412030	54°44′43.61659″ 23°28′14.25382″	63.884	0.0112	65.090	9.814756
8	LAZDIJAI	52V-1021	2412038	54°13′18.96189″ 23°30′43.65627″	129.529	0.0105	131.981	9.814077
9	PETRŪNIŠKIS	85V-0739	2412055	55°43′08.70335″ 26°14′41.29362″	142.250	0.0136	144.924	9.815321
10	RADIKIAI	56V---11	2412065	56°12′13.21889″ 23°34′03.21221″	59.636	0.0134	60.754	9.815793

4. Adjustment of the United Vertical Network of Latvia and Lithuania

In order to unify the geodetic datums of Latvia and Lithuania, to have the reliable basis for geodynamic studies, it was decided to connect both levelling networks. Consequently, the new first order levelling lines Būtingė–Rucava, Joniškis–Eleja and Turmantas–Demene were observed by Latvian and Lithuanian geodesists in 2007–2010. Some data on connecting levelling lines are presented in Tables 5 and 6.

The United Geodetic Vertical Network of Latvia and Lithuania was adjusted using the program HOENA developed by the Leipzig department of Bundesamt für Kartographie und Geodäsie (Schoch 1995). The adjustment of geopotential height differences of the United Levelling Network was performed as a constrained adjustment linked to the datum points presented in Tables 2 and 4.

The misclosures of the connecting loops are presented in Table 7.

The actual misclosures are below the allowable values and that proves the high quality of the geodetic measurements performed both by Latvian and Lithuanian geodesists. Below are the parameters of the adjustment of the United Vertical Network of Latvia and Lithuania:

– Number of fixed points: 26,
– Number of unknowns: 3281,
– Number of measurements: 3397,
– Degrees of freedom: 116,
– Standard deviation (from the results of adjustment): 0.534 kgal·mm/km,

Table 5. Data on benchmarks of connecting levelling lines

UELN code	National benchmark code	Approx. normal height, m	Geodetic latitude	Geodetic longitude
		Būtingė–Rucava		
2412013	26V-6237	9.36900	56 03 09.97802	21 07 09.22163
	26V10238	11.72000	56 04 16.70929	21 07 19.04867
	21L-1684	11.02613	56 04 50.4464	21 07 22.58237
		Joniškis–Eleja		
2412967	56V10051	39.89800	56 20 52.57184	23 38 30.49239
2412066	56S–335	38.89700	56 21 42.24637	23 39 28.86839
	02L-0718	39.24405	56 21 55.1571	23 40 26.33
		Turmantas–Demene		
	03L-0331	137.83819	55 42 44.9058	26 28 14.77
	03L-2285	138.82556	55 42 13.6655	26 28 03.76
2413395	95V-0053	139.20100	55 41 29.00782	26 27 36.01

Table 6. Data on height differences of the connecting lines

Start point	End point	Distance, km	Height differences, m (Lithuanian measurements)	Height differences, m (Latvian measurements)	Geopotential number, kgal·m
			Būtingė–Rucava		
26V-1561	26V-6237	1.76	+3.20416		+3.14513
26V-6237	26V10238	2.08	+2.39474	+2.3948	+2.35063
26V10238	006-1684	1.1		–0.8733	–0.85721
			Joniškis–Eleja		
56V10049	56V10051	1.43	0.56092		+0.55060
56V10051	56S–335	1.88	–1.00096	–1.00014	–0.98255
56S--335	026-0718	1.50		–0.37657	0.36965
			Turmantas–Demene		
046-0331	046-2285	1.21		+0.98737	+0.96913
046-2285	95V-0053	1.60		+0.31743	+0.31157
95V-0053	95S–295	0.35	+1.54210	+1.5421	+1.51363

– A-posteriori standard deviation referred to a levelling distance of 1km (based on actual misclosures): 0.816 kgal·mm,
– The mean value of the standard deviation of adjusted geopotential differences: 0.65 kgal·mm,
– The mean value of the standard deviation of adjusted geopotential heights: 2.75 kgal·mm,
– The greatest value of the standard deviation of adjusted geopotential heights: 4.54 kgal·mm,
– Average redundancy: 0.034.

Table 7. Misclosures of the connecting loops

Loop	Loop perimeter, km	Actual misclosure, kgal·mm	Allowable misclosure, m_0 = 1.0 mm
1	640.7	6.76	49.61
2	548.2	2.47	45.89

Results of the variance component estimation are given in Table 8.

Adjusted geopotential numbers and normal heights of the border points are presented in Table 9.

5. Conclusions

1. The first step was made in preparation for establishment of the United Vertical Network of Latvia and Lithuania. The levelling data of both countries fit each other with a better than 1 mm accuracy. Misclosures of common loops are 6.76 and 2.47 mm.
2. The accuracy of the United Vertical Network of Latvia and Lithuania (the standard deviation is 0.534 kgal·mm/km) is at the same level as that of vertical networks of the greatest part of other countries participating in the UELN project.
3. Differences between the United Vertical Network of Latvia and Lithuania and the UPLN height systems at the border points amount to approx. 15 cm.
4. The adjustment results are basic for high accuracy over boundary civil engineering projects.
5. The adjustment results of the United Vertical Network of Latvia and Lithuania could serve as a basis for the adoption of vertical (height) systems of both countries.

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Table 8. Accuracy of levelling networks of Latvia and Lithuania

Country	Number of measurements	Sum of redundancies	Corrected sum of redundancies	Correction factor	A-posteriori standard deviation kgal·mm/km
Latvia	1904	14.5181	30.293	0.4793	0.6923
Lithuania	1493	18.5848	85.707	0.2168	0.4657
Sum	3397	33.1029	116.000		

Table 9. Normal heights of border benchmarks

Point code	Geopotential number kgal·mm	Normal height m	Standard deviation kgal·mm
26V10238	11.83571	12.05762	0.52
006-1684	10.97960	11.18544	–
56S--335	38.30000	39.01728	0.63
026-0718	38.66980	39.39400	–
046-2285	136.43150	138.99660	–
95V-0053	136.74380	139.31492	0.64

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