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ESTIMATING THE GEOPOTENTIAL VALUE W_0 OF THE LOCAL GEOID BASED ON DATA FROM LOCAL AND GLOBAL NORMAL HEIGHTS OF GPS/LEVELING POINTS IN VIETNAM

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Abstract. Currently, the determination of geopotential value W_0 of local geoid that best fits local mean sea level at the Zero Tide Gauge Station is getting important in building the National Geoid-Based Vertical System. Ha Minh Hoa (2007) and Kotsakis *et al.* (2012) recommended a method, which estimates the geopotential value W_0 of local geoid at the Zero Tide Gauge Station based on equations of relation between the local and global normal heights or between the local and global height anomalies at GPS/leveling points regularly located on the whole territory. The objective of this paper is to determine conditions for estimating the geopotential value W_0 of local geoid at the Zero Tide Gauge Station accomplished for whole territory of Vietnam.

Keywords: local mean sea level, sea surface topography, quasigeoid, global normal height, local normal height, global height anomaly, local height anomaly.

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Introduction

The most important problem for setting up the Geoid-Based Vertical Reference System is to determine a geopotential value W_0 of the local geoid that best fits local mean sea level at the Zero Tide Gauge Station, from which local vertical datum can be defined. Moreover, one of the most important bases to resolve this problem is to use the global geoid with the gravity geopotential $\overline{W}_0 = 6\,2636\,856.0\ \mathrm{m^2 \cdot s^{-2}}$, that has been determined from altimetric data analysis (Burša *et al.* 2002; Petit, Luzum 2010). Currently, there are many approaches to determine a geopotential value W_0 of the local geoid based on the known global geoid with the gravity geopotential $\overline{W}_0 = 6\,263\,6856.0\ \mathrm{m^2 \cdot s^{-2}}$. The value W_0 can be computed by the following formula (Ihde 2004):

$$W_0 = \overline{W}_0 - \gamma_0 \cdot (MDT + TGO),$$

where MDT is the sea surface topography at the Zero Tide Gauge in relation to the global geoid; TGO (Tide Gauge Offset) is deviation between local mean sea level (MSL) and the sea surface topography at the Zero Tide Gauge; γ_0 is the normal gravity on the ellipsoid corresponding with the Zero Tide Gauge Station.

The value of MDT is fully determined from the Mean Dynamic Topography model (MDT) at the Zero Tide Gauge. On the other hand, the value TGO is computed by the formula: $TGO = h_0$ – MSS, where MSS is determined from the Mean Sea Surface model (altimetry-based global MSL model in relation to the ellipsoid) at the Zero Tide Gauge, h_0 is ellipsoidal height of local mean sea level at the Zero Tide Gauge in relation to the ellipsoid and is determined from processing of co-located GPS observations in ITRF (International Terrestrial Reference Frame).

In 2007, Ha Minh Hoa proposed an equation about the relation between the local normal height and the global normal height as follow:

$$\overline{H}_i^{\gamma} = H_i^{\gamma} + \frac{\overline{W}_0 - W_0}{\overline{\gamma}_i}, \tag{1}$$

where index i is a number of an interested point on the Earth surface, W_0 ís the geopotential value of the local geoid at the Zero Tide Gauge, $\overline{W}_0$ is the geopotential value of the global geoid, $\overline{H}_i^{\gamma}$ is the global normal height of point i in relation to the global quasigeoid and determined in the Zero Tide System, H_i^{γ} is the local normal height of a point i in relation to the local quasigeoid and determined in the Zero Tide System (Mäkinen 2008), $\overline{\gamma}_i$ is an average value of the normal gravity along the normal plumb line between the derived point i and the local quasigeoid.

We pay attention to the fact that the quasigeoid coincides with the geoid on the sea and ocean, and differs from the mainland. Moreover, Ha Minh Hoa (2010) proved that the quasigeoid is an image of the Ellipsoid's geoid in the normal gravity field. It was derived from the projection of the Ellipsoid's geoid in the normal gravity field by locating along the normal plumb line. Therefore, the local quasigeoid is closely related to the local geoid and the global quasigeoid is closely related to the global geoid. With a such signification, the local quasigeoid is the equipotential surface (local geoid) on the sea and nearby delta regions. On the mountainous areas, the quasigeoid is the surface, in which the normal gravity U of a point is determined by a formula: U = W_0 – T, where W_0 is the geopotential of the local geoid, T is the disturbing potential of a corresponding point on the Earth surface. The quasigeoid is the base to determine a normal height from a derived point on the Earth surface.

The difference of $\overline{H}_i^{\gamma} - H_i^{\gamma}$ is an actual height of the local quasigeoid corresponding with a point i in relation to the global quasigeoid. In the case of the determined geopotential W_0 of local geoid, the theoretic value of this height is estimated by quantifying $\frac{\overline{W}_0 - W_0}{\overline{\gamma}_i}$. If the quantities of $\frac{\overline{W}_0 - W_0}{\overline{\gamma}_i}$ at all GPS/leveling points are equal, we can see that territory distance between the local quasigeoid and the global quasigeoid is a constant value.

We symbolize h_i as the ellipsoidal height of the GPS/leveling point i which is derived from processing of co-located GPS observations in ITRF with reference ellipsoid WGS84 and transferred to the Zero Tide System (Mäkinen *et al.* 2009). When local GPS/leveling height anomaly $\zeta_i = h_i - H_i^{\gamma}$ of the point i is a distance between the local quasigeoid and the reference ellipsoid by the direction along the normal plumb line corresponding with point i, and global height anomaly $\overline{\zeta}_i = h_i - \overline{H}_i^{\gamma}$ of the point i is a distance between the global quasigeoid and the reference ellipsoid by the direction along the normal plumb line corresponding with point i.

We realize that the global height anomaly $\overline{\zeta}_i$ of the GPS/leveling point i can be calculated as a synthesized value from EGM2008 by using the Harmonic Synthesis Program provided by the EGM development team (NGA). For using of WGS84 as the reference ellipsoid with the associated normal gravity field, the geoid undulations are referenced. This program applies a constant zero-degree term of –41 cm to all geoid undulations computed using EGM2008 with the height_anomaly-to-geoid_undulation correction model (Pavlis *et al.* 2008). The synthesized value from EGM2008 can be transferred to the Zero Tide System (Ekman 1989).

For using the equation (1), we may calculate the global normal height $\overline{H}_i^{\gamma}$ by the formula of $\overline{H}_i^{\gamma} = h_i - \overline{\zeta}_i$. Equation (1) can be transformed to the following equation to show relation between the local and global height anomalies (Ha Minh Hoa 2012a)

$$\zeta_i = \overline{\zeta}_i + \frac{\overline{W}_0 - W_0}{\overline{\gamma}_i}. \quad (2)$$

Comparing the equation (2) with the equation (1) we get following equation

$$\overline{H}_i^{\gamma} - H_i^{\gamma} = \zeta_i - \overline{\zeta}_i\,. \quad (3)$$

In order to determe the geopotential value W_0 of the local geoid, the best solution is to use only first order normal heights. On account of the global geoid with geopotential $\overline{W}_0$ is used for building the Global Gravitational Model (GGM), for example EGM2008, the condition of the absence of any outliers and systematic errors in the set of the differences $\overline{H}_i^{\gamma} - H_i^{\gamma}$ in the equation (1) or $\zeta_i - \overline{\zeta}_i$ in the equation (2) are considered to be a sequence of independent and identically distributed random values, the least squares estimation of equation (1) or equation (2) leads to the best fitting value of geopotential W_0 of the local geoid in terms of a sample (Ha Minh Hoa 2012b).

$$W_0 = \frac{\sum_{i=1}^{M}(W_0)_i}{M}, \quad (4)$$

where M is a number of GPS/levelling points, constituent $(W_0)_i$ is computed by the following formula

$$(W_0)_i = \overline{W}_0 - 10^{-5}\cdot\overline{\gamma}_i\cdot\left(\overline{H}_i^{\gamma} - H_i^{\gamma}\right), \ <\mathrm{m}^2\cdot\mathrm{s}^{-2}> \quad (5)$$

or

$$(W_0)_i = \overline{W}_0 + 10^{-5}\cdot\overline{\gamma}_i\left(\overline{\zeta}_i - \zeta_i\right), \ <\mathrm{m}^2\cdot\mathrm{s}^{-2}> \quad (6)$$

$\overline{\gamma}_i$ was defined by mGal unit, $\overline{\zeta}_i$, $\overline{\overline{\zeta}}_i$ were defined by m unit.

Standard error of best fitting value of geopotential W_0 is estimated by the following formula:

$$m_{W_0} = \sqrt{\frac{\sum_{i=1}^{M}\left((W_0)_i - W_0\right)^2}{M\cdot(M-1)}}. \quad (7)$$

In 2012, Kotsakis *et al.* proposed a similar approach for determing the geopotential value W_0 of the local geoid with the computation of constituent $(W_0)_i$ by equation (6). However in this method, authors proposed the use of the global geoid height $\overline{N}_i$ instead of the global height anomaly $\overline{\zeta}_i$ and the use of the GPS/levelling local geoid height based on the use of the ortho-metric height instead of the GPS-levelled local height anomaly ζ_i.

In this case, the use of the normal gravity $\overline{\gamma}_i$ makes the equation (6) non-rigorous. Normally, in a considered case, the normal gravity $\overline{\gamma}_i$ must be replaced by the average value of the actual gravity $\overline{g}_i$ along the physical plumb line between the derived point i and the geoid in the Earth gravitational field. However, this paper does not present necessary conditions for applying of equation (6).

The objective of this paper is to construct conditions for applying of the equation (1) or the equation (2) for the estimation of the geopotential value W_0 of the local geoid that best fits local mean sea level at the Zero Tide Gauge station. From this result, the local vertical datum can be defined and the case study for whole territory of Vietnam.

1. Determination of conditions for applying of the equation (1) or the equation (2)

The local and global normal heights of a point i are determined by the following formulas:

$$H_i^\gamma = \frac{W_0 - W_i}{\overline{\gamma}_i}, \quad (8)$$

$$\overline{H}_i^\gamma = \frac{\overline{W}_0 - W_i}{\overline{\overline{\gamma}}_i}, \quad (9)$$

where $\overline{\gamma}_i$ is the average value of the normal gravity along the normal plumb line between the derived point i and the local quasigeoid, $\overline{\overline{\gamma}}_i$ is the average value of the normal gravity along the normal plumb line between the derived point i and the global quasigeoid, W_i is the geopotential at the point i.

If this following condition:

$$\frac{1}{\overline{\overline{\gamma}}_i} = \frac{1}{\overline{\gamma}_i} \quad (10)$$

is satisfied, from the equation (8) and the equation (9) we come back to the equation (1).

However, the relationship between the value $\overline{\overline{\gamma}}_i$ and $\overline{\gamma}_i$ is determined by $\overline{\overline{\gamma}}_i = \overline{\gamma}_i + \Delta\gamma$, where the difference $\Delta\gamma = -0.1543\cdot(\overline{H}_i^\gamma - H_i^\gamma)$, then we have the following formula:

$$\frac{1}{\overline{\overline{\gamma}}_i} = \frac{1}{\overline{\gamma}_i}\cdot\left(1 - \frac{\Delta\gamma}{\overline{\overline{\gamma}}_i}\right).$$

So the satisfaction of condition described in (10) depends on height $\overline{H}_i^\gamma - H_i^\gamma$ of the local quasigeoid corresponding with a point i in relation to the global quasigeoid on studied territory. For whole teritory of Vietnam, the difference $\overline{H}_i^\gamma - H_i^\gamma$ of any GPS/leveling point is constant and equal to 0.890 m (Ha Minh Hoa 2012b). With the average value of the normal gravity $\overline{\gamma} = 978\,243$ mGal, we get

$$\frac{1}{\overline{\overline{\gamma}}_i} = \frac{1}{\overline{\gamma}_i}\cdot(1 - 1.4038\cdot 10^{-7}).$$

With 3.143 m local normal height, the geopotential number in the top of Phanxipang Mountain in Vietnam is measured as: $\overline{W}_0 - W_{PXP} = 30\,746.17749\ \text{m}^2\cdot\text{s}^{-2}$, the accepted condition (10) leads to an error of 4 mm in global normal height (9). This error is negligible. The satisfaction of condition (10) is a base for applying the equation (1) or (2) to estimate of the geopotential value W_0 of the local geoid.

The second important condition is related to the absence of any outliers and systematic errors in the set of the differences $\overline{H}_i^\gamma - H_i^\gamma$ in the equation (1) or $\zeta_i - \overline{\zeta}_i$ in the equation (2). A transformation of the values $\overline{H}_i^\gamma$, H_i^γ or ζ_i, $\overline{\zeta}_i$ to the Zero Tide System eliminated the main source of systematic errors, outlier detection and removal in the set of differences $\overline{H}_i^\gamma - H_i^\gamma$ in the equation (1) or $\zeta_i - \overline{\zeta}_i$ in the equation (2) can be calculated according to Smirnov *et al.* (1969).

Because of the existence of the equality described in the equation (3), in the next part we will consider only the differences $\overline{H}_i^\gamma - H_i^\gamma$ in the equation (1). By principle, the differences $\overline{H}_i^\gamma - H_i^\gamma$ in the equation (1) are random. Mean value of the sequence of independent and identically distributed random $\overline{H}_i^\gamma - H_i^\gamma$ (i = 1, 2, ..., M) is calculated by the following formula:

$$\overline{\overline{H}^\gamma - H^\gamma} = \frac{\sum_{i=1}^{M}\left(\overline{H}_i^\gamma - H_i^\gamma\right)}{M}, \quad (11)$$

where the difference $\overline{\overline{H}^\gamma - H^\gamma} - \left(\overline{H}_i^\gamma - H_i^\gamma\right) = V_{\overline{H}_i^\gamma - H_i^\gamma}$ is the residual of value $\overline{H}_i^\gamma - H_i^\gamma$.

We symbolize $m_{\overline{H}^\gamma}$ and m_{H^γ} as Root Mean Square errors (RMS) of the global and local normal heights respectively. When RMS of the differences $\overline{H}_i^{\gamma} - H_i^{\gamma}$ in the equation (1) or $\zeta_i - \overline{\zeta}_i$ in the equation (2) is calculated by this formula: $m_{\overline{H}_i^{\gamma} - H_i^{\gamma}} = \sqrt{m^2_{\overline{H}^{\gamma}} + m^2_{H^{\gamma}}}$ and the error limit is equal to ($2 \cdot m_{\overline{H}_i^{\gamma} - H_i^{\gamma}}$). If all the absolute values of residuals $V_{\overline{H}^{\gamma} - H_i^{\gamma}}$ of differences $\overline{H}_i^{\gamma} - H_i^{\gamma}$ (i = 1, 2, ..., M) do not exceed the error limit, we can accept a assumption that there is an absence of any outliers in the set of the differences $\overline{H}_i^{\gamma} - H_i^{\gamma}$ in the equation (1) or $\zeta_i - \overline{\zeta}_i$ in the equation (2).

In a worst case of processing of GPS data in ITRF, RMS of ellipsoidal height m_h is accepted to be equal to ±0.050 m. RMS of global height anomaly calculated from EGM2008 is equal to ±0.111 m (Pavlis *et al.* 2008) when the biggest RMS of the global normal heights $m_{\overline{H}^{\gamma}}$ is equal to ±0.122 m. With the biggest RMS of the first order local normal heights $m_{H^{\gamma}} = \pm 0.065$ m, the error limit for RMS of the differences $\overline{H}_i^{\gamma} - H_i^{\gamma}$ in the equation (1) or $\zeta_i - \overline{\zeta}_i$ in the equation (2) is equal to 2 × 0.138 m = 0.276 m.

The mean value $\overline{\overline{H}^{\gamma} - H^{\gamma}}$ in the equation (11) expresses the height of the local quasigeoid in relation to the global quasigeoid in the research area and is constant.

The confidence of derived value W_0 in the equation (4) can be estimated by three following conditions:

a) RMS m_{W_0} (7) does not exceed its error limits;

b) The Mean value $\overline{\overline{H}^{\gamma} - H^{\gamma}}$ is equal to $\dfrac{\overline{W}_0 - W_0}{(\gamma_0)_{ZTG}}$, where $(\gamma_0)_{ZTG}$ is the normal gravity on the reference ellipsoid calculated corresponding with the Zero Tide Gauge Station;

c) The derived value W_0 does not cause any systematic errors in the test for a set of differences $dH_j^{\gamma} = \overline{H}_j^{\gamma} - \hat{H}_j^{\gamma}$ (j = 1, 2, ..., L) at GPS/leveling points, that do not be used for determining the value W_0, where

$$\hat{H}_j^{\gamma} = H_j^{\gamma} + \frac{\overline{W}_0 - W_0}{\overline{\gamma}_j}. \tag{12}$$

We will consider a determination of the error limit for RMS m_{W_0} (7). When considering the geopotential $\overline{W}_0$ of the global Geoid as a value without an error, from the formula (1) we can get the RMS of global normal height:

$$m^2_{\overline{H}^{\gamma}} = m^2_{H^{\gamma}} + \frac{1}{\overline{\gamma}^2} \cdot m^2_{W_0}, \tag{13}$$

where $m_{H^{\gamma}}$ is the RMS of local normal height.

We receive the following condition: Determination of the geopotential W_0 of local Geoid with negligible RMS in the formula (13) as $m_{\overline{H}^{\gamma}} = m_{H^{\gamma}}$. Based on the principle of negligible RMS, we have an inequality:

$$\frac{1}{\overline{\gamma}} \cdot m_{W_0} \le \frac{m_{H^{\gamma}}}{3},$$

or

$$m_{W_0} \le \overline{\gamma} \cdot \frac{m_{\hat{H}^{\gamma}}}{3}.$$

For the first order normal heights, their biggest RMS $m_{H^{\gamma}}$ is equal to 0.060 m. For the whole territory of Vietnam, the received average value of the normal gravity is $\overline{\gamma} = 9.783\ \ \mathrm{m \cdot s^{-2}}$. Then, we have:

$$m_{W_0} \le 0.2\ \ \mathrm{m^2 \cdot s^{-2}}. \tag{14}$$

For instant, the error limit RMS m_{W_0} described in the equation (7) is equal to $0.2\ \ \mathrm{m^2 \cdot s^{-2}}$.

To detect an absence of any systematic error in the test to a set of differences $dH_j^{\gamma} = \overline{H}_j^{\gamma} - \hat{H}_j^{\gamma}$ (j = 1, 2, ..., L) at the L testing GPS/leveling points, where normal height $\hat{H}_j^{\gamma}$ was calculated by the formula (12), we check following inequality:

$$\left| \sum_{j=1}^{L} \Delta H_j^{\gamma} \right| \le 0.25 \cdot \sum_{j=1}^{L} \left| \Delta H_j^{\gamma} \right|. \tag{15}$$

If condition (15) is satisfied, we can see that differences $dH_j^{\gamma} = \overline{H}_j^{\gamma} - \hat{H}_j^{\gamma}$ (j = 1, 2, ..., L) does not contain any systematic errors.

2. Case studies in Vietnam

To determine the geopotential value W_0 of the local geoid that best fits local mean sea level at the Tide Gauge Station HON DAU (Zero Tide Gauge Station in Vietnam), we used 180 first order benchmarks with co-located GPS observations (GPS/first order leveling points), in which 35 GPS/first order leveling points were used for calculating the best fitting value W_0 by using the formula (4), the remaining 145 GPS/first order leveling points were used for estimating confidence of the derived value W_0. Processing of the co-located GPS observations had been accomplished in ITRF2005 corresponding with ellipsoid WGS84 by using Bernese 5.0 software. The global height anomalies of all GPS/first order leveling points had been calculated from EGM2008 by using a specialist software provided from EGM development team (NGA). We had calculated the global normal heights $\overline{H}^{\gamma}$ of all 180 GPS/first order leveling points. The local normal heights H^{γ}

of the those GPS/first order leveling points are given in the local vertical system HP72. All the global and local normal heights of all 180 GPS/first order leveling points had been transformed into the Zero Tide System. The outlier detection in the set of all differences $(\bar{H}^{\gamma})_Z - (H^{\gamma})_Z$ at the 180 GPS/first order leveling points had been accomplished according to Smirnov *et al.* (1969) and showed that there is not an outlier in above mentioned differences $(\bar{H}^{\gamma})_Z - (H^{\gamma})_Z$, where Z shows the Zero Tide System.

The gravity geopotential $\bar{W}_0 = 62\,636\,856.0\ \mathrm{m}^2 \cdot \mathrm{s}^{-2}$ of global geoid had been used for calculating $(W_0)_i$ (i = 1, 2, ..., 35) at every GPS/first order leveling point i by the formula (5). Table 1 describes the results for determining the best fitting value W_0 of local geoid that best fits local mean sea level at the Tide Gauge Station HON DAU based on 35 GPS/first order leveling points.

The best fitting geopotential value W_0 of local geoid is equal to 62 636 847.2911 $\mathrm{m}^2 \cdot \mathrm{s}^{-2}$ with RMS:

$$m_{W_0} = \pm\sqrt{\frac{\sum_{i=1}^{35}((W_0)_i - W_0)^2}{M \cdot (M-1)}} = \pm\sqrt{\frac{39.79403355}{34.35}} = \pm 0.183\ \mathrm{m}^2 \cdot \mathrm{s}^{-2}.$$

Table 1. Determination of the best fitting value W_0 of local geoid at the tide gauge station HON DAU

No i	Point number	$(\bar{H}_i^{\gamma})_Z - (H_i^{\gamma})_Z$ (m)	$(W_0)_i$ (m²/s²)	$(W_0)_i - W_0$ (m²/s²)
1	I(BH-LS)97	0.983	62636846.3793	–0.9118
2	I(BH-TH)122A	0.896	62636847.2314	–0.0597
3	I(BMT-APD)30	1.049	62636845.7386	–1.5525
4	I(BH-TH)119	0.916	62636847.0359	–0.2552
5	I(BH-HN)33	0.899	62636847.2014	–0.0897
6	I(BH-HN)39	0.904	62636847.1524	–0.1387
7	I(BH-HN)42	0.876	62636847.4265	0.1354
8	I(BH-HN)48	1.013	62636846.0857	–1.2054
9	I(HN-HP)11A	0.734	62636848.8164	1.5253
10	I(HN-HP)2A	1.003	62636846.1836	–1.1075
11	I(HN-HP)5	1.037	62636845.8509	–1.4402
12	I(HN-VL)10A	0.797	62636848.2000	0.9089
13	I(HN-HP)7	0.949	62636846.7122	–0.5789
14	I(HN-VL)4-1	0.913	62636847.0646	–0.2265
15	I(HN-VL)6-1	0.884	62636847.3485	0.0574
16	I(HP-NB)14A	0.768	62636848.4839	1.1928
17	I(LS-HN)22	0.773	62636848.4345	1.1434
18	I(LS-HN)29	0.845	62636847.7299	0.4388
19	I(LS-HN)36	0.932	62636846.8785	–0.4126
20	I(VL-HT)325-1	0.965	62636846.5604	–0.7307
21	I(BH-HN)16A	0.963	62636846.5748	–0.7163
22	I(HN-VL)28-1	0.899	62636847.2022	–0.0889
23	I(HN-VL)95	0.728	62636848.8766	1.5855
24	I(HN-VL)45-1	0.920	62636846.9971	–0.294
25	I(VL-HT)152-1	0.844	62636847.7433	0.4522
26	I(VL-HT)121	0.949	62636846.7157	–0.5754
27	I(VL-HT)73	1.062	62636845.6094	–1.6817
28	I(VL-HT)95	1.044	62636845.7859	–1.5052
29	LS01	0.638	62636849.7559	2.4648
30	TB01	1.010	62636846.1155	–1.1756
31	QN01	0.861	62636847.5759	0.2848
32	QNG1	0.781	62636848.3589	1.0678
33	PY01	0.641	62636849.7291	2.438
34	KH01	0.781	62636848.3597	1.0686
35	BP01	0.892	62636847.2742	–0.0169
Mean value		0.890	**62636847.2911**	

With its absolute value is smaller than error limit of 0.2 $m^2 \cdot s^{-2}$.

Because all the absolute values of residuals of defferences $(\bar{H}_i^\gamma)_z - (H_i^\gamma)_z$ (i = 1, 2, ..., 35) do not exceed error limit of 0.276 m, we can accept an assumption that there is not an outlier in above mentioned differences.

For the Tide Gauge Station HON DAU, the normal gravity on ellipsoid WGS84 is $(\gamma_0)_{ZTG} = 9.786762046\ \text{m} \cdot \text{s}^{-2}$. Then the mean value $\overline{\bar{H}^\gamma - H^\gamma} = 0.890$ m is equal to

$$\frac{\bar{W}_0 - W_0}{(\gamma_0)_{ZTG}} = \frac{8.7089}{9.786762046} = 0.890 \text{ m.}$$

In order to check a constant height of the local quasigeoid in relation to global quasigeoid in whole territory of Vietnam, we have a following formula:

$$\bar{H}_i^\gamma - H_i^\gamma = \frac{\bar{W}_0 - W_0}{10^{-5} \cdot \bar{\gamma}_i} = \frac{8.7089 \text{ m}^2 \cdot \text{s}^{-2}}{10^{-5} \cdot \bar{\gamma}_i}.$$

The experements were calculated for some points located in different regions of Vietnam. The estimation results are presented in Table 2 below.

Table 2. Estimation of height of local quasigeoid in relation to global quasigeoid in whole territory of Vietnam

Point name	$\bar{\gamma}$	$\frac{8.7089 \text{ m}^2 \cdot \text{s}^{-2}}{10^{-5} \cdot \bar{\gamma}_i}$
	mGal	m
High mountainous regions		
Sa Pa	978547.64500	0.890
Cao Bang	978770.65010	0.890
Dien Bien	978644.09999	0.890
Lao Bao	978421.26903	0.890
Buon Ma Thuot	978207.80151	0.890
Thao Nguyen	978540.32946	0.890
Chieng Mung	978609.20810	0.890
Lay Nua	978720.04737	0.890
Ha Giang	978793.60098	0.890
Yen Bai	978732.78786	0.890
The delta regions		
Lang	978695.92035	0.890
Vinh	978561.10701	0.890
Da Nang	978427.31490	0.890
Nha Trang	978261.10788	0.890
Ho Chi Minh	978212.68242	0.890
Islands		
Phu Quoc	978194.52390	0.890
Bach Long Vi	978642.82107	0.890
Con Dao	978149.50599	0.890

The experimental results show that the height of local quasigeoid in relation to global quasigeoid in whole territory of Vietnam is a constant, and equal to 0.890 m.

To estimate confidence of derived value W_0, we check the inequality (15) at the 145 GPS/first order leveling points and get

$$\left| \sum_{j=1}^{145} \Delta H_j^\gamma \right| = 3.972 \le 0.25.$$

$$\sum_{i=1}^{145} \left| \Delta H_j^\gamma \right| = 0.25 \times 17.147 = 4.287.$$

All above mentioned checking results show the confidence of derived geopotential value W_0 = 62 636 847.2911 $m^2 \cdot s^{-2}$ of local geoid that best fits local mean sea level at the Tide Gauge Station HON DAU in Vietnam.

Conclusions

In the current conditions of Vietnam, due to the insufficience of both land and sea gravity data, the determination of the best fitting geopotential value W_0 of the local geoid at the Tide Gauge Station HON DAU by using the equation of the relation between the local normal height and the global normal height (1), or the equation of the relation between the local and global height anomalies local and global normal heights (2) constructed at GPS/first order leveling points is the most appropriate. However, for the differences between the local and global normal heights or between the local and global height anomalies derived at GPS/leveling points must satisfy some conditions described in Section 2.

The determination of the best fitting geopotential value W_0 = 62 636 847.2911 $m^2 \cdot s^{-2}$ of the local geoid at the Tide Gauge Station HON DAU completely satisfies the required accuracy for the construction of geoid-based vertical system in Vietnam.

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