



DFHRS-BASED COMPUTATION OF QUASI-GEOID OF LATVIA

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Abstract. In geodesy, civil engineering and related fields high accuracy coordinate determination is needed, for that reason GNSS technologies plays important role. Transformation from GNSS derived ellipsoidal heights to orthometric or normal heights requires a high accuracy geoid or quasi-geoid model, respectively the accuracy of the currently used Latvian gravimetric quasi-geoid model LV'98 is 6–8 cm. The objective of this work was to calculate an improved quasi-geoid (QGeoid) for Latvia. The computation was performed by applying the DFHRS software. This paper discusses obtained geoid height reference surface, its comparisons to other geoid models, fitting point statistics and quality control based on independent measurements.

Keywords: Geoid/Quasi-Geoid-Computation, GNSS Height Determination, Levelling.

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Introduction

The objective of this work was to calculate the QGeoid height reference surface for Latvia. The computation was performed by applying the DFHRS software of Karlsruhe University of Applied Sciences.

The obtained geoid height reference surface was then compared with Latvian gravimetric geoid model LV'98 which is broadly used by land surveyors for more than 10 years. It is based on gravimetric measurements, data digitized from gravimetric maps and satellite altimetry data over Baltic Sea, its computation was performed by applying GRAVSOFTE software. The estimated accuracy of LV'98 geoid model is 6–8 cm (Kaminskis 2010).

Comparisons were made also between the geoid height reference surface obtained by using DFHRS software and EGG97, EGM2008, Eigen5c, Eigen6c, GOCE GO_CONS_GFC_2_DIR_R3 models, and between LV'98 and above mentioned regional and global models. Appropriate transformation was applied because the global models are not fitted to national height system.

1. DFHRS software

The DFHBF (Digitale Finite-Element Höhen-Bezugs-Fläche) or DFHRS (Digital Finite-Element Height Ref-

erence Surface) software (www.dfhbf.de), developed at the Karlsruhe University of Applied Sciences, Faculty of Geomatics (Jäger 1999) was used for computation of QGeoid height reference surface (HRS) of Latvia.

In the DFHRS concept the area is divided into smaller finite elements – meshes. The QGeoid HRS l is calculated by a polynomial of degree l in term of (x, y) coordinates in each mesh indicated by index k (Jäger 1999):

$$N_{DFHRS} = \left\{ \begin{array}{l} N(p_k) = \sum_{i=0}^l \sum_{j=0}^{l-i} a_{ij,k} \cdot y^i \cdot x^j = f^T \cdot p_k \\ C_{0,1,2}(p_c; p_d) \end{array} \right\}, \quad (1)$$

where f is a design matrix and p_k is a polynomial parameter matrix. The corresponding polynomial coefficients are introduced as $a_{ij,k}$ over m meshes ($i = 0, l$; $j = 0, l$; $k = 1, m$).

The surface between two neighboring meshes c and d should be continuous, so the continuity conditions are considered between elements to have a continuous surface with C_1 , C_1 and C_2 continuity levels at the border line. The continuity level type C_0 implies equal func-

tional values along each common mesh border. Level C_1 implies equal tangential planes and level C_2 implies equal curvature along the common borders of the QGeoid HRS model. Each group of meshes forms a patch. To reduce the effect of medium or long wave length systematic shape deflections in the QGeoid height observations N and the vertical deflections (ξ, η) from QGeoid or geopotential models (GPM), these observations are subdivided in a number of patches. Patches are formed depending on distribution of fitting points, each patch should contain at least 4 fitting points, which are necessary for a datum parameter estimation. The patches are related to individual sets of datum parameters, in order to model the long-wavelength systematic errors from QGeoid or GMP.

The calculated continuous polynomial parameters are stored together with the mesh topology information in a DFHRS-DB database. The DFHRS computed height N is provided by means of this database, which contains the QGeoid HRS parameters together with mesh and patch design information.

The mathematical model for computation of DFHRS-DB parameters is described by (Jäger et al. 2012).

2. Computation process

As an input data (N, ξ, η) the European Gravimetric Geoid Model 1997 (EGG97) was used. EGG97 is based on high-resolution gravity and terrain data available in 1997 as well as the global geopotential model EGM96 (Denker, Torge 1998). The geoid undulations and vertical deflections for 25 points in each mesh from EGG97 model were introduced as observations in the DFHBF approach and software.

102 GPS/levelling points $(B, L, h | H)$ – serving both as fitting points for the QGeoid surface as well as the datum parameter-estimation of the patches – with known ellipsoidal GNSS heights h and normal heights H were introduced. Most of the fitting points are located within territory of Latvia, 3 points in Estonia and 17 points in Lithuania.

The designed mesh size is 5×5 km, so the total number of meshes is 4601 in the computation area. The patch size varies depending on distribution of reference points (from 80×40 km to 150×100 km). In Fig. 1 the thin blue lines represent meshes, the thick blue lines – the patch borders and the green triangles – the fitting points.

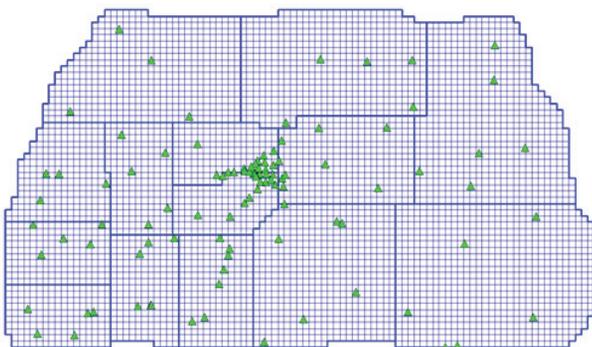


Fig. 1. Computation design for Latvia DFHRS-DB

DFHRS software provides internal quality control of the fitting points. The functional models mean an overdetermined system of equations with respect to the parameters, and require parameter estimations for redundant equation systems. The DFHRS software solves the respective parameter estimation by a Least Squares Adjustment. Based on Least Squares Adjustment the quality control and the quality indicators are set up as statistical tests (data snooping) for all single observations in the DFHBF-software.

The computation of Latvia QGeoid HRS was repeated three times, removing fitting points with detected residuals exceeding 3 cm after first and second computation. Most of the removed fitting points are located in North-West part of Latvia, in Kurzeme region (Fig. 2).

3rd order polynomials for the HRS meshes parameterization were used on all computations. Both geoid undulations (115025) and vertical deflections (230050) from EGG97 model were included as mesh observations. The number of continuity conditions was 65227, the number of unknown polynomial parameters was 46010 and the redundancy number was 364311.

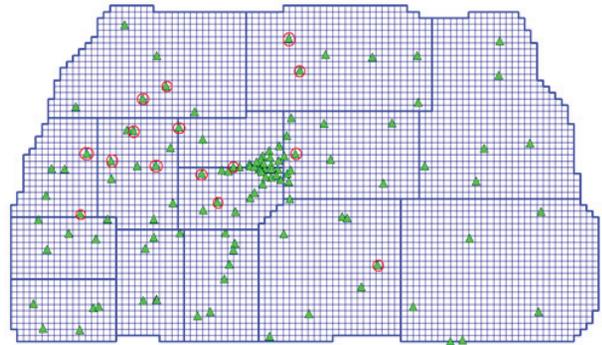


Fig. 2. Removed fitting points (red circles)

3. QGeoid computation results

The resulting QGeoid model is presented on Fig. 3. QGeoid heights in the territory of Latvia vary from 19.20 m in North-East part to 24.50 m in South-West part.

For the quality control of obtained QGeoid HRS result, it was tested using the same 102 GPS/levelling points which were used during DFHRS computation. The same test was performed for LV'98 QGeoid model, 2σ (96 GPS/levelling points) results are shown in Table 1. Residual RMS of 1.6 cm for Latvian QGeoid HRS was obtained. The same RMS appears for LV'98 QGeoid model, however the minimum and maximum values are larger than those in DFHRS solution.

Table 1. Fitting point statistics (2σ – 95% of fitting points)

	Min (m)	Max (m)	RMSE (m)
DFHRS	-0,036	0,034	0,016
LV'98	-0,042	0,040	0,016

For comparison between LV'98 model and DFHRS solution (Fig. 4), minimum difference is -25.7 cm, maximum: 21.1 cm, RMS: 5.8 cm.

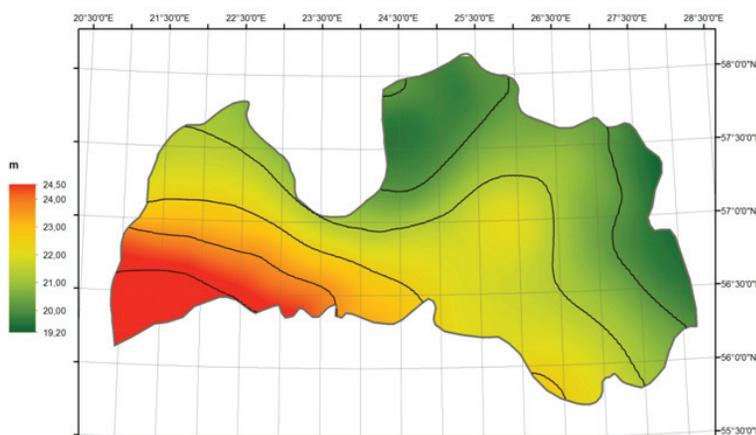


Fig. 3. Obtained QGeoid height reference surface for the territory of Latvia

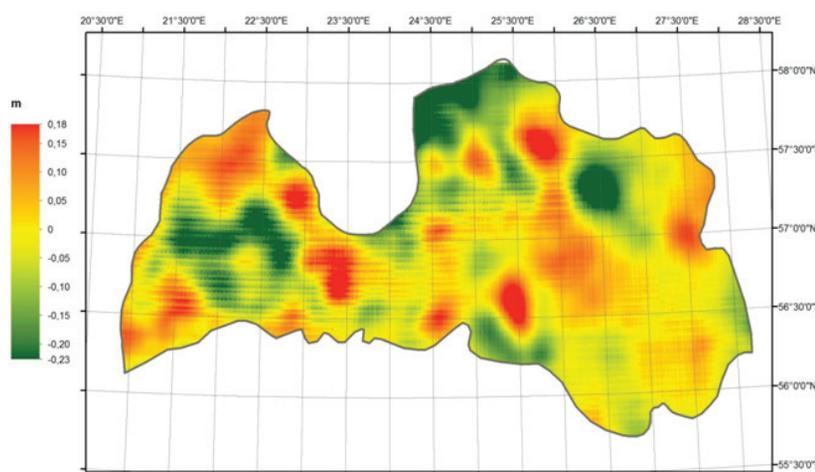


Fig. 4. Difference between LV'98 and DFHRS solution

4. Comparison with EGG97 and Global Geopotential Models

Because the global models are not fitted to national height system, following transformation was performed:

$$\begin{aligned}
 h - H &= N_G - \left[\cos(L) \cdot \cos(B) \right] \cdot \\
 (u - u_G) &+ \left[\cos(B) \cdot \sin(L) \right] \cdot (v - v_G) + \left[\sin(B) \right] \cdot \\
 (w - w_G) &+ \left[e^2 \cdot N \cdot \sin(B) \cdot \cos(B) \cdot \sin(L) \right] \cdot \\
 (e_x - e_{x,G}) &+ \left[-e^2 \cdot N \cdot \sin(B) \cdot \cos(B) \cdot \cos(L) \right] \cdot \\
 (e_y - e_{y,G}) &+ \left[h + W^2 \cdot N \right] \cdot \Delta m + \left[N_G + W^2 \cdot N \right] \cdot \Delta m_G \quad (2)
 \end{aligned}$$

where point position is represented by geographical latitude and longitude (B, L), h is ellipsoidal height, H is orthometric height, N_G is geoid height, N is normal radius of curvature, $W = a / N$, $e^2 = 2f - f^2$ ($a =$ main axis $f =$ flattening of reference ellipsoid GRS80), three translations ($u - u_G, v - v_G, w - w_G$), two rotations ($e_x - e_{x,G}, e_y - e_{y,G}$) in horizontal plane and scale difference ($\Delta m, \Delta m_G$) are present (Jäger 1999).

For the transformation of European Gravimetric Geoid Model 1997 and global geopotential models to national height system, 13 fitting points evenly distributed within territory of Latvia were introduced.

After the fitting of EGG97 and GGM's, the comparisons between EGG97, EGM2008 (Pavlis *et al.* 2008), Eigen5c (Förste *et al.* 2008), Eigen6c (ICGEM), GOCE GO_CONS_GFC_2_DIR_R3 (Brunisma *et al.* 2010) models and LV'98, as well DFHRS solution, were made. The results are shown in Table 2.

The comparison with global models shows that LV'98 model and DFHBF solution has the best agreement with EGG97 and EGM2008 – RMS is 4–6 cm. Because EGG97 model was implemented in DFHRS solution, its RMS is 1.7 cm. GO_CONS_GFC_2_DIR_R3 model contain satellite only information, derived from the tracking of artificial Earth satellites – Goce, Grace and Lageos. This satellite only model has the worst agreement with both LV'98 and DFHRS solution. Some of the comparisons are shown in Figs. 5 and 6.

Similar statistic results were obtained using the same 102 GPS/levelling points which were used during DFHRS computation for datum definition. 2σ (96 GPS/levelling points) results are shown in Table 3.

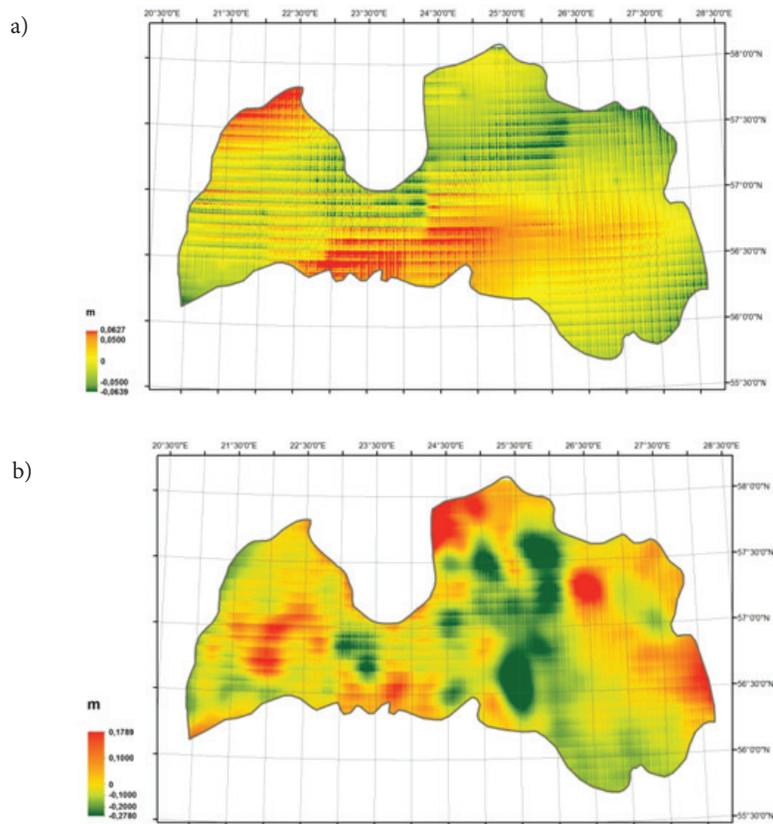


Fig. 5. Differences between DFHRS and EGG97 (a), DFHRS and EGM2008 (b)

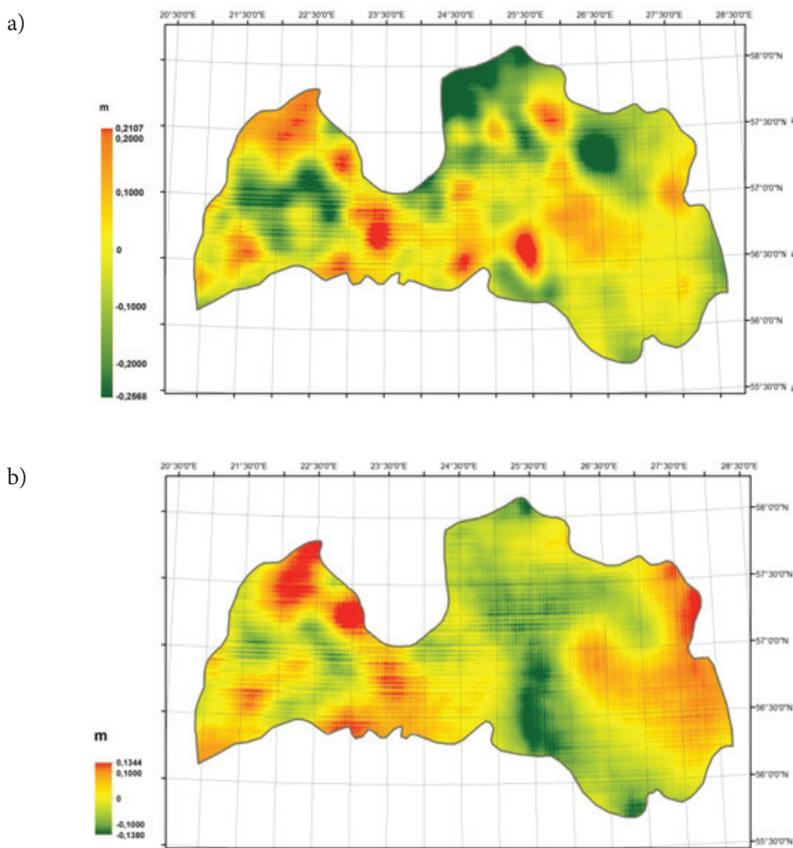


Fig. 6. Differences between LV'98 and EGG97 (a), LV'98 and EGM2008 (b)

Table 2. Comparison with EGG97 and Global Geopotential Models

	Min (m)	Max (m)	Average (m)	RMS (m)
DFHRS-EGG97	-0,064	0,063	-0,013	0,017
DFHRS-EGM2008	-0,278	0,179	-0,046	0,052
DFHRS-Eigen6c	-0,233	0,188	0,005	0,064
DFHRS-Eigen5c	-0,206	0,672	0,130	0,115
DFHRS- GO_CONS_GFC_2_DIR_R3	-0,460	1,030	0,381	0,266
LV'98-EGG97	-0,257	0,211	0,000	0,058
LV'98-EGM2008	-0,138	0,134	-0,033	0,044
LV'98-Eigen6c	-0,203	0,201	0,018	0,063
LV'98-Eigen5c	-0,211	0,480	0,142	0,123
LV'98- GO_CONS_GFC_2_DIR_R3	-0,461	1,077	0,393	0,272

Table 3. Fitting point statistics for EGG97 and Global Geopotential Models (2σ – 95% of fitting points)

	Min (m)	Max (m)	RMS (m)
EGG97	-0,078	0,074	0,037
EGM2008	-0,085	0,079	0,042
Eigen6c	-0,151	0,125	0,055
Eigen5c	-0,162	0,317	0,153
GO_CONS_GFC_2_DIR_R3	-0,662	0,745	0,346

5. Quality control based on independent measurements

Table 4 shows three levelling control points, which were not used for the computation of Latvian QGeoid HRS. An example of quality control of Moldova QGeoid HRS is presented at (Jäger 2010). The GNSS control measurements were performed on 14 December

2012. by geodesists of Latvia University of Agriculture, Faculty of Rural Engineering, as a static 4 hour measurements. $H_{levelling}$ values in BAS-77 (Baltic Height System 1977) are obtained from Latvian Geospatial Information agency database. Point No 1636 is located in North-West part of Latvia, point No 1155 is located in West central part and point No 1727 is located in central part of Latvia. Table 4 gives an overview of the results of GPS control measurement comparison of $H_{levelling}$ and $h_{ellipsoidal}$ with N_{DFHRS} . Differences between $H_{levelling}$ and $H_{GNSS} = h_{ellipsoidal} - N_{DFHRS}$ are between 0 cm to 2.6 cm.

6. Detection of height deformations in the area of Riga using the DFHRS database

11 levelling control points, which were not used for the computation of Latvian QGeoid HRS, were selected in

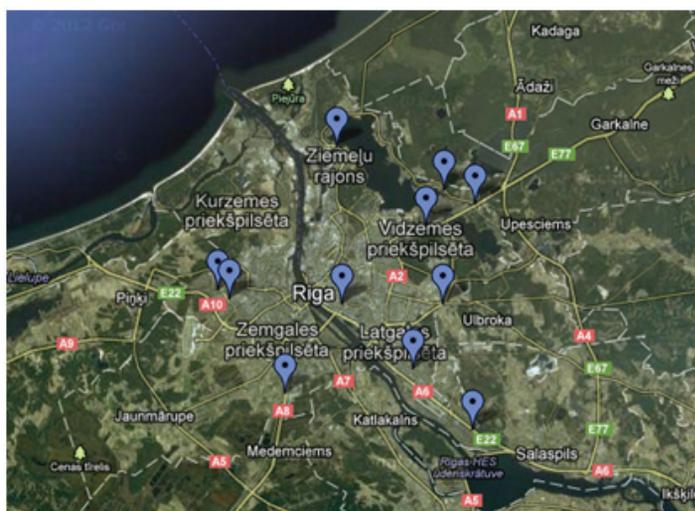


Fig. 7. Selected independent levelling control points in Riga city

Table 4. 1st order leveling benchmarks for quality control

1 st order levelling benchmark name, No	Year of levelling	Latitude	Longitude	$h_{ellipsoidal}$ (m)	$H_{levelling}$ (m)	N_{DFHRS} (m)	Difference to N_{DFHRS} (m)
Mērsrags, 1636	2007	57,38516	22,97266	27,620	6,890	20,704	0,026
Vāne, 1155	2007	56,94707	22,44686	116,975	94,480	22,495	0,000
Eleja, 1727	2005	56,41508	23,69842	55,359	32,380	23,001	-0,022

Table 5. Levelling benchmark measurement comparison to DFHRS computed QGeoid HRS values

Point No	Latitude	Longitude	$h_{\text{ellipsoidal}}$ (m)	$H_{\text{levelling}(1977)}$ (m)	N_{DFHRS} (m)	Difference to N_{DFHRS} (m)
5715	56,90561	24,21107	32,890	11,811	21,116	-0,037
1193	56,86987	24,27417	31,844	10,566	21,258	0,020
915	56,94540	24,01676	32,804	11,823	21,036	-0,055
3389	56,89100	24,07539	33,490	12,314	21,213	-0,038
8540	57,03301	24,13018	30,232	9,623	20,682	-0,072
37	57,00825	24,24445	27,707	7,042	20,728	-0,063
938	57,00033	24,27682	34,027	13,328	20,756	-0,058
3336	56,94287	24,24295	30,359	9,439	20,976	-0,056
T8	56,94285	24,13518	25,691	4,746	20,997	-0,052
0895	56,95117	24,00392	29,418	8,415	21,021	-0,017
868	56,98750	24,22432	29,513	8,736	20,814	-0,037
					Mean Systematic Difference	-0,0489

Riga city territory (Fig. 7). These points are considered as the most stable Riga city levelling network points (ground benchmarks), $H_{\text{levelling}(1977)}$ values are taken from 2nd order levelling network catalogue of years 1975–1977, $h_{\text{ellipsoidal}}$ values were obtained during RTK measurements in 2010 (Balodis et al. 2011).

Table 5 gives an overview on the sites of the GNSS RTK measurement and levelling catalogue value comparison to DFHRS computed QGeoid HRS values. The mean difference of this comparison reaches -4.9 cm. However, since 1977, Riga levelling network has not been controlled. According the investigation of (Silabriedis 2012) the Riga leveling network has experienced significant deformations. Table 5 shows that height differences between $H_{\text{GNSS}} = h_{\text{ellipsoidal}} - N_{\text{DFHRS}}$ and $H_{\text{levelling}(1977)}$ reflect the height deformations of 5 cm subsidence since years 1975–1977.

Conclusions

QGeoid height reference surface for Latvia of RMS 1.6 cm was obtained using DFHBF software. In case of poor coverage of fitting point data it is possible to change input parameters (mesh and patch size) of DFHBF software to obtain better accuracy. High accuracy geoid height reference surface can be achieved by minimum number of observations (102 fitting points). The DFHBF software version 5 (www.dfhbf.de) will also be able to handle terrestrial gravity observations in the integrated Least-Squares-Adjustment approach, which can – in opposite to the Stokes approach – be controlled each by data-snooping. The comparisons with Global Geopotential Models and fitting point quality assessment show that the high resolution and integrated EGM2008 has the agreement with both LV'98 and new DFHBF based solution.

Currently the project of digital zenith camera and its control software for vertical deflection measurements is under development at the University of Latvia, Institute of Geodesy and Geoinformation (Ābele et al. 2012). After obtaining first vertical deflection measurements, these

will be used for an update of the new Latvian QGeoid model solution, using again the DFHBF software.

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