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COMPARISON OF DIFFERENT MERCATOR PROJECTION SYSTEMS FOR THE TOPOGRAPHICAL SURVEY OF THE BUCHANAN-TOKADEH (YEKEPA) RAILWAY, LIBERIA, AFRICA

Juan Alberto IBARRA GRANDA[1, 2], Edgar SOARES DA SILVA[3]

[1]*Instituto Geográfico Agustín Codazzi (IGAC), Bogotá, Colombia*
[2]*External researcher, Universidad Nacional Mayor de San Marcos, Lima, Peru*
[3]*Freelancer, Consulting*

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Abstract. For this project of the Buchanan-Tokadeh (Yekepa) railway located in the Republic of Liberia, Africa, it is about carrying out comparative analysis of precisions between two projection systems, the Local Transverse Mercator (LTM), divided into 15 zones; and Hotine Oblique Mercator (HOM); as well as the projection heights of their Local Topographic Planes (LTP) both in orthometric heights to determine the scale factor, as well as ellipsoidal ones to extend the semi-axes of the ellipsoid of the WGS84 datum to give an scale factor equal to 1 of these systems, along 245 km of route, taking as reference tolerances precisions that must be within 1/40,000 or 2.5 cms per kilometer, and also that the difference in height does not exceed 150 meters. Getting results by performing survey traverse polygon made with a total station, in a way that the deformations using an LTM divided into 15 zones and the HOM expanding the semi-axes of the reference ellipsoid of the Datum WGS84 complying with the aforementioned standards.

Keywords: LTM, HOM, LTP, projection height, mean sea level, ellipsoidal height, scale factor, WGS84 datum, deformation.

✉Corresponding author. E-mails: ***cartografia1992@gmail.com; juan.ibarrag@unmsm.edu.pe***

1. Introduction

In some countries, their official mapping agencies have adopted map projection systems that are not suitable for large scale mapping representations due to severe deformations, such as the Universal Transverse Mercator (UTM) projection used in the Republic of Liberia, which is located in Zone 29 North (Bundoo, 2013).

For this reason, there are methodologies that allow to reduce the deformations of a cartographic projection system for linear works, whether for railroads, roads, etc.; one of them is based on the Local Topographic Plane (LTP), who define it as: "A local projection system on a two-dimensional Cartesian plane at the average height where the representation is made (on a portion of the Earth's surface), in which we incorporate cartographic standards, is called LTP, orthogonal grid for fully topographic purposes" (Castillo Becerra, 2015, p. 13).

According to the previous announcement, the LTP depends on the heights above mean sea level and/or ellipsoid where the distances measured on the ground are similar to the projected ones. For this, an average height is needed so that the deformations are minimal, called projection height. This system can be coupled with any cartographic projection.

There are some research antecedents or standards for these linear projects. These will be briefly described in this research.

According Chilean Roads Manual (Ministry of Public Works of Chile & Viability Directorate, 2018), for these projects they use local projections called LTM (Local Transverse Mercator), in which their scale factors are based on the heights of the LTP (kh) at the central meridian (MCL). These projected planes are positioned approximately at ground level. The LTM cover an extension of ½ degree on each side of the MCL with an accuracy at the extremes of the zone of 1/30,000 or 3.3 cm/km, which is the maximum tolerable accuracy considering the terrestrial radius of 6,378,000 meters, as well as altitudes above sea level, and provided that the differences in LTP heights do not exceed 250 meters. However, the Brazilian standard NBR 14166 (1998) indicates that the height difference limit for a LTP should not exceed 150 meters, i.e., approximately 2.5 cm/km or 1/40,000.

Another type of cartographic projection studied for this type of project was the Hotine Oblique Mercator (HOM) by Enríquez Turiño (2009), which consisted of a theoretical route for a high-speed railroad from the southwest of the Iberian Peninsula to the border of France in the northeast, starting from Faro (Portugal), through the cities of Huelva, Seville, Cordoba, Ciudad Real, Toledo, Madrid, Guadalajara, Zaragoza, Lleida, Barcelona, ending in Gerona; using the method called Variant B, that is, central point and azimuth of the central line.

On the other hand, Santitamnont et al. (2021) conducted a study titled "A Study on WGS-TM Map Projection of the Thai-Chinese High-Speed Rail", focuses on the Transverse Mercator. The authors found incomplete and inaccurate information regarding the three previously used projection systems, including ellipsoidal heights (hpp) for a coverage of 81 control points to define these systems, the researchers experimented with the ellipsoidal heights by extending the semi-axes major and minor of the WGS84 datum ellipsoid. The results were satisfactory when compared with the collected coordinate data. However, this method is considered experimental for linear projects and is not widely used. Therefore, the scale factor was determined iteratively using the trial-and-error method and similar results were obtained.

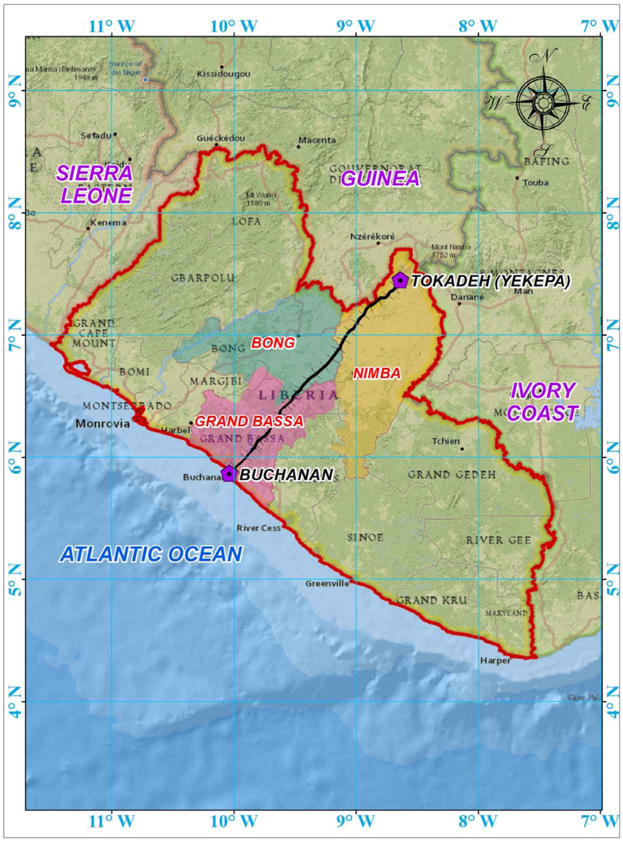

Figure 1. Location of the Buchanan–Tokadeh railroad (source: Esri, Garmin, HERE, UNEP-WCMC, USGS, NASA, ESA, METI, NRCan, GEBCO, NOAA, iPC HDX, & National Geographic, 2024)

2. Materials and methods

2.1. Area of study

The railroad spans 245 kilometers in the Republic of Liberia, Africa, connecting Port Buchanan and Tokadeh (Yekepa) through the counties of Grand Bassa, Bong, and Nimba. It is situated between meridians 8°38′ W and 10°3′ W, and parallels 5°52′ N and 7°27′ N (see Figure 1).

2.2. Data

The data set comprised a 12.5 meters resolution ALOS PALSAR digital elevation model (DEM), developed by the Alaska Satellite Facility project. A PGM format raster file containing geoidal undulations information from the EGM 2008 model of 1 arcminute resolution was downloaded from the GeographicLib website. Finally, eight survey traverse polygons were measured with a total station and GPS, strategically chosen along the railroad route to observe the behavior of the deformations to be analyzed.

2.3. Methodology

Using ArcGIS 10 software, points in shape format were placed at 500-meter intervals near the railway track area. This was done to extract the altimetric information from the points on the DEM ALOS PALSAR. The information was then exported from the shape format to text, which included geographical coordinates and height. This information was used to process the longitudinal profile of the terrain in order to construct the local topographic plane (LTP), which will be described later (see Figure 2).

Figure 2. Longitudinal railroad profile

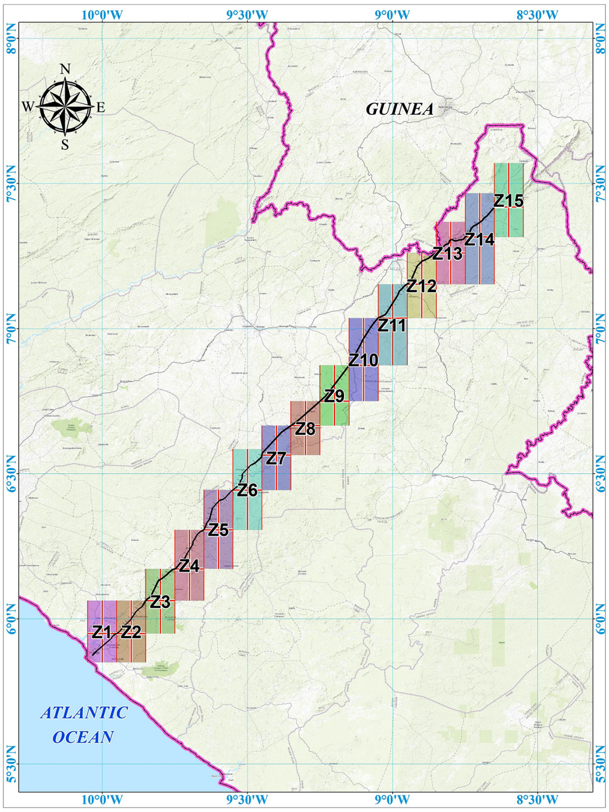

Figure 3. Location of the 15 LTM zones (source: own elaboration based on Esri, HERE, Garmin, Intermap, Increment P Corp., GEBCO, USGS, FAO, NPS, NRCan, GeoBase, IGN, Kadaster NL, Ordnance Survey, Esri Japan, METI, Esri China (Hong Kong), © OpenStreetMap contributors, GIS User Community, & HDX, 2024)

It was delimited 15 zones of the Local Transverse Mercator projection (LTM) (see Figure 3), each zone has 6′ of amplitude, that is, approximately 10 km, whose central meridian divides the zone in 3′on both sides of it. The reason or criterion to establish that amplitude, arose from the reading on Road Manual of Paraguay which expresses the following: "The base polygonal will be constituted by fixed vertices (monumented) that will be the control and closure points of all the topographic works that are later carried out in the final design, these points should be placed every 10 km and the same should be properly georeferenced with the specifications defined in the Engineering manual "CONTROL AND TOPOGRAPHIC SURVEYING EM 1110-1-1005" of the U.S. Corps of Engineers" (Ministry of Transport and Communications of Paraguay, 2019, p. 285).

For the calculation of the zone scaling factors, four LTP were determined as shown (see Figure 4).

The first LTP takes as reference the mean sea level or geoid, up to the boundary between zones Z3 and Z4, with an approximate altitude of 135 meters above sea level, because if the boundary between zones Z4 and Z5 is considered with an altitude of 180 meters above sea level, it would exceed the reference tolerance of the standard mentioned in the introduction, which is 150 meters.

In the second LTP, we start from the boundary between zones Z3 and Z4, with an approximate height difference of a maximum of 110 meters. Therefore, the projection height is 244.90 meters above sea level, so that in the other zones tend to reduce until the boundary between zones Z12 and Z13 with an altitude of 280 meters above sea level, in which the height difference would be 35.1 meters.

The Third LTP only includes the Z13 zone, as the altitudes range from approximately 280 to 395 meters above mean sea level, a difference of 115 meters. Therefore, the projection height of 327.755 masl was selected.

Starting from the limit between zones Z13 and Z14, the last LTP covers up to the last Z15 zone, with a maximum altitude of 475 meters above mean sea level, resulting in a range of 80 meters. Therefore, the projection height of 464.42 meters above mean sea level was considered.

The scale factors (Ko) of the LTM zones will be determined using the height factor formula (Kh) according to the Chilean Roads Manual (Ministry of Public Works of Chile & Viability Directorate, 2018). These scale factors will also be used for the HOM:

$$Ko = Kh = \frac{R + H}{R}, \tag{1}$$

where: R – mean radius of the earth 6 378 000 meters; H – orthometric altitude (mean sea level).

Once the scale factors have been calculated, the other parameters of the zones are completed as shown in Table 1.

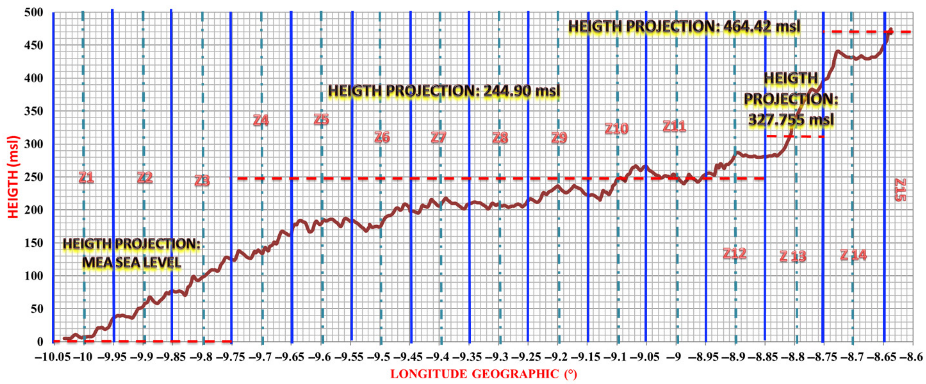

Figure 4. Terrain profile with the 15 zones LTM 6′ and 4 LTP

Table 1. Origins of the 15 LTM 6′ zones

ZONE	HEIGTH PROJECTION	SCALE FACTOR (Ko)	MC	PARALELL ORIGIN	HEIGTHS DIFFERENCE	EAST (m)	NORTH (m)
Z1	0	1	10°00′ W	05°56′57.6492″ N	5–135	500000.000	657868.424
Z2	0	1	09°54′ W	05°56′57.6492″ N		511072.392	657868.424
Z3	0	1	09°48′W	06°03′46.17″ N		522142.491	670417.551
Z4	244.9	1.0000383976	09°42′W	06°10′19″N	135–280	533210.559	682484.730
Z5	244.9	1.0000383976	09°36′W	06°18′24.3″N		544276.012	697393.099
Z6	244.9	1.0000383976	09°30′W	06°26′37.662″N		555338.527	712549.210
Z7	244.9	1.0000383976	09°24′W	06°33′51.9588″N		566398.404	725890.901
Z8	244.9	1.0000383976	09°18′W	06°39′56.7072″N		577456.029	737096.100
Z9	244.9	1.0000383976	09°12′W	06°45′0.3348″N		588511.752	746423.682
Z10	244.9	1.0000383976	09°06′W	06°52′24.42″N		599564.651	760066.244
Z11	244.9	1.0000383976	09°00′W	07°02′09.9348″N		610613.747	778053.716
Z12	244.9	1.0000383976	08°54′W	07°09′12.5928″N		621660.046	791038.177
Z13	327.755	1.0000513883	08°48′W	07°15′37.116°N	280–395	632703.829	802851.157
Z14	464.42	1.0000728159	08°42′W	07°25′7.3848″W	395–475	643746.447	808975.844
Z15	464.42	1.0000728159	08°36′W	07°18′56.4768″W		654786.633	820371.024

For the construction of the Hotine Oblique Mercator (HOM) system, the variant B method was used (Snyder, 1987), which consists of the central point of the projection and the geodetic azimuth of the center line. For this, four central points were determined that are located on the center line with an azimuth of 41°33′40.3224″, of which the projection parameters are shown, with the same scale factors previously calculated (Figure 5).

The projection parameters, with the same scale factors previously calculated, are shown in Table 2.

The semi-axes of the WGS 84 Datum ellipsoid have also been extended so that the scale factor of these systems is the unit, for this purpose the geoidal undulations have been calculated to determine the ellipsoidal heights of projection of the 4 LTP'S, supported on the geoidal model EGM 2008 of 1′ of arc (Karney, 2023), leaving the parameters established for both systems (see Tables 3 and 4).

According to Santitamnont et al. (2021):

$$a' = a + h; \tag{2}$$

$$b' = b + h, \tag{3}$$

where: a′ and b′ are the extended semi-axes of the ellipsoid; a – semi-major axis of the ellipsoid; b – minor semi-axis of the ellipsoid; h – Ellipsoidal height.

To calculate deformation, it is necessary to consider the combined scaling factor (*kc*). This factor represents the ratio of ground distance to the projected distance (Ivars, 2014), of the systems in both LTM and HOM

$$kc = \frac{project\ \ distance}{ground\ \ distance}. \tag{4}$$

To express deformation, a distance of 1000 meters (equivalent to 100000 centimeters) is used as a reference. The expression is the absolute value of cm per km (Marcos de França & Walter Uhlman, 2007)

$$\text{Deformation} = |\, kc \times 100000 - 100000 \,|. \tag{5}$$

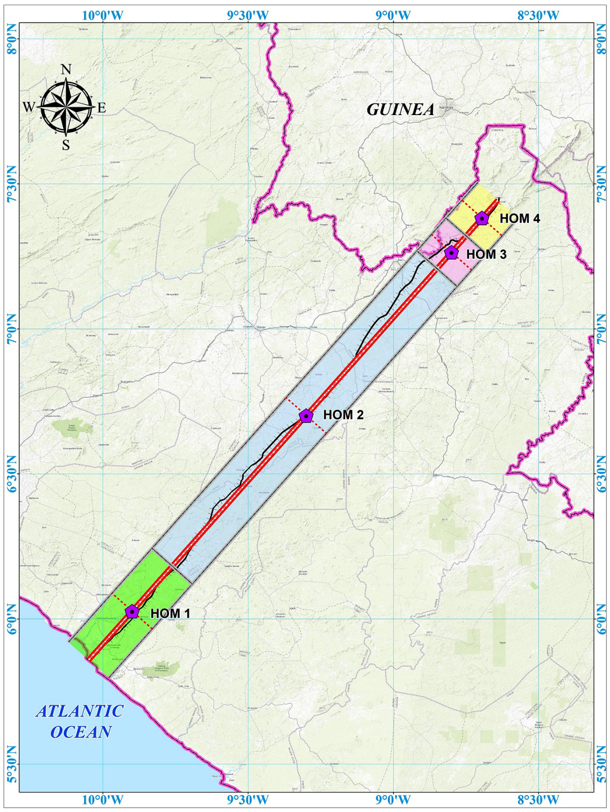

Figure 5. HOM projection of the railway (source: own elaboration based on Esri, HERE, Garmin, Intermap, Increment P Corp., GEBCO, USGS, FAO, NPS, NRCan, GeoBase, IGN, Kadaster NL, Ordnance Survey, Esri Japan, METI, Esri China (Hong Kong), © OpenStreetMap contributors, GIS User Community, & HDX, 2024)

Table 2. Parameters of the HOM projection of the railway

PARAMETERS	HOM 1	HOM 2	HOM 3	HOM 4
LONGITUDE POINT CENTRAL	06°39′ 5.0796″ W	06°39′ 5.0796″ W	06°39′ 5.0796″ W	06°39′ 5.0796″ W
LATITUDE POINT CENTRAL	09°20′ 40.3224″ N	09°20′ 40.3224″ N	09°20′ 40.3224″ N	09°20′ 40.3224″ N
AZIMUT LINE CENTRAL	41°32′39.2496″	41°32′39.2496″	41°32′39.2496″	41°32′39.2496″
SCALE FACTOR (Ko)	1	1.0000383976	1.0000513883	1.0000728159
FALSE EAST	461918.101	461918.101	461918.101	461918.101
FALSE NORTH	735227.103	735227.103	735227.103	735227.103
DATUM	WGS84	WGS84	WGS84	WGS84
LATITUDE LIMITS	05°47′50.473″N -	06″07′14.450″ N -	07°08′46.742°N -	07°15′32.404″N-
	06″14′37.304″ N	07°16′9.595°N	07°22′55.258″N	07° 30′ 19.685″ N

Table 3. Origins of the 15 LTM 6′ zone

ZONE	MAJOR SEMIAXIS	MINOR SEMIAXIS	HEIGTH PROJECTION	SCALE FACTOR (Ko)	MC	PARALELL ORIGIN	EAST (m)	NORTH (m)
1	6378166.950	6356782.264	29.5	1	10°00′W	05°56′57.6492″N	500000.000	657868.424
2	6378167.070	6356782.384	30.07	1	09°54′W	05°56′57.6492″N	511072.392	657868.424
3	6378167.02	6356782.334	30.02	1	09°48′W	06°03′46.17″ N	522142.491	670417.551
4	6378412.09	6357027.404	275.09	1	09°42′W	06°10′19″N	533210.559	682484.730
5	6378412.5	6357027.814	275.5	1	09°36′W	06°18′24.3″N	544276.012	697393.099
6	6378412.84	6357028.154	275.84	1	09°30′W	06°26′37.662″N	555338.527	712549.210
7	6378413.12	6357028.434	276.12	1	09°24′W	06°33′51.9588″N	566398.404	725890.901
8	6378413.31	6357028.624	276.31	1	09°18′W	06°39′56.7072″N	577456.029	737096.100
9	6378413.44	6357028.754	276.44	1	09°12′W	06°45′0.3348″N	588511.752	746423.682
10	6378413.72	6357029.034	276.72	1	09°06′W	06°52′24.42″N	599564.651	760066.244
11	6378414.19	6357029.504	277.19	1	09°00′W	07°02′09.9348″N	610613.747	778053.716
12	6378414.5	6357029.814	277.5	1	08°54′W	07°09′12.5928″N	621660.046	791038.177
13	6378497.655	6357112.969	360.655	1	08°48′W	07°15′37.116°N	632703.829	802851.157
14	6378634.44	6357249.754	497.44	1	08°42′W	07°25′7.3848″W	643746.447	808975.844
15	6378634.75	6357250.064	497.75	1	08°36′W	07°18′56.4768″W	654786.633	820371.024

Table 4. Parameters of the HOM projection of the railroad

PARAMETERS	HOM 1	HOM 2	HOM 3	HOM 4
LONGITUDE CENTRAL POINT	06°39′ 5.0796″ W	06°39′ 5.0796″ W	06°39′ 5.0796″ W	06°39′ 5.0796″ W
LATITUDE CENTRAL POINT	09°20′ 40.3224″ N	09°20′ 40.3224″ N	09°20′ 40.3224″ N	09°20′ 40.3224″ N
AZIMUT LINE CENTRAL	41°32′39.2496″	41°32′39.2496″	41°32′39.2496″	41°32′39.2496″
SCALE FACTOR (Ko)	1	1	1	1
FALSE EAST	461918.101	461918.101	461918.101	461918.101
FALSE WEST	735227.103	735227.103	735227.103	735227.103
HEIGTH PROJECTION	31.38	276.28	359.135	495.8
DATUM	WGS84	WGS84	WGS84	WGS84
SEMIAXIS MAJOR	6378168.38	6378413.28	6378496.135	6378632.8
SEMIAXIS MINOR	6356783.894	6357028.794	6357111.649	6357248.314
LIMITS LATITUDES	05°47′50.473″N -	06″07′14.450″ N -	07°08′46.742°N -	07°15′32.404″N-
	06″14′37.304″ N	07°16′9.595°N	07°22′55.258″N	07° 30′ 19.685″ N

3. Results and discussion

If it is observed in Figure 6, the survey polygon traverse RB04-RT07, between km 4+535.61 and 7+490.1, approximately of 3 km; the deformation is small in both LTM Z1 zone and HOM 1, if the semi-axes of the ellipsoid of the WGS84 datum are enlarged according to their projection ellipsoidal heights (29.5 and 31.38 meters, respectively) compared to the scale factor (which is 1) that is at mean sea level (geoid); with a maximum approximate deformation of 0.16 cm/km and 0.63 cm/km, i.e. accuracies of 1/631 220 and 1/158 878, respectively; while the minimum is approximately 0.08 and 0.55 cm/km, i.e. accuracies of 1/1 195 956 and 1/180 334, respectively, which are within the tolerances of the accuracies of the standard NBR 14166/1998.

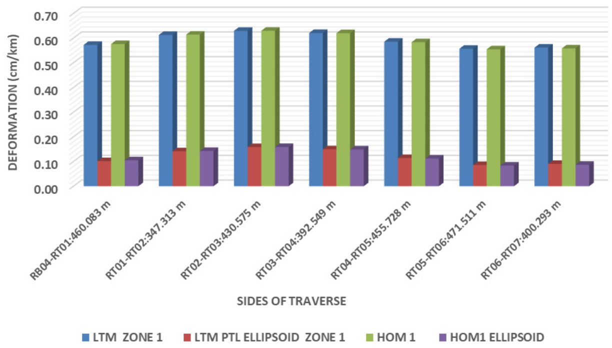

Figure 6. Deformation in the RB04-RT07 survey traverse polygon

In the survey traverse polygon RB39-RB43 (see Figure 7), there is a distance of approximately 3.2 kilometers between km 42+652.35 and 45+897.96. Both systems (LTM zone Z3 and HOM 1) show an increasing deformation with the same scale factor as the previous one. This is due to the fact that the altitudes are no longer close to the mean sea level or the ellipsoid. The maximum and minimum deformations are 2.19 and 1.68 cm/km, the accuracies are 1/45,740 and 1/59,672 respectively. When extending the semi-axes of the ellipsoid of the WGS84 datum, with altitudes of ellipsoidal projections of 30.02 meters for LTM zone Z3 and 31.38 meters for the HOM 1, the deformations are smaller compared to the previous ones. The maximum and minimum deformations are 2.08 cm/km and 1.61 cm/km respectively, with precisions of 1/48,050 and 1/62,085. However, all these values fall within the tolerances of the norm.

Continuing the analysis, in the survey traverse polygon RB 56-RB63 (see Figure 8), between kilometer

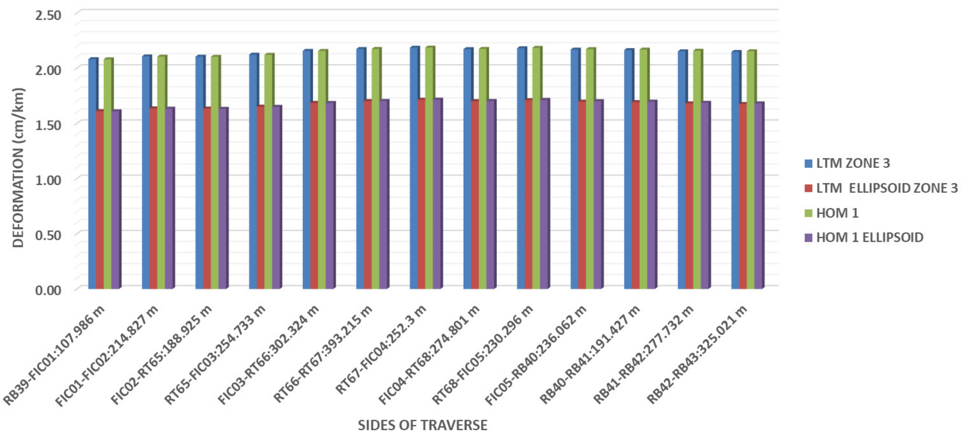

Figure 7. Deformation in the RB39-RB-43 survey traverse polygon

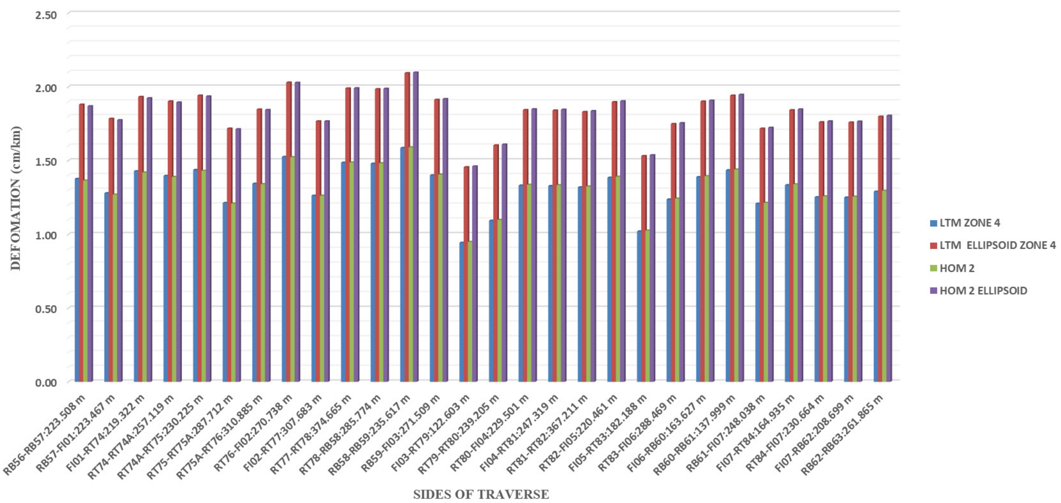

Figure 8. Deformation in the survey traverse polygon RB56-RB63

51+294.8 and 58+101.2, which is 6.8 kilometers away, the LTM system zone 4 and HOM 2 have a scale factor of 1.0000383976 instead of 1 due to the implementation of the LTP with a projection height of 244.9 meters above mean sea level, which includes up to zone Z12. The maximum deformation is 1.59 cm/km with a precision of 1/63,042, while the minimum is 1.02 cm/km with a precision of 1/98,039. When expanding the semi-axes of the ellipsoid of the WGS84 datum, the maximum deformation is 2.10 cm/km with a precision of 1/47,732, and the minimum is 1.53 cm/km with a precision of 1/65,156. Although these deformations are greater than the previous ones, all values are within the tolerance of the norm.

In the survey traverse polygon RBC16A-RBC20D (see Figure 9), located between kilometers 105+458.47 and 109+844.62, with an approximate distance of 4.4 kilometers, which corresponds to the LTM zone 7 and HOM 2, with the same scale factor of the previous survey traverse polygon, the maximum deformation of the sides of the traverse polygon is 1.18 cm/km, with a precision of 1/85 097, and the minimum deformation of 0.22 cm/km, a precision of 1/453 560; while with the extension of the semi-axes of the ellipsoid of the WGS84 datum for both systems, the maximum is 1.67 cm/km with a precision of 1/59 928, while the minimum is 1 cm/km, precision of 1/99 838. When comparing these deformations, it is observed that by expanding the semi-axes of the ellipsoid of the WGS84 datum they are greater than the systems that have the scale factor of 1.0000383976, however, the trend of these deformations is decreasing if they are observed with the previous traverse polygon and also continue to remain within the NBR 14166/1998 standard.

In the survey traverse polygon RGN31A-RGN11 (see Figure 10), it includes kilometers 199+977.12 and 209+116.78, with a distance of 9.2 kilometers, it is located in Zone 12 of the LTM system and also corresponds to the HOM 2 system with a scale factor also of 1.0000383976, it can be seen that the deformations continue to decrease compared to the previous surveys traverses polygons analyzed, this is because the altitudes of the traverse sides are between 264,757 and 288,429 meters above sea level, which do not differ by more than 50 m with the projection height of the LTP (244.9 meters above sea level), the same happens if the semi-axes of the ellipsoid of the WGS84 datum are expanded, however the latter registers lower deformations for both systems than those that are

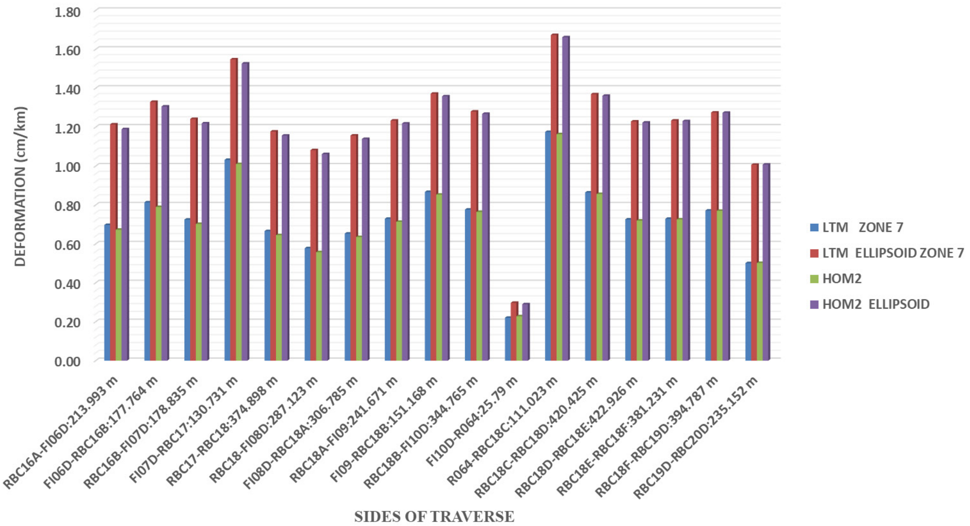

Figure 9. Deformation in the RBC16A-RBC20D survey traverse polygon

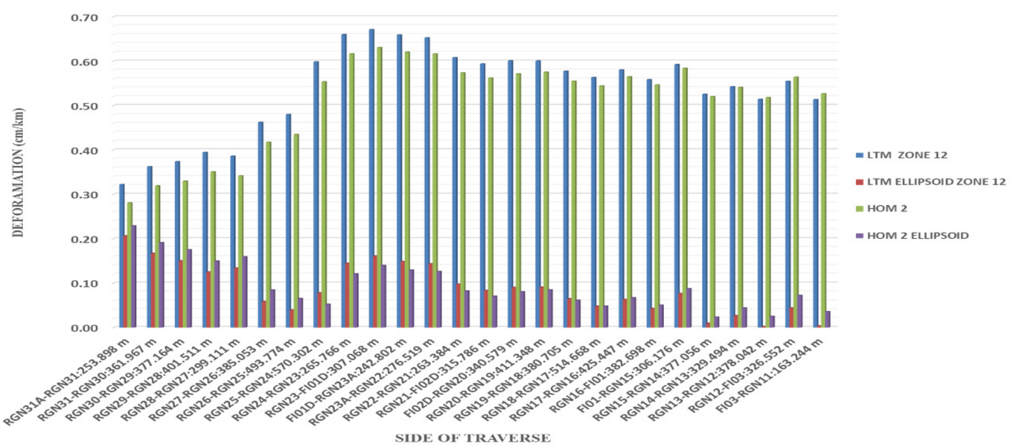

Figure 10. Deformation in the RGN31A-RGN11 survey traverse polygon

referred to the mean sea level, having the maximum deformation 0.21 cm/km with a precision of 1/483 991, and the minimum 0.004 cm/km, precision of 1/2 588 4775 for both the LTM and the HOM, compared to the previous systems described, which have a maximum deformation of 0.67 cm/km, that is, a precision of 1/149 017 and the minimum 0.28 cm/km, precision 1/356 278. All these results are within the tolerances of the norm.

In the LTM system, the Z13 zone is used with the LTP whose projection height is 327.755 meters above sea level, and with the scale factor of 1.00005146, the survey traverse polygon RT01-K509 with a distance of 5.8 kilometers (see Figure 11), located between kilometers 217+406.32 and 223+ 201.20, the deformations are minimum reaching 0.22 cm/km and maximum 1.07 cm/km, that is, with precisions of 1/462 364 and 1/93 403 respectively; the same trend for the HOM 3 projection because it has the same scale factor. Expanding the semi-axes of the ellipsoid of the WGS84 datum, based on the ellipsoidal projection height of 360.655 meters for the LTM and 359.135 meters for the HOM 3, the minimum and maximum deformations are approximately 0.06 and 0.54 cm/km, precisions of 1/1 627 543 and 1/186 885 respectively. It is observed that the deformations are low, therefore, the accuracies are greater compared to the projection height at mean sea level. On the other hand, this analysis shows that all these values are also within the tolerances of the NBR 14166/1998 standard.

In the Z14 and Z15 zones of the LTM system, the change of scale factor is made and a new LTP is implanted whose scale factor is 1.000072919 with orthometric height of projection of 464.42 above sea level, the survey traverse polygon K506-RS03, of 7.7 kilometers approximately of distance (see Figure 12), is located between kilometers 226+201.5 and 231+201. 2, in the Z14 zone and also corresponds to the projection HOM 4. The deformations in the distances are approximately in the maximum 1.06 cm/km and the minimum is 0.33 cm/km that correspond with accuracies of 1/93 919 and 1/307 813 respectively, from which it is appreciated that the accuracies are high in comparison when enlargement is given in the semi-axes of the ellipsoid of the WGS84 datum, with a height ellipsoidal

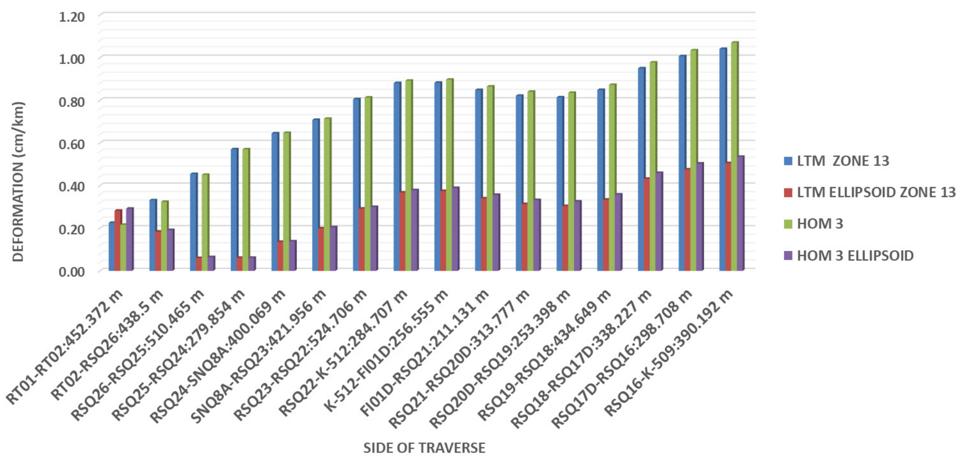

Figure 11. Deformation in the RT01-K509 survey traverse polygon

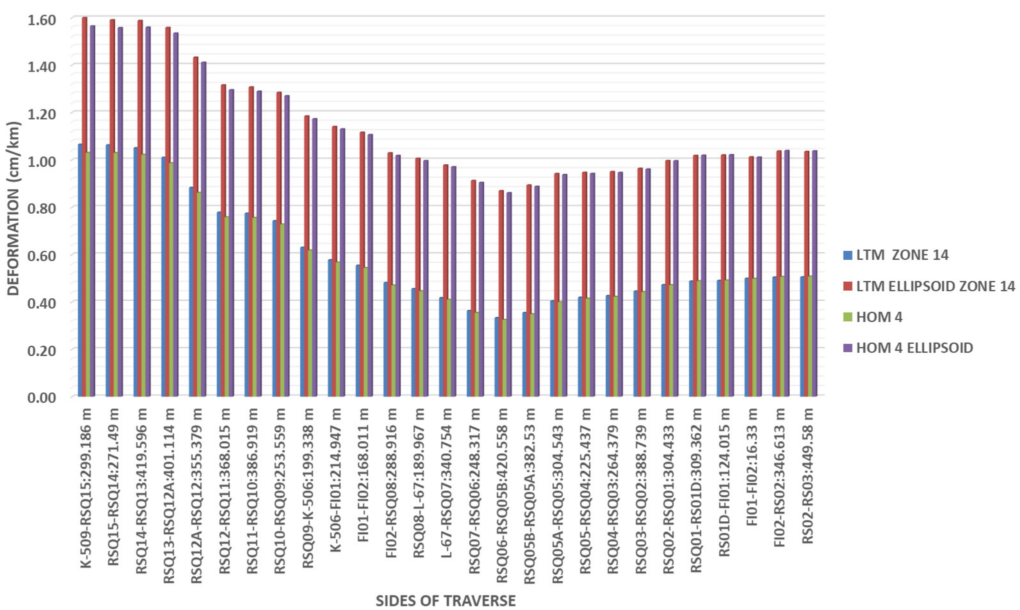

Figure 12. Deformation in the K-506-RS03 survey traverse polygon

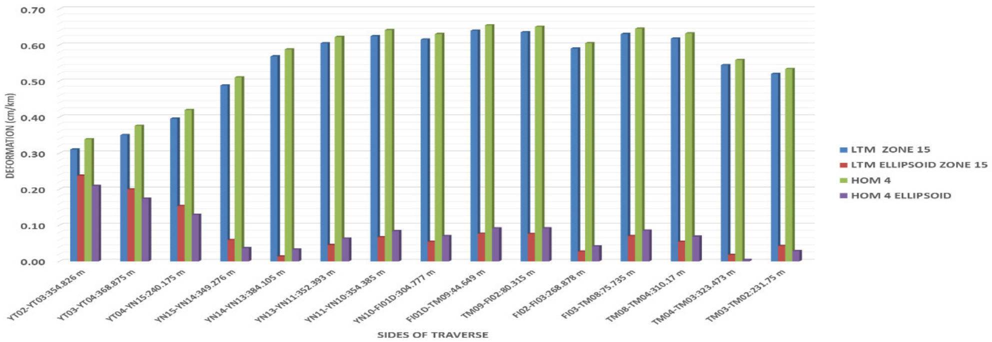

Figure 13. Deformation in the YT02-TM02 survey traverse polygon

projection for the LTM Huso 14 of 497.44 meters and in the HOM 4 of 495.8 meters, have approximate maximum deformations of 1.6 cm/km and minimum of 0.87 cm/km with accuracies of 1/62 521 and 1/116 241 respectively; they are still within the tolerance of norm.

In the survey traverse polygon YT02-TM02 of approximately 4 km distance, located between kilometers 239+805.422 and 243+833 in the last LTM zone Z15, also corresponds to the HOM 4 projection (see Figure 13). The deformations at the maximum are approximately 0.64 cm/km and the minimum is 0.31 cm/km which correspond to accuracies of 1/156 328 and 1/322 471 respectively. When enlarging the semi-axis of the ellipsoid of the WGS84 datum, with an ellipsoidal projection height of 497.75 meters for the LTM and 495.8 meters for the HOM 4, where the approximate deformations are approximately 0.24 cm/km at the maximum and 0.02 cm/km at the minimum, corresponding to accuracies 1/421 291 and 1/5 589 707 respectively; it is observed that the last deformations are less than the previous ones and continue to remain within the tolerances of the norms.

It should be noted that Santitamnont et al., in their analyzes of the aforementioned railway project, with respect to the Combined Scale Factor, the absolute maximum value of the calculated deformations was 1.7 cm/km, having projection heights of 0, 195 and 160 meters, they did not consider their geoidal undulations for the calculation of the scale factors of the Transverse Mercator systems, they directly assumed ellipsoidal heights. Most of the points they measured have very low deformations, less than 1cm/km except what was mentioned above, this is due to the relief of the terrain and the location of the central meridians of the Transverse Mercator systems. In the case of Liberia, the relief is different, taking into account its projection heights referred to mean sea level, such as values of 0, 244.9,327.755 and 464.64 meters, for example, it shows that the relief is increasing altitudinally, which is why established LTM zones of lower amplitude (6 minutes) so that the deformations are the lowest possible despite obtaining the maximum 2.19 cm/km, however using the ellipsoidal heights for the extension of the semi-axes of the ellipsoid of the WGS84 datum (which if geoid undulations were used to calculate ellipsoidal heights), the maximum deformation reached is 2.1 cm/km. Likewise, observing this behavior with the LTM zones when implementing the HOM system based on the 4 LTP, the deformations are similar and averages were obtained, under the scale factor method (altitude above mean sea level), 0.97 cm/km were obtained; while by the other method of widening the semi-axes it was lower with 0.74 cm/km; Although it can be seen in the previous graphs of 3 survey traverse such as RB56-RB63, RBC16A-RBC20D and K-506-RS03; The deformations were greater compared to those calculated through the scale factor.

4. Conclusions

From the results obtained, the use of the methodology of extending the semi-axes of the ellipsoid of the WGS84 datum with ellipsoidal heights are optimal, both for the LTM and for the HOM; because the highest deformation value that resulted was 2.10 cm/km compared to the deformations obtained according to the scale factor of these systems, which was 2.19 cm/km, which are tolerable for the Brazilian standard NBR14166, although in some survey traverse higher accuracies are observed using the scale factor, with altitudes above mean sea level or geoid.

The use of LTM zones by 6-minute-wide spindles, although it is true that it is possible to work with good accuracies, however, the use of the HOM that gives similar results to the previous one, is more integrated compared to the other projection, due to the creation of a single oblique spindle that is rotated in reference to the railroad and is only delimited in 4 LTP, according to its projection heights of its central points.

As expressed by Santitamnont et al., the extension of the semi-axes of the ellipsoid of the WGS84 datum, by means of ellipsoidal heights, is not very well known or popular for these types of projects, even more so for the Hotine Oblique Mercator (HOM) projection.

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Author contributions

Edgar Soares da Silva has contributed to the constructions of surveys traverse's polygon's by collecting measurement data, using GPS and total station equipment. Juan Alberto Ibarra Granda has contributed to the construction of the railway cartographic projection systems, as well as the analysis and interpretation of these systems based on these data and the preparation of the manuscript.

Disclosure statement

The authors of the manuscript have no interest in issues financial, professional or personal related to the publication of this document, but on the contrary the purpose is to share the results of the research with the scientific community.

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