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A COMPARISON OF TRANSFORMATION MODELS BETWEEN GEODETIC REFERENCE FRAMES: CASE STUDY IN ILLIZI REGION (ALGERIA)

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Abstract. The Illizi region is an important petroleum zone in Algeria, where various seismic surveys have been conducted. The merging of adjacent surveys is not possible due to incompatible data linked to the geodetic networks. In this study, the transformation of coordinates, from the global system (WGS84) to the local system based on the Clarke 1880 A spheroid, is carried out based on a set of 57 control points well distributed over the study area with coordinates determined in both the global and local systems. Five approaches were used to determine the transformation parameters between the two systems, namely: Geocentric Translation Model, Bursa-Wolf Transformation, Molodensky-Badekas Transformation, Abridged Molodensky transformation and Multiple Regression Equations (MRE). From statistics on the determined parameters and considering its advantage of reversibility, the Bursa-Wolf Transformation Model is the most suitable model to be used to transform coordinates between the two systems in the study area. Small amount of residuals in transformed coordinates using this model indicates acceptable Bursa-Wolf parameter estimation. An improvement in the results was observed after removing of the outliers control points detected using a statistical test. For the validation of the estimated parameters, external control points were used. The results show acceptable RMS in transformed coordinates of these points.

Keywords: geodetic reference system, residuals, coordinates transformation, transformation parameters, global system, local system.

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1. Introduction

In geodesy, it is essential to represent all the collected data in various ways in a common geodetic reference system. In the past, these data were expressed in arbitrarily defined local systems, which made it impossible to combine them in a single reference frame. With the development of satellite navigation systems, the position is expressed in a unified global system namely WGS84 system, thus avoiding the need for multiple geodetic reference frames (Mitsakaki, 2004).

Most countries have not migrated to this global system and cannot directly apply GNSS measurements without transforming them into a local system (Ziggah et al., 2017). Coordinates provided by the world geodetic system (WGS84) often have to be transformed into local geodetic coordinate systems (Paláncz et al., 2010). The Illizi region is a concrete example where GNSS data are often transformed to the local reference frame in order to merge them with the old geodetic data.

Illizi is located in South-East Algeria, on the border with Libya. It is considered as an important industrial zone, where several energy and industrial companies operate.

Several GPS networks have been observed in this region, but the coordinates of the points forming the backbone of these networks are not compatible with the true WGS84 geodetic datum. In fact, the quality of each GPS network depends on the nature and accuracy of the used control points. In addition, the coordinates of these networks have been transformed to the local datum by applying several sets of transformation parameters that are not necessarily accurate. A major problem then arises as soon as we want to merge two adjacent seismic studies, whether under the WGS84 geodetic datum or under the local geodetic datum.

In this study, we explain how it is possible to determine exhaustive transformation parameters between the WGS84 and the local datum using the available control points with coordinates determined in both the global and local systems. Giving a set of candidate models for the data, the preferred model for Illizi region is chosen with the minimum RMS of differences between the original local coordinates of control points and its corresponding transformed WGS84 coordinates.

2. Materials and methods

2.1. Geodetic data

1. *Local geodetic triangulation 1960-61*: This triangulation, carried out between 1960 and 1961, comprises a main network (66 points) and a secondary network (50 points). Planimetric coordinates are expressed in the UTM projection associated to the Clarke 1880A local ellipsoid. These points have been linked in altimetry. It was highlighted that the various triangulations that still exist in the southern regions of Algeria have not been adjusted for as a whole, so there may be a discrepancy between two neighboring triangulations. It is advisable to avoid carrying out operations based on points from different origins without first estimating this discrepancy.
2. *GNSS network 2021*: This network was surveyed using GNSS techniques. It consists of 57 legacy points from the Illizi geodetic triangulation network, which serve as dual-reference points between the WGS84 global datum and the local geodetic datum (see Figure 1). The geodetic tie of the network was performed from three stations determined with high accuracy under ITRF 2014.
3. As the number of observed GNSS points was substantial, the GNSS observations were rigorously planned and the observation strategy was fine-tuned (choice of favorable constellations, choice of antennas after check/calibration, occupancy strategy, minimum mask angle, field check of sustainability of the old points, availability of material and human resources, etc.).

As several GNSS receivers were used simultaneously, a computation of internal and external closures (in relation to the three control points) was systematically carried out after processing of the collected data in static mode using TBC (Trimble Business Centre) software. Final processing of the observed baselines was carried out using precise ephemeris. Network adjustment was performed and the geodetic coordinates of points were adjusted under WGS84. Residuals on the adjusted coordinates were in sub-centimetric order in the Easting and Northing components. Residuals in the vertical component were from 2 to 3 cm.

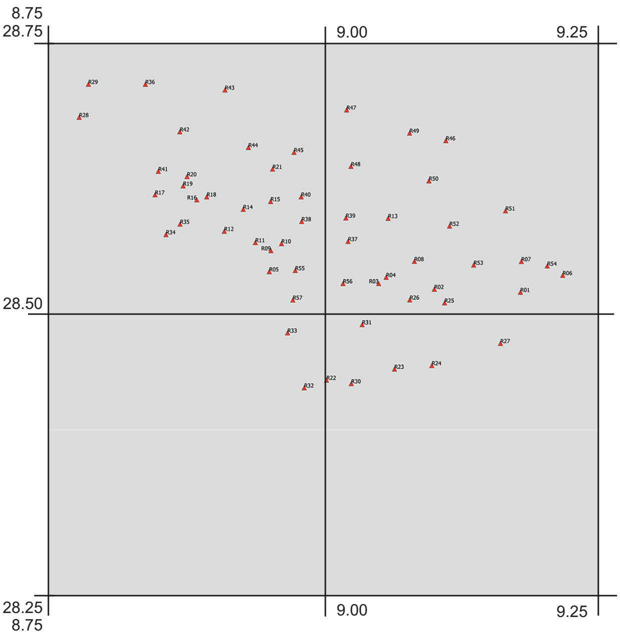

Figure 1. Spatial distribution of the 57 geodetic points used for developing the transformation models

2.2. Methods

Given a number of points with coordinates in two different spatial reference frames (known as control points); the problem is to find a good model for the transformation between these two frames. In geodesy, we use a variety of 3D transformation models. The most commonly used models are Geocentric Translation Model, Seven Parameter Similarity Transformation, Molodensky-Badekas Transformation, Abridged Molodensky transformation and Multiple Regression Equations (MRE).

We start from the *Geocentric Translation Model*. In this model, we assume that the axes of the ellipsoids associated to the two datums are parallel and that there is no scale difference between the source and target coordinate reference system.

Then geocentric coordinate reference systems may be related to each other through three translations known as shifts T_X, T_Y and T_Z in the sense of source geocentric coordinate system to target geocentric coordinate system:

$$X_T = X_S + T_X; \tag{1}$$

$$Y_T = Y_S + T_Y; \tag{2}$$

$$Z_T = Z_S + T_Z, \tag{3}$$

where (X_S, Y_S, Z_S) are the coordinates of the control point in the source geocentric coordinate system and (X_T, Y_T, Z_T) are the coordinates of the point in the target geocentric coordinate system.

The *Seven Parameter Similarity Transformation*, also known as *Bursa-Wolf* 7 Parameter model, is derived by assuming that the axes of source and target systems are not parallel and the two systems have different scales (Bursa, 1962; Wolf, 1963):

$$\begin{pmatrix} X_T \\ Y_T \\ Z_T \end{pmatrix} = (1+\Delta S) \times \begin{pmatrix} 1 & +R_Z & -R_Y \\ -R_Z & 1 & +R_X \\ +R_Y & -R_X & 1 \end{pmatrix} \times \begin{pmatrix} X_S \\ Y_S \\ Z_S \end{pmatrix} + \begin{pmatrix} T_X \\ T_Y \\ T_Z \end{pmatrix}, \tag{4}$$

where (R_X, R_Y, R_Z) represent the rotations to be applied to the point's vector and ΔS is the scale correction to be made to the position vector in the source coordinate reference system in order to obtain the correct scale in the target coordinate reference system.

In the *Molodensky-Badekas* transformation, instead of the rotations being derived from the geocentric coordinate reference system origin, they may be derived from a location within the control points used in the determination. Three additional parameters are then required, making 10 parameters in total. The equation is (Molodensky et al., 1962; Badekas, 1969):

$$\begin{pmatrix} X_T \\ Y_T \\ Z_T \end{pmatrix} = (1+\Delta S)\times\begin{pmatrix} 1 & +R_Z & -R_Y \\ -R_Z & 1 & +R_X \\ +R_Y & -R_X & 1 \end{pmatrix}\times \begin{pmatrix} X_S - X_P \\ Y_S - Y_P \\ Z_S - Z_P \end{pmatrix} + \begin{pmatrix} X_P \\ Y_P \\ Z_P \end{pmatrix} + \begin{pmatrix} T_X \\ T_Y \\ T_Z \end{pmatrix}, \quad (5)$$

where (X_P, Y_P, Z_P) are the Cartesian coordinates of the centroid point, given in the source coordinate reference system.

The Molodensky transformation is a complex equation for the shift in latitude, longitude and height. Abridged version of these equations, which yields the result called a five parameter transformation, is given as follows (Defense Mapping Agency [DMA], 1990):

$$\phi_T = \phi_S + d\phi; \quad (6)$$

$$\lambda_T = \lambda_S + d\lambda; \quad (7)$$

$$h_T = h_S + dh, \quad (8)$$

where

$$d\phi" = \frac{(-T_X \sin\phi_S \cos\lambda_S - T_Y \sin\phi_S \sin\lambda_S + T_Z \cos\phi_S +}{(\rho_S \sin 1")} \frac{\left[a_S \times df + f_S \times da\right]\sin 2\phi_S)}{}; \quad (9)$$

$$d\lambda" = \frac{1}{(\nu_S \cos\phi_S \sin 1")}\left(-T_X \sin\lambda_S - T_Y \cos\lambda_S\right); \quad (10)$$

$$dh = T_X \cos\phi_S \cos\lambda_S + T_Y \cos\phi_S \sin\lambda_S + T_Z \sin\phi_S + \left(a_S * df + f_S * da\right)\sin^2\phi_S - da, \quad (11)$$

where (ϕ_S, λ_S, h_S) and (ϕ_T, λ_T, h_T) represent the latitude, longitude and ellipsoid height in the source and target reference systems, ρ_S and ν_S are the meridian and prime vertical radii of curvature at the given latitude ϕ_S, *da* is the difference in the semi-major axes of the target and source ellipsoids and *df* is the difference in the flattening of the two ellipsoids.

The *abridged Molodensky transformation* is simple to implement, requiring only the 3 shifts between the input and output frame, and the corresponding differences between the semi major axes and flattening parameters of the reference ellipsoids. The equations do not contain the ellipsoidal heights h_S of points to be transformed.

The *multiple regression equations* (MRE) are ad hoc equations for transforming two-dimensional geographical coordinates between geodetic datums. Since they offer a means of modelling distortions, they are capable of a more accurate fit to datum-shift datasets than more basic direct methods (Ruffhead, 2022; Appelbaum, 1982). MRE take the general form (DMA, 1987):

$$d\phi" = A_0 + A_1U + A_2V + A_3U^2 + A_4UV + A_5V^2 + \ldots + A_{55}U^9V + A_{56}U^8V^2 + \ldots + A_{99}U^9V^9; \quad (12)$$

$$d\lambda" = B_0 + B_1U + B_2V + B_3U^2 + B_4UV + B_5V^2 + \ldots + B_{55}U^9V + B_{56}U^8V^2 + \ldots + B_{99}U^9V^9, \quad (13)$$

where $A_0, A_1, \ldots, A_{nn}, B_0, B_1, \ldots, B_{nn}$ are the coefficients determined in the development and U and V are the scaled latitude and longitude given by:

$$U = K\left(\phi_{S_{in\deg}} - \phi_m\right); \quad (14)$$

$$V = K\left(\lambda_{S_{in\deg}} - \lambda_m\right), \quad (15)$$

where ϕ_m, λ_m are the latitude and longitude, in decimal degrees, of the point at the middle of the area of validity and K is the degrees-to-radians conversion factor. A widespread practice, notably in (National Imagery and Mapping Agency, 2004), is to set K to $(n\pi/180)$ where n is a small integer. In our case, we take $K = 9\pi/180$.

3. Results and discussion

During the phase of classic geodesy measurements or recovery of old coordinate data, errors can occur. These errors may be due to a measuring tool problem, a reading error or an entry error. These doubtful coordinate values (outliers) should not be included in the processing so as not to disrupt the adequacy of our transformation models. For this purpose, a number of more or less powerful statistical tests are available to detect outliers. The most widely used of these tests is the one based on the definition, around the empirical mean of the data series, of a random interval (depending on the n data) with a high probability (confidence interval).

The approach taken to estimate the transformation parameters is then as follows:

- Identification of control points with known positions in both coordinate systems (global and local).
- Conversion of geographic coordinates (ϕ, λ, h) under WGS84 of double points into UTM plane coordinates (Easting, Northing) and three-dimensional Cartesian coordinates (*X*, *Y*, *Z*).
- Conversion of UTM (Easting, Northing) plane coordinates and orthometric heights under the local coordinate system into geographic coordinates (ϕ, λ) and three-dimensional Cartesian coordinates (*X*, *Y*, *Z*). The ellipsoidal height relative to Clarke 1880 A, ellipsoid associated with the North Sahara system, is taken to be equal to the orthometric height.
- Estimation of transformation parameters from the WGS84 coordinate system to the local coordinate system according to the chosen model.
- Transformation of WGS84 coordinates into local coordinates: geographic, UTM plane and three-dimensional Cartesian, using estimated parameters.

- Estimation of the differences (residuals) between the source local geographic, UTM plane and three-dimensional Cartesian coordinates and those obtained by transformation from WGS84 as well as the RMS of the differences for each component.
- Definition of a 95% confidence interval for the Easting differences series and detection of outlier points whose differences are outside this interval.
- Definition of a 95% confidence interval for the Northing differences series and detection of outlier points whose differences are outside this interval.
- If necessary, eliminate control points deemed outliers in Easting and/or in Northing and re-estimate transformation parameters.

The rest of this section presents the assessment results of the transformation parameters according to the five models previously presented.

Using the all-57 control points, the evaluation of the performance of each coordinate transformation method is based on the analysis of residuals generated during parameter estimation. As an example, Table 1 shows the residuals in 3D Cartesian coordinates *X*, *Y* and *Z* components and in UTM plane coordinates between the source local coordinates and those obtained from the coordinates transformation based on the Bursa-Wolf model.

As shown in Table 1, the residuals are from –35.4 cm to +22.2 cm in Easting component and from –198.2 cm to +86.1 cm in Northing component and from –11.1 cm to +9.8 cm in height component. The RMS values for Easting, Northing and height components are ±7.8 cm, ±17.7 cm and ±2.1 cm, respectively. These low RMS values indicate acceptable Bursa-Wolf parameters estimates.

Table 2 summarizes the obtained RMS values using the different transformation models in 3D Cartesian coordinates and in UTM components. RMS values are ranged from ±7.8 cm to ±19.2 cm in Easting component and from ±17.7 cm to ±30.3 cm in Northing component. The relatively high RMS values in Northing component indicate the possible presence of outlier values on the source local coordinates of some control points.

For each model, a statistical test based on a 95% confidence interval is applied separately for the Easting and Northing residuals series to detect possible outliers. As shown in Table 3, this test revealed that there are two points depicting largest coordinate differences, particularly in Northing component, and therefore are detected as outliers for the whole of transformation models, namely the R05 and R32 points.

A second processing was carried out in order to estimate more exhaustive transformation parameters than those previously determined. For that:

- The two doubtful points (R05 and R32) were eliminated from the treatment,
- 49 control points out of the 55 deemed good were used for the estimation of the parameters,
- The remaining six (06) control points out of the 55 were used as test points (external validation of the estimated transformation parameters). The identifiers of these 06 points are: R09 (near R05), R22 (near R32), R01 (extremity of the zone), H (extremity of the zone), R03 (middle of the zone) and R48 (middle of the area).

Table 1. Residuals on transformed coordinates using Bursa-Wolf model

Station	X (m)	Y (m)	Z (m)	Easting (m)	Northing (m)	Height (m)
R01	0.007	–0.217	0.076	–0.215	0.080	0.011
R02	–0.046	–0.002	0.140	0.005	0.144	0.027
R03	–0.031	0.134	0.044	0.137	0.043	0.013
R04	0.066	–0.167	–0.049	–0.175	–0.062	0.011
R05	–0.414	–0.424	0.723	–0.354	0.861	–0.072
R06	0.048	–0.239	0.005	–0.243	–0.001	0.010
R07	0.054	–0.222	–0.054	–0.227	–0.057	–0.011
R08	–0.075	0.040	0.096	0.051	0.116	–0.014
R09	–0.014	0.127	0.021	0.128	0.016	0.015
R10	–0.023	0.052	0.047	0.055	0.048	0.010
R11	–0.008	0.128	0.028	0.127	0.019	0.024
R12	–0.011	0.137	0.000	0.137	–0.005	0.009
R13	–0.076	0.061	0.085	0.072	0.105	–0.017
R14	0.025	0.130	–0.062	0.124	–0.075	0.010
R15	–0.039	0.130	0.065	0.135	0.066	0.015
R16	0.008	0.073	–0.016	0.071	–0.023	0.009
R17	–0.066	–0.071	0.049	–0.060	0.079	–0.044
R18	–0.035	0.144	–0.026	0.147	–0.017	–0.023
R19	–0.014	–0.053	0.057	–0.050	0.060	0.008
R20	0.001	0.034	–0.008	0.034	–0.010	0.002

End of Table 1

Station	X (m)	Y (m)	Z (m)	Easting (m)	Northing (m)	Height (m)
R21	–0.125	0.067	–0.024	0.085	0.033	–0.111
R22	0.007	0.099	–0.068	0.097	–0.070	–0.013
R23	–0.079	0.100	0.087	0.111	0.106	–0.013
R24	–0.076	–0.043	0.197	–0.031	0.211	0.022
R25	–0.051	–0.044	0.143	–0.035	0.153	0.018
R26	–0.011	0.057	0.100	0.058	0.088	0.046
R27	–0.059	–0.133	0.204	–0.122	0.217	0.028
R28	0.068	–0.137	–0.058	–0.146	–0.073	0.012
R29	–0.005	–0.160	0.080	–0.157	0.085	0.012
R30	–0.108	0.103	0.114	0.119	0.144	–0.025
R31	–0.040	0.136	0.088	0.141	0.086	0.027
R32	0.939	0.189	–1.737	0.040	–1.982	0.014
R33	0.106	–0.121	–0.154	–0.136	–0.176	0.002
R34	0.100	–0.301	–0.190	–0.313	–0.192	–0.045
R35	–0.034	0.022	–0.042	0.027	–0.023	–0.047
R36	–0.004	0.029	0.086	0.029	0.075	0.041
R37	–0.062	0.115	0.079	0.123	0.090	–0.001
R38	–0.052	0.072	0.073	0.079	0.083	0.000
R39	–0.104	0.093	0.029	0.109	0.068	–0.064
R40	–0.057	0.090	0.088	0.098	0.097	0.005
R41	0.081	–0.045	0.072	–0.057	0.028	0.098
R42	0.030	0.075	–0.005	0.070	–0.025	0.034
R43	–0.005	0.225	0.053	0.222	0.031	0.052
R44	0.002	0.116	–0.002	0.114	–0.012	0.017
R45	–0.107	0.106	–0.058	0.122	–0.008	–0.106
R46	0.063	–0.088	–0.057	–0.096	–0.073	0.015
R47	0.016	0.132	–0.050	0.127	–0.062	0.008
R48	–0.040	0.098	0.016	0.103	0.026	–0.013
R49	0.035	–0.023	–0.032	–0.028	–0.043	0.011
R50	0.003	–0.091	–0.043	–0.090	–0.033	–0.030
R51	0.024	–0.095	–0.104	–0.098	–0.096	–0.043
R52	0.089	–0.035	–0.160	–0.049	–0.180	–0.005
R53	0.014	–0.126	0.025	–0.126	0.025	0.006
R54	0.058	–0.198	–0.014	–0.205	–0.025	0.016
R55	–0.020	0.006	0.097	0.009	0.094	0.030
R56	–0.013	0.106	0.067	0.107	0.057	0.036
R57	0.059	–0.191	–0.121	–0.198	–0.120	–0.033
RMS (m)	0.086	0.080	0.155	0.078	0.177	0.021

Table 2. RMS on transformed coordinates

	X (m)	Y (m)	Z (m)	Easting (m)	Northing (m)	Height (m)
Geocentric Translation	0.102	0.201	0.208	0.192	0.236	0.035
Bursa-Wolf	0.086	0.080	0.155	0.078	0.177	0.021
Molodensky-Badekas	0.087	0.081	0.156	0.079	0.178	0.021
Abridged Molodensky	0.101	0.201	0.208	0.192	0.236	0.035
MRE 1st Order	–	–	–	0.133	0.303	–
MRE 2nd Order	–	–	–	0.097	0.266	–
MRE 3rd Order	–	–	–	0.092	0.226	–
MRE 4th Order	–	–	–	0.086	0.199	–

Table 3. Detected outliers and gaps between source and transformed local UTM coordinates

Model	Outlier in Easting		Outlier in Northing	
	Station	Residual (m)	Station	Residual (m)
Geocentric Translation	R01	–0.660	R05	0.920
	R06	–0.817	R32	–1.547
	R07	–0.709		
	R54	–0.750		
Bursa-Wolf	R05	–0.354	R05	0.861
	R34	–0.313	R32	–1.982
Molodensky-Badekas	R05 R34	–0.354 –0.313	R05 R32	0.861 –1.982
Abridged Molodensky	R01 R06 R07 R54	–0.659 –0.815 –0.708 –0.748	R05 R32	0.920 –1.548
MRE 1st Order	R05 R34	–0.368 –0.355	R05 R32	0.911 –1.853
MRE 2nd Order	R04 R05 R34 R57	–0.224 –0.374 –0.182 –0.223	R05 R30 R32	0.911 0.504 –1.360
MRE 3rd Order	R04 R05 R57	–0.189 –0.338 –0.172	R05 R22 R30 R32	0.786 0.555 0.530 –0.855
MRE 4th Order	R04 R05	–0.226 –0.307	R05 R22 R32	0.693 0.517 –0.474

The results in term of residuals and RMS have improved for all models. As an example of transformation model, Table 4 shows the obtained residuals in 3D Cartesian coordinates X, Y and Z components and in UTM plane coordinates between the source local coordinates and those obtained from the coordinates transformation based on the Bursa-Wolf model after removing the two outliers. The RMS values went from ±7.8 cm to ±6.9 cm in Easting and from ±17.7 cm to ±5.1 cm in Northing. For the Cartesian coordinates, RMS values decreased from 8.6 cm to 3.3 cm in X, from 8.0 cm to 6.7 cm in Y and from 15.5 cm to 4.8 cm in Z.

Table 5 shows the assessment results from coordinate's transformation of the six test points based on the Bursa-Wolf model. From the Table 5, the RMS of coordinate differences in 3D Cartesian components, X, Y and Z can be achieved at ±2.6 cm, ±15.3 cm and ±7.5 cm for the three components, respectively. These stations still exhibit acceptable RMS.

Table 6 resumes the RMS on transformed coordinates of the 49 control points per model. The RMS values after removing of the two outlier points are slightly better than the previous ones (see Table 2); the RMS values are decreased for all models. A comparison of the RMS of the different models, in Easting and Northing components, shows that the most appropriate approach for transformation from WGS84 to local datum in Illizi region is that of Borsa-Wolf model, followed by Molodensky-Badekas model.

Table 7 resumes the obtained RMS on transformed coordinates of the six test points using different models

Table 4. Residuals on transformed coordinates using Bursa-Wolf model after removing outlier points

Station	X (m)	Y (m)	Z (m)	Easting (m)	Northing (m)	Height (m)
R02	–0.032	0.035	0.103	0.039	0.103	0.027
R04	0.081	–0.147	–0.083	–0.158	–0.100	0.011
R06	0.053	–0.158	–0.026	–0.164	–0.036	0.011
R07	0.059	–0.155	–0.080	–0.162	–0.087	–0.009
R08	–0.065	0.069	0.068	0.078	0.085	–0.015
R10	–0.009	0.035	0.024	0.036	0.022	0.009
R11	0.007	0.102	0.005	0.100	–0.007	0.023
R12	0.004	0.099	–0.019	0.097	–0.026	0.008
R13	–0.071	0.080	0.072	0.090	0.090	–0.016
R14	0.035	0.098	–0.074	0.091	–0.088	0.008
R15	–0.031	0.108	0.056	0.112	0.056	0.014
R16	0.019	0.024	–0.025	0.021	–0.032	0.008
R17	–0.055	–0.133	0.041	–0.123	0.072	–0.047
R18	–0.025	0.099	–0.034	0.101	–0.025	–0.024
R19	–0.005	–0.107	0.053	–0.105	0.056	0.006
R20	0.009	–0.019	–0.009	–0.020	–0.011	0.001
R21	–0.123	0.044	–0.021	0.062	0.036	–0.111
R23	–0.050	0.126	0.020	0.132	0.031	–0.016
R24	–0.050	–0.004	0.132	0.004	0.139	0.019
R25	–0.035	–0.003	0.101	0.003	0.105	0.017
R26	0.007	0.086	0.058	0.084	0.041	0.045
R27	–0.040	–0.071	0.148	–0.064	0.154	0.026
R28	0.070	–0.229	–0.039	–0.237	–0.050	0.011

End of Table 4

Station	X (m)	Y (m)	Z (m)	Easting (m)	Northing (m)	Height (m)
R30	–0.074	0.116	0.042	0.126	0.063	–0.028
R31	–0.016	0.150	0.037	0.151	0.029	0.025
R33	0.134	–0.132	–0.209	–0.151	–0.237	–0.002
R34	0.118	–0.359	–0.211	–0.373	–0.215	–0.047
R35	–0.018	–0.031	–0.060	0.028	–0.042	–0.049
R36	–0.010	–0.041	0.117	–0.039	0.110	0.041
R37	–0.051	0.121	0.057	0.127	0.065	–0.001
R38	–0.042	0.061	0.058	0.067	0.066	0.000
R39	–0.097	0.097	0.016	0.112	0.053	–0.063
R40	–0.052	0.078	0.082	0.085	0.091	0.005
R41	0.090	–0.108	0.072	–0.120	0.028	0.098
R42	0.030	0.019	0.010	0.014	–0.007	0.033
R43	–0.014	0.183	0.083	0.182	0.065	0.053
R44	0.001	0.083	0.008	0.081	0.000	0.016
R45	–0.109	0.090	–0.049	0.106	0.002	–0.105
R46	0.051	–0.053	–0.041	–0.060	–0.056	0.017
R47	0.003	0.132	–0.025	0.129	–0.034	0.009
R49	0.023	0.000	–0.014	–0.003	–0.023	0.013
R50	–0.001	–0.060	–0.042	–0.059	–0.032	–0.029
R51	0.021	–0.036	–0.113	–0.039	–0.107	–0.041
R52	0.091	0.005	–0.175	–0.010	–0.197	–0.004
R53	0.022	–0.076	–0.003	–0.078	–0.007	0.007
R54	0.062	–0.123	–0.042	–0.132	–0.057	0.017
R55	–0.002	–0.005	0.065	–0.004	0.058	0.028
R56	0.005	0.111	0.030	0.109	0.015	0.034
R57	0.082	–0.202	–0.165	–0.212	–0.169	–0.036
RMS (m)	0.033	0.067	0.048	0.069	0.051	0.021

Table 5. Residuals on transformed coordinates of testing points using Bursa-Wolf model after removing outlier points

Station	X (m)	Y (m)	Z (m)	Easting (m)	Northing (m)	Height (m)
R01	0.017	–0.150	0.039	–0.151	0.038	0.012
R03	–0.015	0.152	0.008	0.152	0.002	0.012
R09	0.002	0.106	–0.005	0.105	–0.013	0.014
R22	0.042	0.103	–0.139	0.096	–0.149	–0.016
R29	–0.009	–0.251	0.110	–0.247	0.120	0.011
R48	–0.043	0.102	0.021	0.107	0.031	–0.013
RMS (m)	0.026	0.153	0.075	0.152	0.081	0.013

Table 6. RMS on transformed coordinates after removing outlier points

Model	X (m)	Y (m)	Z (m)	Easting (m)	Northing (m)	Height (m)
Geocentric Translation	0.075	0.202	0.173	0.194	0.193	0.035
Bursa-Wolf	0.033	0.067	0.048	0.069	0.051	0.021
Molodensky-Badekas	0.034	0.068	0.049	0.070	0.052	0.022
Abridged Molodensky	0.075	0.201	0.172	0.194	0.192	0.035
MRE 1st Order	–	–	–	0.121	0.089	–
MRE 2nd Order	–	–	–	0.082	0.069	–
MRE 3rd Order	–	–	–	0.078	0.065	–
MRE4thOrder	–	–	–	0.074	0.062	–

Table 7. RMS on transformed coordinates of testing points after removing outlier points

Model	X (m)	Y (m)	Z (m)	Easting (m)	Northing (m)	Height (m)
GeocentricTranslation	0.136	0.346	0.320	0.332	0.357	0.049
Bursa-Wolf	0.026	0.153	0.075	0.152	0.081	0.013
Molodensky-Badekas	0.026	0.153	0.075	0.152	0.081	0.013
AbridgedMolodensky	0.136	0.345	0.319	0.332	0.356	0.050
MRE 1^{st} Order	–	–	–	0.154	0.085	–
MRE 2^{nd} Order	–	–	–	0.090	0.040	–
MRE 3^{rd} Order	–	–	–	0.077	0.046	–
MRE 4^{th} Order	–	–	–	0.068	0.060	–

after elimination of the outlier points. The RMS values are low, which means that the estimated parameters according different models are acceptable.

4. Conclusions

This study demonstrated analysis on the performance of five models for coordinate transformation at Illizi region. The experimental work used two geodetic datums which are WGS84 and local geodetic triangulation. The analysis was carried out on a set of 57 control points that are known in both datum.

The residuals from the estimation of the transformation parameters according to the five models were acceptable, with a slight improvement after removing of two detected points as outliers using a statistical test.

After the series of tests and residuals analysis, it was concluded that the geocentric Bursa-Wolf model is the most appropriate approach for coordinate transformation from WGS84 to local datum in the study area. The accuracy of Bursa-Wolf estimations exhibits RMS value at ±6.9 cm and ±5.1 cm in UTM Easting and Northing components respectively. The test stations show RMS up to ±15 cm.

In conclusions, the experiment in the Illizi region already shows that it is entirely possible to resolve the problem linked to the referencing of the various geodetic networks to meet the needs in positioning and navigation.

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Author contributions

Mr. Haddad was responsible for the conceptualization and development of the transformation programs, performed the computational analyses, interpreted the theoretical framework, and wrote the theoretical sections of the manuscript. Ms. Gahlouz contributed to the processing and analysis of the data, interpreted the results, and drafted the "Results and Interpretation" section. Both authors critically reviewed the manuscript and approved the final version for publication.

Disclosure statement

The authors declare that they have no known financial or commercial conflicts of interest that could have influenced the research reported in this paper.

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