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TO DETERMINATION OF THE HEIGHTS ON GEODYNAMIC AND TECHNOGENIC POLYGONS

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Abstract. When performing high-precision geometric leveling on geodynamic and technogenic polygons, the problem of selecting a system of heights and obtaining reliable data arises, taking into account, first of all, the heterogeneity of the gravitational field along the lines of leveling. The research and development of a methodological approach to solve the above issues is the purpose of this publication. For the research process, the method of mathematical processing of a wide spectrum of geodetic and gravimetric measurements, differential and mathematical analysis, methods of conducting high-precision leveling and gravimetric works are used. During the process of field geodetic and gravimetric measurements, the values of gravitational acceleration in the mountain, foothill and plain areas were obtained, which made possible, on the basis of the mathematical dependences deduced by the author, to find corrections in the measured excesses for the non-parallelism of the level surfaces. On the basis of unique experimental level-gravimetric observations and their corresponding mathematical elaboration, numerical characteristics of gravitational acceleration for various forms of relief have been obtained. In the final case, the obtained analytic dependencies give an opportunity to take into account the influence of the non-parallelism of the level surfaces in the measured excesses or the heights of high-precision geometric leveling, without taking into account the gravitational field of the normal Earth. The proposed methodological approach to the determination of heights by high-precision geometric leveling on geodynamic and technogenic polygons may take into account the influence of the non-parallelism of the level surfaces according to the derived formulas and should be effectively used at present.

Keywords: height, gravity potential, excess, gravitational acceleration, correction.

Introduction

An important issue requiring periodic monitoring is the requirements to ensure the safe operation of various engineering facilities, as well as environmental protection.

Different kinds of monitoring are carried out to solve the above problem. One of such effective monitoring methods can be geodetic, which allows to determine the vertical and horizontal movements of engineering objects and the earth's surface and on this basis to establish certain types of deformations and their processes in general. For this purpose, special geodetic networks are being built on geodynamic and technogenic polygons. One of the important components of such networks is the vertical component, which determines the high-altitude changes in the position of the research objects, which, in turn, requires reliable and accurate obtaining of the final results, taking into account the real gravimetric field of the Earth.

The actual problem of taking into account the influence of a real gravimetric field on the precision of high-precision geometric leveling is devoted to this publication.

1. Analysis of recent research works and publications

In the latter period, considerable attention is paid to the study of the results of geodetic measurements in real conditions on the topographic surface of the Earth. In particular, study (Vasillev, 2016) presents the results of high-precision geometric leveling using gravimetric data in the deposits of carbohydrate extraction.

Another scientific work presents the results of the monitoring of deformation processes using the methods of geodesy and gravimetry on geodynamic polygons (Brovar, 1983).

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A number of scientific publications are devoted to the study of the frequency of the placement of gravimetric points along the leveling lines, depending on the required precision of leveling, terrain, density of underground layers, etc. (Dvulit & Smelianets, 2014; Ilkiv, 1972; Yurkina & Korotkova, 1965).

Particular attention deserves the attention to the theory of heights in the gravitational field of the Earth (Dvulit, 2002), as well as the study of the function and role of the gravitational field in solving various problems of engineering geodesy.

In theory and practice of high-precision geometric leveling, three basic systems of heights are considered: normal, ortometric and dynamic.

The first two systems of heights have a common disadvantage, namely, the level surface has different heights, since different values of gravitational acceleration or normal gravity, which is a function of latitude, are different on it.

Proceeding from the considerations that the heights of the points on the same level surface are the same and the starting point of the height count is closest to the integral value of the measured excesses, in the engineering practice a dynamic system of heights is widely used. In this system there are no aforementioned disadvantages of a normal and orthometric system of heights.

2. Problem statement

The purpose of this publication is to develop, on the basis of experimental data, a methodological approach on taking into account the gravitational field in high-precision leveling on geodynamic and technogenic polygons with the application of heights that can be considered as dynamic.

3. Main research material

When constructing special geodetic networks on geodynamic or technogenic polygons, in general, a local (conditional) coordinate system is established.

Let's consider a separate leveling line from the starting point O with a conditional mark of 0.000 m to the point A with a mark defined by the results of geometric leveling.

Let's record for this line the difference in potentials (Dvulit, 2002):

$$W_A - W_0 = \int_0^A g dh, \tag{1}$$

where W_0, W_A – the potential of the gravity, respectively, at the starting and endpoints; g – value of gravitational acceleration from p.O to p.A; dh – measured excess from p.O to p.A.

The integral of the right-hand side of formula (1) is represented as:

$$\sum_{i=1}^n \frac{g_i + g_{i+1}}{2} h_i,$$

where g_i is the measured value of the gravitational acceleration at the point i, h_i – the excess at the i-th station of leveling.

So we have:

$$W_A - W_O = \int_0^A g dh = \sum_{i=1}^n \frac{g_i + g_{i+1}}{2} h_i. \tag{2}$$

On the other hand, the potential of gravity at a fixed point A can be represented as a Taylor series. It could be limited to the second member (Brovar, 1983):

$$W_A = W_o + g_A H_A + \frac{1}{2}\frac{dg}{dn} H_A^2 + \dots \tag{3}$$

or

$$W_A - W_o = g_A H_A + \frac{1}{2}\frac{dg}{dn} H_A^2 + \dots, \tag{4}$$

where H_A – height of point A; $\frac{dg}{dn}$ – change of gravitational acceleration.

Equating the right sides of expressions (2) and (4) we obtain:

$$\sum_{i=1}^n \frac{g_i + g_{i+1}}{2} h_i = g_A H_A + \frac{1}{2}\frac{dg}{dn} H_A^2. \tag{5}$$

Hence

$$H_A = \frac{1}{g_A}\left[\sum_{i=1}^n \frac{g_i + g_{i+1}}{2} h_i - \frac{1}{2}\frac{dg}{dn} H_A^2\right]. \tag{6}$$

The value of gravitational acceleration for a single level line can be represented:

$$g_A = g_0 + \frac{dg}{dn} H_A, \tag{7}$$

where g_0 – gravitational acceleration at the point O.

The first term of formula (6) for such a line can be represented as:

$$\frac{1}{g_A}\sum_{i=1}^n \frac{g_i + g_{i+1}}{2} h_i = \frac{1}{g_A} g_c h_b, \tag{8}$$

where g_c – mean value of gravitational acceleration between level points; h_b – measured excess between points of leveling.

We will transform the right-hand side of formula (8). We have:

$$\frac{1}{g_A} g_c h_b = \frac{h_b}{g_A}\left(g_A + g_c - g_A\right) =$$
$$h_b + \frac{g_c - g_A}{g_A} h_b. \tag{9}$$

Taking into consideration (9) the Eq. (6) will be:

$$H_A = h_b + \frac{g_{\bar{n}} - g_A}{g_A} h_b - \frac{1}{2 g_A}\frac{dg}{dn} H_A^2. \tag{10}$$

Thus, the height of the leveling point of the isolated leveling line is equal to the measured excess, taking into account the corresponding correction f for the non-parallelism of the level surfaces due mainly to the heterogeneity of masses within the Earth.

The expression for determining correction f has the form:

$$f = \frac{g_c - g_A}{g_A} h_b - \frac{1}{2g_A} \frac{dg}{dn} H_A^2 . \tag{11}$$

When calculating the correction f in our isolated leveling line, we can accept that $h_b \approx H_A$ and $g_A \approx g_o$.

Considering $\Delta g = g_o - g_A$, as well as $g_c - g_A = -\frac{\Delta g}{2}$, we obtain

$$f = \frac{1}{g_o} h_b \left(-\frac{\Delta g}{2} - \frac{dg}{dn} \frac{h_b}{2} \right) . \tag{12}$$

Taking into consideration that:

$$\frac{dg}{dn} \frac{h_b}{2} = \Delta g \Big/ 2 , \tag{13}$$

we have

$$f = -\frac{1}{2g_o} h_b \Delta g . \tag{14}$$

The final formula for calculating heights in the conditional system (the height of the starting point is $H_O = 0.0000$ m) for the point i will look like:

$$H_i = \int_O^i dh_i - \frac{1}{2g_o} \Delta g_i \int_O^i dh_i , \tag{15}$$

where dh_i – measured excesses in the line of leveling to the point i; Δg_i – the difference between the gravitational acceleration between its final value at the leveling point and its value at the starting point.

The use of formula (15) is difficult, since it requires considerable material and physical costs necessary for measuring the gravitational acceleration at each point of setting the leveling rod. In this case, the problem to establish the optimal number of gravimetric measurements that would satisfy the requirements of the main task with regard to the accuracy of determining the height of points or their vertical displacement with the required precision arises.

For this purpose, it is necessary to carry out studies on the density of gravimetric measurements on the line of leveling, based on the physical and geographical location of the work area and the required accuracy of obtaining the final results.

In the scientific publication of prof. Dvulit P. and Smelyanets O. (Dvulit & Smelianets, 2014) a detailed analysis of the methods for calculating the frequency of placement of gravimetric points along the lines of geometric leveling is made. In accordance with the authors' analysis after the research of prof. Pelinen L. in mountain areas (highlands) for leveling class I the distance between gravimetric points should not exceed 3 km, for the II class – 6 km and the III class – 17 km. For medium-highs, respectively, 12, 25 and 75 km and plains – 50, 100 and 300 km. If we take the accuracy of the calculation of the correction for the effect of gravitational acceleration as 0.1 mm, then at leveling I class the distance between the gravimetric points in the highlands should not exceed 1 km, and in others – about 5 km.

Polish researcher Bokun Yu. proposed to perform gravimetric observations in mountain areas in 1.5–2 km, foothill areas – 2–3 km and plains – 4–6 km.

Neumann Yu. proposed optimal distances between gravimetric points:
  – for leveling I class: high altitude areas - 1.1 km, foothill areas – 5 km, plain areas – 24 km;
  – for leveling II class: high altitude areas – 1.4 km, foothill areas – 11 km, plain areas – 32 km;
  – for leveling III class respectively – 2.2, 16 and 50 km.

On the basis of analytical analysis, the authors came to the conclusion that in the mountain areas at the geometric leveling I class, gravimetric points should be located 1 km, II class – 2 km and III class – up to 10 km.

For the foothills and plain areas, gravimetric points along the leveling lines I class should be located in 2 km, and for II class – approximately 4–6 km.

Here there are the examples of a possible implementation of this methodological approach for determining the corrections in the height of the points of the Carpathian Geodynamic Polygon.

During the period 1965–1975 on the Carpathian Geodynamic Polygon under the guidance of prof. Migal M. the unique experimental studies were carried out to determine the parameters of the gravitational field in mountain conditions and their effect on precision of the leveling. To this end, a special high-altitude network of three leveling lines of II class was constructed, where the gravimetric measurements of gravitational acceleration at each point of the leveling line were made (Ilkiv, 1972). The starting point for the three heights for leveling lines was O ($H_o = 0.000$ m). All three leveling lines lay in different directions: high-altitude, foothill, and almost plain.

The first level line was projected from the leveling mark at the railway station to the point of triangulation on the top of the mountain. This leveling-gravimetric line included 723 leveling benchmarks in which the leveling rods were installed, and simultaneously with the leveling works the values of gravitational acceleration were carried out at these points. The length of the leveling line was 17.3 km, the difference in the heights of the end and starting points was $h \approx 1185$ m. The gravimetric network consisted of 25 reference gravimetric points, located at a distance of 0.7–1.6 km from each other. In this case, a conditional system of gravitational acceleration was established, the reference point of which was the mark of the railway station ($g_0 = 980\ 000$ mGal).

The second leveling line included 207 points. The length of this line was 5.9 km, and the difference between the heights of the finite points $h \approx 515$ m.

The third leveling line of 2.6 km in length and the difference in the height of the finite points $h \approx 280$ m included 95 leveling benchmarks.

For the second and third leveling lines, special gravimetric networks were constructed in the same way as for the first line.

For all three leveling lines the permissible and received inconsistencies were calculated:

They were:

– for the first leveling line – $f_h = -43$ mm;

– for the second – $f_h = -19$ mm;

– for the third – $f_h = -1$ mm.

The values of received inconsistencies correspond to their permissible values for leveling II class.

Mean square error of measurement of gravitational acceleration in ordinary gravimetric points (point of installation of leveling rods) was $m_p = 0.24$ mGal .

For all leveling-gravimetric lines the value of gravitational acceleration $\dfrac{dg}{dn}$ is calculated. They were: for the first line – 0.214 mGal/m; for the second one – 0.204 mGal/m; for the third – 197 mGal/m. Their mean square errors $m\left(\dfrac{dg}{dn}\right)$ are equal to:

– for the first line – 0.006 mGal/m;

– for the second line – 0.005 mGal/m;

– for the third line – 0.010 mGal/m.

The obtained results of experimental leveling-gravimetric works served as the basis for the implementation of theoretical calculations on the determination of heights on geodynamic and tectonic polygons, which excludes the use of formulas for "normal" Earth. In Table 1, we give the calculation of the corrections f for the above-mentioned three-level lines. Calculations are made according to formula (14).

Table 1. Values of corrections f in leveling lines

Part of the formula	Title of leveling line		
	1	2	3
h_b , m	1185.15	514.60	283.53
g_o , mGal	980 000	980 000	980 000
Δg , mGal	253.62	104.97	60.67
f, m	0.1534	0.0276	0.0081

The analysis of the results presented in Table 1 allows us to conclude that in mountain regions ($h > 1000$ m) the correction in measured excesses of high-precision geometric leveling for the non-parallelism of the level surfaces is significant and reaches more than 10 cm. In the foothills and plain areas, the value of this correction varies from a few tens of millimeters to 1.

Note that the value f depends on the value Δg , which in its turn is a function of its representation in the leveling line.

Experimental studies have shown that in the same area of high precision gravimetric works, the value of the change of the vertical gradient of gravitational acceleration $\dfrac{dg}{dn}$ is different.

In order to study the effect of individual values $\dfrac{dg}{dn}$

for the correction f, in Table 2 the values of these corrections for all three leveling lines are given. In this case, we assume that for each leveling line $\dfrac{dg}{dn}$ is equal to 0.214 mGal, 0.204 mGal, 0.197 mGal respectively.

Table 2. Values of these corrections f, m

Values $\dfrac{dg}{dn}$ mGal/m	Title of leveling line		
	1	2	3
0.214	0.1534	0.0289	0.0088
0.204	0.1462	0.0276	0.0083
0.197	0.1412	0.0266	0.0081

Results of the calculations listed in Table 2 will allow us to conclude that in highland (mountain) areas the value of the vertical gradient of gravitational acceleration substantially affects the accuracy of geometric leveling and, in this case, the change $\dfrac{dg}{dn}$ by 0.007 mGal leads to a change in excess (correction in excess) by 12 mm.

In the foothill area, the change $\dfrac{dg}{dn}$ also results the change in the correction value in excess up to 2 mm and practically does not affect the value f in plain areas. Thus, on the basis of the experimental data obtained and their analysis, it can be concluded that in determining the absolute values of heights it is necessary for each individual leveling line to determine the values of the change in the gravitational acceleration, that is, to carry out a complete cycle of gravimetric measurements. For plains and mountain areas, you can use the mean value $\dfrac{dg}{dn}$ for a given region to calculate the corrections f. This statement does not apply to regions with underground gas storages, neotectonic movements of the earth's surface and underground slabs, exploration of carbohydrates and other mineral resources.

In order to determine the effect of the correction f on the results of leveling in certain sections of the high-altitude area, the leveling line 1 with an excess equal to 1185.15 m of the endpoint over the initial one was divided for calculations into 11 sections. Sections were set in such a way that the change of the vertical gradient $\dfrac{dg}{dn}$ in the section was approximately the same. The results of correction calculations in the sections are given in Table 3.

The analysis of the above results shows that in sections 0–66 and 66–88, where the measured excesses reach 200 m, and $\Delta g \approx 35$ mGal. The magnitude of the correction f in this case lies in the boundaries of 3 mm, which allows us to conclude that this correction can be neglected.

In sections 88–183, 183–353 and 353–396 with a limiting value of excess which is up to 1000 m and a change in the gravitational acceleration up to 200 mGal, the correction f is quite significant and reaches 100 mm, requiring its mandatory determination and consideration.

Table 3. Values of corrections *f* in the sections of leveling line 1, m

Title of leveling line	Δg, mGal	h_b, m	f, m
0–66	23.73	118.42	0.0014
66–88	34.06	168.21	0.0029
88–183	77.86	359.76	0.0143
183–353	154.50	728.24	0.0574
353–396	164.59	776.61	0.0652
396–497	207.95	967.07	0.1026
497–526	211.98	1023.01	0.1159
526–560	218.91	1005.91	0.1123
560–636	229.39	1061.66	0.1242
636–690	246.25	1132.66	0.1423
690–A	253.62	1185.15	0.1534

With excesses between the endpoints of more than 1000 m and a change of the gravitational acceleration of about 250 mGal, the correction *f* reaches values greater than 150 mm, which is essential in high-precision geometric leveling.

When determining the densities of gravimetric points along the leveling line, we should adhere to the recommendations set in the scientific articles (Dvulit & Smelianets, 2014; Ilkiv, 1972).

Conclusions

In this paper, on the basis of the widespread use of experimental data, studies have been carried out to take into account the influence of non-parallelism of level surfaces in measured excesses for high-altitude, foothill and plain areas.

It was established that in mountain areas, when excesses between the endpoints exceed 1000 m, the correction in excesses because of non-parallelism of level surfaces may reach 150 mm or more; in foothills with excesses up to 1000 m – about 100 mm; in plains with excesses up to 200 m – several millimeters.

The method of using local (dynamic) heights is proposed, which does not require information about the normal gravitational field of the Earth. This method can be effectively used on geodynamic and technogenic polygons, where the marks of the points can be determined in the conventional system of heights.

Disclosure statement

The authors declare no conflict of interest.

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