GEODESY AND CARTOGRAPHY
ISSN 2029-6991 / eISSN 2029-7009
2017 VOLUME 43(4): 147–157
https://doi.org/10.3846/20296991.2017.1412615

UDK 528.21

IMPACT OF DATUM TRANSFORMATION ON LOCAL VARIATIONS OF GEOMETRIC GEOID IN NIGER

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Received 15 October 2017; accepted 27 November 2017

Abstract. In this study, we have conducted an investigation on the impact of the coordinates' transformation on local variations of geometric geoid. The study area is limited by 1°43′12″ to 4°00′37″ East and 13°01′57″ to 14°31′20″ North in the southwest of the Niger Republic. We used 39 network GPS/levelling points established by the Japan International Cooperation Agency (JICA) and the National Geographic Institute of Niger (IGNN), including the DOPPLER point ANG302/no.65. Using other coordinates of point no. 65 provided by IGNN, we transformed the points into WGS84 and computed a new geometric geoid model. The comparison of the new model with EGM2008 geoid up to d/o 2160 gives the STD of 15 cm and the RMS of 16cm. Local variations of the geometric geoids, were compared to that of EGM2008 geoid. The comparison through basic statistics, trend lines and 3D overlaps, showed a similar trend between the geometric geoid from the transformed coordinates and that of EGM2008. On the contrary, the JICA-IGNN geometric geoid generated an opposite and exaggerated trend. The Jarque-Bera test confirms that the three samples follow a normal distribution at the significance level α = 5%. The equality of variances between EGM2008 and JICA-IGNN geoids has been rejected by the Fisher's F-Test/two-tailed at α = 10%. However the test confirms the variances equality between EGM2008 and the transformed geometric geoid at α = 5% and α = 10%. The two-tailed Student's T-Test at α = 5% also confirms the equality of means between EGM2008 geoid and transformed geometric geoid samples.

Keywords: geodetic datum, coordinates transformation, Geoid, GPS/Levelling, EGM2008, Niger.

Introduction

The historical and permanent task of the geodesy involves defining, realizing and releasing geographic terrestrial references, with several technical and scientific applications such as mapping, surveying, navigation (Levallois *et al.* 2001). The spatial techniques of global and 3D datum replaces the old 2D geodetic datum development process. It has capabilities to provide heights above the ellipsoid. The joint use of spirit levelling and ellipsoidal heights defines GNSS/levelling points from which geometric geoid models are determined. However, the cartographic systems of several countries are still in the 2D and local datum. In order to maintain the consistency of national cartographic systems, the coordinates from GPS in the global datum WGS84 are transformed into a local datum. Besides, studies are done in some countries to determine the transformation parameters between local and global geodetic systems (Orupabo *et al.* 2014).

The topographic map project of the *Djerma Ganda* and *Dallols* regions, in southwest part of the Niger Republic was carried out by the Japan International Cooperation Agency (JICA) and the National Geographic Institute (IGNN) (JICA & IGNN 1996). A 39-points GPS/Levelling network had been built, including the DOPPLER point ANG302/no.65 as origin.

Coordinates' transformation had been performed and a relative geometric geoid model had been computed, referenced to point no.65. The points had been used in two GPS Levelling campaigns (Favre 2000; Thibaud *et al.* 2011). In Niger Republic, the geodetic system is two-dimensional and based on Clarke 1880 ellipsoid and the Universal Transverse Mercator (UTM) projection. Notice that the entire cartographic system of the National Geographic Institute (IGN-France) in Africa was based on Clarke 1880 ellipsoid as recommended at the International Conference of Bukavu (Congo-Zaire) in 1953 (INCT 2007).

The aim of this study is to investigate the coordinates' transformations impact on the local variations of geometric geoid. The study area is limited by 1°43′12″ to 4°00′37″ East and 13°01′57″ to 14°31′20″ North. Sec. 1 of this paper presents the theory on geodetic and coordinates systems transformation. Sec. 2 introduces the study area and the data. The methodology is summarized in Sec. 3. The discussion over the results is given in Sec. 4; and the conclusion and perspectives are mentioned in the last section.

1. Theory on geodetic systems and coordinates systems' transformation

The geodetic datum is the digital realization of a geodetic reference system and the geodetic network is its practical realization, for example by a set of materialized points. The origin point of a network is the point where, after the prior choice of a geodetic ellipsoid, the geographical astronomic and geodetic coordinates are equal, the ellipsoidal height is equal to orthometric one, conventionally. The geoid and the ellipsoid are

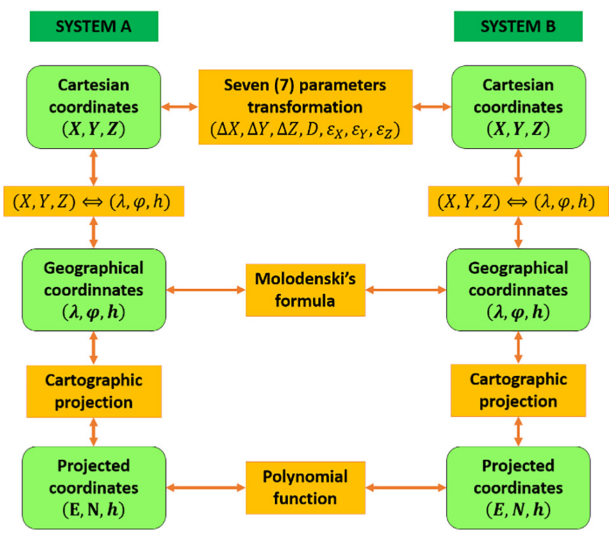

Fig. 1. Coordinates transformation process in geodesy (Dufour 2001)

parallel, or tangent at the origin level, this is conform to do a spatial similarity (translation and rotation in space), in order to nullify the vertical deflection.

The two-dimensional geodetic systems are derived from terrestrial measurements, based on the use of an origin point and provide only 2D coordinates on the ellipsoid (λ, φ) or projected ones (E, N). The 3D systems are determined by spatial measurements and provide coordinates in space, (X, Y, Z) or (λ, φ, *h*), centred at the Earth's mass centre.

Nowadays with the development of Global Navigation Satellite System (GNSS), the acquisition of data becomes easy, so that we can establish 3D geodetic systems. The need for transformation between local systems and even local systems to global systems and *vice versa* remains a concern of the geodetic community. Figure 1 presents the synthesis of the coordinates' transformation process in geodesy, between two geodetic systems A and B. There are three common methods for geodetic systems transformation:

- The spatial similarity, the easiest and most common for Cartesian geocentric coordinates, where ΔX, ΔY, ΔZ are the shifts along X, Y, Z axis respectively from System A to System B, D is the errors associated with scale parameter between the two Systems and ε_X, ε_Y, ε_Z are rotational elements;
- The Molodensky's formula for transformations between geographical coordinates;
- The polynomial model for transformations between projected coordinates.

In general, the datum shift requires measurements on several control points to better estimate the parameters of transformation. Here are some examples between local datum and WGS84: 45 points from North Sahara datum in Algeria (INCT 2007), 9 points from Merchich datum in Morocco and 59 points from Minna datum in Nigeria (Orupabo *et al.* 2014).

For a given ellipsoid, Cartesian geocentric coordinates (X, Y, Z) can be converted into geographical coordinates (λ, φ, h) by the following formulas (Dufour 2001):

$$\begin{cases} X=\left(N+h\right)\cos\lambda\cos\varphi \\ Y=\left(N+h\right)\sin\lambda\ cons\varphi \\ Z=\left[N\left(1-e^2\right)+h\right]\sin\varphi \end{cases}, \quad (1)$$

where λ and φ are the geographic longitude and latitude respectively, *h* – the ellipsoidal height, *a* – the semi-major axis, *b* – the semi-minor axis, $e=\sqrt{\dfrac{a^2-b^2}{a^2}}$ the first eccentricity,

$N = \dfrac{a}{\sqrt{1-(e\times\sin\varphi)^2}}$ the normal curvature radius at a considered point.

On the other hand, the following formulas transforms geocentric into geographical coordinates (Dufour 2001):

$$\begin{cases} \lambda = \tan^{-1}\left(\dfrac{X}{Y}\right) \\ \varphi = \tan^{-1}\left[\dfrac{Z\left(1-f^2\right)+e^2 a\sin^3\mu}{(1-f)\left(\sqrt{X^2+Y^2}-e^2 a\cos^3\mu\right)}\right] \\ h = \sqrt{X^2+Y^2}\cos\varphi + Z\sin\varphi - a\sqrt{1-(e\sin\varphi)^2} \end{cases}, \quad (2)$$

with $\mu = \tan^{-1}\left[\dfrac{Z}{\sqrt{X^2+Y^2}}\left((1-f)+\dfrac{e^2 a}{R}\right)\right]$,

$R = \sqrt{X^2+Y^2+Z^2}$ and $f = 1-\sqrt{1-e^2}$ the flattening.

2. Data

The data used are the geographical and geocentric Cartesian coordinates, orthometric heights of the GPS observation network established by JICA, the coordinates of the same origin point provided by IGNN and the grid of geoid heights computed from EGM2008 (Pavlis *et al.* 2008).

2.1. GPS/Levelling points network by JICA-IGNN

The point no.65 also known as ANG302, from the African Doppler Survey (ADOS) (Kumar 1983), was chosen as origin point to determine the coordinates' transformation parameters for the shifting into local geodetic datum. The height above WGS84 ellipsoid was set equal to the orthometric height. The coordinates' transformation approach followed by JICA-IGNN is summarized in Figure 2. The coordinates of the point no.65 are presented in Table 1. Table 2 shows the statistics of the geocentric coordinates as well as the orthometric and ellipsoidal heights of all 39 points. Figure 3 shows the study area and the GPS/Levelling network.

A relative model of geometric geoid had been computed at the GPS/levelling points on WGS84 using the formula (Heiskanen, Moritz 1967):

$$N = h - H, \quad (3)$$

with N the relative geoid height which is equal to zero at point no. 65, h the ellipsoidal height and H the orthometric height. The statistics of the relative geometric geoid are shown in Table 3, its grid is presented in Figure 4.

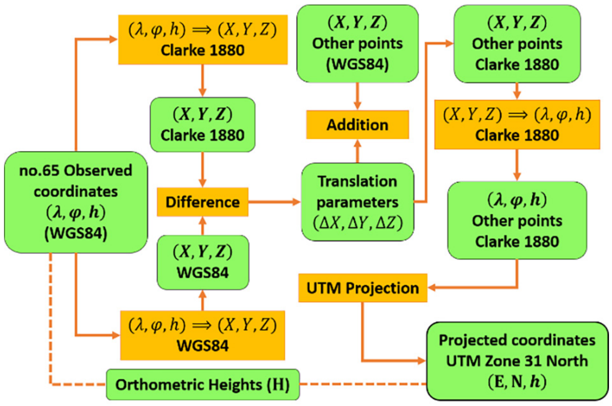

Fig. 2. Process of JICA-IGNN (JICA & IGN-N 1996)

Table 1. Coordinates of the point no.65 on WGS84 released by JICA-IGNN

X (m)	Y (m)	Z (m)	$h = H$ (m)
6 205 259.6597	339 171.4629	1 431 651.2409	242.33

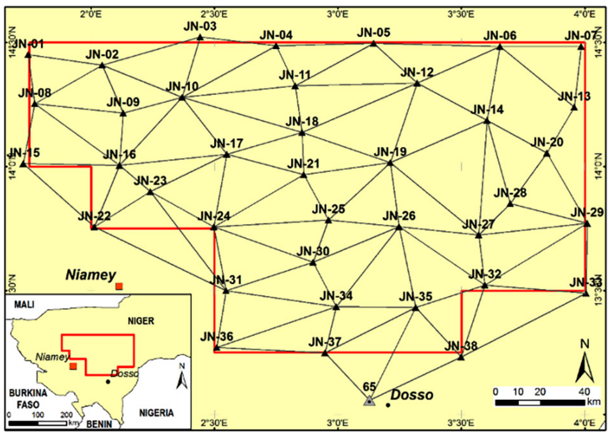

Fig. 3. Location map of the study area and GPS/Levelling Network (redesigned)

Table 2. Statistics of coordinates on WGS84 released by JICA-IGNN

	X (m)	Y (m)	Z (m)	h (m)	H (m)
MIN	618 240.517	186 355.619	1 431 651.241	190.576	191.074
MAX	6 205 259.660	433 342.597	1 589 062.095	273.612	275.693
MEAN	6 040 469.414	317 429.697	1 526 126.629	235.905	237.091
STD	891 168.149	73 348.501	44 266.002	21.731	21.892
RMS	6 104 186.088	325 582.043	1 526 752.018	236.878	238.074

Table 3. Statistics of relative variations of JICA-IGNN geometric geoid

MIN	MAX	MEAN	STD	RMS
–3.2	0.19	–1.19	0.91	1.49

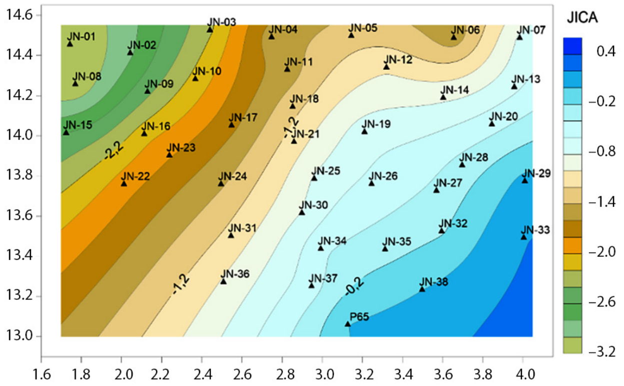

Fig. 4. Relative geometric geoid in meters on WGS84 (reinterpolated)

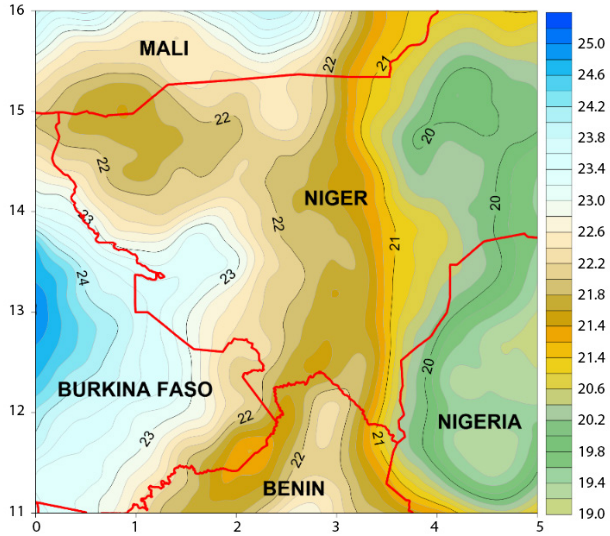

Fig. 5. Geoid Undulations from EGM2008 at d/o 2190. *Source*: Ibrahim Yahaya *et al.* 2017.

2.2. Data from the National Geographic Institute – Niger

In the late 1980s (1988), the National Geographic Institute of Niger (IGNN) had measured the point no.65 by accurate GPS observations. The resulting coordinates in WGS84 are presented in Table 4. The same coordinates are used as references by the joint commission of border delimitation between Niger and Burkina Faso in 2016.

The coordinates are different from those presented in the JICA-IGNN report (cf. Sect. 2.1).

2.3. Earth Geopotential Model 2008 (EGM2008)

The Earth Geopotential Model 2008 (EGM2008) had been developed under the leadership of the National Geospatial-Intelligence Agency (NGA) (Pavlis *et al.* 2008). It is complete to spherical harmonic degree and order 2159, and contains additional coefficients extending to degree 2190 and order 2159. It incorporates data from GRACE satellite mission, marine gravity anomalies derived from satellite altimetry, and a comprehensive set of terrestrial gravity anomalies. Since its release, EGM2008 is likely to become the standard geopotential model used for many applications, such as geoid and gravity anomalies computation (Okiwelu *et al.* 2011; Ibrahim Yahaya *et al.* 2015). It is also used for the evaluation with GPS/Levelling points and/or terrestrial gravity data, and as reference model for validating Global Geopotential Models (El Brirchi, El Azzab 2012; Godah, Krynski 2015; Benahmed Daho 2010). There are also applications in geology and geophysics (Evariste *et al.* 2014).

EGM2008 and several other models are available on the website of the International Centre for Global Earth Models (ICGEM), that provides also a calculation service for spherical harmonic functionals (Barthelmes, Köhler 2012). We compiled a database of gravity anomalies and geoid heights from these GGMs (Ibrahim Yahaya *et al.* 2017). Figure 5 shows the grid of geoid heights from EGM2008, evaluated at its maximum degree and order, it will be used in this study.

3. Methodology

We proceeded by coordinates' transformation into WGS84 datum, the computation of new geometric geoid model from transformed coordinates and its undulations are compared with those of EGM2008 geoid grid as preliminary assessment. We compared the local variations of the geometric geoid model released by the JICA, the model from transformed coordinate versus EGM2008 as reference model. The comparison is based on basic statistics and visual interpretation. To go further, we applied the hypothesis tests for comparing two populations. The trial version of XLSTAT Software (Addinsoft 2017) had been used for the hypothesis tests.

3.1. Coordinates' transformation

For the control, the geocentric coordinates (X, Y, Z) of point no. 65 were recomputed from geographic coordinates (λ, φ, h) by applying (1) and using the parameters of the ellipsoid WGS84 shown in Table 5.

Therefore, we calculated the difference between the coordinates of point no. 65 released by IGNN

Table 4. Coordinates of point no.65 in WGS84 released by IGNN

X	*Y*	*Z*	λ	φ	*h*	*H*
6 205 280.30	339 048.53	1 431 688.20	3.127457889	13.0577392	264.220	242.333

Table 5. Parameters of the WGS84 ellipsoid

a	*b*	*1/f*	*e*	*e'*
6 378 137.00	6 356 752.3142	298.2572221	0.081819191	0.082094438

$\left(X_{IGNN}^{65}, Y_{IGNN}^{65}, Z_{IGNN}^{65}\right)$ and those released by JICA-IGNN $\left(X_{JICA}^{65}, Y_{JICA}^{65}, Z_{JICA}^{65}\right)$ to determine the parameters of translation (ΔX, ΔY, ΔZ):

$$\begin{cases} \Delta X = X_{IGNN}^{65} - X_{JICA}^{65} \\ \Delta Y = Y_{IGNN}^{65} - Y_{JICA}^{65} \\ \Delta Z = Z_{IGNN}^{65} - Z_{JICA}^{65} \end{cases}. \quad (4)$$

We added the translation parameters to the geocentric coordinates of the 38 remaining points respectively to get the transformed coordinates:

$$\begin{cases} X_{Transf} = X_{JICA} + \Delta X \\ Y_{Transf} = Y_{JICA} + \Delta Y \\ Z_{Transf} = Z_{JICA} + \Delta Z \end{cases}. \quad (5)$$

The geocentric coordinates were transformed to geographical using (2). Then, we used (3) to compute the geometric geoid heights with the orthometric heights and the transformed ellipsoidal heights.

3.2. Points values extraction

We used the Spatial Analyst tool in ArcGIS to extract the point values from the geoid height grid of EGM2008 at GPS/Levelling points, both the transformed coordinates and those provided by JICA-IGNN.

3.3. Comparing EGM2008 with transformed GPS/Levelling points

We compared the geoid heights form EGM2008 and the transformed geometric geoid heights at all 39 points, including no. 65. The comparison is based on the difference:

$$\Delta N = N_{GPS} - N_{EGM2008}. \quad (6)$$

The statistics in terms Minimum, Maximum, Mean, Standard Deviation and Root Mean Square are presented.

3.4. Comparison of local variations of geometric geoid with global geoid form EGM2008

If we set a reference point with a geoid height N_{REF}, the relative variation ΔN_i of the same quantity at other points with geoid height N_i can be computed as follows:

$$\Delta N_i = N_i - N_{REF}. \quad (7)$$

1. Comparison by basic statistics, trend line and 3D representations

In the data provided by JICA and IGNN, the point no. 65 had already been chosen as reference, its geoid height is equal to zero. We also set the same point as reference, and then computed local relative variations of EGM2008's geoid and the transformed geometric geoid at the GPS/Levelling points. The basic statistics of relative values are computed in terms of Minimum, Maximum, Mean, Variance, Standard Deviation and Root Mean Square. We also presented the graph of the geoids' variations at all points and the 3D representations for visual interpretations.

2. Comparisons by hypothesis tests

The two-sample comparison of variances or Fisher's F-Test and two-sample T-Test or the Student's test for means' comparison are applied. We chose EGM2008 geoid as reference model, and its distributions were compared with those of the JICA-IGNN and the transformed geometric models. The two tests require that the distributions are normal and the Fisher's F-Test is prior to the Student's T-Test. In our case, the three samples have the same size, 39.

– Normality test by Jarque-Bera Test

We first made the normality test or Jarque-Bera Test (Jarque, Bera 1987) on the three distributions in order to confirm if they all follow the normal distribution. The null hypothesis H_0 is the assumption that the variable from which the sample was extracted follows a normal distribution and the alternative hypothesis H_a for the contrary. We set 5% as significance level.

– Two-sample comparison of variances or Fisher's F-Test: Two-tailed test

If the distribution of the two samples is normal, then the variable from which the ratio of variances was extracted follows the Fisher-Snedecor distribution with (38, 38) degrees of freedom. The null hypothesis H_0 is that the ratio between the variances is equal to 1 and the alternative hypothesis H_a for the contrary. We set two significance levels: 5% and 10%.

– Two-sample T-Test or the Student's test: two tailed test

If the variances are equal, the variable follows the Student's distribution with 76 degrees of freedom. The null hypothesis H_0 is the assumption that the difference between the means is equal to zero, the alternative hypothesis H_a for the contrary. We set 5% as significance level.

$$\frac{\left(\overline{X_2}-\overline{X_1}\right)}{s^*\sqrt{\frac{1}{39}+\frac{1}{39}}} \rightarrow T(76), \tag{8}$$

with $\overline{X_1}$ and $\overline{X_2}$ the means and s^* the estimated variance. The full methodology is summarized in Figure 6.

4. Results and discussions

4.1. Transformed coordinates

The recomputed Cartesian coordinates of point no. 65 are given in Table 6. The values are compared with those released by IGNN, see Table 4. The values are similar. Table 7 presents the parameters of translation at point no. 65. Table 8 shows the statistics of the Cartesian geocentric and geographical coordinates of the 39 points as well as those of the geometric geoid height, in the WGS84 datum, after the coordinate's transformation.

The geometric geoid heights vary from 20.067 m to 22.767 m in the study area, with 21.597 m and 0.745 m as mean and STD values respectively. The results of the comparisons are presented in the following sections.

4.2. Comparing EGM2008 with transformed GPS/Levelling points

The statistics of the geoid heights and the differences are presented in Table 9. The standard deviation of 15 cm and the mean of 4 cm show that the geometric geoid from transformed GPS/Levelling points fits slightly to the global model from EGM2008. As an assessment, this result is better than those obtained in surrounding countries, 24 cm in Morocco (El Brirchi, El Azzab 2011) and 21.2 cm in Algeria (Benahmed Daho 2009). The RMS of 38 cm is obtained in Khartoum area (Abdalla *et al.* 2012).

Table 6. Verification of coordinates at point no. 65

w	N	X	Y	Z
0.99983	6 379 227.045	6 205 280.296	339 048.528	1 431 688.195

Table 7. Parameters of translation at point no. 65

	X	Y	Z
IGNN	6 205 280.3000	339 048.5300	1 431 688.2000
JICA	6 205 259.6597	339 171.4629	1 431 651.2409
(ΔX, ΔY, ΔZ)	20.6403	–122.9329	36.9591

Table 8. Statistics of coordinates and geometric geoid after transformation

	X_{Transf}	Y_{Transf}	Z_{Transf}	λ_{Transf}	φ_{Transf}	h_{Transf}	H	N_{Transf}
MIN	6 162 065.486	186 232.686	1 431 688.200	1.7241	13.0600	212.952	191.074	20.067
MAX	6 205 280.300	433 219.664	1 589 099.054	4.0091	14.5258	298.160	275.693	22.767
MEAN	6 183 131.080	317 283.688	1 526 163.589	2.9373	13.9390	258.688	237.091	21.597
STD	11 006.794	73 337.498	44 266.002	0.6789	0.4123	21.727	21.892	0.745
RMS	6 183 140.625	325 437.274	1 526 788.962	3.0128	13.9449	259.575	238.074	21.609

Table 9. Statistics of geoid heights and differences at transformed GPS/Levelling points

MODELS	MIN	MAX	MEAN	STD	RMS
N_{Transf}	20.067	22.767	21.597	0.745	21.609
$N_{EGM2008}$	20.248	22.717	21.555	0.649	21.564
ΔN	–0.45	0.24	–0.04	0.15	0.16

Table 10. Statistics of geoids' variations

MODELS	MIN	MAX	MEAN	VAR	STD	RMS
JICA-IGNN	–3.2	0.19	–1.19	0.82	0.91	1.49
EGM2008	–1.41	1.06	–0.11	0.42	0.65	0.65
Transformed geoid	–1.82	0.88	–0.29	0.55	0.74	0.79

4.3. Comparison of local variations of geometric geoid with global geoid form EGM2008

Table 10 presents the statistics of local variation of EGM2008 geoid and the relative geometric geoid before and after coordinates' transformation. According to basic statistics, the JICA-IGNN model has a predominance of negative values with –1.19 m as mean value, the highest STD = 0.91 m and values range.

The relative variations of EGM2008 and transformed geometric geoids grid referenced to point no.65 are shown in Figure 7 and Figure 8 respectively.

The Figure 9 presents the graph of relative geoids' variations. The values are sorted by ascending order according to EGM2008 variations at GPS/Levelling points. The Figure 10 and Figure 11 show the 3D overlap of EGM2008 and JICA-IGNN geometric geoid, and that of EGM2008 and transformed geometric geoid, respectively.

We notice on Figure 9 and Figure 10 that the variation trends are different between the JICA-IGNN geometric model and EGM2008 geoids, the first one

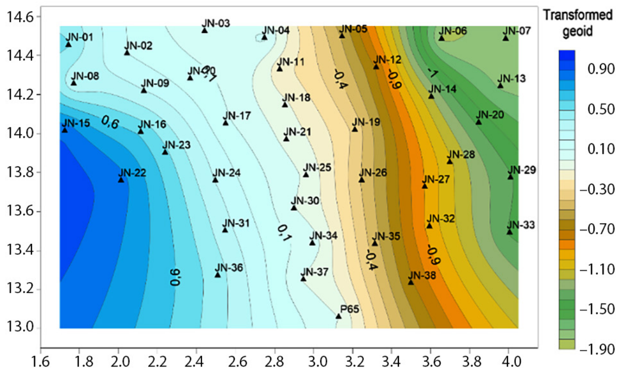

Fig. 8. Relative geometric geoid from transformed coordinates

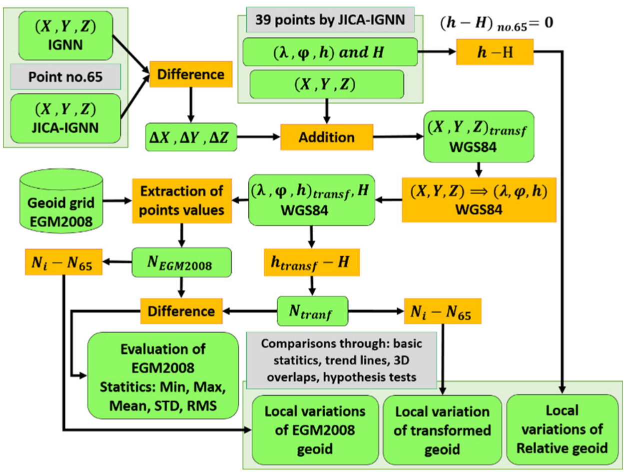

Fig. 6. Full methodology process

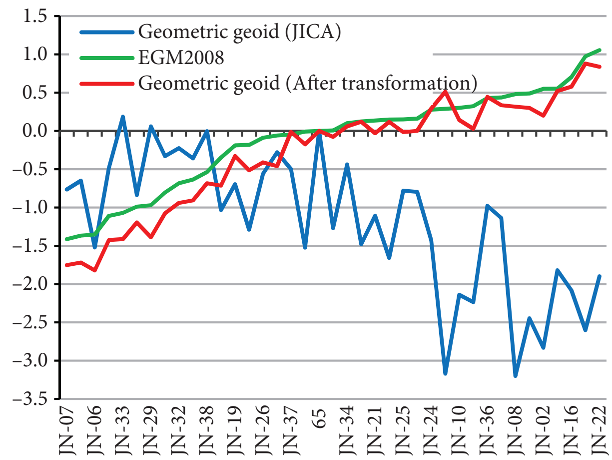

Fig. 9. Relative geoids' variations

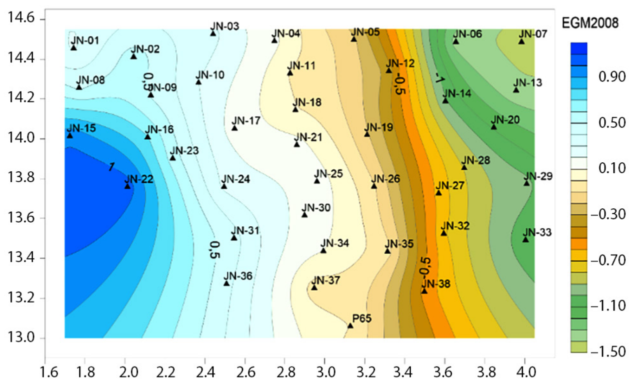

Fig. 7. Relative geoid of EGM2008 referenced to point no. 65

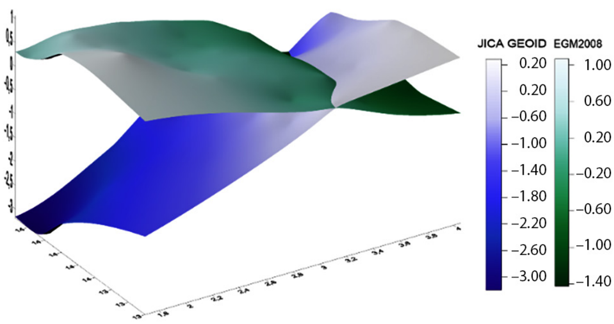

Fig. 10. 3D view of EGM2008 (green) and JICA-IGNN geometric (blue) geoids

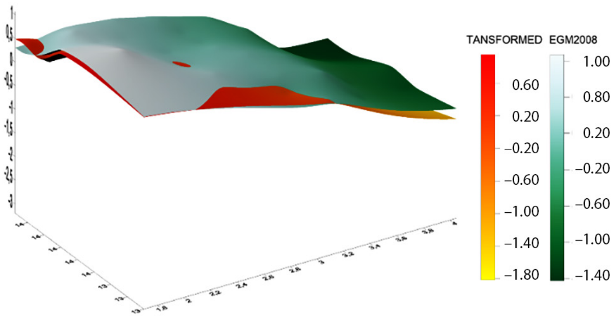

Fig. 11. 3D view of EGM2008 (green) and transformed geometric (red) geoids

has more significant variations. The transformation established by JICA-IGNN (JICA & IGN-N 1996) at point no. 65 has reversed and exaggerated the geoid heights variations in the study area. According to Figure 9 and Figure 11, the transformed geometric model and EGM2008 geoids have similar trends and the graphs are slightly close. The coordinates' transformation has restored the geometric geoid trend, its variations are similar to EGM2008 global geoid, even though EGM2008 appears smoother.

1. Jarque-Bera test

At the significance level α = 5%, the observed values are 2.589, 2.532, 3.025, the p-values(Two-tailed) are 0.274, 0.282, 0.220 respectively for JICA-IGNN, EGM2008 and Transformed geometric geoids, and the critical value is 5.991. The respective computed p-value, for all distributions, is greater than the significance level α = 5%, the null hypothesis H_0 is accepted. The risk to reject H_0 while it is true, is 27.40%, 28.2% and 22% respectively for JICA-IGNN, EGM2008 and Transformed coordinates' geoids. In Figure 12, the empiric cumulative distribution functions are close to the bisecting line. The Jarque-Bera confirms the normality assumption for the three samples.

2. Fisher's F-test/ Two-tailed

The results of EGM2008 comparisons with the other two models are as follows:

– JICA-IGNN versus EGM2008: Observed value F_{obs} = 1.892, F_{crit} = 1.892, p_value = 0.0528. The computed p-value is greater than the significance level α = 5%, the null hypothesis H_0 is accepted. The risk to reject the null hypothesis H_0 while it is true is 5.28%.
– EGM2008 versus Transformed geoid: Observed value F_{obs} = 1.279, Critical value F_{crit} = 1.907, p_value = 0.452. As the computed p-value is greater than the significance level α = 5%, one cannot reject the null hypothesis H_0. The risk to reject the null hypothesis H_0 while it is true is 45.17%.

At the significance of α = 5%, we can see on the distribution that the observed value is very close to the critical one, then we augmented the significance level α, to 10%. Figure 13 presents the result.

– JICA-IGNN versus EGM2008: Observed value F_{obs} = 1.892, Critical value F_{crit} = 1.717, p_value = 0.0528. As the computed p-value is lower than the significance level α = 10%, the alternative hypothesis H_a is accepted.

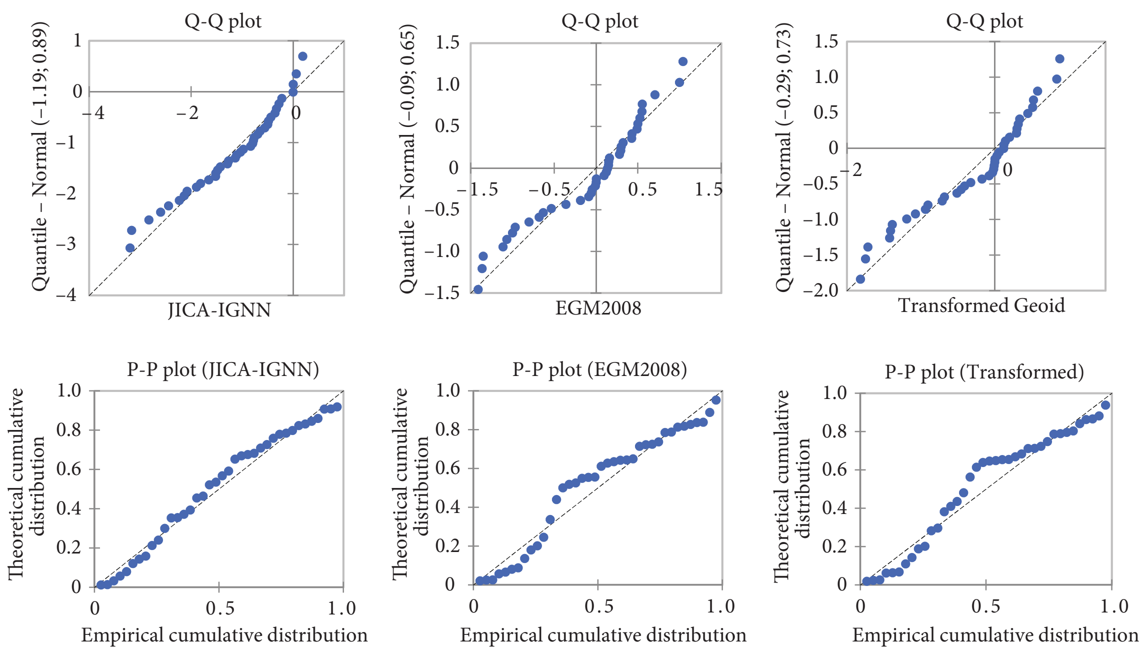

Fig. 12. Normal Quantile-Quantile, Probability-Probability plots

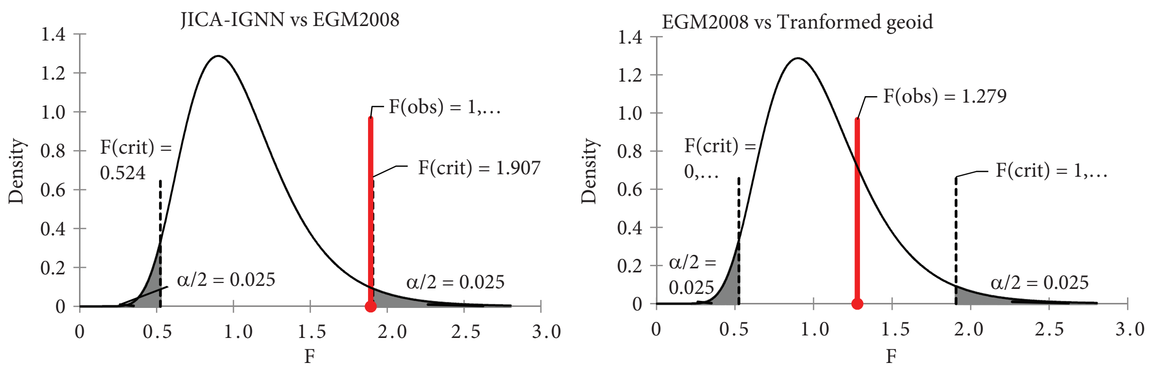

Fig. 13. Fisher-Snedecor Distributions, significance level 5%

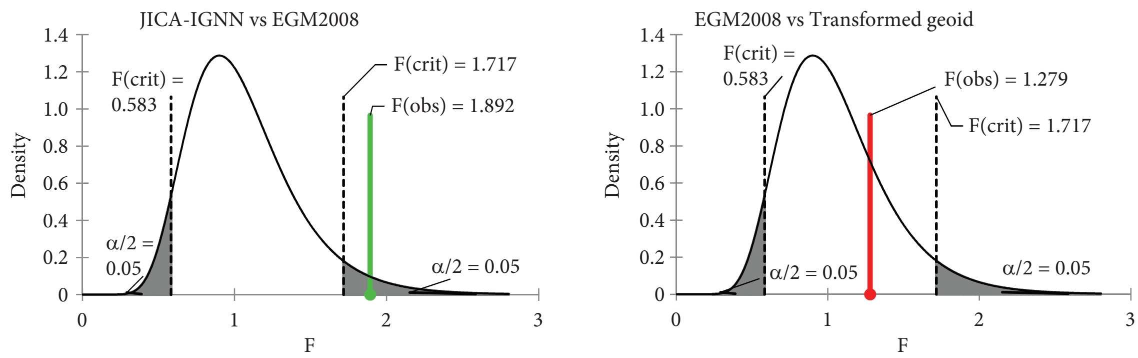

Fig. 14. Fisher-Snedecor Distributions, significance level 10%

- EGM2008 versus Transformed geoid: Observed value F_{obs} = 1.279, Critical value F_{crit} = 1.717, p_value = 0.4517. As the computed p_value is greater than the significance level α = 10%, the null hypothesis H_0 is accepted. The risk to reject the null hypothesis H_0 while it is true is 45.17%.

At the significance level of 10%, the observed value for the comparison between EGM2008 and JICA-IGNN geometric geoids is in the reject zone. We concluded at this level that the variances of EGM2008 and Transformed geometric geoid are equal. Therefore, only their means can be compared. The results are illustrated in Figure 14.

- Two-tailed T-test between EGM2008 and Transformed geoid: Difference = 0.202, Observed value $t_{obs} = -1.268$, Critical value $|t|_{crit}$ = 1.992, p_value = 0.2087. As the computed p-value is greater than the significance level α = 5%, the null hypothesis H_0 is accepted. The risk to reject the null hypothesis H_0 while it is true is 20.87%. The Student's distribution graph in Figure 15 confirms also the acceptance of H_0.

The Student's T-Test confirms the equality of means between the EGM2008 and transformed geometric geoid samples.

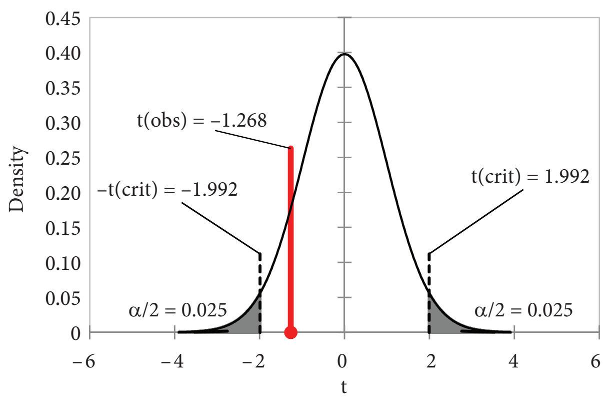

Fig. 15. Student's Distribution for Two-tailed test, significance level 5%

Conclusions

In this study, we transformed coordinates of 39 GPS/levelling points located in the southwest part of the Niger Republic into the WGS84. The differences between the transformed geometric and EGM2008 geoids give – 4 cm, 15 cm and 16 cm as mean value, STD and RMS respectively. The analysis of local variations referenced to the origin point no. 65 through basic statistics, trend lines and the 3D grids superposition, showed that the transformed geometric model and EGM2008 geoids have the similar trend. On a contrary, variations in the geometric geoid released by JICA-IGNN are reversed and exaggerated. The Jarque-Bera test confirms that the local variations of geoids follow normal distributions at the significance level α = 5%. By the Fisher's F-Test, the variances equality between the EGM2008 geoid and JICA-IGNN geometric model has been rejected at a significance level α = 10%. The same test confirms the variances equality between EGM2008 geoid and transformed model at the significance levels α = 5% and α = 10%. The two-tailed Student's T-Test with the significance levels α = 5% also confirms the equality of means between the EGM2008 and transformed geometric geoid samples. The coordinates' transformation has restored the true local variations of the geometric model of geoid. The transformed GPS/Levelling points can be used for the local assessment of other Global Geopotential Models (GGMs).

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