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USE OF TLS TECHNOLOGY IN HIGHWAY CONSTRUCTION

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Abstract. This article deals with issues related to the measurement of TLS technology, or 3D scanning in road construction. Based on the data and results obtained, a technological procedure for the use of TLS technology on highways and A-roads will be drawn up, mainly for monitoring the transition areas of bridges, which currently does not exist in the Czech Republic.

A smooth connection between two different structures in the transition areas should provide a comfortable crossing of the bridge structure. In order to unambiguously determine the movements in these areas, it is necessary to eliminate any inaccuracies that may affect the final result. For this reason, it was necessary to use a combination of traditional geodetic methods and special geodesy methods. In addition, several innovative methods were used, which emerged in this work based on newly emerging facts. All these operations and the presentation of the results will be described in this work.

Keywords: terrestrial laser scanning, bridge transition zone, technological procedure, transport construction.

Introduction

The paper aims to find a suitable mathematical model that would include all calculations affecting the determination of a priori accuracy analysis for subsequent surveying of highway bridge transition zones using the technology of terrestrial laser scanning (TLS).

Many international authors have already dealt with the issue of TLS in their studies, whether it was the determination of systematic measurement errors (Lichti, 2017), the creation of an error model by TLS (Gordon & Lichti, 2007), or the creation of a digital terrain model (DMT), again using the TLS method (O'Banion & Olsen, 2019). Regarding TLS technology in transport construction, articles about its use for determining the condition of bridges (Guldur et al., 2015) and for motorway construction (Johnson & Johnson, 2012), where even the influence of the age of asphalt layers on the accuracy of measurements is described, have already been published. However, this paper focuses on a specific structural part, which is the transition area of bridges. It is a relatively insignificant part of the road in terms of area but structurally very complex, where most of the defects and deformations that affect the driving comfort occur.

Research results indicate that selecting and implementing land surveying using the TLS technology and supplementary equipment is advisable. The carried out analyses aim to render results allowing for better interpretation of the obtained data of required accuracy. The accuracy is vital for the unambiguous determination of movements and deformations in the transition zones occurring in millimetres.

The results will also be used as a basis for the unification of methodological procedures to measure transition areas of bridge structures with TLS technology on motorways and A-roads. Such a separate technological procedure for monitoring transition areas has not yet been approved or developed.

1. Area of study

The highway bridge transition zones suffer from the manifestations of differential settlement of the structure and the adjacent road bank. Therefore, the transition zones require special structural designs, working processes and checks. Despite all the precautions, sufficient homogeneity is not always achieved, and deformations occur in the zones. Figure 1 shows a schematic representation of a bridge transition zone.

The bridges No. 56-018a.1 and 56-018a.2 are located in Ostrava, district of Moravská Ostrava a Přívoz, in the cadastral district Přívoz, the Czech Republic (Figure 2).

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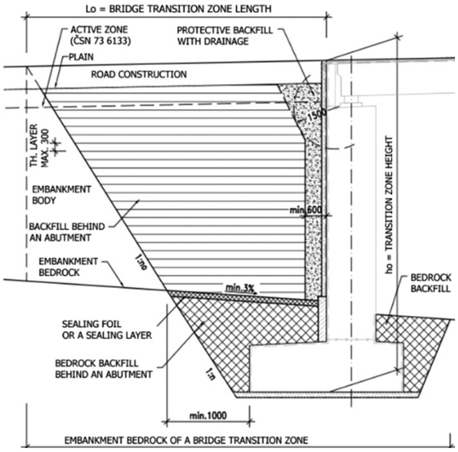

Figure 1. Bridge transition zone

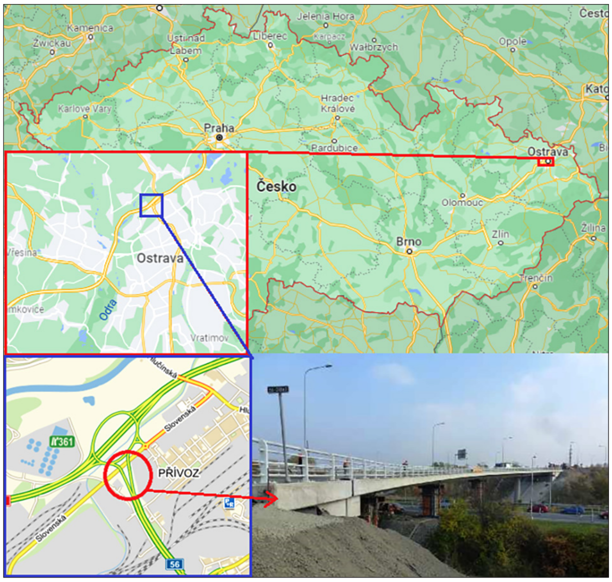

Figure 2. Locality

The bridges make parts of A-road No. 56, connecting the city centre and motorway D1. The bridges serve as access roads to a grade-separated roundabout with the motorway D1 mentioned above.

The bridges were erected within the construction of "Motorway D47, section 4708.2 Ostrava, Rudná – Hrušov, the second construction" in 2008. Their designation is 56-018a.1 (SO 228) and 56-018a.2 (SO 229).


Figure 3. Bridge structures reg. No. 56-018a.1 (above)

The construction of the bridges (Figure 3) was started and completed in 2008. They were put in operation in the spring 2009. They are concrete bridges of 5 compartments. The basic supporting member is a monolithic continuous beam of a trapezoidal section and fully restrained cantilevers. The bridge prestress is formed by 19 rope tendons. The interaxial spacing of the main girders is 18.16 m.

The bridges have the following parameters:

- superstructure: 94.90 m;
- bridge length: 114.50 m;
- width: 10 m;
- load-bearing structure length: 96.81 m.

In 2015 the bridges were repaired due to the constant development of the banks leading to disrupted statics. The heterogeneous material used in the embankment began to push against the load-bearing structure, which began to deviate (Figure 4).


Figure 4. Deviation of the bridge bearings in bridge No. 56-018a.1

A solution was proposed based on expert designer and structural engineer's statements and the financial demands of bridge repairs. The solution lies in excavating and backfilling the transition zones (Figure 5), constructing new approach slabs, bridge bolts, locking parapets and bearings. The bridge had to be lifted using a press (Figure 6), rectified and re-implanted for such works. All the operations during the reconstruction are supposed to guarantee a sufficient life of the given bridges and their trouble-free use.

Due to the constant consolidation of embankments below the bridge transition zones, the excavated transition zones were selected for the precise measurements using


Figure 5. The excavated bridge transition zone

Figure 6. Uplifting a bridge using presses

the TLS technology as TLS is able to plot irregular and constant movement.

To determine the position and altitude in real time, it is also possible to use the Global Navigation Satellite Systems (GNSS) technology – see Labant et al. (2017) for details.

2. Data and method

2.1. Accuracy analyses

Accuracy analyses are vital parts of layout tasks in civil engineering and engineering geodesy. Accuracy analyses aim to assess the accuracy of target layout parameters when the requirements are clearly determined by the project, standard or contract conditions, and the binding compliance with these requirements.

Basic analysis is carried out with the assumption of random errors, and consequently, systematic errors are introduced. In general, accuracy analyses may be classified as analytical, where the law of mean error accumulation is applied, and empirical, i.e. simulation methods. Next, we distinguish accuracy analyses before surveying (a priori), during surveying, and after surveying (posteriori).

The goal of the a priori accuracy analysis is to identify the accuracy requirements for the partial and target parameters, determination of mean errors of partial and target parameters, choice of laying out technology and means, determination of verification surveying mean error, and the choice of technology and means for the verification surveying.

Accuracy analyses in surveying verify whether the given geometric parameters of required accuracy are obtained. Hypothesis significance testing is used, where the significance level is selected with respect to the used confidence interval. It tests whether the hypotheses of a surveying task probability model are adhered to, whether the surveyed quantities are obtained with required statistical properties and planned accuracy. However, no test may prove the validity of a hypothesis; we may only, with specific risk, claim whether there is a reason to reject the hypothesis or not.

The obtained laying-out accuracy is assessed via a comparison of a control geometric member's measured deviation, the difference in laying outs with a permissible laying-out deviation or mean setting-out error. This assessment serves as a basis for accuracy analyses after surveying.

2.2. Accuracy determination via a priori surveying analysis

According to the basic formulation of balancing problems, it is generally true that when repeated measurements of the same quantity are made, different results will be obtained, i.e., that individual measurements are influenced by many elements that cannot be completely eliminated, such as imperfections in our perceptual senses, instruments, climatic conditions, etc. By using a more accurate instrument, choosing the right conditions for measurement, and by experience, we can only reduce this influence and thus achieve the desired accuracy of the measurement (Štroner & Křemen, 2017).

The measured quantities are usually lengths, angles, heights, or directions, which are expressed by a number and a unit. Each of these quantities l_i, repeatedly measured, includes in its result the real error ε_i, which has affected the result, either negatively or positively, against the actual value L_i of the quantity. Accordingly, the real error:

$$\varepsilon_i = L_i - l_i, \tag{1}$$

where: ε_i – the real error of the measurand, L_i – the actual (true) value of the given quantity, l_i – measured quantity.

This real error is made up of gross errors and random and systematic errors, which are also referred to as inevitable. Errors and gross errors will be eliminated by appropriate measurement procedures, checks, etc., and will, therefore, not be considered further in the calculation. The real error is, therefore, made up of random and systematic errors.

$$\varepsilon_i = \Delta_i + c_i, \tag{2}$$

where: ε_i – the real error of the measurand, Δ_i – systematic errors, c_i – random errors.

Given the same measurement conditions, i.e., the same climatic conditions, same instrument, same person, same design or manufacturing imperfection, repeated measurements are subject to the same significant systematic errors with some degree of dependence between them. The magnitude of these errors cannot be calculated or statistically determined, i.e. they are not statistical. Therefore, they can be defined by mathematical corrections, a suitable measurement procedure or calibration of the instrument.

Random errors cannot be suppressed when measuring the same quantity under the same conditions. Their magnitude can vary and can take on either negative or positive values that are around zero. These errors do not follow any laws, are not interdependent, cannot be justified or predicted. However, the practice has shown that their magnitude and sign follow a normal distribution for larger sets of measurements of the same kind. Since they are statistical, their accuracy characteristics can be expressed in terms of the magnitude of the expected variance of the measurement, including the probability of error within a given interval. Currently, the standard deviation σ or the mean error m is most commonly used in surveying. The mean error m defines the interval of the normal distribution. Random errors with a normal distribution are characterized as follows:

- probability of positive or negative error of a certain size is the same;
- small errors are more common than large ones;
- if the error exceeds a given limit, we consider it to be gross.

In order to define the gross errors from a set of measurements, it is necessary to determine a limit, which is the standard deviation σ of a measurement, or the size of the base interval <–σ; +σ>. The multiple of this interval is called the confidence coefficient up. Its choice is significant. If up is too small, our measurements will be uneconomical, and thus measurements that are OK will be excluded. Conversely, if the u_p coefficient is chosen to be large, the risk of gross errors in the measurement increases. The most common choice is u_p = 2 (95% probability of error) or u_p = 2.5 (99% probability of error). The result is called the permissible deviation and is denoted by δ_i, then:

$$\delta_i = u_p \cdot m_i, \tag{3}$$

where: δ_i – the permissible measurement deviation, u_p – reliability coefficient, m_i – mean measurement error (standard deviation).

The difference to the full probability (= 1) is called the significance level or significance risk α. Then:

$$\alpha = 1 - P, \tag{4}$$

where: α – the significance level, P – probability.

The significance level tells us how much of the results (in %) should exceed a given threshold. These values will be excluded as gross errors.

In order to achieve the maximum magnitude of the measurement error, it is necessary to know the limit deviation, which is obtained in connection with the choice of the reliability coefficient u_p.

At the same time, it should be emphasized that none of the accuracy characteristics listed above includes the effect of systematic errors, which are another part of the calculation of the complete mean error m, whereby:

$$m^2 = \sigma^2 + m_\Delta^2 = m_c^2 + m_\Delta^2, \tag{5}$$

where: m – the full mean error, σ – basic standard deviation, m_Δ – mean systematic error, m_c – mean accidental error.

In order to achieve the required results, it is first necessary to determine the required accuracy or the permissible deviation. This accuracy is given by the Czech technical standard ČSN 73 0212-4 Geodetic accuracy in construction (Czech Standardization Institute, 1994), and according to the above-mentioned technical standard, the permissible deviation of heights for motorways, A-roads and B-roads is set at $\pm 20 \cdot 10^{-3} m$, in the case of the occurrence of a cemented construction layer on the ground plane in the transition area.

Therefore: δ_z is a characteristic of the accuracy of geometric elements (permissible deviation of the z coordinate), while for geometric parameters for which the tolerance of the height component Δz is prescribed, it is determined by the condition:

$$\delta_z \le 0.2 \cdot \Delta z; \tag{6}$$

$$\delta_z \le 0.2 \cdot 40; \tag{7}$$

$$\delta_z \le 8 \cdot 10^{-3} m. \tag{8}$$

The relationship between the limit deviation (δ_z) and the mean measurement error (m_z) is:

$$\delta_z \le u_p \cdot m_z; \tag{9}$$

$$m_z \le \frac{\delta_z}{u_p}; \tag{10}$$

$$m_z \le 3.2 \cdot 10^{-3} m, \tag{11}$$

where: u_p = 2.5 is the confidence factor.

In this case, the geometric parameters are easily controllable, so $u_p = 2$ could have been chosen, but the effect of systematic errors must be included in the calculation.

In the previous calculation, only the height error was determined, which is standard for the measurement. However, the positional deviations are necessary for the unambiguous determination of the height error at a given location, so it is necessary to maintain the same accuracy in determining the position of the point, i.e. in the direction of the x and y coordinate axes. For this reason, the allowed deviations m_x and m_y will be chosen as for the height component.

All of these deviations indicate the magnitude of the individual semi-axes of the error ellipsoid always in the direction of a certain measurement error, i.e. in the two transverse directions that indicate the horizontal and zenith angle error and in the direction of the length measurement. These quantities can be represented graphically by an error ellipsoid, i.e. as an area connecting points with the same probability density, i.e. the error ellipsoid defines the area of the probability of occurrence of the position of a certain point.

Just to complete the accuracy characteristics, in addition to the two-dimensional mean errors, the spatial accuracy (3D) will also be expressed using the coordinate mean error (in space) $m_{x,y,z}$ or the spatial mean error m_{pr}:

$$m_{pr}^2 = m_x^2 + m_y^2 + m_z^2; \tag{12}$$

$$m_{x,y,z}^2 = \frac{m_x^2 + m_y^2 + m_z^2}{3} = \frac{m_{pr}^2}{3}. \tag{13}$$

Although these accuracies are expressed as a single number, they represent accuracy in three dimensions, i.e. in all coordinate axes by the same value.

Thus, it is a simplification for both mean errors (coordinate and spatial), neglecting the three components and their mutual covariance and replacing the error ellipsoid with an error sphere whose radius is the quadratic mean semiaxes of the ellipsoid. A graphical representation of the two mean errors would be two concentric spheres, with the sphere describing the coordinate mean error $m_{x,y,z}$ having a radius √3 times smaller.

The applicability of a one-dimensional quantity to express spatial accuracy (mean spatial and coordinate error) is determined by the more the ratio of the individual mean errors m_x, m_y and m_z differs from one, the less accurate their interpretation.

All of the above deviations are intended to represent the maximum interval of all inaccuracies that may affect the result.

By comparing the results calculated by the a priori accuracy analysis with the required accuracy, it will be possible to state whether the predicted accuracy of the selected methods and instruments is sufficient.

2.3. Instrumentation

In the case of a priori accuracy analysis surveying, it is important to select suitable instrumentation. Concerning the required accuracy, a universal Trimble S8 total station was selected for the terrestrial surveying, mainly due to the declared mean error of the measured angle and mean error of the measured length, and a Leica NOVA MS60 multistation with laser scanning – see the parameters in Table 1. Where D is measured length.

Table 1. Parameters of used geodetic instruments

Parameters	Trimble S8	Leica NOVA MS60
Mean error of the measured angle	1″/0.3 mgon	1″/0.3 mgon
Distance meter range to a prism	1.5 m–5500 m	1.5 m–10 000 m
Mean error of the measured length (prism)	±(0.8 mm + 1 ppmD)	±(1 mm + 1.5 ppmD)
Distance meter range (no prism)	1.5 m–1000 m	1.5 m–2000 m
Mean error of the measured length (no prism)	±(2 mm + 2 ppmD)	±(2 mm + 2 ppmD)

3. A priori accuracy analysis

In order to perform an a priori accuracy analysis, an error model will be created that best describes the requirements of this measurement.

This analysis has been divided into several stages due to the existence of many factors that enter the mathematical model and can affect the resulting accuracy:

1. Determination of accuracy of the primary surveying network – spatial polar method;
2. Determination of accuracy of the secondary surveying network – spatial polar method;
3. Accuracy of free station method;
4. Calculation of accuracy of points determined by laser scanning.

Primary surveying network:

The primary surveying network (PSN) is identical with the local setting-out net or its part. It is a topographical and altimetric setting-out network.

The setting-out network (Figure 7) was built in connection with the construction of the motorway. It includes the points 8200, 8202, 8222 and 8224 marked by heavy monumentation (observation pillars with forced centring). The setting-out network points may be understood as the initial, stable and sufficiently accurate points, and thus the initial points in the calculations of accuracy analysis (points of the primary surveying network) will be considered errorless. The given calculations will provide the internal accuracy of surveying, while the stability of the primary surveying network points will be supported by successive surveying.

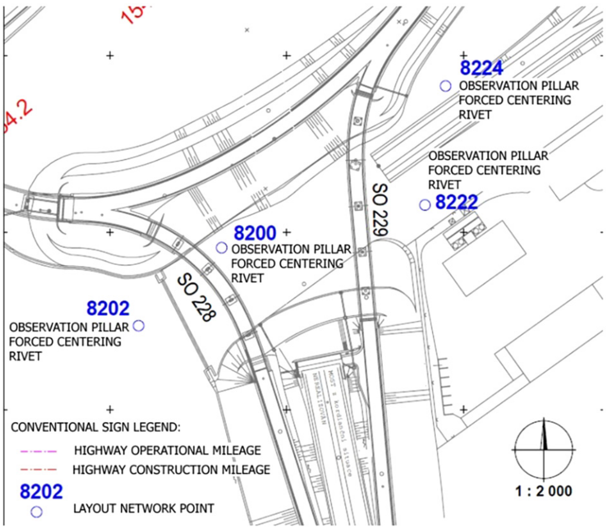

Figure 7. Primary surveying network

The points of the primary network were connected to the reference system with the local network character, which is connected with S-JTSK and Bpv. However, this was done maintaining the shape and dimensions of the local network, i.e. without necessary corrections and reductions.

Secondary surveying network:

In order to ensure a better configuration of the surveying network points, the primary surveying network was complemented with the secondary surveying network (SSN), namely the monumentation of minimum 3 points above each bearing of both the bridges. These points had to be clearly visible from the upper part of the embankment for the subsequent surveying of the transition zones. The points were monumented on lamp posts, either by a reflective label or sighting nail in the lamp post's base. The maximum altitude of the reflective label on the lamp post was 1.3 m from the concrete base to eliminate lamp post vibrations.

A priori analysis of the accuracy of the secondary surveying network:

First of all, it was necessary to design this network in order to achieve the improvement of its configuration mentioned above. In the modelling and, therefore, in the calculation of the priori accuracy, approximate coordinates of the points of this network were considered. Thus, no field measurements were needed at this time. However, it is already advisable to copy the actual situation in this design so that the transfer of these facts to the field differs from the design to the smallest possible extent.

In practice, the initial determination of the secondary surveying network accuracy is made with the determination of the model with all known input values with the fewest number of repetitions (Group 1), and a further decision is made on whether the number of repetitions is sufficient based on a comparison of accuracies. However, our case is different in that the precision of the starting points (primary system points) is not given. Only the precision of the secondary system points based on them is known. Therefore, it will be appropriate to perform the measurements in two groups to gain a margin to achieve the desired precision. According to (Harazim, 2014), it is evident that the accuracy of lengths and directions does not change significantly with an increasing number of repetitions higher than 2. Thus it would be inefficient and uneconomical to consider a more significant number of repetitions.

For the subsequent adjustment, the PSN points 8200, 8202, 8222 and 8224 were chosen as fixed points. Measurements between these points do not affect the adjustment result and only affect the accuracy characteristics.

By giving the coordinates of the "fixed" PSN points, it was proposed to adjust the SSN as a fixed-connection network. However, for our measurements, a local network was chosen, so in the end, it was adjusted as a free network with fixed points specified. Thus, the network virtually becomes a fixed-connection network, but no deformation occurs. In fact, by specifying fixed points (coordinates), the conditions for the unknowns will be added to the solution. Thus, it will be essentially an adjustment of the mediating conditions with the conditions for the unknowns, i.e., the unknown quantities sought are not measured directly but are obtained through other measured quantities that are in a known functional relationship with the unknown quantities and are bound by other conditions (Štroner & Hampacher, 2011).

The least squares method (LSM) was chosen for the subsequent calculation, which is the most commonly used method in geodesy, giving the smallest mean error of estimation of unknown quantities under certain assumptions. This issue is explained in more detail in (Hampacher & Radouch, 1997). The result of the adjustment is then the adjusted coordinates and their accuracy characteristics.

Therefore, the relationship is:

$$\overline{l} = \overline{l}\left(x\quad\right). \tag{14}$$

Again, this arises from the traditional formulation of mediating adjusting problems, where the measured quantities (mostly lengths, angles, heights, or directions) are expressed by a number and a unit, specifically horizontal lengths, horizontal and zenith angles (mediating quantities). Each of these quantities, l, repeatedly measured, includes the actual error ε which affected the result, either negatively or positively, against the actual value L of the quantity. Accordingly, the actual error, which is also described in detail in Chapter 3, is:

$$L = l + \varepsilon = L\left(\overline{x}^T\right), \tag{15}$$

where: ε – the real error of the measurand, L – the actual (true) value of the given quantity.

At the moment, however, we cannot determine ε, and hence not L. Therefore, it will be necessary to find an approximation for L, which we will call the equilibrium value and denote by $\overline{l}$. Then the following will hold:

$$\overline{l} = l + v = \overline{l}\left(x^T\right) \tag{16}$$

or

$$v = \overline{l}\left(x^T\right) - l \tag{17}$$

for which the LSM condition must be met:

$$\left[pvv\right] = v^T \cdot P \cdot v = \min, \tag{18}$$

where: v – the normalized repair, or repair vector of geometric quantities, p – weight or matrix of weights (P).

However, in order to obtain simple equations to calculate the unknowns of interest, it is necessary to linearize them using Taylor development with a restriction to first-order terms. After substituting in and introducing a new notation, we obtain the so-called *correction equation:*

$$v = A \cdot dx - l', \tag{19}$$

where: A – the experiment plan matrix (derivative matrix), l' – vector of reduced measurements.

As mentioned above, the mediating quantities are bound by other conditions, in this case, by fixing the coordinates of the "fixed" points (PSN points). Therefore, the following relation must apply:

$$\varphi(x^T) = 0, \tag{20}$$

which arises again from the formulation of tasks, but in this case, conditional ones. To derive this relation, a similar procedure is followed as for adjusting the mediating measurements.

Again, after satisfying the LSM condition, subsequent linearization and introduction of a new label, we get the so-called *transformed conditional equation*:

$$B^T \cdot dx + b = 0, \quad (21)$$

where: B – the matrix of linearized terms, b – vector of absolute terms.

This gives us a system of standard equations, which can be written as:

$$\begin{pmatrix} A^T \cdot P \cdot A & B \\ B^T & 0 \end{pmatrix} \cdot \begin{pmatrix} dx \\ k \end{pmatrix} + \begin{pmatrix} A^T \cdot P \cdot l' \\ b \end{pmatrix} = 0, \quad (22)$$

where: k – the correlate.

The equilibrium unknown x is then determined:

$$x = x_0 + dx, \quad (23)$$

where: x_0 – the approximate value of the unknown variables, dx – vector of increments compared to approximate values.

The weight matrix P will have the form:

$$P = \begin{pmatrix} p_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & p_n \end{pmatrix}, \quad (24)$$

where individual p_i weights are selected for independent measurements according to:

$$p_i = \frac{\sigma_0^2}{\sigma_i^2}, \quad (25)$$

where: σ_0 – the chosen constant, σ_i – standard deviation of the i-th measurement.

Derivation matrix A:

$$A_{i,j} = \frac{\partial f_i}{\partial x_j}, \quad (26)$$

where the individual elements of this matrix have for the i-th measurement l_i the j-th unknown X_j, f_i is the functional relationship between the measurement l_i and the determined coordinates. Therefore, the covariance matrix determining the accuracy of the result will be:

$$M = \sigma_0^2 \begin{pmatrix} A^T \cdot P \cdot A & B \\ B^T & 0 \end{pmatrix}^{-1}. \quad (27)$$

As mentioned before, it was not necessary to have any field measurements at this time. The values of the approximate coordinates and the quantities required from them were used for the calculation. The approximate grid configuration gives the shape of matrix A. The accuracy of the measured quantities gives the weight matrix P.

Therefore, the accuracy of the resulting model is determined by the accuracy of the individual measured variables and the accuracy of other factors that can affect the resulting accuracy calculation. Therefore, it was also necessary to include other aspects in the calculation, such as the accuracy of the substrate, inaccuracies in the alignment (centring) of the instrument and the target, and the accuracy of the determination of the instrument height. The mean centring error for severe forced instrument centring was used in our calculation $m_c = 10^{-5} m$ and the accuracy of the instrument height determination was chosen $m_i = 10^{-4} m$. The mean error in the determination of the target centration m_e and the determination of the target height m_t was chosen to be error-free, in the case of using a reflective label. In the case of using a reflective mini prism, it was $m_e = m_t = 70 \cdot 10^{-5} m$.

The mean errors given by the manufacturer of the respective instrumentation were used for the accuracy characterizing the measurement of individual variables.

Table 2. Final accuracies of the secondary surveying network points

Point No.	m_x [mm]	m_y [mm]	$m_{x,y}$ [mm]	m_{xy} [mm]	m_p [mm]	m_z [mm]	m_{pr} [mm]	$m_{x,y,z}$ [mm]
2288001	1.04	1.15	1.55	–0.04	0.78	1.01	1.85	0.62
2284001	0.54	0.38	0.66	–0.01	0.33	0.23	0.70	0.23
2284002	0.50	0.36	0.62	–0.08	0.31	0.17	0.64	0.21
2298002	1.15	1.14	1.62	–0.74	0.81	1.08	1.95	0.65
2294003	0.55	0.45	0.71	–0.25	0.36	0.28	0.76	0.25
2294004	0.64	0.55	0.84	–0.01	0.42	0.45	0.96	0.32
206735	1.12	1.13	1.59	0.22	0.80	1.06	1.91	0.64
206736	1.08	1.15	1.58	0.04	0.79	1.03	1.88	0.63
206737	1.06	1.16	1.57	0.01	0.79	1.03	1.88	0.63
2284007	0.41	0.56	0.69	0.03	0.35	0.27	0.74	0.25
2284008	0.51	0.53	0.74	0.22	0.37	0.35	0.81	0.27
206835	1.18	1.26	1.73	–0.48	0.86	1.18	2.09	0.70
206836	1.18	1.19	1.68	–0.40	0.84	1.14	2.03	0.68
206837	1.18	1.15	1.65	–0.35	0.82	1.10	1.98	0.66
2294005	0.64	0.66	0.92	–0.40	0.46	0.54	1.07	0.36
2294006	0.64	0.77	0.71	–0.47	0.50	0.63	1.18	0.39

The influence of the substrate has not been considered at this time, as the points of the primary survey net were chosen as error-free (see the chapter Primary surveying network).

Table 2 shows the expected accuracies of the individual points of the secondary measurement system in [mm].

These points will be used as a guide to determine the coordinates of the free stations.

Accuracy of the free station method:

The free station method requires a sufficiently dense initial survey net while providing considerable freedom for further work in the area. Redundant measurements ensure simple control.

The principle of the position thus determined is that its coordinates are determined by measuring distances, horizontal and zenith angles from the determined free station to points of known coordinates (Štroner, 2012).

Many approximate methods can be used to calculate the coordinates of the free station, but the best calculation is obtained by the least squares method (LSM).

The calculation will be carried out as follows. From station S (Figure 8), the horizontal directions φ_1 to φ_3 (with mean error m_φ) and distances d_1 to d_3 (with mean error m_d) are surveyed using known orientation points. The unknowns are the coordinates of the position S [X_S Y_S Z_S] and the orientation shift of the observed set O, the vector of unknowns.

$$X = \left(X_S \; Y_S \; Z_S \; O\right)^T. \tag{28}$$

Measurement vector:

$$l = \left(\varphi_1 \, \varphi_2 \, \varphi_3 \, d_1 \, d_2 \, d_3 \, z_1 \, z_2 \, z_3\right). \tag{29}$$

Furthermore, t is a function of the unknowns:

$$t = f\left(x\right). \tag{30}$$

And for a given measurement, then:

$$\varphi_{Si} = \arctan\left(\frac{Y_i - Y_S}{X_i - X_S}\right); \tag{31}$$

$$d_{Si} = \sqrt{\left(X_i - X_s\right)^2 + \left(Y_i - Y_s\right)^2}; \tag{32}$$

$$z_{Si} = Z_S + d_i \cdot \cot g \, z_i. \tag{33}$$

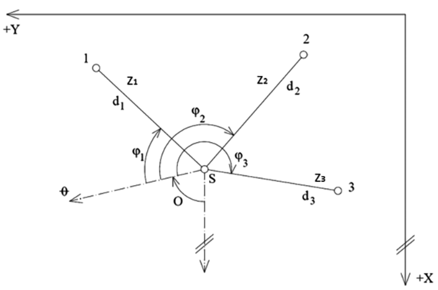

Figure 8. Scheme to determine a temporary station

Along with the measured values, it is necessary to know the approximate values of the unknowns X_0, which are uncompensated from the measured values and known coordinates. It will always be an iterative calculation as the relationships are not linear.

The matrix of derivatives of functional relations by unknowns A: (the shape of this matrix is derived from the approximate network configuration).

$$A = \begin{pmatrix} \frac{\partial\varphi_1}{\partial X_S} & \frac{\partial\varphi_1}{\partial Y_S} & \frac{\partial\varphi_1}{\partial Z_S} & \frac{\partial\varphi_1}{\partial O} \\ \frac{\partial\varphi_2}{\partial X_S} & \frac{\partial\varphi_2}{\partial Y_S} & \frac{\partial\varphi_2}{\partial Z_S} & \frac{\partial\varphi_2}{\partial O} \\ \frac{\partial\varphi_3}{\partial X_S} & \frac{\partial\varphi_3}{\partial Y_S} & \frac{\partial\varphi_3}{\partial Z_S} & \frac{\partial\varphi_3}{\partial O} \\ \frac{\partial d_1}{\partial X_S} & \frac{\partial d_1}{\partial Y_S} & \frac{\partial d_1}{\partial Z_S} & \frac{\partial d_1}{\partial O} \\ \frac{\partial d_2}{\partial X_S} & \frac{\partial d_2}{\partial Y_S} & \frac{\partial d_2}{\partial Z_S} & \frac{\partial d_2}{\partial O} \\ \frac{\partial d_3}{\partial X_S} & \frac{\partial d_3}{\partial Y_S} & \frac{\partial d_3}{\partial Z_S} & \frac{\partial d_3}{\partial O} \\ \frac{\partial z_1}{\partial X_S} & \frac{\partial z_1}{\partial Y_S} & \frac{\partial z_1}{\partial Z_S} & \frac{\partial z_1}{\partial O} \\ \frac{\partial z_2}{\partial X_S} & \frac{\partial z_2}{\partial Y_S} & \frac{\partial z_2}{\partial Z_S} & \frac{\partial z_2}{\partial O} \\ \frac{\partial z_3}{\partial X_S} & \frac{\partial z_3}{\partial Y_S} & \frac{\partial z_3}{\partial Z_S} & \frac{\partial z_3}{\partial O} \end{pmatrix} = \begin{pmatrix} \frac{\partial\varphi_1}{\partial X_S} & \frac{\partial\varphi_1}{\partial Y_S} & \frac{\partial\varphi_1}{\partial Z_S} & 1 \\ \frac{\partial\varphi_2}{\partial X_S} & \frac{\partial\varphi_2}{\partial Y_S} & \frac{\partial\varphi_2}{\partial Z_S} & 1 \\ \frac{\partial\varphi_3}{\partial X_S} & \frac{\partial\varphi_3}{\partial Y_S} & \frac{\partial\varphi_3}{\partial Z_S} & 1 \\ \frac{\partial d_1}{\partial X_S} & \frac{\partial d_1}{\partial Y_S} & \frac{\partial d_1}{\partial Z_S} & 0 \\ \frac{\partial d_2}{\partial X_S} & \frac{\partial d_2}{\partial Y_S} & \frac{\partial d_2}{\partial Z_S} & 0 \\ \frac{\partial d_3}{\partial X_S} & \frac{\partial d_3}{\partial Y_S} & \frac{\partial d_3}{\partial Z_S} & 0 \\ \frac{\partial z_1}{\partial X_S} & \frac{\partial z_1}{\partial Y_S} & \frac{\partial z_1}{\partial Z_S} & 1 \\ \frac{\partial z_2}{\partial X_S} & \frac{\partial z_2}{\partial Y_S} & \frac{\partial z_2}{\partial Z_S} & 1 \\ \frac{\partial z_3}{\partial X_S} & \frac{\partial z_3}{\partial Y_S} & \frac{\partial z_3}{\partial Z_S} & 1 \end{pmatrix}. \tag{34}$$

The weight matrix P (the accuracy of the measured quantities determines the shape of the weight matrix):

$$P = diag\left(p_1 \, p_2 \, p_3 \ldots p_n\right); \tag{35}$$

$$p_i = \frac{m_0^2}{m_i^2}, \tag{36}$$

where: m_0 – the a priori mean unit error, m_i – mean error of individual measurements.

It is useful to define the mean errors m; in a way that best describes the situation at hand. In addition to the accuracy of the measurements, the accuracy of the orientation points and the accuracy of their centring over the monumentation mark must be included in the calculation. Therefore, the resulting mean direction error will have the following form:

$$m_{\varphi_u}^2 = m_\varphi^2 + m_{XY_O}^2 + m_{C_O}^2, \tag{37}$$

where: m_{XY_O} – the precision of the coordinates of the orientation point, m_{C_O} – accuracy of the orientation point centring, and similarly for the mean length error:

$$m_{d_u}^2 = m_d^2 + m_{XY_O}^2 + m_{C_O}^2. \tag{38}$$

And for the medium zenith angle error:

$$m_{z_u}^2 = m_z^2 + m_{Z_O}^2 + m_{t_O}^2, \quad (39)$$

where. m_{Z_O} – the accuracy of the height coordinates of the orientation point, m_{t_O} – the accuracy of determining the height of the orientation point.

This adjustment gives the adjusted coordinates and orientation shift and the mean errors of the adjusted unknowns in the form of a covariance matrix, which we obtain based on the accuracies of the measurements that enter the calculation. Again, we follow the basic formulation of the equilibration problem.

Vector of reduced measurements l':

$$l' = f\left(X_0\right) - l. \quad (40)$$

Then the calculation of the increments of the unknown dx has the following form:

$$dx = -\left(A^T \cdot P \cdot A\right)^{-1} \cdot A^T \cdot P \cdot l'. \quad (41)$$

Repair vector v:

$$v = A \cdot dx + l'. \quad (42)$$

Aposterior mean unit error:

$$S_0 = \sqrt{\frac{v^T \cdot P \cdot v}{n-k}}. \quad (43)$$

A covariance matrix of the adjusted unknowns:

$$\Sigma = S_0^2 \cdot \left(A^T \cdot P \cdot A\right)^{-1}. \quad (44)$$

On the diagonal of the covariance matrix, there are mean error squares m_{Xs}, m_{Ys}, m_O.

$$m_{Xs} = \sqrt{\Sigma_{1,1}}; \quad (45)$$

$$m_{Ys} = \sqrt{\Sigma_{2,2}}; \quad (46)$$

$$m_{Zs} = \sqrt{\Sigma_{3,3}}; \quad (47)$$

$$m_O = \sqrt{\Sigma_{4,4}}. \quad (48)$$

Then for the mean positional error:

$$m_p = \sqrt{\left(\Sigma_{1,1} + \Sigma_{2,2} + \Sigma_{4,4}\right)} \quad (49)$$

and for the mean coordinate error:

$$m_{x,y} = \sqrt{0.5 \cdot \left(\Sigma_{1,1} + \Sigma_{2,2} + \Sigma_{4,4}\right)} = 0.5 \cdot m_p. \quad (50)$$

Several principles must be observed when choosing a free station:

- higher quality results are obtained when measuring directions and lengths;
- it is important to proceed from the measured values of at least 2 points, but it is more advantageous to survey at least 4 points;
- the best configuration of points is when the orientation points are evenly distributed along the circumference of an imaginary circle, in the centre of which there is a free station; very good results may be obtained even at the very narrow configuration;
- the credibility of the result may be achieved via a proper selection of weights.

The coordinates of the free stations will be approximately determined again for their subsequent calculation of accuracy.

For the calculation, we chose a simulation with the worst configuration of visible points above the transition zones, i.e. more remote orientation points and points close to one another, i.e. orientation points with a small horizontal angle.

For the analysis of accuracy, one free station whose accuracy will show the worst values will be chosen for each transition zone

Table 3 indicates the accuracy in the determination of free stations for the transition zones above the bearings.

The accuracy of the calculation of the free station itself was impaired by the effect of the accuracy of the orientation points, i.e. the secondary system points.

External influence on the surveying using terrestrial laser scanning system:

The principle of surveying using terrestrial laser scanning systems is identical to electronic tachymeters or aerial laser scanning systems, so-called lidars. It is the case of measuring a transit time from sending a laser pulse to the arrival of a reflected pulse, from a reflector to the detector of the signal. The transit time, or its half, recalculated to the speed of light in the real environment must be corrected by a deceleration of the light propagation in the real environment as opposed to vacuum depending on the real meteorological conditions.

The speed V of the electromagnetic radiation in the air is given by the relation:

$$V = \frac{V_0}{n}, \quad (51)$$

Table 3. The accuracy of the determination of free stations above the bearings

Point No.	m_x [mm]	m_y [mm]	$m_{x,y}$ [mm]	m_{xy} [mm]	m_p [mm]	m_z [mm]	m_{pr} [mm]	$m_{x,y,z}$ [mm]
2416001	1.48	1.80	1.17	0.09	2.33	1.58	2.82	0.94
2416005	1.22	1.24	0.87	0.09	1.74	0.91	1.96	0.65
2416007	1.05	1.02	0.73	0.05	1.46	0.71	1.63	0.54
2416009	1.76	1.84	1.27	–0.07	2.55	1.57	2.99	1.00

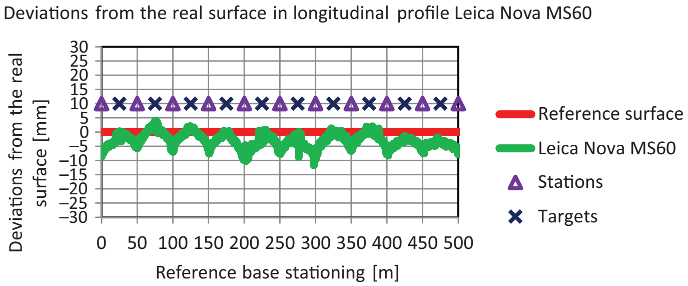

Figure 9. Deviations from the real surface in a longitudinal cross-section

where: V_0 – the speed of light in vacuum, n – the air refraction.

The speed of the modulated waves changes in dependence on the atmospheric conditions during measurements, which leads to corresponding changes in the modulated wavelengths and thus to the basic correction of measurement in the optical measurement of lengths. The refraction of electromagnetic waves in the air is a function of air temperature, atmospheric pressure and water vapour pressure.

The following equation is used for refraction in light emission in the real environment:

$$n_g = 1 + \left[287.604 + \frac{4.8864}{\lambda^2} + \frac{0.068}{\lambda^4} \right] \cdot 10^{-6}, \qquad (52)$$

where: λ is the wavelength of a carrier beam $[\mu\text{m}]$.

The equation applies under the assumption that the measurement is carried out using diodes that generate light based on light emission from diodes GaAs or GaSb in the standard environment, i.e. $0\,°\text{C}, 1013.2472\, HPa\, a\, 0.03\% CO_2$.

Due to changes in the temperature, pressure and moisture, the air refraction n_a is given by the equation:

$$n_a = 1 + \frac{0.359474\left(n_g - 1\right)p}{273.1 + t} - \frac{1.5026 e \cdot 10^{-6}}{273.2 + t}, \qquad (53)$$

where: p is the atmospheric pressure $[\text{mmHg}]$, t – the temperature of air [°C], e – the vapour pressure $[\text{mmHg}]$.

The method of correction of the measured distance assuming a homogeneous environment is, therefore, the role of the above equations. For more information on the optical index of refraction in dependence on pressure, temperature and other conditions see Owens (1967).

Calculation of accuracy of points determined by laser scanning:

The determination of the accuracy of points obtained via laser scanning is more complex than that due to a number of input factors influencing the final accuracy. Some errors will be eliminated, and some partially simplified.

The basis for calculating the accuracy of points determined by laser scanning is the determination of spatial coordinates from surveying the horizontal and zenith angles and the distances using non-prism surveying, i.e. spatial polar method.

In non-selective data collection using laser scanning, we obtain measured points in the form of the so-called point clouds comprising hundreds to thousands of points in a certain raster. The raster never has regular dimensions due to measuring non-identical angles and distances of points. It means that a point cloud will be denser closer to the station. For this reason, interpretation may be more difficult when modelling in more remote areas. This drawback may be eliminated by dividing the measured areas and making the raster denser in more remote distances. In this case, points will be 5 cm away in the closer area and 3 cm away in more remote distances.

The accuracy of measuring a point cloud depends on the material reflectivity to measure distances using a rangefinder, i.e. the quality of points will differ when measuring during a hot sunny day or heavy rain. Thus, it is important to survey during dry weather and suitable temperatures.

As the transition zone makes part of a line structure or A-road, it does not have large slope ratios in the area. For the sake of simplicity, we can say that it is a plane. In such a case, rangefinder rays fall under a more favourable angle in the vicinity of the instrument, and thus better reflection of the rangefinder occurs. During modelling of such parts, they may get "elevated". This error is documented in Figure 9, which was obtained by a multistation Leica Nova MS60 on a reference base.

To eliminate this error, we use control points (Figure 10), which are located on the borders of the individual surveyed sites. This way, the deformed measurements get adjusted again.

The calculation of the accuracy of the discrete points determined by laser scanning would be uneconomical, unnecessary, partially inconclusive and time-consuming.

Figure 10. Ground control point

Table 4. Gives the calculated a priori scanning accuracies based on error accumulation law

Point No.	m_x [mm]	m_y [mm]	$m_{x,y}$ [mm]	m_{xy} [mm]	m_p [mm]	m_z [mm]	m_{pr} [mm]	$m_{x,y,z}$ [mm]
101	2.59	2.04	2.36	1.65	3.30	2.23	3.98	1.33

Therefore, a point will be selected where we assume the most significant error and least accuracy.

The accuracy of the measured length most influences the determination of point accuracy. The maximum distance of points measured by laser scanning does not exceed 35 m. It is due to testing the method before its use in practice. When using a rangefinder for non-prism surveying of lengths, angles of the falling laser beams on the surface represent an important factor affecting the surveying accuracy. The biggest error logically burdens such a distant point, and thus, the maximum distance of 35 m from the station guarantees relatively good angles of fall, thus increasing the quality of determining the position and altitude of point clouds. The mean errors will be calculated for the longest measured length of a detailed survey point, i.e. the most remote point of laser scanning (Table 4).

Conclusions

The error model calculations included the errors that have the greatest impact on the resulting accuracy based on the chosen methods. These are centring errors and errors in the determination of instrument or target height, reflection errors, etc. These errors are related to the method of the monumentation of points, determining the target or instrument height, choice of methods, or instrumentation. For this reason, methods have been chosen which are minimally or not affected by these errors. These include, for example, the use of reflective labels to monument points or the determination of the position of points using free stations, where the effect of these errors does not enter into the calculation at all.

The individual tasks for the determination of a priori accuracy were performed under the least favourable computational model, i.e. that the field measurements will be performed according to the principles that ensure the quality and accuracy of the required data.

The results of the a priori accuracy analysis calculations show that, despite the complexity of the task, the chosen measurement methods and procedures are sufficiently accurate and applicable, depending on the comparison of the calculated accuracy of the laser scanning points with the required measurement error limit. Thus, very high accuracies, i.e. in the order of mm or hundredths of mm, can be achieved for the measurement of transition areas of bridge structures using the TLS technology.

Based on the established facts, obtained data and results, it was decided that the methods and calculations are satisfactory, and further research can be continued to develop a technological procedure for measuring bridge transition areas using the TLS technology on motorways and A-roads, which will be subsequently submitted to the working group at the Road and Motorway Directorate of the Czech Republic.

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