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LEAST-SQUARES MODIFICATION OF STOKES' FORMULA WITH EGM08

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Abstract. Least-squares modification is an optimal method of modifying Stokes' formula. This method can be categorized as a generalization of the spectral combination methods as it considers the truncation error of the integral formulas in its combination process. In short, this method involves the modification parameters based on minimizing the error of terrestrial gravimetric data, satellite data and the truncation error of the integral. In this respect, the choice of the geopotential model definitely plays an important role. This paper uses the recent combined geopotential model EGM08 for generating the spectra of gravity anomaly and its error. Numerical results show that EGM08 improves least-squares modification by about 10 cm comparing to the traditional way.

Keywords: biased estimator, unbiased estimator, degree variance, global root mean squares error, optimization, spectral combination.

1. Introduction

The geoid is an equipotential surface which is the best approximation of the mean sea level. The geoid is a physical shape of the Earth and is extended through the continents. This surface is considered as vertical datum on heights. One of the most well-known methods of determining such a surface is using Stokes' integral the formula of which needs a global coverage of terrestrial gravimetric data. Molodenski *et al.* (1962) were the first who suggested modifying the kernel of this integral in order to minimize the effect of the far zone data. In fact, modification changes the spectrum of the kernel to be more sensitive to the data around the computation point rather than to the far zone data. Following Molodenski *et al.* (1962), many other modification methods were proposed, such as Wong and Gore (1969), Meissl (1971), Colombo (1981), Vanicek and Kleusberg (1987) etc. These methods are categorized as deterministic ones in which the truncation error of the integral is minimized. Later, Sjöberg (1980 and 1981) proposed a stochastic method to combine the integral formula with geopotential models and named this method spectral combination. Sjöberg (1984a) introduced the spectral combination method that included the error of a geopotential model and the truncation error of the integral and called it least-squares modification (LSM) which helped Sjöberg (1984b) with finding ways of including the error spectra of terrestrial gravimetric data. This method of modification considering the error of the geopotential model, terrestrial data and the truncation error of the integral is categorized as a stochastic approach to modification. Sjöberg theoretically (1986) compared LSM and other deterministic methods.

Sjöberg (1991) further investigated LSM considering a different degree of modification and the geopotential model. He partitioned his method into two parts a) biased LSM (BLSM) and b) unbiased LSM (ULSM). Consequently, these stochastic methods were investigated and tested by other researchers, like Nahavandchi (1999) and Hunegnaw (2001). Sjöberg (2003) finally presented a general model for LSM and optimum LSM (OLSM). Ågren (2004) also used ULSM in his computation for his works on geoid determination. Ellmann (2005) numerically tested and compared these three stochastic methods.

The above introduced information about stochastic modification shows that this method is sensitive to the spectra of geopotential coefficients and its errors as well as to terrestrial data. Therefore, a type of the geopotential model and the quality of terrestrial data play an important role in the LSM process. However, the error spectra of the models are restricted to a certain degree and order which does not satisfy LSM. Thus, some analytical models are used to generate high degree spectra, e.g. Kaula (1963) model, Tscherning/Rapp (TR) (1974) model or Jekeli and Moritz model (Jekeli 1978). However, the recent geopotential model EGM08 was developed to degree and order 2160 and can be a successor for such analytical models. Also this model contains useful information due to the high degree spectra of the error. This paper investigates this replacement and compares both cases of using the analytical model and *EGM08*. Besides, it considers the global root mean square error (RMSE) of the integral geoid estimators modified by BLSM, ULSM and OLSM based on both models. Such a study has not been reported until now and it is useful for researching LSM in the future.

2. The *EGM08*

EGM2008 has been released by the NGA (National Geospatial-Intelligence Agency, US) EGM Development team. The long wavelength structure of this model was extracted from GRACE (Tapley 2005) data to degree and order 60 (Kenyon *et al.* 2007) and terrestrial gravity anomalies through the whole work with $5' \times 5'$ resolution for short wavelength structure. First, a primarily model (EGM05) was developed by Pavlis *et al.* (2004) to test the feasibility of such developments. Based on the acceptable results of this model having relatively good agreements with satellite altimetry, GPS/Leveling and deflection of vertical data, they proceeded with the further development of EGM08. In order to estimate the propagated error in the data on the geopotential coefficients, Pavlis and Saleh (2004) used the discredited integral formulas as a functional relation between the obtained data (e.g. terrestrial gravity anomaly, airborne gravimetry, satellite altimetry, marine gravity) and the coefficients. They stated that such estimation was sufficient for high degree coefficients. The effect of terrain was also considered based on the shuttle radar topography mission (SRTM) (Werner 2001) and a new topographic database DTM2006 was developed by Pavlis *et al.* (2006) and based on SRTM and BEDMAP. To clarify ice and water column thinness, information on bathymetry from altimetry and ship sounding data, the former version DTM2002 was presented.

3. Least-squares modification of Stokes' formula

The geoid can be expressed by the below presented integral well-known Stokes' formula (Heiskanen and Moritz 1967):

$$N(P) = \frac{R}{4\pi\gamma} \iint_{\sigma} S(\psi) \Delta g(Q) d\sigma, \tag{1}$$

where: R is the radius of a reference sphere, γ is normal gravity at computation point P, r is the geocentric distance at ψ, is the geocentric angle between P and integration point Q with the following expression:

$$\cos\psi = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\lambda' - \lambda), \tag{2}$$

where: θ and λ are the co-latitude and longitude of P and θ' and λ' are that of integration point Q. σ is the unit sphere, Δg(Q) is the gravity anomaly at sea level and

$$S(\psi) = \sum_{n=2}^{\infty} \frac{2n+1}{2} \Omega_n P_n(\cos\psi), \tag{3}$$

is a spectral form of Stokes' function with the spectrum:

$$\Omega_n = \frac{2}{n-1}. \tag{4}$$

Equation (1) shows that the integration should be performed globally, which means that Δg(Q) with a global coverage is required. Therefore, we should look for an approach to modify the integral in such a way that the contribution of the far zone data is minimized. Different methods for modifying Stokes' formula have been presented; however, in this case, concentration is on stochastic approaches as in Sjöberg (1984a) and (1984b).

A general estimator of the geoid presented based on LSM by Sjöberg (2003). Let us start a discussion applying this geoid estimator:

$$\tilde{N}(P) = \frac{R}{4\pi\gamma} \iint_{\sigma 0} S^L(\psi) \Delta g^{\mathrm{T}}(Q) d\sigma + \frac{R}{2\gamma} \sum_{n=2}^{M} b_n \Delta g_n^{\mathrm{EGM}}(P), \tag{5}$$

where: $L = M$ is the maximum degree of modification and the geopotential model that also may differ. In this case, the estimator will be unbiased through degree M, for more details see Sjöberg (1991), b_n is a parameter that differs according to the type of the estimator, and

$$S^L(\psi) = S(\psi) - \sum_{n=2}^{L} \frac{2n+1}{2} s_n P_n(\cos\psi), \tag{6}$$

is the modified Stokes' kernel function and s_n are the estimated modification parameters. The closed form of Stokes' function is (Heiskanen and Moritz 1967: 94, Eq. 2-164):

$$S(\psi) = \frac{1}{\sin(\psi/2)} - 6\sin\left(\frac{\psi}{2}\right) + 1 - \cos\psi\left\{5 + 3\ln\left[\sin\left(\frac{\psi}{2}\right) + \sin^2\left(\frac{\psi}{2}\right)\right]\right\}. \tag{7}$$

In order to show the sources gravity anomaly is derived from, we separate them into Δg^{T} for terrestrial and Δg^{EGM} for geopotential model based data and $\Delta g_n^{\mathrm{EGM}}$ is the spectrum of Δg^{EGM} in Eq. (5). Modification parameters s_n are obtained solving the following system of equations:

$$\sum_{r=2}^{M} a_{kr} s_r = h_k, \quad k = 2,3,\dots, \mathrm{M}, \tag{8}$$

where: a_{kr} and h_k change choosing LSM. This system of equations is constructed taking a derivative of the global RMSE of the estimator with respect to s_n. The BLSM parameters are derived from setting $b_n = s_n$ in the geoid estimator Eq. (5) and solving the system of equations Eq. (8) with the following elements (Sjöberg 2003):

$$a_{rk} = (\sigma_r^2 + dc_r)\delta_{kr} - E_{kr}(\psi_0)\sigma_r^2 - E_{rk}(\psi_0)\sigma_k^2 + \sum_{n=2}^{\infty} E_{nr}(\psi_0)E_{nk}(\psi_0)(\sigma_n^2 + c_n), \tag{9a}$$

$$h_k = \left[\grave{\mathrm{U}}_k - Q_k(\psi_0)\right]\sigma_k^2 + \sum_{n=2}^{\infty}\left[Q_n(\psi_0)(\sigma_n^2 + c_n) - \Omega_k\sigma_n^2\right]E_{nk}(\psi_0), \tag{9b}$$

where:

$$E_{rk}(\psi_0) = \frac{2k+1}{2} e_{rk}(\psi_0), \tag{9c}$$

$$e_{rk}(\psi_0) = \int_{\psi_0}^{\pi} P_r(\cos\psi)P_k(\cos\psi)\sin\psi d\psi, \tag{9d}$$

$$Q_k(\psi) = \int_{\psi_0}^{\pi} S(\psi)P_k(\cos\psi)\sin\psi d\psi, \tag{9e}$$

and σ_k^2 is the error spectrum of terrestrial gravimetric data and dc_k is the error spectrum of gravity anomaly obtained from the existing geopotential model. For more details on the computation of σ_k^2 the reader is referred e.g. to Sjöberg (1986), Ågren (2004) and Ellmann (2005). $e_{rk}(\psi_0)$ are the well-known Paul's coefficients (Paul 1978) and $Q_k(\psi_0)$ is the truncation coefficient of Stokes' integral, that can be evaluated recursively (Hagiwara 1972). The spectrum of gravity anomaly is evaluated using the geopotential model for those degrees below the maximum degree of modification. The analytical models like Kaula, TR are used for the degrees above that maximum degree. According to Ellmann (2005) and Ågren (2004), the TR model is superior with respect to others from the modification aspect. Relying on their conclusion, we use this model in our numerical studies through this paper.

The global RMSE of the BLSM geoid estimator is:

$$\delta\tilde{N}^2 = \frac{R^2}{4\gamma^2}\sum_{n=2}^{L}\left\{\left[\Omega_n - s_n - Q_n^L(\psi_0)\right]\sigma_n^2 + \left[Q_n^L(\psi_0)\right]^2 c_n + s_n^2 dc_n\right\} + \frac{R^2}{4\gamma^2}\sum_{n=m+1}^{\infty}\left\{\left[\Omega_n - Q_n^L(\psi_0)\right]^2\sigma_n^2 + \left[Q_n^L(\psi_0)\right]^2 c_n\right\}, \tag{9f}$$

where:

$$Q_k^L(\psi_0) = Q_k(\psi_0) - \sum_{r=2}^{L}\frac{2r+1}{2} s_r e_{kr}(\psi_0). \tag{9g}$$

ULSM parameters are derived, if we select $b_n = Q_n^L(\psi_0) + s_n$ and solve the system of Eq. (8) with the following elements (Sjöberg 2003):

$$a_{rk} = d_r\delta_{kr} - E_{rk}(\psi_0)d_r - E_{kr}(\psi_0)d_k + \sum_{n=2}^{\infty} E_{nr}(\psi_0)E_{nk}(\psi_0)d_n, \tag{10a}$$

$$h_k = \Omega_k\sigma_k^2 - Q_k(\psi_0)d_k + \sum_{n=2}^{\infty}\left[Q_n(\psi_0)d_n - \Omega_n\sigma_n^2\right]\times E_{nk}(\psi_0), \tag{10b}$$

where:

$$d_n = \sigma_n^2 + dc_n. \tag{10c}$$

The global RMSE of this estimator is:

$$\delta\tilde{N}^2 = \frac{R^2}{4\gamma^2}\sum_{n=2}^{\infty}\left\{\left\{\Omega_n - s_n^* - Q_n^L(\psi_0)\right\}^2\sigma_n^2 + \left[Q_n^L(\psi_0) + s_n^*\right]^2\right\}dc_n^*, \tag{10d}$$

where:

$$s_n^* = \begin{cases} s_n & n \le L \\ 0 & n > L \end{cases} \text{ and } dc_n^* = \begin{cases} dc_n & n \le L \\ 0 & n > L \end{cases}. \tag{10e}$$

The OLSM parameters of the estimator are derived if we set $b_n = \left[Q_n^L(\psi_0) + s_n\right]c_n/(c_n + dc_n)$ and solve Eq. (8) with the following elements (Sjöberg 2003):

$$a_{rk} = C_r\delta_{kr} - E_{rk}(\psi_0)C_r - E_{kr}(\psi_0)C_k + , \sum_{n=2}^{\infty} E_{nr}(\psi_0)E_{nk}(\psi_0)C_n, \tag{11a}$$

$$h_k = \Omega_k\sigma_k^2 - Q_k(\psi_0)C_k + \sum_{n=2}^{\infty}\left[Q_n(\psi_0)C_n - \Omega_n\sigma_n^2\right]\times E_{nk}^0(\psi_0), \tag{11b}$$

where:

$$C_k = \sigma_k^2 + \begin{cases} c_k dc_k/(c_k + dc_k) & 2 \le k \le M \\ c_k & k > M \end{cases}. \tag{11c}$$

The global RMSE of the estimator will be:

$$\delta\tilde{N}^2 = \frac{R^2}{4\gamma^2}\sum_{n=2}^{\infty}\left\{\left[b_n^* - s_n^* - Q_n^L(\psi_0)\right]^2 c_n + \left[\Omega_n - s_n^* - Q_n^L(\psi_0)\right]^2\sigma_n^2\right\} + \frac{R^2}{4\gamma^2}\sum_{n=2}^{M} b_n^2 dc_n. \tag{11d}$$

4. Numerical investigations

Two geopotential models EGM96 (Lemoine *et al.* 1998) and EGM08 are used to generate the signal and error spectra of gravity anomaly. Correlation length 0.1° is considered to generate the error spectrum of terrestrial gravimetric data. In this case, we assume that the error of terrestrial data is 5 mGal. Any geopotential model is restricted to a certain degree of spherical harmonics. As we know, the maximum degree of EGM96 and EGM08 are 360 and 2160 respectively. Therefore, the signal and error spectra are limited to these degrees. In order to combine Stokes' integral with the geopotential model, the integral is written in a spectral form and the spectra of the geopotential model and the integral formula are combined. The degree of modification in this study is L = M = 150 which means that we use geopotential models EGM96 and EGM08 taking into account such degree and order. In case we use EGM96, higher spectra than degree 150 are extracted from the TR model to degree 2000 and when using EGM08, they are extracted from the model itself. In order to show differences between these two types of data, we plot their corresponding spectra in Fig. 1a showing that Kaula and TR models follow a similar pattern with increasing the signal degree and the signal spectra of the models are larger than that of EGM08. Fig. 1a also discloses that the error spectra of EGM08 are larger than its corresponding signal degree when degree 1800 is reached. Figure 1b illustrates variances in the error degree of EGM08 and EGM96. As mentioned above, the modification degree is 150, and therefore in this figure, we plotted EGM96 error considering this degree called a traditional method. Since the error spectra of EGM96 are not available coming after its maximum degree, we also use the signal itself as the error spectra. This is a traditional way of using the geopotential model in all LSM methods. Ellmann (2005) selects the TR model following the degree of modification as the error spectra and therefore we can see a jump in the error spectra of the geopotential model in Fig. 1b. Taking into account personal communication with Ellamnn, the author did not find it problematic in the LSM methods. As the figure presents, the error spectra of EGM08 are smaller than those of EGM96.

As shown in Fig. 1, EGM08 is superior to the traditional method of LSM in which a combination of EGM96 and the TR model is used. First, EGM08 has been computed based on real high resolution data all over the globe. Second, the error spectra of the geopotential coefficients are smaller than those of EGM96.

In this case, we concentrate on LSM methods and compare the results of modification methods based on TR and EGM08. Further, thinking of our goal of modification, we consider the geocentric angle 3°. The system of equations, Eq. (8), will be ill-conditioned for estimating the modification parameters based on ULSM and OLSM. The reason for this instability was theoretically expressed by Ågren (2004) and numerically displayed by Ellmann (2005). However, the consequence of this instability is totally harmless and with a simple regularization, the system of equations can be solved. Here we take advantage of the truncated singular value decomposition to stabilize the system of equations. Figure 2 shows the modified Stokes' kernel based on TR and EGM08 models with respect to the original kernel. As the figure presents, the kernel modified by EGM08 decreases faster than that modified based on the TR model. It holds for all LSM methods. Figures 2b and 2c indicate that the kernel modified by the TR model decreased before reaching the end of cap size (geocentric angle 3°) while this is not the case for the EGM08 model, although the consequence of this matter may not be important.

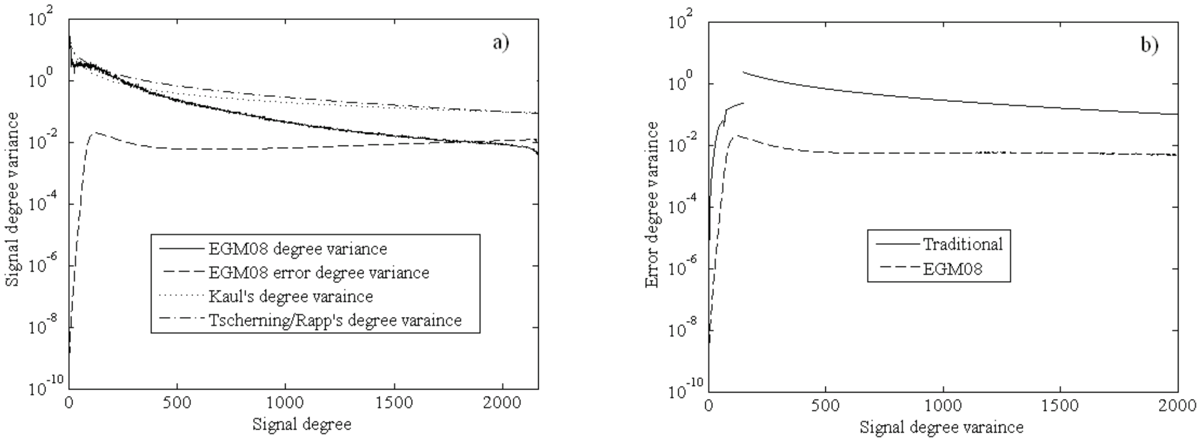

Fig. 1. Signal degree variances of EGM08, some analytical models (a), error degree variances of EGM08, EGM96 and the TR model (b)

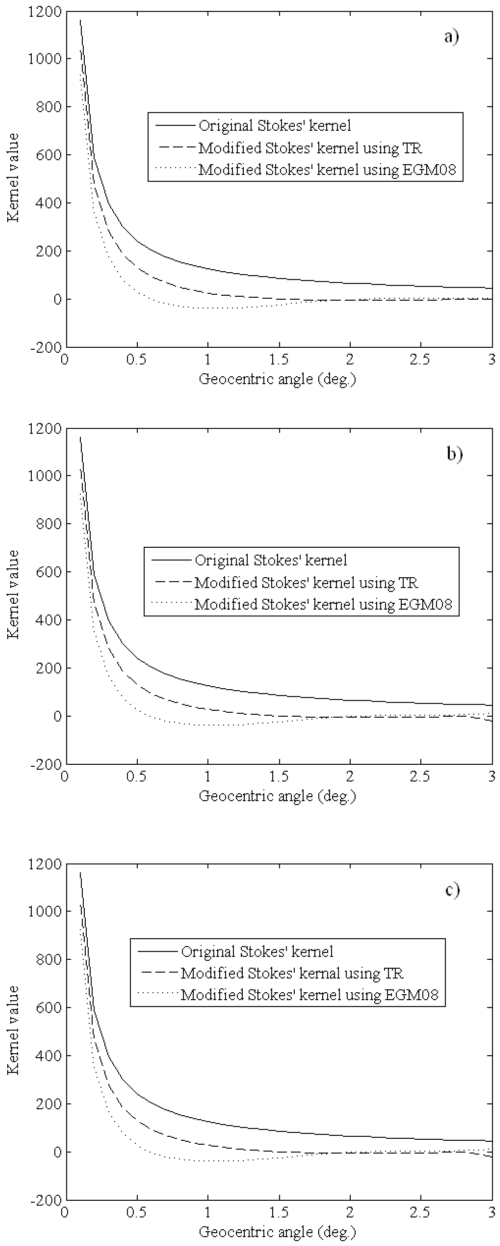

Fig. 2. The modified kernels based on the TR and EGM08 models: a) BLSM, b) ULSM and c) OLSM

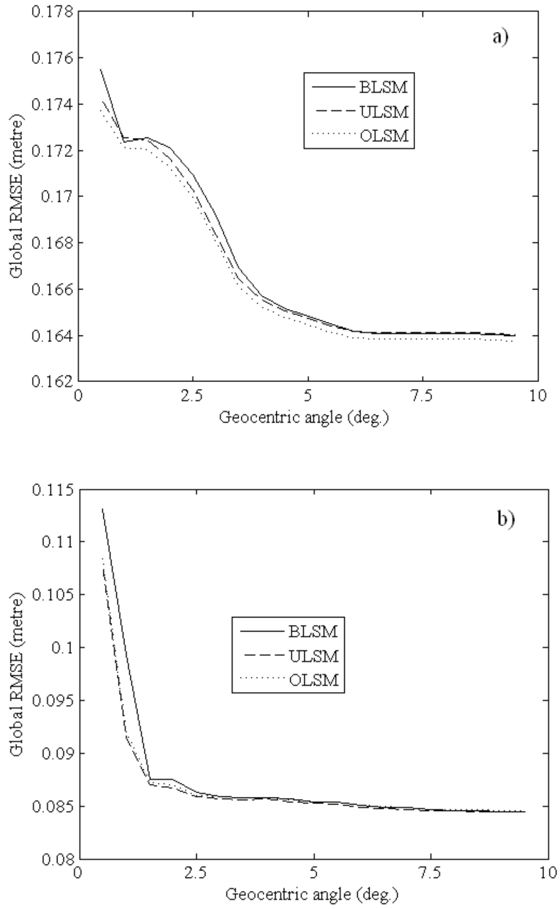

Fig. 3. The global RMSE of BLSM, ULSM and OLSM estimators modified using: a) TR and b) EGM08 models

Figure 3 shows the global RMSE of the estimators modified applying BLSM, ULSM and OLSM methods. The computations are preformed from the geocentric angle 3° to 10° and Fig. 3a shows the global RSME of the estimators modified using the TR model for different geocentric angles. The figure also displays that BLSM and OLSM propagate the largest and the smallest error respectively and ULSM is a method in the middle of these two methods. However, differences are very small and may be negligible in practice. Figure 3b illustrates a similar plot for the estimators by using EGM08 to generate the spectra. This figure demonstrates insignificant difference between ULSM and OLSM. Difference between BLSM and the others can reach 5 mm which is significant in precise geoid determination aspects. An important point is that Fig. 3 presents difference between the error of the modified estimators based on the TR model and EGM08. Moreover, the figure discloses that in case of using EGM08, the errors are considerably reduced. Roughly speaking, EGM08 improves the estimators twice.

Up to now, the main source of these improvements has not been clear. Thus, in order to investigate it, let us assume zero error in terrestrial data which means that we absolutely rely on the quality of terrestrial data. The LSM process is repeated based on this assumption and the global RMSE of the modified estimators is visualized. In such a case, the main sources of the errors are from the geopotential model and the truncation of the integral formula. Figure 4a shows that the global RMSE is between 1 cm and 13 cm. A comparison between this figure and Fig. 3a indicates that errors in terrestrial data are significant. One can see in Fig. 4b that the errors are reduced by about 2 cm.

In this case, let us consider no error in the geopotential model where the only error source will be the truncation error of the integral formula. Figs. 5a and 5b point to insignificant improvement in the estimators. We know that truncation error depends on the spectra of gravity anomaly generated either from the existing

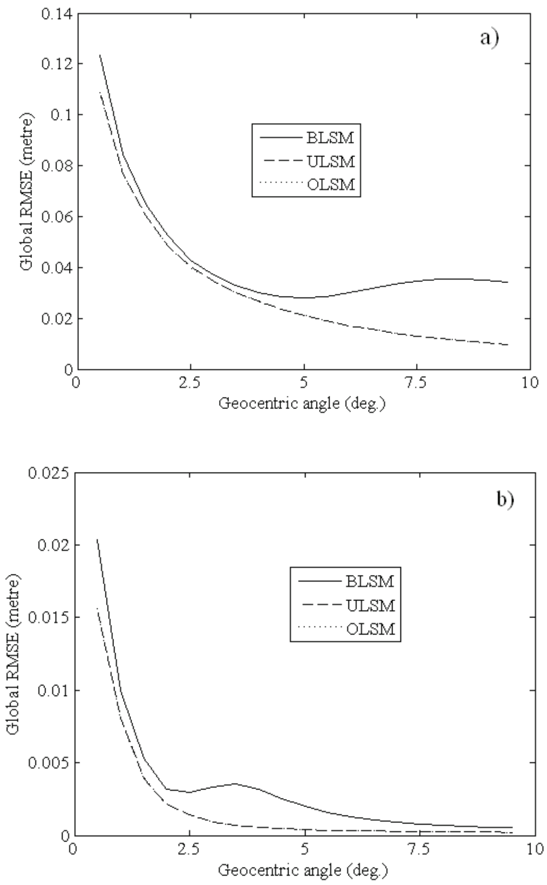

Fig. 4. The global RMSE of BLSM, ULSM and OLSM estimators modified using: a) TR and b) EGM08 models with zero error in terrestrial data

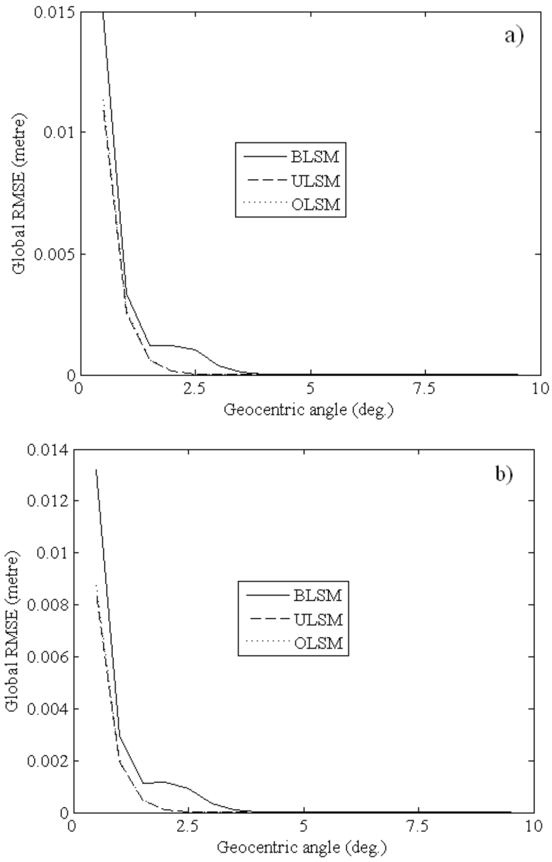

Fig. 5. The global RMSE of BLSM, ULSM and OLSM estimators modified using: a) TR and b) EGM08 models with zero error in terrestrial data and geopotential model

analytical model (e.g. the TR model) or from a high degree geopotential model (e.g. the EGM08). However, improvements in estimating the global RMSE are below the level of 1 cm and are significant for determining a precise geoid model. The carried out numerical studies reveal that the main reason for improving the modified estimators is mostly related to data error rather than to truncation error. The error of the geopotential model has the main role in this respect and it is quite reasonable as the long wavelength structure of the geoid is constructed from such a model and definitely the error of this important portion of the gravity field can significantly change the error of the estimator and correspondingly the geoid. The study also shows that the use of the TR model and EGM08 for generating the signal spectra of gravity anomaly insignificantly reduces truncation error.

5. Conclusions

EGM08 solves the problem of lacking spectra for LSM methods, and therefore there is no need using the analytical models that only follow the general pattern of the spectra of gravity anomaly. This study has disclosed that in LSM, the signal spectra of gravity anomalies can reduce the global RMSE of the estimators by a few millimeters. In conclusion, improving LSM is mostly related to the small standard error of the geopotential coefficients and advantage of the high degree spectra of gravity anomaly is mostly related to the truncation error of Stokes' integral that can be significant in precise geoid determination. The study also displays that EGM08 reduces the global RMSE of the estimator to 10 cm.

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Research interests: Physical and Space Geodesy, gravity gradiometry, mathematical statistics, optimization and design of geodetic networks.