GEODEZIJA IR KARTOGRAFIJA / GEODESY AND CARTOGRAPHY
2010
36(2): 85–90

UDK 528.232
doi: 10.3846/gc.2010.14

APŽVALGINIAI STRAIPSNIAI

DISCUSSION OF THE EFFECTS OF SELECTING INCORRECT ELLIPSOID PARAMETERS

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Received 03 04 2010; accepted 01 04 2010

Abstract. GNSS positioning has become an integral part of many functions within society. Conveniently, the World Geodetic System 1984 (WGS84) ellipsoid is associated with GNSS surveying since its definition has been developed over the years based on concerted efforts using combinations of VLBI, satellite/lunar laser and high precision GPS campaigns. These campaigns have enabled better and better determinations of the position of the "zeroes" in three dimensions of a rotating Earth centred coordinate axis system. The object here is to investigate the sensitivity of user coordinates on the projection to the varying values that are, or may be being, applied as the ellipsoidal flattening during the coordinate conversion processes.

Keywords: ellipsoid, GNSS, WGS84, three dimensional coordinate system.

1. Introduction

GNSS positioning has become an integral part of many functions within society. Conveniently, the World Geodetic System 1984 (WGS84) ellipsoid is associated with GNSS surveying since its definition has been developed over the years based on concerted efforts using combinations of VLBI, satellite/lunar laser and high precision GPS campaigns. These campaigns have enabled better and better determinations of the position of the "zeroes" in three dimensions of a rotating Earth centred coordinate axis system.

On the other hand, a definition of the centre of the Earth in this way is of limited use until some way is found to realise Earth surface positions with respect to it. Many further campaigns have been used to assist with this realisation. For Norway, the outcome has been that the final realisation is called EUREF89, and, according to (5) uses the GRS80 ellipsoid with the following defining parameters:

Semi-major axis (a): 6378137.0 metres,
Reciprocal flattening (1/f): 298.257222101.

These parameters are then expected to be used nationally within Norway, and are built in to software released by the Norwegian Mapping Authority which is subsequently installed in commercial Norwegian survey software. It must be remembered that these parameters are fundamental for converting between Earth-centred three-dimensional coordinates, and geographical coordinates, and then further to projection coordinates.

GNSS survey equipments however are not produced in Norway for Norwegian users. These instruments are advanced, and there are relatively few producers worldwide, and even fewer that are commercially well represented in Norway. The result is that there are mostly only three different marques of high precision GNSS receivers in use in Norway. It is thus interesting to notice that the parameters normally used by these different receiver types are not exactly equal to each other, as shown in the following Table 1.

Table 1. Ellipsoid parameters from different sources

Source	a	1/f
Marque-1	6378137.0	298.25722210088
Norwegian Mapping Authority (NMA)	6378137.0	298.257222101
Marque-2	6378137.0	298.25722
Marque-3	6378137.0	298.257223563

Clearly there are some differences. Clearly also, users are properly advised by their respective suppliers. However, with the introduction of a new national projection system in Norway (Haakonsen 2008) which is specially designed to fulfil the needs of the civil engineering industry, there are likely to be enough conversions back and forth between systems that it is considered timely to further examine these differences. The object here is to investigate the sensitivity of user coordinates on the projection to the varying values that are, or may be being, applied as the ellipsoidal flattening during the coordinate conversion processes.

2. The Conversion Processes

It is perhaps relevant to recall the computational route that must be navigated in order to arrive at projection coordinates, although, naturally, these things do not require much thought in these times due to the availability of advanced and sophisticated software tools.

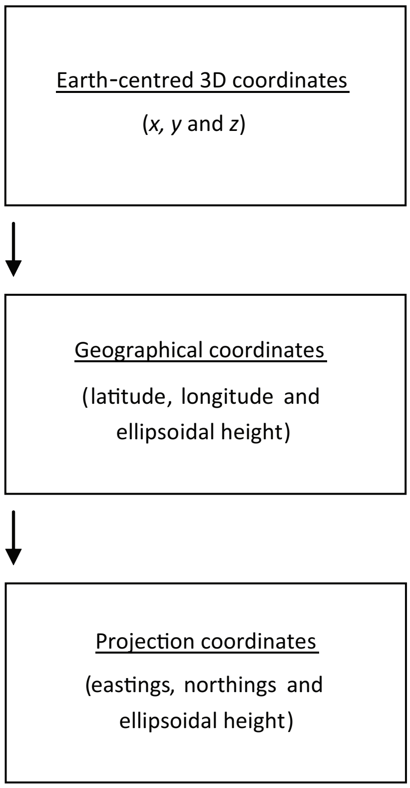

Fig. 1. The process to convert the result of GNSS observations to projection coordinates

Firstly, decisions need to be made to determine which ellipsoid and projection system is to be used. Subsequently, the needs of the practical surveyor concerning height measured against a spirit level (or compensator) must also be brought to account. This latter matter, which involves the determination of geoid information, needs to be remembered, although it will not be examined closely here.

Selection of ellipsoid and projection system is taken here to imply that their respective defining parameters are ascertained, and that the computational connections between them are also established. The process to convert the result of GNSS observations to projection coordinates can then be generally described as in the following diagram (Fig. 1), reading top-down.

When, as in Norway, a second projection system is used for separate purposes, there is the option of developing a transformation procedure. Transformations are normally invoked when local datum systems are selected, often because accuracy requirements may become advanced and because therefore transformation routines are able to take care of any remaining distortions in either of the eventual systems.

Where however, the second projection system is implemented nationally and with clear defining parameters, again as in Norway, then the correct and safe method of obtaining projection coordinates is either to follow the above diagram directly to the other system, or to carry out a conversion, in which case the process is as described below (Fig. 2).

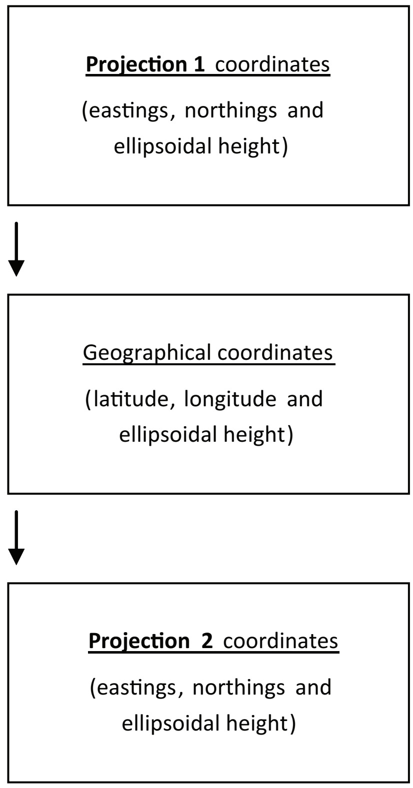

Fig. 2. The process of obtaining projection coordinates in Norway

2.1. The Equations converting to Latitude, Longitude and Height

The expressions for conversion from Earth-centred coordinates (x, y and z)to geographical coordinates are well known. They require knowledge of the selected ellipsoid in the form of the semi-major axis length (a) and either the flattening (f) or the second eccentricity (e^2). The latter two are exactly connected in accordance with the standard geometry of an ellipse.

Thus, given a and f,

$$e^2 = 2f - f^2 \text{ exactly.} \tag{1}$$

It follows that the radius of curvature in the prime vertical at a given latitude φ is given by:

$$\nu = \frac{a}{\sqrt{(1-e^2\sin^2\varphi)}}. \tag{2}$$

Now it is possible to compute the final latitude φ, longitude λ and ellipsoidal height H:

$$\lambda = \tan^{-1}\left(\frac{y}{x}\right); \tag{3}$$

$$\varphi = \tan^{-1}\left(\frac{(z+e^2\nu\sin\varphi)}{\sqrt{(x^2+y^2)}}\right). \tag{4}$$

Clearly, the value obtained for the latitude φ depends on the provisional value that has been selected. Therefore, expression (Defence … 1989 needs to be iterated using improved values of the meridional radius given by expression (Bomford 1971) each time. Fortunately, convergence is normally quite rapid, and then the ellipsoidal height H comes from:

$$H = \frac{\sqrt{(x^2+y^2)}}{\cos\varphi} - \nu. \tag{5}$$

It needs to be remembered incidentally that the height extracted from (States … 2003) is the height over the selected ellipsoid. This is normally denoted by the capital, while the lower case h conventionally denotes the height over the geoid. The relationship between these two is through the geoid separation N as follows:

$$H = N + h. \tag{6}$$

Closer examination of the above will show that the chosen ellipsoid is not required to obtain longitude. Ellipsoid parameters are, however, required to obtain latitude and ellipsoidal height. In both cases, the semi-major axis and the flattening and appear in the expression for the prime vertical radius of curvature (expression (Bomford 1971) and subsequently in the expressions for latitude and height (Defence … 1989; States … 2003).

2.2. Testing different values of Flattening

Some simple tests have been carried out to examine the effects of the slightly varying values of the ellipsoid's flattening. These consisted of converting Earth-centred coordinates in a 15 degree grid in the first longitude quadrant, and then extracting the differences from the full definition of GRS80. As already stated, no longitude difference was expected, while latitude differences were converted immediately into millimetres using the meridional radius of curvature from GRS80.

The differences in all cases were predictably dependent on latitude and ranged as shown in the following Table 2.

Not surprisingly, when the latitude approaches the equator, then the trigonometrical sine of the latitude becomes very small, and the expressions become more and more analogous to spherical formulae. Thus the zero values in the above table can be misleading. In the following Table 3, the smallest and largest differences are presented for a 15 degree grid between latitudes 15 and 75, and longitudes 0 to 90 degrees.

2.3. Converting to the Transverse Mercator Projection

The formulæ for converting geographical coordinates to TM projection coordinates also require that the ellipsoid size parameters are defined. These are used to evaluate both the radius of curvature in the prime vertical (ν as above) and the radius of curvature in the meridian ρ. Thereafter, the processing is rather complex, and is very well documented – see (Redfearn 1948). (The sequence of formulæ is repeated in the appendix.) It is enough to note at this point that the expressions concerned involve expansion series which include various ascending powers of the ellipsoid's eccentricity e. This latter is directly connected to the ellipsoid's flattening by (Redfearn 1948) above.

Table 2. The differences in latitude and height

	Latitude (mm)		Height (mm)	
Difference	Smallest	Largest	Smallest	Largest
Marque-1 – NMA	0	$+8.52\times10^{-6}$	0	$+8.46\times10^{-6}$
Marque-1 – Marque-2	0	–0.151	0	–0.140
Marque-1 – Marque-3	0	+0.105	0	+0.098

Table 3. The smallest and largest differences in latitude and height

	Latitude (mm)		Height (mm)	
Difference	Smallest	Largest	Smallest	Largest
Marque-1 – NMA	$+4.25\times10^{-6}$	$+8.52\times10^{-6}$	$+1.14\times10^{-6}$	$+8.46 \times 10^{-6}$
Marque-1 – Marque-2	–0.076	–0.151	–0.010	–0.140
Marque-1 – Marque-3	+0.053	+0.105	+0.007	+0.098

Further, it is perhaps not surprising to note that the reverse conversion, from TM to geographical also involves the same ellipsoidal parameters incorporated into a sequence of series expansions.

Using the evidence from the first tests converting Earth-centred to geographical coordinates, worst case coordinates were further converted to UTM coordinates using the Redfearn algorithms (Redfearn 1948). There, the differences between the various values of flattening emerged to be in the range –0.2 to +0.2 mm.

3. Conclusion so far

Summarising, then, it seems that the varying values of the ellipsoid's flattening as used by different marques of software and hardware systems, can lead to positional errors of the order of 0.2 mm numerically in geographical latitude. This, when further converted into TM coordinates can add about another 0.2 mm numerically. Whether these accumulate or eliminate each other depends of course on the sign of the differences.

4. Effects in Satellite Surveying and Commentary

Conventional modern satellite surveying is performed more and more frequently using real time methods. These techniques demand the provision of observational data from one or more fixed reference or base stations where the precise coordinates are known.

Meanwhile, it goes without saying that the satellite system itself functions in Earth-centred coordinates. Thus, the end user coordinates must at some point be converted through the processes briefly described above. The question then becomes – where do the known coordinates of the reference station(s) come from? At the same time, there is nothing to say that all the receiver equipments will be of the same marque, and this implies that the available "hard-wired" values of the ellipsoid's flattening need not be the same for all users, although informed users will of course take care that their instrument set-up is correct.

It is therefore not difficult to imagine a scenario where these relatively tiny errors start can start to accumulate to a point where they may begin to have practical consequences:

- Imagine a reference station established at a "known" point with given coordinates in UTM.
- These coordinates are converted to geographical – perhaps using the incorrect ellipsoid flattening – collecting some small error perhaps up to 0.2 mm.
- These are then further converted to Earth centred coordinates using the same, or indeed another (wrong) ellipsoid flattening – perhaps accumulating (or concealing) another 0.2 mm.
- The reference station does what reference stations do, and transmits its numbers over to the roving receiver.
- The rover meanwhile is also measuring its position in Earth-centred coordinates, receiving correction information from the reference (which is based on conversions from UTM using one or even maybe two different variants of ellipsoid)
- The rover is already set-up with what are thought by the operator to be the correct ellipsoid These may not be the same as the one (or even two) ellipsoids that have been used by the reference.
- So now the rover proceeds to convert back to geographical – the third opportunity for choosing the wrong ellipsoid parameters.
- And then it convert into UTM – with therefore a fourth opportunity.

That then completes the conventional survey operation. However, construction industry needs might dictate further conversion either to a national projection as in Norway, or to a local datum system. Indeed, that local system might not be that well related to the spirit level bubble, as in offshore platform applications. The objective here is to produce results that are as nearly uncontaminated by scale factors and other such hindrances so that they measure "truth" as evidenced by the simple tape measure. Here, the whole error accumulation process can restart itself, either through continuing to choose the incorrect ellipsoid parameters, or by selecting an insufficiently robust transformation system.

A yet further opportunity for injecting error into the whole process can arise from the perceived need to operate in orthometric height systems. This is natural enough, in that orthometric heights follow equi-potential surfaces and are directly reflected and visible in the spirit level bubble or instrument compensator.l Converting from ellipsoidal to orthometric height implies some knowledge of the local geoid. Not surprisingly, the local geoid separation, while generally known, might not agree between users at the millimetre level, or even at the centimetre level.

It would thus be defendable to argue that variations in the geoid models used can lead to more gross errors. On the other hand, as geoid models improve, so the significance of introduced ellipsoid model errors will increase.

5. Afterthought

Meanwhile, given the number of opportunities for injecting the odd fraction of a millimetre, it is surely a matter of professional integrity for surveyors to ensure that:

- They have an appropriate understanding of the underlying issues which are otherwise camouflaged within user software systems, and
- Their system set-up parameters are all lined up and in agreement with each other.

6. Appendix

Geographical to Grid Conversion by Redfearn (1948)

Except where otherwise indicated, formulæ are extracted from Bomford, and use Bomford's notation (1971) and (1980). Throughout:

- Suffix "0" indicates that the symbol refers or relates to the origin of the projection, or to one of the defining parameters of the projection.
- The suffix " ′ " indicates that the symbol is computed on the Footprint Latitude.

Northing and Easting from Latitude and Longitude ("Forward Problem")

Notation used:

N, E = Plane coordinates in the projection.

m = Projection scale at a point.

m_0 = Projection scale on the Central Meridian.

M = Meridian arc in metres (also known as the meridian distance) from the projection origin – on the Equator for UTM. (See below for method for computing meridian distance.)

$\nu = \dfrac{a}{\sqrt{(1-e^2\sin^2\varphi)}}$ – Radius of Curvature in the Prime Vertical.

$\rho = \dfrac{a(1-e^2)}{(1-e^2\sin^2\varphi)^{3/2}}$ – Radius of Curvature in the Meridian.

a = Semi-major axis of the ellipsoid.

e^2 = The square of the ellipsoid eccentricity.

φ, λ = Latitude and Longitude. Positive (+) North and East.

$\omega = \lambda - \lambda_0$ in radians.

$\psi = \nu/\rho$.

$t = \tan\varphi$.

$\gamma =$ Convergence, clockwise from the meridian to the North axis.

Meridian Arc:

$$M = \int_0^\varphi \rho\, d\varphi = a(A_0\varphi - A_2\sin 2\varphi +$$

$$A_4\sin 4\varphi - A_6\sin 6\varphi \ldots),$$

where:

$$A_0 = 1 - \frac{e^2}{4} - \frac{3e^4}{64} - \frac{5e^6}{256} - \ldots;$$

$$A_2 = \frac{3}{8}\left(e^2 + \frac{e^4}{4} + \frac{15e^6}{128} + \ldots\right);$$

$$A_4 = \frac{15}{256}\left(e^4 + \frac{3e^6}{4} + \ldots\right);$$

$$A_6 = \frac{35}{3072}e^6 + \ldots$$

Note that this provides the meridian arc length from the equator. It would perhaps be more correct to use the notation M_0. Where the meridian arc is required between two latitudes off the Equator, then it is necessary to compute the meridian arc from the Equator for both latitudes, and then take the difference. Clearly, if one of the latitudes is in the southern hemisphere, then the sign of the latitude will take care of this difference correctly.

Northing: (Norwegian "X")

$$N = m_0 \begin{bmatrix} M + \nu\sin\varphi\dfrac{\omega^2}{2}\cos\varphi + \\ \nu\sin\varphi\dfrac{\omega^4}{24}\cos^3\varphi(4\psi^2+\psi-t^2) + \\ \nu\sin\varphi\dfrac{\omega^6}{720}\cos^5\varphi\left\{8\psi^4(11-24t^2)-28\psi^3\cdot\right. \\ \left.(1-6t^2)+\psi^2(1-32t^2)-\psi(2t^2)+t^4\right\} + \\ \nu\sin\varphi\dfrac{\omega^8}{40\,320}\cos^7\varphi(1385-3111t^2 + \\ 543t^4 - t^6). \end{bmatrix}$$

Note that the value of N here is the North ordinate from the origin which is the intersection of the Central Meridian and the Equator, and the False Northing therefore needs to be added in later.

Easting: (Norwegian "Y")

$$E = m_0 \begin{bmatrix} \nu\omega\cos\varphi + \nu\dfrac{\omega^3}{6}\cos^3\varphi(\psi - t^2) + \\ +\nu\dfrac{\omega^5}{120}\cos^5\varphi\left\{4\psi^3(1-6t^2)+\psi^2(1+8t^2)-\right. \\ \left.\psi(2t^2)+t^4\right\} + \nu\dfrac{\omega^7}{5040}\cos^7\varphi(61-479t^2 + \\ 179t^4 - t^6). \end{bmatrix}$$

Note that the value of E here is the East ordinate from the origin which is the intersection of the Central Meridian and the Equator, and the False Easting therefore needs to be added in later.

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