STATYBINĖS KONSTRUKCIJOS IR TECHNOLOGIJOS
ENGINEERING STRUCTURES AND TECHNOLOGIES
2009
1(4): 166–171

Lietuvos statybininkų asociacija

doi: 10.3846/skt.2009.20

DISCRETE ANALYSIS FOR SINGLE-PYLON SUSPENSION BRIDGES

Martti Kiisa[1], Juhan Idnurm[2], Siim Idnurm[3]

Tallinn University of Technology, Ehitajate tee 5, 19086 Tallinn, Estonia
E-mail: [1]martti.kiisa@mnt.ee; [2]juhan.idnurm@ttu.ee; [3]siim.idnurm@ttu.ee

Received 02 10 2009; accepted 14 11 2009

Abstract. This paper presents calculation methods, developed at Tallinn University of Technology (Idnurm 2004; Kulbach 2007), for the single-pylon suspension bridges stiffened by a girder. Classical suspension bridge consists of a geometrically non-linear cable, connected by hangers with an elastic linear stiffening girder, pylons in both ends of the bridge and anchor cables. Alternate form of a suspension bridge is a bridge, with only one pylon on the middle of the span and suspension cable is connected to the abutments or the ends of the stiffening girder. In the calculation of suspension bridges, the geometrically non-linear behaviour of the parabolic cable is the main problem. The linear methods of analysis suit only for small spans. A geometrically non-linear continual model is especially useful for classical loading cases – a uniformly distributed load on the whole or half span. But the modern traffic models consist of concentrated and uniformly distributed loads. The discrete model of a suspension bridge allows us to apply all kinds of loads, such as distributed or concentrated ones. The assumptions of the discrete method described here are: linear elastic strain-stress dependence of the material and absence of horizontal displacements of hangers. Hanger elongation is taken into account.

Keywords: cable structure, girder-stiffened structure, suspension bridge, long-span structures, discrete analysis, geometric non-linearity.

1. Introduction

Here is presented calculation methods for single-pylon suspension structures, stiffened by girders (Figure 1).

The common assumptions about the linear elastic strain-stress dependence of materials and absence of elongations of hangers are taken into account. The cables are regarded as geometrically nonlinear rods without bending rigidity, and stiffening girder – as bended linear bar. The mutual action between the carrying cables and stiffening members are regarded as a nodal contact load in supporting nodes of hangers.

Discrete analysis is based on condition of equilibriums, made for the nodal points of the cable. Under uniformly distributed load the cable will take parabolic form. In reality, the cable is loaded by concentrated forces and it takes the form of a string polygon. The conditions of equilibrium are written for every node of polygon and elongation of the cable was determined using equation of deformations compatibility for every section of the cable. These conditions form nonlinear

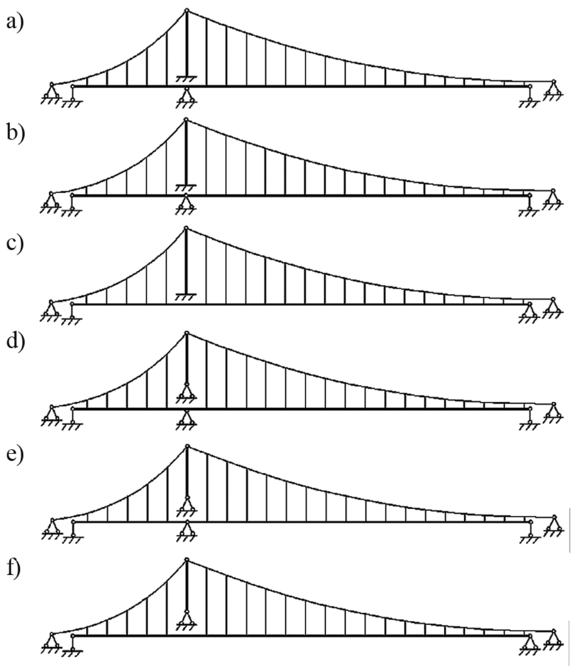

Fig. 1. The schemes of the single-pylon suspension bridges stiffened by a girder

ISSN 2029-2317 print / ISSN 2029-2325 online
http://www.skt.vgtu.lt

equation system, which give after solving all nodes displacements and internal forces for cables and for stiffening girder.

Here is used the scheme described as follows:

1. The initial balance of the cable (before the following loading) is described.
2. The final balance of the cable is described (the non-connected pieces of the stiffening girder are hanging on the cable and the cable is deformed).
3. The final balance of the whole structure is described (the stiffening girder is consistent and in initial solution takes approximately 10% of all load, 90% of load is imposed to the cable).

2. Basic equations

2.1. Discrete model for elastic cable

The initial state of equilibrium of the cable loaded by a concentrated load is shown in Figure 2.

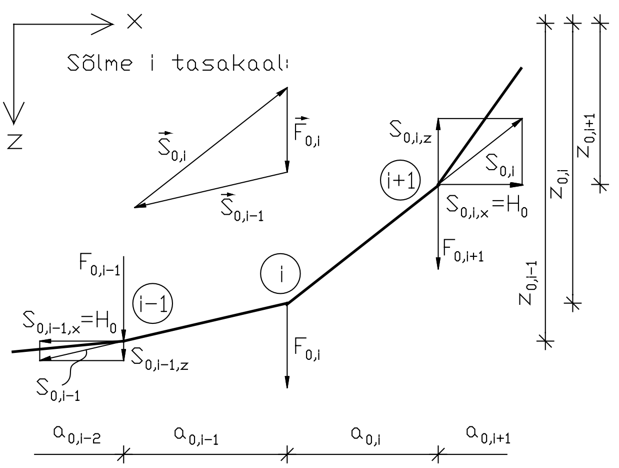

Fig. 2. A initial discrete scheme of the cable in the state of equilibrium

From the equilibrium considerations of forces we may write for every node (Kulbach, Õiger 1986)

$$F_{0,i} + H_0 \frac{z_{0,i+1} - z_{0,i}}{a_{0,i}} + H_0 \frac{z_{0,i-1} - z_{0,i}}{a_{0,i-1}} = 0, \tag{1}$$

where $F_{0,i}$ – initial external force; H_0 – initial force horizontal component of the cable (cable force); $z_{0,i-1}$, $z_{0,i}$, $z_{0,i+1}$ – initial vertical coordinates of the cable nodes; $a_{0,i-1}$, $a_{0,i}$ – horizontal distance between hangers.

For a cable which has supporting nodes on different level we may calculate H_0 as (Kulbach, Õiger 1986)

$$H_0 = \frac{a_{0,0} \sum_{i=1}^{n} F_{0,i}(L_0 - x_{0,i})}{L_0(z_{0,1} - z_{0,0}) + a_{0,0}(z_{0,0} - z_{0,n+1})}, \tag{2}$$

where L_0 – span of the cable.

By the action of the temporary loads ΔF_i (Figure 3), the equilibrium equation for the node i is expressed as (Kulbach, Õiger 1986)

$$F_i + H \frac{z_{0,i+1} + w_{i+1} - z_{0,i} - w_i}{a_{0,i}} +$$

$$H \frac{z_{0,i-1} + w_{i-1} \quad z_{0,i} \quad w_i}{a_{0,i-1}} = 0, \tag{3}$$

where F_i, – final external nodal forces ($F_i = F_{0,i} + \Delta F_i$); H – thrust from temporary and initial load; w_{i-1}, w_i, w_{i+1} – vertical displacements of the nodes.

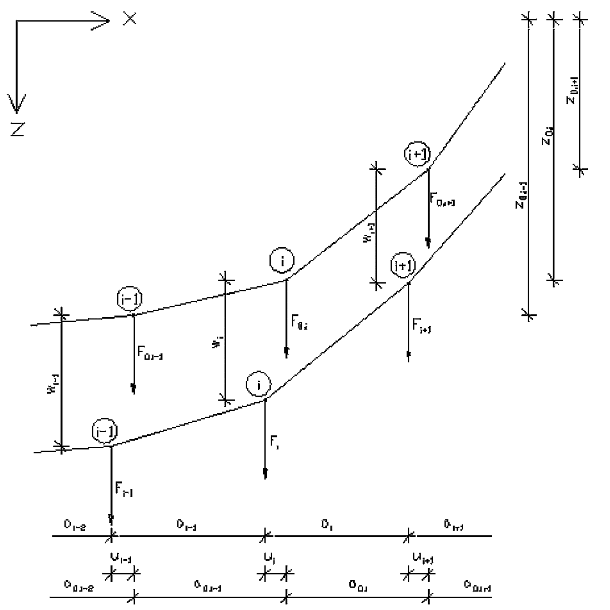

Fig. 3. Deformation of the cable under additional load

There are two unknown parameters in Eq. (3): w_i and H. Thus we need another equation for calculating them. We can use the compatibility condition of the relative elongation of the cable. The relative elongation of the cable is expressed as (Kulbach, Õiger 1986)

$$\varepsilon_i = \frac{1}{1 + \left(\frac{z_{0,i+1} - z_{0,i}}{a_{0,i}} \right)^2} \times$$

$$\left[\frac{u_{i+1} - u_i}{a_{0,i}} + \frac{w_{i+1} - w_i}{a_{0,i}} \left(\frac{z_{0,i+1} - z_{0,i}}{a_{0,i}} + \frac{w_{i+1} - w_i}{2a_{0,i}} \right) \right] \tag{4}$$

and from condition of linear deformation

$$\varepsilon_i = \frac{H - H_0}{EA}\sqrt{1+\left(\frac{z_{0,i+1}-z_{0,i}}{a_{0,i}}\right)^2}, \tag{5}$$

where EA – stiffness of the cable in tension; u_{i-1}, u_i, u_{i+1} – horizontal displacements of the nodes.

Taking into account (4) and (5), this compatibility condition may be presented as (Kulbach, Õiger 1986)

$$\frac{u_{i+1}-u_u}{a_{0,i}} = \frac{H-H_0}{EA}\left[1+\left(\frac{z_{0,i+1}-z_{0,i}}{a_{0,i}}\right)^2\right]^{3/2} -$$
$$\frac{w_{i+1}-w_i}{a_{0,i}}\left(\frac{z_{0,i+1}-z_{0,i}}{a_{0,i}}+\frac{w_{i+1}-w_i}{2a_{0,i}}\right). \tag{6}$$

Horizontal displacements of the internal nodes u_i may be eliminated by means of summation of the equations of deformation compatibility (6) and after replacing (Kulbach, Õiger 1986)

$$\sum_{i=0}^{n}\left(u_{i+1}-u_i\right) = u_{n+1}-u_0, \tag{7}$$

where u_{n+1} u_0 – horizontal displacements of the support nodes of the cables.

We may write the Eq. (6) in the form (Kulbach, Õiger 1986)

$$\frac{H-H_0}{EA}\left\{\sum_{i=0}^{n} a_{0,i}\left[1+\left(\frac{z_{0,i+1}-z_{0,i}}{a_{0,i}}\right)^2\right]^{\frac{3}{2}} - \frac{EA}{H-H_0}\left(u_{n+1}-u_0\right)\right\} =$$
$$\sum_{i=1}^{n}\left(w_{i+1}-w_i\right)\left(\frac{z_{0,i+1}-z_{0,i}}{a_{0,i}}+\frac{w_{i+1}-w_i}{2a_{0,i}}\right). \tag{8}$$

In case of single-pylon suspension bridge, we have two spans, which are carried by the main cable. Because we have two spans, we must calculate two different girder-stiffened cable systems, which are connected to the central pylon. Then, for the first span, we have horizontal displacements for cable ends: $u_{0,1} = 0$; $u_{n+1,1}$ is unknown. For the second span, $u_{0,2} = u_{n+1,1}$; $u_{n+1,0} = 0$.

For initial form of the bridge, when a bridge is symmetrical, we can obtain $H_I = H_{II}$. In loaded state, the central pylon has horizontal deformation and if the bottom support of pylon is fixed, then the final cable forces $H_I \neq H_{II}$; if bottom support of the pylon has released, then $H_I = H_{II}$ (Figure 4). The relations between H and H_{II} can be described, when bending rigidity of pylons is constant, as (Idnurm, J., Kiisa, Idnurm, S. 2009)

$$\frac{H_{II}h_p^3}{6} - \frac{H_I h_p^3}{6} - E_p I_p u_p = 0, \tag{9}$$

where h_p – height of the central pylon; E_pI_p – pylon's bending rigidity; u_p – horizontal displacement in top of the pylon.

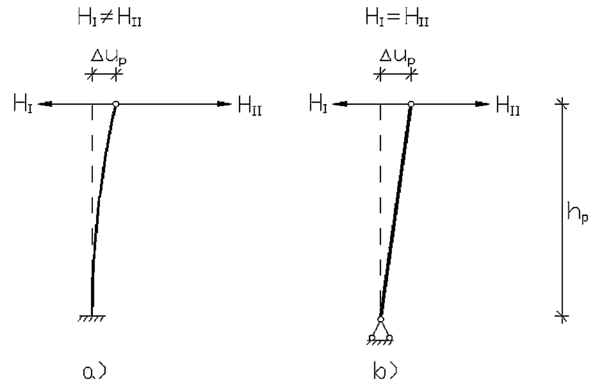

Fig. 4. The calculation scheme of the cable internal forces depending on the characterization of the pylon

Single pylon suspension bridge has two unknown cable forces H_I, H_{II} and one unknown horizontal displacement u_p. For this kind of bridge, we can calculate all displacements of internal nodes of both cable and contact forces in hangers, if H_I, H_{II} and u_p is determined. The best way is to take initial displacement $u_p = 0$ and solve this system is to calculate both spans separately. After solving the systems, we can calculate new value of u_p using condition (9), and repeat this process until change of pylon top displacement $\Delta u_p = 0$. If bottom support of the pylon has released, then we can't calculate u_p directly, but then we can search the value of u_p, to find condition $H_I = H_{II}$.

2.2. Analysis of the stiffening girder

The different schemes of a girder are presented in Figures 5–7. Let us consider only the stiffening girder. The equation that describes the deflection of the girder can be written as follows (Jürgenson 1985):

$$w(x) = w_0 + \varphi_0 x - \sum_{j=1}^{m} M_j \frac{\left(x-x_{M,j}\right)^2}{2E_bI_b}\mathcal{H}\left(x-x_{M,j}\right) +$$
$$\sum_{k=1}^{n} P_k \frac{\left(x-x_{P,k}\right)^3}{6E_bI_b}\mathcal{H}\left(x-x_{P,k}\right) +$$
$$\sum_{l=1}^{q} p_l \frac{\left(x-x_{p,l,1}\right)^4}{24E_bI_b}\mathcal{H}\left(x-x_{p,l,1}\right) -$$
$$\sum_{l=1}^{q} p_l \frac{\left(x-x_{p,l,2}\right)^4}{24E_bI_b}\mathcal{H}\left(x-x_{p,l,2}\right), \tag{10}$$

where w_0 – initial deflection of rotation at the first point of the girder; φ_0 – angle of rotation at the first point of the girder; M_j – concentrated moments; P_k – concentrated forces (internal forces in the hangers, vertical support reactions and external loads); p_l – uniformly distributed loads; $E_b I_b$ – rigidity of the stiffening girder in bending; $\mathcal{H}(x)$ – the *Heaviside*'s function.

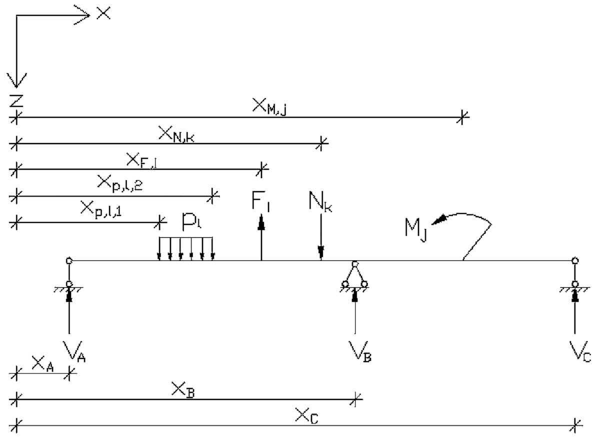

Fig. 5. The stiffening girder as 2-span continuous beam

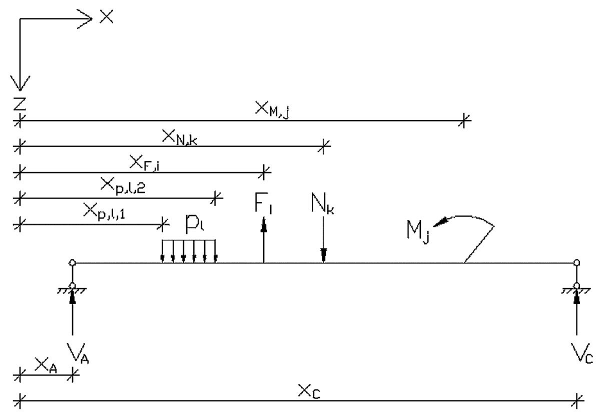

Fig. 6. The stiffening girder as single beam

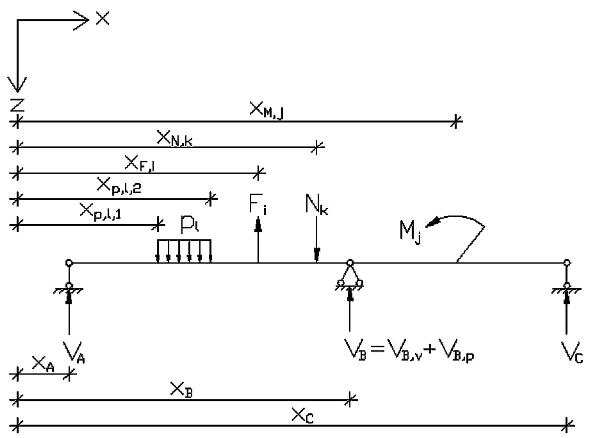

Fig. 7. The stiffening girder as 2-span single beam

Equation (10) can be used for calculating deflection from the sum of applied external concentrated moments, concentrated forces and from the uniformly distributed loads. The constants w_0 and φ_0 are all different on schemes described in Figures 5–7. They can be calculated using Equation (10) in points, where displacements are known (on supports).

2.3. Discrete model for the girder stiffened cable

There are different solutions described to calculate the stiffening girder and the cable as a integrated construction (Idnurm 2004). The new solution plan is defined as follows:

1. The cable and the stiffening girder can be calculated separately. The final solution is found when vertical displacements of the cable and stiffening girder are the same in points which are connected with the same hanger.
2. The first step is to calculate the cable initial forces H_0 and the shape of the cable z_i using only self-loads (the non-connected blocks of the stiffening girder are hanging on the cable).
3. 10% of the successive load is imposed to the girder and vertical displacements w_i of the points where hangers are connected will be found (the contact forces of the hangers are not taken into account).
4. The cable initial forces H_I and H_{II} can be found in loaded condition.
5. The inside forces of the hangers can be calculated using the equilibrium considerations of forces of cable nodes.
6. Vertical displacements w_i of the points where hangers are connected will be found again using all loads placed on the girder and also the contact forces of the hangers.
7. The final step is to compare displacements found in subsection 6 with displacements found in subsection 3 using the next formula: $\sum_{i=1}^{n}\left(w_{i,new}-w_{i,old}\right)^2$. If this sum total is near the 0, then calculated inside forces and displacements are final and solution is finished. If the sum total is not 0, the calculation process must be repeated from the subsection 3 using loads which are corrected by comparison with mentioned 10%.

3. Numerical results

Comparison of results for classical suspension bridge from different calculation methods are presented in

(Idnurm 2004; Kulbach, Idnurm, J., Idnurm, S. 2002; Kulbach 1995, 1998, 1999, 2007). There the results from analytical (continual) method (Aare, Kulbach 1984) and discrete method give similar results, which are more exact than results, got from linear finite element method (because of finite element method defines the cable as many single segments which are connected).

For the numerical results we used single-pylon suspension bridge with spans 90 m. Height of the pylon is taken 20 m.

In the calculation, three different calculation methods were used: discrete methodology, linear finite element method (FEM), non-linear FEM. For FEM is used commercial software Staad/Pro v8i. The loads and basis of modelling using FEM are the same as for the analytical (continual) method and discrete method.

The calculations were carried out with the combinations of the cross-sections of two different cables, with cross-section areas 31800 mm^2 and 45800 mm^2. The modulus of elasticity for the cable was taken 170 GPa, for the hangers and girder the modulus of elasticity was 210 GPA. The moment of inertia of the girder was taken $3.38 \cdot 10^{10}$ mm^4.

The prestressing load, which consist self-weight of the girder and deck structure is $p_0 = 30$ kN/m (at that moment there are no deflections of the girder and the cable because the cable is prestressed), additional self-weight load is $p_1 = 20$ kN/m, and traffic load $p_t = 30$ kN/m.

The geometry and loads for bridge is presented in Figure 8. Total load p for this calculation is taken between $p_{20} = 20$ kN/m to $p_{50} = 50$ kN/m. Then, maximum displacement from traffic load $p_t = 30$ kN/m can be calculated $w_t = w$ from $p_{50} - w$ from p_{20}.

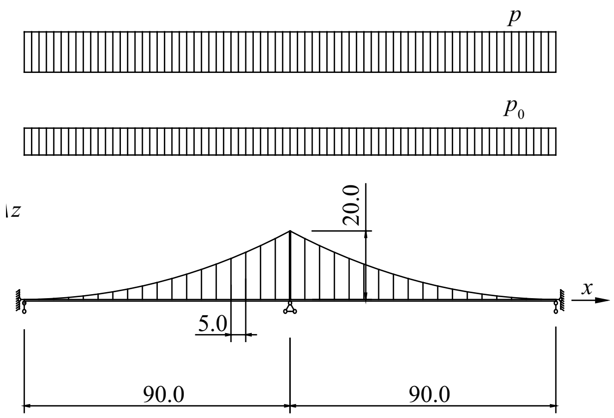

Fig. 8. Calculation scheme for the single-pylon bridge

Maximum deflections of the girder are presented in Table 1.

Table 1. Maximum deflections of the girder, m

p, kN/m	A = 31800 mm^2			A = 45800 mm^2		
	Discrete	Linear	Non-linear	Discrete	Linear	Non-linear
20	–0.193	–0.215	–0.213	–0.150	–0.163	–0.161
30	–0.283	–0.323	–0.301	–0.220	–0.245	–0.239
40	–0.370	–0.430	–0.386	–0.289	–0.326	–0.305
50	–0.454	–0.538	–0.467	–0.355	–0.407	–0.369

Comparison of displacements is presented in Figure 9.

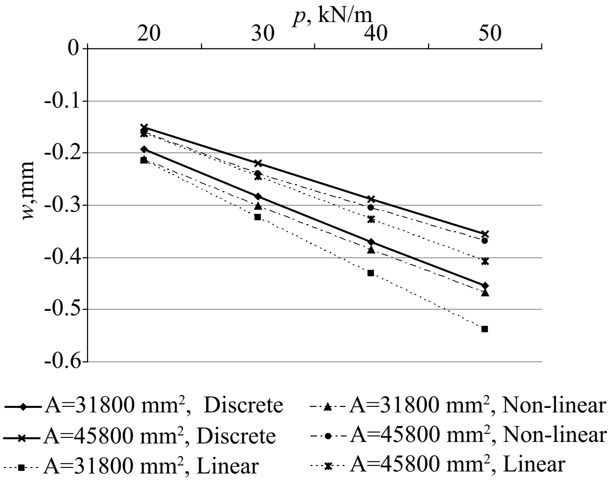

Fig. 9. Maximum displacements of the stiffening girder

The discrete method gives smallest deflection of the girder. Smallest displacement from traffic load $w_t = -0.208$ m come from non-linear scheme when cross-section of the cable is 45800 mm^2. Displacement from the traffic load in case of discrete scheme $w_t = 0.205$ mm is practically similar as displacement from non-linear scheme. Displacements from linear scheme are noticeably larger than displacements from non-linear or discrete scheme.

In case of self-anchoring scheme, displacement from load $p_1 = 20$ kN/m was $w_{p1} = 0.188$ m, displacement from load $p = p_1 + p_t = 50$ kN/m was $w_p = 0.395$ m and displacement from traffic load only was $w_t = 0.395 - 0.188 = 0.207$ m. This is practically same as displacements from bridge, where cables are anchored to the abutments.

4. Conclusion

The geometrically nonlinear equations for discrete analysis of single-pylon suspension bridge presented in this paper enable adequate determination of deflections and inner forces for these structures. Numerical examples demonstrated a very good accordance between results of discrete method and non-linear analysis. If discrete loadings are predicted for the suspension bridge, then the discrete calculation model should be used for the calculation of the bridge. The continual model fits well in situations where the influence of discrete loadings is insignificant. The continual method also has the advantage of good and quick convergence over the discrete model.

References

Aare, J.; Kulbach, V. 1984. Accurate and approximate analysis of statical behaviour of suspension bridges, *Journal of Struct. Mech*. 17(3): 1–12.

Idnurm, J. 2004. *Discrete Analysis of Cable-Supported Bridges.* PhD Thesis, Tallinn University of Technology, Tallinn.

Idnurm, J.; Kiisa, M.; Idnurm, S. 2009. Discrete analysis for classical and single-pylon suspension bridges, in *Proceedings of the Twelfth International Conference on Civil, Structural and Environmental Engineering Computing*, B. H. V. Topping; L. F. Costa Neves; R. C. Barros (Editors). Civil-Comp Press, Stirlingshire, United Kingdom, 218. doi:10.4203/ccp.91.218

Jürgenson, A. 1985. *Tugevusõpetus* [Strength of Materials]. Tallinn: Valgus. 458 p.

Kulbach, V. 1995. Statical analysis of girder- or cable-stiffened suspended structures, in *Proceedings of the Estonian Academy of Sciences Engineering* 1: 2–19.

Kulbach, V. 1998. Half span loading of cable structures, *Journal of Constructional Steel Research* 49(2): 167–180. doi:10.1016/S0143-974X(98)00215-6

Kulbach, V. 1999. Design of different suspension bridges, in *Proceedings of Conference Eurosteel ´99*, 395–398.

Kulbach, V. 2007. *Cable Structures. Design and Static Analysis*. Estonian Academy Publishers, Tallinn.

Kulbach, V.; Idnurm, J.; Idnurm, S. 2002. Discrete and Continuous modeling of suspension bridge, in *Proceedings of the Estonian Academy of Sciences Engineering* 2: 121–133.

Kulbach, V.; Õiger, K. 1986. *Static Analysis of Suspended Systems* [Статический расчет висячих систем]. Tallinn: TPI. 114 p. (in Russian)

KABAMOJO TILTO SU VIENU PILONU DISKREČIOJI ANALIZĖ

M. Kiisa, J. Idnurm, S. Idnurm

Santrauka. Straipsnyje pateikiamas skaičiavimo metodas, pasiūlytas Talino technikos universiteto mokslininkų (Idnurm 2004; Kulbach 2007) ir skirtas kabamųjų tiltų su vienu pilonu standumo sijai skaičiuoti. Klasikiniai kabamieji tiltai susideda iš geometriškai nietiesiško lyno, kuris tampriomis atotampomis sujungtas su standumo sija. Paprastai lynas būna pakabintas tarp dviejų pilonų, inkaruotų inkariniais lynais. Alternatyvi kabamųjų tiltų konstrukcija yra tokia, kai naudojamas tik vienas pilonas tilto viduryje, o pagrindinis geometriškai netiesiškas lynas yra inkaruojamas standumo sijos galuose. Skaičiuojant kabamuosius tiltus, pagrindinė problema yra parabolinio lyno geometrinio netiesiškumo elgsena. Tiesinė analizė tinkama tik mažiems tarpatramiams. Geometrinio netiesiškumo modelis ypač naudingas klasikinės apkrovos atveju – vienodai išskirstytai apkrovai visam arba pusei tarpatramio. Tačiau šiuolaikinius transporto modelius sudaro koncentruotos ir vienodai išskirstytos apkrovos. Diskretusis kabamojo tilto modelis leidžia mums taikyti visų tipų apkrovas, tokias kaip koncentruotos ir išskirstytos. Diskrečiojo modelio prielaidos yra tokios: taikoma medžiagų ir horizontalių atotampų poslinkių tiesinė įtempių ir deformacijų priklausomybė. Atotampų pailgėjimas taip pat yra įvertintas.

Reikšminiai žodžiai: lynų konstrukcijos, standumo sijos, kabamasis lynas, didelių tarpatramių konstrukcijos, diskrečioji analizė, geometrinis netiesiškumas.

Martti KIISA. PhD student and MSc at the Department of Transportation, Tallinn University of Technology. Research interests: girder- stiffened cable and suspension bridges.

Juhan IDNURM. PhD, Assoc. Prof. at the Department of Transportation, Tallinn University of Technology. Research interests: girder-stiffened cable and suspension bridges.

Siim IDNURM. PhD, Prof. at the Department of Transportation, Tallinn University of Technology. Research interests: girder-stiffened cable and suspension bridges.