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RELIABILITY AND DURABILITY ASSESSMENT OF CONCRETE AND COMPOSITE STRUCTURES

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Abstract. Methods of probability design and statistical control of reinforced concrete and composite structures lead to material economy and increase in reliability of buildings and constructions. Limit states of construction are to be caused by loads and material characteristics as well as by unfavourable environmental conditions. Reinforced concrete is heterogeneous material cast-in-site under variable conditions with variable materials and reinforced with environmentally sensitive material. The fluctuation of strength characteristics of constituent materials is taken into consideration in construction codes through the concept of characteristic strength and other properties such as diffusivity and permeability. As far as durability of structure is concerned, the probable fluctuation related to stress state in a material is additional to fluctuation related to the material characteristics. Methods of assessment of load-bearing capacity and durability of a single concrete and composite structural elements and reliability of statically determined and undetermined systems are developed and discussed.

Keywords: probability design, reliability, durability, reinforced concrete, composite column, strength.

1. Introduction

Simple practical methods of probability design and statistical control of reinforced concrete (RC) and composite structures lead to material economy and increase in reliability of buildings and constructions. Limit states of construction are to be caused by loads, material characteristics and by unfavourable environmental conditions. In durability calculations the lifetime of structures before attaining the limit state is referred to a technical resource and economical category.

Reinforced concrete is heterogeneous material cast-in-site under variable conditions with variable materials and reinforced with environmentally sensitive material. It is inevitable that there is a fluctuation in strength characteristics of constituent materials. This aspect is always taken into consideration in construction codes through the concept of characteristic strength and other properties such as diffusivity and permeability. As far as durability of structure is concerned, fluctuation related to stress state in a material is additional to fluctuation related to the material characteristics. Therefore when long exploitation life is required, it appears logical that probabilistic aspect is included in the prediction of the lifetime of structures. The purpose of this study is to present simple methods of assessment of load bearing capacity and durability of a single concrete and composite structural elements and reliability of statically determined and undetermined framed systems. Proposed methods are based only on dispersion of main mechanical and geometrical characteristics of constituent materials and structural elements.

2. Methods of reliability and durability assessment

2.1. Load bearing analysis of structures

Calculation model of simple structures (beams, columns, etc.) can be represented by flow-chart consisting of a few conditioned elements characterising the probability of the load carrying efficiency in different zones of the structure. In the case of complicated and composite structures (trusses, frames, towers etc.) calculation model consists of many conditioned elements (Kudzys 1985). The assessment of load-bearing capacity provides the probability of ultimate limit state absence, while suitability assessment prevents serviceability limit states.

Reliability analysis of statically determined systems can be easily performed on the base of distribution of material characteristics and relationships between constructive parameters. The complexity of analysis of statically undetermined systems is caused by correct estimation of internal forces and their redistribution. Therefore, in this case the reliability analysis gives only approximate quality assessment of the structure (Bolotin 1982; Nowak, Collins 2000). It is necessary to note, that only using probability methods can assess the effect of environment on the behaviour of statically determined and undetermined structures.

When assessing the safety, suitability and durability of the structure, models consisting of conditioned elements in the sense of reliability can be connected in series, parallel or have mixed system, which contains elements in series and in parallel. Moreover, all stochastically dependent elements have to be combined into subsystems. Every separate system can form conditioned elements that present probabilistic characteristics of dangerous zones of the structure and are caused by the same load effects and material characteristics (Bolotin 1982).

In the case of system consisting of m individual stochastically independent conditioned elements with reliabilities $P_{1,suc}$, $P_{2,suc}$, …, $P_{m,suc}$ the system will fail when at least one element fails. The faultless maintenance of the system with elements connected in series can be determined as product of all individual reliability probabilities. The reliability of the system is:

$$P_{suc} = \prod_{k=1}^{m} P_{k,suc} \; . \tag{1}$$

When the reliability of all elements is equal to P_0, the reliability of the system can be determined in the following way:

$$P_{suc} = P_0^m \; . \tag{2}$$

Obviously, the reliability of system consisting of elements connected in the chain can be much lower that of a single system member.

The system with parallel connection of conditioned elements has to be used in the analysis of fault probabilities. In supporting an externally applied load, the failed members of a given parallel structure support no load, while all surviving members will share the applied load according to a specific manner. In the case of system consisting of n individual stochastically independent conditioned elements with failure probabilities $P_{1,fail}$, $P_{2,fail}$, …, $P_{n,fail}$ the probability of system failure can be determined as

$$P_{fail} = \prod_{k=1}^{n} P_{k,fail} \; . \tag{3}$$

Hereof the system reliability with elements in parallel is

$$P_{suc} = 1 - \prod_{k=1}^{n} (1 - P_{k,suc}) \; . \tag{4}$$

When the reliability of all elements is equal to P_0, the reliability of the system can be determined in a simple way:

$$P_{suc} = 1 - (1 - P_0)^n \; . \tag{5}$$

A parallel connection of elements provides the system reliability factors to be considerably increased. By using this model for statically undetermined structures the reliability of such structures appears to be higher than that of statically determinate ones. Therefore, it is advisable to use cast in place reinforced concrete for the primary and complex structures.

2.2. Durability calculation of structures

The system reliability measure is the probability of random events, on condition that structure meet the requirements of serviceability, strength and stability throughout its design working life T_*, without significant loss of utility or unforeseen maintenance (Sarja 2000; Litzner, Becker 1999). Wherewith the reliability $P_{suc}(T_*)$, determined for all design working life T_*, there is necessary to know the reliability $P_{suc}(t)$ for the moment t, where $0 \le t \le T_*$. The external actions and their effects undergo change in the time, but maintenance conditions of the structure deteriorate, i.e. the reliability $P_{suc}(t)$ is a decreasing function of time t (Bolotin 1982). Changes of the reliability can be characterised by the frequency of failure occurrences or faults intensity λ.

The reliability of a structure can be determined on the base of faults intensity λ in the following way:

$$P_{suc}(t) = e^{\left[-\int_0^t \lambda(\tau) d\tau \right]} \; . \tag{6}$$

In the case when $\lambda(t) = \text{const}$, the reliability can be expressed by exponential law:

$$P_{suc}(t) = e^{-\lambda t} . \quad (7)$$

There is possibility to use this law for approximation of the maintenance conditions of the structure.

In the case when the reliability functions of all elements are equal and can be expressed in the form (7), the reliability of a system consisting of elements in chains can be determined by using expression:

$$P_{suc} = e^{-m\lambda t} , \quad (8)$$

but the expected value of the system durability, expressed as maintenance time T, can be determined as

$$\langle T \rangle = \int_0^{\infty} P_{suc}(t)dt = \frac{1}{m\lambda} . \quad (9)$$

Accordingly, the mean value of system durability decreases with the number of elements.

In analogous way, the durability of a system consisting of elements in parallel can be determined by using expression:

$$\langle T \rangle = \int_0^{\infty} \left[1-(1-e^{-\lambda t})^n \right] dt . \quad (10)$$

and after integration the expected value of the system durability is

$$\langle T \rangle = \langle T_0 \rangle \sum_{k=1}^{n} \frac{1}{k} , \quad (11)$$

where $\langle T_0 \rangle$ is expected value of single element durability.

3. Numerical analysis of elements

3.1. Reliability of RC beam

In general case, the load carrying capacity of a beam depends on dispersion of strength characteristics of materials and actions as well as maintenance and environmental effects. The loss of reliability can occur, mainly, in the case when high load effects coincide with low resistance of the element. By using characteristic concrete strength f_{ck} the moment resistance of a simple beam can be expressed as

$$M_{Rk} = \mu b d^2 f_{ck} , \quad (12)$$

where μ is relative bending moment, b and d are width and effective dept of a beam, respectively (Eurocode 2, 2004).

After determination of reinforcement cross section A_s, by using moment M_{Rk} and strength of reinforcement f_{yk}, the moment resistance of the beam can be determined in the following way:

$$M_{Rk} = \left[\frac{A_s f_{ck}}{d} \left(1 - \frac{A_s f_{yk}}{2b f_{ck} d} \right) \right] b d^2 f_{ck} =$$

$$d f_{yk} A_s - \left(\frac{\left(f_{yk} \right)^2 A_s^2}{b f_{ck}} \right). \quad (13)$$

Expression (13) can be written in functional form:

$$M_{Rk} = \varphi(b, d, f_{ck}, f_{yk}) . \quad (14)$$

By using designation x_i for parameters in (14) the expression can be written in the form

$$M_{Rk} = \varphi(x_i) . \quad (15)$$

The variation of actual load-bearing capacity can be determined depending on the variation of factors x_i. Standard deviation of actual moment M_{act} caused by factors x_i is determined as

$$s_i^{M_{act}} = \left| \frac{\partial \varphi}{\partial x_i} \right| s_i , \quad (16)$$

where s_i is standard deviation of factor x_i.

In the case of small deviations of parameters, the mean quadratic deviation of moment resistance M_{act} taking into account the dispersion of all factors x_i is

$$s_{M_{act}} = \sqrt{\sum_i \left(s_i^{M_{act}} \right)^2} . \quad (17)$$

For unfavourable effects of parameters, which determine the moment resistance, the probability of beam failure can be expressed as

$$P_{fail} = \frac{1}{2} - \frac{1}{2} \Phi \left(\frac{M_{Rk} - M_{Ek}}{s_{M_{act}}} \right), \quad (18)$$

where Φ is probability integral and M_{Ek} – characteristic value of acting moment.

Let us examine reinforced concrete beam of cross section 50×25 cm loaded by moment M_{Ek} = 250 kNm. The cross section of needed tension reinforcement is A_s = 20 cm^2 and effective depth d = 47 cm.

In numerical analysis it is assumed that fluctuation of strength and geometric characteristics is ±5%. In the case of moment resistance of the beam M_{Rk} = 310 kNm and taking into account the mean quadratic deviation of moment resistance $s_{M_{act}}$ = 2160 kNm, the probability of beam failure is:

$$P_{fail} = \frac{1}{2} - \frac{1}{2}\Phi\left(\frac{310-250}{21.60}\right) =$$

$$\frac{1}{2} - \frac{1}{2}\Phi(2.78) = 0{,}0028, \quad (19)$$

i.e. 0.28%.

That means that in unfavourable case less than 3 beams of 1000 could fail.

3.2. Reliability analysis of composite column

In the case of composite columns special attention has to be paid to the dispersion of geometrical and material properties and it's influence on load-bearing capacity of the column. By using characteristic strengths of steel tube, concrete and reinforcement and neglecting the confinement effect of steel tube, the plastic resistance of composite column can be expressed as sum of all constituents (Eurocode 4, 2005; Bergman *et al.* 1995):

$$N_{pl.Rk} = A_a f_{yk} + A_c f_{ck} + A_s f_{sk}\,, \quad (20)$$

where A_a, A_c, and A_s are cross section areas of construction steel, concrete and reinforcement, respectively. The effect of interaction between steel tube and concrete core in composite column is studied in (Kvedaras, Kudzys 2006).

The equation (20) can be written in the form

$$N_{pl.Rk} = \varphi(A_a, A_c, A_s, f_{yk}, f_{ck}, f_{sk}) \quad (21)$$

or in compact form

$$N_{act} = \varphi(x_i)\,, \quad (22)$$

where the variation of actual load-bearing capacity N_{act} can be determined depending on the variation of factors x_i. Standard deviation of N_{act} caused by factors x_i is determined as

$$s_i^{N_{act}} = \left|\frac{\partial\phi}{\partial x_i}\right| s_i\,, \quad (23)$$

where s_i is standard deviation of factor x_i.

Assuming that the dispersion of all mentioned above characteristics (deviation from the nominal value) is ±5%, the total standard deviation of load bearing capacity N is ±6.2%. Results of dispersion analysis of composite column load bearing capacity are summarized in Table 1. The probability P, that actual load bearing capacity N_{act} is less than design capacity N_{Rd}, can be determined as

$$P(N_{act} - \Delta N < N_d) = \frac{1}{2} - \frac{1}{2}\Phi\left(\frac{N_{act} - N_d}{s_{(N_{act}-\Delta N)}}\right), \quad (24)$$

where Φ is probability integral; $s_{(N_{act}-\Delta N)}$ – standard deviation of load carrying capacity. It is determined that in the case of dispersion ±5% the probability of a failure is P = 0.065%, for dispersion ±10% – P = 6.8%.

On the basis of constitutive relationships for material components, the stress state in a composite column is determined taking into account the dependence of the modulus of elasticity and Poisson's ratio on the stress level in concrete (Neville 1981). The effect of confinement acts at a high stress level when the structural steel behaves in tension and the concrete in compression. The main load limiting factors are concrete design strength f_{cd} and D/t ratio. Considering load limiting factors for high strength concrete (class C35/45) and steel grade S235 the load bearing capacity of the composite column increases by 18% and for thin wall hollow section (D/t = 90) the steel economy is for 50%.

The ultimate limit state of materials is not attained for all the parts simultaneously. In order to improve the working conditions of a composite element and to prevent the possibility of a failure because of a small thickness of structural steel, fire and because of a dispersion of material properties, the appropriate strength classes of concrete and steel should be used.

3.3. Durability analysis of RC beam

The durability design concerns serviceability limit states such as corrosion due to chloride penetration, corrosion due to carbonation, surface deterioration, frost attack etc. The most significant durability problem is

Table 1. Results of dispersion analysis of composite column load bearing capacity

Dispersion, %	Standard deviation composite column capacity s_i^N, kN						Actual total deviation $s_{N_{act}}$ kN
	s_D^N	s_t^N	$s_{d_s}^N$	$s_{f_{yk}}^N$	$s_{f_{ck}}^N$	$s_{f_{sk}}^N$	
5	127	45	62	53	48	38	169
10	249	102	148	121	116	102	365

the chloride penetration that results in a corrosion of steel embedded in concrete. After depassivation or onset of steel corrosion, it may take several years before any visual sign of deterioration such as cracking and spalling will occur, and it may still take long time before the structural capacity or integrity becomes significantly reduced (Capra *et al.* 2003).

Reduction of the probability of corrosion can be achieved by improving the quality of the concrete cover, either by a well-considered mix design or by application of a coating as well as by using a special type of reinforcing steel.

In the case of carbonation or chloride-induced corrosion the determination of durability and service life of concrete structures, the reliability of the predicted durability and service life depends on the accuracy with which both the properties of the concrete, including aging effects, and the thickness of the concrete cover had been estimated. Simple and advanced models are used for predicting degradation processes, like carbonation of the concrete and ingress of chloride ions caused corrosion of steel reinforcement.

It is experimentally established that chemical reactions usually pass according to exponential law (7). In durability design it is necessary to determine such initial geometrical characteristic of a structure or component which in the moment $t = T$ is not less than design value.

In the case of RC beam the initial cross section of tension reinforcement $A_{0,s}$ can be determined by using expression

$$A_{0,s} = A_{d,s} e^{\lambda t} . \quad (25)$$

Assuming experimentally determined faults intensity $\lambda = 0.01$ 1/year and design cross section of steel rebar $A_{d,s} = 25$ cm^2, the reliability of the RC beam after lifetime $T = 30$ years is provided when initial cross section is

$$A_{0,s} = A_{d,s} e^{\lambda T} = 25 \times e^{0.01 \times 30} = 34 \text{ cm}^2 . \quad (26)$$

By introducing advanced technology and reducing imposed load there is possibility to extend lifetime of the structure. In this case the design value of the cross section of steel reinforcement is less. On the basis of expression (25) it is possible to determine the remaining maintenance time (Bolotin 1982):

$$T = \frac{1}{\lambda} \ln \frac{A^*_{0,s}}{A^*_{d,s}} = \frac{1}{0.01} \times \ln \frac{25}{20} = 22 \text{ years} , \quad (27)$$

where $A^*_{d,s} = 20\, cm^2$ is the design cross section of steel rebar for diminished load.

The use of exponential law for the description of aging processes, deterioration of the structure and their elements is the first approximation, which is based on theoretical and experimental investigation. Obviously, the assessment has to be grounded on the wide bases of classified experimental results as well as inspection and testing of existing structures.

The probabilistic service life design methods do not only allow to design a structure in the planning phase for a given target service life design (Schiessl 2005). It can be advantageously applied also for existing structures to estimate the remaining service life or by repeated application in certain time intervals to improve the precision of the service life prediction until a given safety index will be reached.

4. Conclusions

The probability-based assessment of structural safety of structural members is performed and the following main conclusions can be drawn:

1. In the reliability analysis of RC beam, when the fluctuation of strength and geometric characteristics is ±5%, in the unfavourable case the probability of beam failure is 0.28%.
2. The estimation of composite column resistance shows that, when dispersion of all design factors is ±5%, the probability of column failure is 0.065%, for dispersion ±10% – 6.8%.
3. Based on probability approach and by using approximate faults intensity law the assessment of RC beam durability is performed.
4. Proposed simple methods allow estimate the risk level of reinforced concrete and composite elements taking into account experimentally determined dispersion of mechanical and geometrical characteristics of constituent materials and structural elements.

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BETONINIŲ IR KOMPOZITINIŲ KONSTRUKCIJŲ PATIKIMUMO BEI ILGAAMŽIŠKUMO VERTINIMAS

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Santrauka. Tikimybinių metodų ir statistinės kontrolės taikymas gelžbetoninėms ir kompozitinėms konstrukcijoms leidžia šias konstrukcijas projektuoti ekonomiškiau bei užtikrinti pastatų ir konstrukcijų patikimumą. Statybinių konstrukcijų ribinius būvius lemia medžiagų ir apkrovų charakteristikos bei nepalankios aplinkos sąlygos. Gelžbetonis yra heterogeninė, statybos aikštelėje liejama medžiaga su dideliu variacijos koeficientu, o plienas – aplinkai jautri statybinė medžiaga. Medžiagos stiprumo variacija ir kitos savybės (difuzija, pralaidumas) įvertinama projektavimo normose imant medžiagos charakteristinę reikšmę. Straipsnyje pasiūlyti ir aptarti betoninių bei kompozitinių konstrukcijų laikomosios galios statistiškai apibrėžtų ir neapibrėžtų sistemų vertinimo metodai skaičiuojant ilgaamžiškumą ir patikimumą.

Reikšminiai žodžiai: tikimybinis skaičiavimas, patikimumas, ilgaamžiškumas, gelžbetonis, kompozitinė kolona, stipris.

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