



ANALYSIS OF SHEAR DESIGN RECOMMENDATIONS FOR FRP REINFORCED CONCRETE BEAMS

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Abstract. Research shows that most shear design models for concrete beams reinforced with FRP reinforcement provide conservative results that leads to excessive amounts of reinforcement and increased overall cost of such construction. This paper presents comparative analysis of current shear design models for concrete beams reinforced with longitudinal FRP reinforcement and FRP stirrups. New analytical shear design model, developed by Valivonis et al., has been included in the analysis. A database with 88 specimens reinforced with FRP reinforcement was compiled in order to verify the accuracy of the proposed model by Valivonis et al. It is shown that proposed shear design model yields quite accurate and consistent results as an average of V_{exp} / V_{pred} values is 0.98 and coefficient of variation is 26.0% for this model.

Keywords: shear strength, fiber reinforced polymer bar, stirrup, reinforced concrete.

Introduction

Working life of steel reinforced concrete structures is shortened by steel reinforcement corrosion. Maintenance and repair of such damaged structures is very expensive. Corrosion is particularly dangerous for structures in aggressive marine environment or unprotected from the effect of deicing salts. Working life of such structures is highly dependent on the durability of reinforcement. Fiber-reinforced polymer (FRP) reinforcement is more durable than steel reinforcement so it can be considered as more advantageous alternative to steel reinforcement when using it to reinforce concrete structures exposed to aggressive environment. Reinforcement, located closest to the surface of the element (e.g. shear reinforcement in beams), is the most vulnerable to aggressive environmental effects. For this reason, employment of FRP shear reinforcement in such cases is even more meaningful. Use of FRP reinforcement in the manufacture of such structures can potentially extend its working life and reduce overall life cycle cost. For example, it is reported that billions of dollars are spent every year in North America for repair and replacement of pile systems (Benmokrane, Ali, Mohamed, Robert, & ElSafty, 2016; Mohamed, Afifi, & Benmokrane, 2014). Also it was determined that repair of all steel reinforced concrete structures in Canada would

cost about 74 billion dollars (Natural Sciences and Engineering Research Council of Canada, 2010).

Up to now FRP reinforcement was successfully used as main structural reinforcement in construction of various concrete bridges, underground parking lots, tunnels (Nanni & Faza, 2002; Mohamed & Benmokrane, 2013). Therefore research, development and practical use of concrete structures reinforced with FRP reinforcement recently has gained more and more interest in the field of civil engineering.

1. Review of the current design provisions

The main differences of FRP reinforcement compared to steel reinforcement are lower modulus of elasticity and linear elastic behavior up to rupture. Also when using FRP reinforcement as shear stirrups in beams, it must be evaluated that tensile strength of the bent part of the FRP reinforcement bar is significantly lower than that of the straight part (Shehata, Morphy, & Rizkalla, 2000; El-Sayed, El-Salakawy, & Benmokrane, 2007). Shear strength of FRP reinforced elements is affected by the mentioned mechanical properties of FRP reinforcement. For this reason, these mechanical properties should be taken into account in the shear design equations.

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So far a number of detailed studies have been carried out in order to perform comprehensive shear analysis of concrete structures reinforced with FRP reinforcement (Shehata et al., 2000; Razaqpur & Spadea, 2014; Alam & Hussein, 2012; Bentz, Massam, & Collins, 2010; Ahmed, El-Salakawy, & Benmokrane, 2010; Fico, Prota, & Manfredi, 2008; El-Sayed & Benmokrane, 2008; El-Sayed, El-Salakawy, & Benmokrane, 2006; Alkhrdaji, Wideman, Belarbi, & Nanni, 2001; Guadagnini, Pilakoutas, & Waldron, 2006; Tottori & Wakui, 1993). Calculation methods and design standards were created to determine element's shear capacity. These include ACI 440.1R-06 (American Concrete Institute, 2006), CNR-DT 203/2006 (Italian Research Council, 2007), CSA S806-12 (Canadian Standards Association, 2012), CSA S6-14 (Canadian Standards Association, 2014) and JSCE (Japan Society of Civil Engineering, 1997). Shear design models in these standards are based on a theory that shear capacity of reinforced concrete beam consists of concrete shear resistance V_c and FRP transverse reinforcement shear resistance V_f . However, research shows that these models provide conservative shear capacity predictions of FRP reinforced elements. Therefore new design models are constantly being developed and researchers are looking for more precise methods for determining shear resistance.

For instance, Hoult, Sherwood, Bentz, and Collins (2008) developed shear design model for FRP reinforced beams based on modified compression field theory (MCFT). The results of Hoult *et al.* research showed that shear behavior of FRP reinforced beams without shear reinforcement is very similar to that of steel reinforced beams. This was later confirmed by the results of Bentz et al. (2010) research. Also, it is worth mentioning that Perera, Arteaga, and Diego (2010) developed new calculation method based on artificial intelligence techniques.

Through this section, shear strength calculation equations as specified by the ACI 440.1R-06, CNR-DT 203/2006, CSA S806-12, CSA S6-14, Oller, Mari, Bairan, and Cladera (2015), Hegger, Niewels, and Kurth (2009), Nehdi, El Chabib, and Aly Said (2007) will be reviewed.

According to ACI 440.1R-06, CNR-DT 203/2006, CSA S806-12, CSA S6-14, Hegger et al. (2009) and Nehdi et al. (2007) shear design models, shear resistance is calculated using this equation:

$$V_u = V_c + V_f, \quad (1)$$

where: V_c – concrete contribution to shear capacity; V_f – FRP contribution to the shear capacity.

ACI 44.1R-06

New design model for concrete shear resistance was proposed by Tureyen and Frosch (2002). ACI 440.1R-06 adopted this method and concrete shear resistance according to it is expressed as:

$$V_c = \frac{2}{5} \sqrt{f_c} b_w c; \quad (2)$$

$$c = kd; \quad (3)$$

$$k = \sqrt{2\rho_f n_f + (\rho_f n_f)^2} - \rho_f n_f; \quad (4)$$

$$n_f = E_f / E_c, \quad (5)$$

where b_w – beam width; c – depth of compression zone at cracked transformed section; d – effective depth; k – coefficient, which accounts the decreasing depth of neutral axis; E_f – modulus of elasticity of longitudinal FRP reinforcement; E_c – modulus of elasticity of concrete.

Equation (2) is shear capacity equation for steel reinforcement, modified by the empirically determined factor which accounts for the axial stiffness of the FRP reinforcement.

Equation for shear resistance provided by shear reinforcement V_f is based on a modified truss analogy. It is assumed that total shear is carried by the shear reinforcement in this model. Shear capacity provided by FRP shear reinforcement:

$$V_f = \frac{f_{fw} A_{fw} d}{s}; \quad (6)$$

$$f_{fw} = 0.004 E_{fw} \leq f_{fb}; \quad (7)$$

$$f_{fb} = \left(0.05 \cdot \frac{r_b}{d_b} + 0.3 \right) f_{fwu}, \quad (8)$$

where f_{fw} – stress level in the FRP shear reinforcement at ultimate state; A_{fw} – area of FRP shear reinforcement; s – spacing of shear reinforcement; f_{fb} – tensile strength of FRP bent bar; r_b – bending radius of FRP bar; d_b – diameter of the FRP bar in the bent portion.

CNR-DT 203/2006

According to CNR-DT 203/2006, limitation of shear capacity of FRP reinforced elements using FRP stirrups shall be satisfied:

$$V_u \leq V_{u,max}, \quad (9)$$

where $V_{u,max}$ – concrete contribution corresponding to shear failure due to crushing of the web.

Formula for calculating concrete shear resistance is a modified version of the Eurocode 2 shear equation for conventional steel RC members. Concrete shear resistance according to CNR-DT 203/2006:

$$V_c = 1.3 \cdot \left(\frac{E_f}{E_s} \right)^{1/2} \cdot \tau \cdot k \cdot (1.2 + 40\rho_f) \cdot b_w \cdot d; \quad (10)$$

$$\tau = 0.25 f_{ct}; \quad (11)$$

$$k = 1.6 - d \geq 1, \quad (12)$$

where: E_s – modulus of elasticity of steel reinforcement; τ – shear stress; ρ_f – longitudinal FRP reinforcement ratio.

The main difference between CNR-DT 203/2006 and Eurocode 2 equations is that additional limitation shall be satisfied. This limitation is presented because experimental results showed that the shear contribution of FRP shear reinforcement is lower than steel in the case of RC beams

with FRP stirrups:

$$1.3 \cdot \left(\frac{E_f}{E_s} \right)^{1/2} \leq 1. \quad (13)$$

Shear capacity provided by FRP shear reinforcement can be calculated using the following equation:

$$V_f = \frac{A_{fw} \cdot f_{fr} \cdot d}{s}; \quad (14)$$

$$f_{fr} = f_{fwu} \cdot \gamma_{f,\Phi}, \quad (15)$$

where f_{fr} – reduced tensile strength of the FRP reinforcement; f_{fwu} – tensile strength of FRP shear reinforcement; $\gamma_{f,\Phi}$ – partial factor to account for the bending effect (shall be set equal to 2 when no specific experimental tests are performed).

$V_{u,max}$ can be calculated as follows:

$$V_{u,max} = \frac{\alpha_{cw} b_w z v_1 f_c}{ctg\theta + tg\theta}; \quad (16)$$

$$z = 0.9d; \quad (17)$$

$$v_1 = 0.6 \left[1 - \left(\frac{f_c}{250} \right) \right]; \quad (18)$$

$$ctg\theta = \sqrt{\frac{\alpha_{cw} b_w v_1 f_c s}{A_{fw} f_{fw}} - 1}, \quad (19)$$

where θ – angle between the concrete compression strut and the beam axis perpendicular to the shear force; α_{cw} – coefficient taking into account the state of the stress in the compression chord; v_1 – strength reduction factor for concrete cracked in shear.

CSA S806-12

Canadian Standards Association uses shear design model which is theoretically derived and based on strut-and-tie model for FRP reinforcement. According to CSA S806-12 design recommendations, shear capacity of FRP reinforced concrete beam is also limited:

$$V_u \leq 0.22 \phi_c f_c b_w z, \quad (20)$$

where ϕ_c – resistance factor for concrete.

Concrete shear capacity can be evaluated using this equation:

$$V_c = 0.05 \lambda \phi_c \cdot k_m \cdot k_r \cdot f_c^{1/3} \cdot b_w \cdot z; \quad (21)$$

$$k_m = \sqrt{d/a} \leq 1.0; \quad (22)$$

$$k_r = 1 + \left(E_f \rho_f \right)^{1/3}, \quad (23)$$

where λ – factor accounting for concrete density; a is shear span.

FRP reinforcement shear capacity is given by:

$$V_f = \left(\frac{\phi_f A_{fw} f_{fw} z}{s} \right) ctg\theta; \quad (24)$$

$$f_{fw} = 0.005 E_{fw} \leq 0.4 f_{fwu}; \quad (25)$$

$$\theta = 30 + 7000 \varepsilon_x; \quad (26)$$

$$\varepsilon_x = \frac{(M/z) + V}{2(E_f A_f)} \leq 0.003, \quad (27)$$

where ϕ_f – resistance factor for FRP reinforcement; ε_x – longitudinal strain at midheight of the cross section; M – bending moment; V – shear load; A_f – area of FRP longitudinal reinforcement.

CSA S6-14

Canadian Highway Bridge Design Code also uses strut-and-tie models for shear capacity calculation of concrete beams. Concrete shear resistance according to CSA S6-14:

$$V_c = 2.5 \left[\frac{0.4}{(1 + 1500 \varepsilon_x)} \cdot \frac{1300}{(1000 + s_{ze})} \right] f_{cr} b_w z; \quad (28)$$

$$\varepsilon_x = \frac{(M/z) + V}{2(E_f A_f)} \leq 0.003; \quad (29)$$

$$s_{ze} = 300 \text{ mm}; \quad (30)$$

$$f_{cr} = 0.4 \sqrt{f_c}, \quad (31)$$

where: s_{ze} – effective crack spacing for members without stirrups; f_{cr} – cracking strength of concrete.

Shear capacity provided by FRP shear reinforcement:

$$V_f = \left(\frac{\phi_f A_{fw} f_{fw} z}{s} \right) ctg\theta; \quad (32)$$

$$f_{fw} = 0.004 E_{fw} \leq f_{fb}; \quad (33)$$

$$f_{fb} = \frac{(0.05 r_b / d_b + 0.3) f_{fwu}}{1.5}; \quad (34)$$

$$\theta = (30 + 7000 \varepsilon_x) \left(0.88 + \frac{s_{ze}}{2500} \right). \quad (35)$$

Hegger et al.

Equation of concrete shear resistance is similar to the equation of shear design model given in Eurocode 2 with some parameters that evaluate unique FRP longitudinal reinforcement characteristics. These parameters were determined through experimental tests. Concrete shear resistance is expressed as:

$$V_c = k_f \cdot \beta \cdot 0.205 \cdot \kappa \cdot (100 \cdot \rho_f \cdot \frac{E_f}{E_s} \cdot f_c)^{1/3} \cdot b_w \cdot d; \quad (36)$$

$$k_f = 1 - 10 \cdot \rho_{fw} \cdot \frac{E_{fw}}{E_c}; \quad (37)$$

$$\beta = 3 \cdot \frac{d}{a}; \quad (38)$$

$$\kappa = 1 + \sqrt{200/d}. \quad (39)$$

FRP transverse reinforcement shear capacity depends on a limit stirrup strain based on the results of existing experimental work. Shear capacity provided by FRP shear reinforcement:

$$V_f = \min \left(\frac{A_{fw} \cdot f_{fw} \cdot z \cdot ctg\theta}{s}; b_w \cdot z \cdot \alpha_c \cdot f_c \cdot \frac{1}{ctg\theta + tg\theta} \right); \quad (40)$$

$$f_{fw} = \min(0.4 \cdot f_{fwu}; E_{fw} \cdot \varepsilon_{fwu}); \quad (41)$$

$$\varepsilon_{fwu} = 3 + \frac{0.015}{(\rho_{fw} \cdot E_{fw} / E_c)}; \quad (42)$$

$$\alpha_c = 0.2. \quad (43)$$

Nehdi et al.

Nehdi et al. (2007) developed equations to calculate the shear capacity of FRP reinforced concrete beams based on the genetic algorithms approach. In training and testing of the model $f_c, b_w, d, a/d, E_f/E_s, \rho_f, \rho_{fw}, f_{fwu}$ were entered as input variables. Study performed by these authors showed that the axial rigidity of FRP longitudinal bars is best represented by a cubic root function and that the contribution of FRP stirrups to shear strength is a square root function of the stirrups ultimate capacity rather than a linear function as proposed by current shear provisions.

Concrete shear resistance is calculated according to this equation:

$$V_c = 2.1 \left(\frac{f_c \rho_f d E_f}{a E_s} \right)^{0.3} b_w d, \text{ when } a/d \geq 2.5; \quad (44)$$

$$V_c = 2.1 \left(\frac{f_c \rho_f d E_f}{a E_s} \right)^{0.3} b_w d \cdot \frac{2.5d}{a}, \text{ when } a/d < 2.5. \quad (45)$$

Transverse FRP reinforcement shear resistance:

$$V_f = 0.5 (\rho_{fw} f_{fwu})^{0.5} b_w d. \quad (46)$$

Oller et al.

Oller et al. (2015) developed shear design model that is based on the principles of structural mechanics and on the observed experimental behaviour of FRP reinforced concrete beams with FRP transverse reinforcement. Each shear transfer mechanism has been included and evaluated in this model.

Ultimate shear strength of FRP reinforced concrete beam is calculated using this equation:

$$V_u = V_{cc} + V_w + V_t = f_{ct} \cdot b_w \cdot d \cdot (v_{cc} + v_w + v_t), \quad (47)$$

where: V_{cc} – shear resisted by the un-cracked concrete zone; V_w – shear resisted by tensile stresses transferred along the crack; V_t – shear resisted by transverse reinforcement crossing the diagonal critical shear crack; v_{cc}, v_w, v_t – dimensionless forms of shear resisted by the un-cracked concrete zone, by tensile stresses transferred along the crack and by transverse reinforcement crossing the diagonal critical shear crack, respectively.

Shear resistance of un-cracked concrete zone:

$$v_{cc} = \zeta \cdot (1.072 - 0.01 \cdot \alpha) \cdot ((0.98 + 0.22 \cdot v_t) \cdot \xi + 0.05); \quad (48)$$

$$\zeta = 1.2 - 0.2 \cdot a \geq 0.65; \quad (49)$$

$$\alpha = E_f / E_c; \quad (50)$$

$$\xi = \alpha \cdot \rho_f \cdot \left(-1 + \sqrt{1 + \frac{2}{\alpha \cdot \rho_f}} \right), \quad (51)$$

where: ζ – coefficient which accounts the size effect on the shear failure; ξ – relative neutral axis depth.

Equation for the shear transferred by the crack:

$$v_w = \frac{0.386}{\varepsilon_{fw,m}} \cdot \frac{f_{ct}}{E_c} \cdot \left(1 + \frac{8 \cdot G_f \cdot E_c}{f_{ct}^2 \cdot d} \right); \quad (52)$$

$$\varepsilon_{fw,m} = 0.225 \cdot \varepsilon_{fwu}; \quad (53)$$

$$G_f = 0.028 \cdot f_c^{0.18} \cdot d_{max}^{0.32}, \quad (54)$$

where: $\varepsilon_{fw,m}$ – mean strain at the stirrups crossing the crack; f_{ct} – concrete tensile strength; G_f – fracture energy of concrete; ε_{fwu} – ultimate strain of the transverse FRP stirrups; d_{max} – maximum aggregate size.

The contribution of the FRP stirrups can be expressed as:

$$v_t = \frac{\rho_{fw} \cdot 0.85 \cdot E_{fw} \cdot \varepsilon_{fw,m}}{f_{ct}}. \quad (55)$$

2. Proposed shear design model

Proposed shear design model by Valivonis, Budvytis, Atutis, M., Atutis, E., and Juknevičius (2015) will be included in the comparison and analysis of calculation methods. This shear design model is described in more detail in the paper of Valivonis et al. (2015). Here only final equations for calculating shear resistance are given.

This design model is based on the assumption that shear strength consists of concrete shear capacity and FRP transverse reinforcement shear capacity. Following equation is recommended for calculating concrete shear capacity:

$$V_c = \frac{\varphi_{c2} \varphi_f f_{ct} b_w d^2}{a} \geq \varphi_{c3} \varphi_f f_{ct} b_w d; \quad (56)$$

$$\varphi_f = 0.4 \cdot \left(\frac{E_f}{E_s} \right)^{\rho_f}; \quad (57)$$

$$a \leq \frac{\varphi_{c4}}{\varphi_{c3}} d; \quad (58)$$

$$\varphi_{c2} = 2.0; \varphi_{c3} = 0.45; \varphi_{c4} = 1.5,$$

where: $\varphi_{c2}, \varphi_{c3}, \varphi_{c4}$ – coefficients which estimate concrete's properties; φ_f – coefficient which estimate the influence of FRP flexural reinforcement for the concrete shear resistance.

FRP transverse reinforcement shear capacity is expressed as:

$$V_f = v_{fw} a_0; \quad (59)$$

$$v_{fw} = \frac{f_{fw} A_{fw}}{s}; \quad (60)$$

$$a_0 = \sqrt{\frac{\varphi_{c2} f_{ct} b_w d^2}{v_{fw}}}; \quad (61)$$

$$d \leq a_0 \leq \min \left\{ \frac{2d}{a} \right\}; \quad (62)$$

$$f_{fw} = \varepsilon_{fw} \cdot E_{fw}; \quad (63)$$

$$\varepsilon_{fw} = \sqrt{\left(\frac{h}{0.85} \right)^{-1.5} f_c \frac{\rho_f E_f}{\rho_{fw} E_{fw}}} \cdot 10^{-4} \leq 0.0045, \quad (64)$$

where a_0 – critical projection of shear cracking zone, where FRP shear reinforcement contributes to shear strength of concrete beam.

3. Database of beams reinforced with flexural and shear FRP reinforcement

The performance of the proposed model for predicting the ultimate shear capacity is evaluated in this section together with other reviewed shear design models. In this analysis, only beams with longitudinal and shear FRP reinforcement were analyzed.

As shown in Table 1, 88 specimens with FRP shear reinforcement were analyzed. The collected database included beams reinforced with different amounts and types of reinforcement – aramid, carbon and glass FRP reinforcement.

4. Comparison of shear strength predictions with experimental results

In order to determine the relative accuracy of the proposed shear design model and to compare it with the other available shear design models, comparison of experimental and theoretical shear strength values was performed. Table 2 presents the mean, the standard deviation and the coefficient of variation of the ratio of experimental to predicted shear strengths of the specimens. Also the results of this analysis are shown in Figures 1–8, where correlation between the experimental shear force V_{exp} and the theoretical prediction V_{pred} is presented.

According to the data in the Table 2, the mean value of V_{exp}/V_{pred} is 0.98 with standard deviation of 0.26 and coefficient of variation of 26.0% for proposed shear design model of FRP reinforced concrete beams. According to these results, systematic error is 0.02 and random error is equal to 0.26 for this calculation model. These results are similar to the results of CSA S806-12 and Hegger et al. (2009) shear design models. Statistical analysis results show that proposed model, CSA S806-12 and Hegger et al. (2009) models are the most accurate models.

Also it should be noted that model of CNR-DT 203/2006 gives the least standard deviation of 0.16. This parameter is used to quantify the amount of dispersion of a set of data values. It means that despite of the fact that compiled database included specimens reinforced with different types and different amounts of FRP reinforcement, dispersion of V_{exp}/V_{pred} values is quite small.

It can be observed that shear design models of ACI 440.1R-06 and CSA S6-14 are quite conservative with

very high mean value, standard deviation and coefficient of variation of V_{exp}/V_{pred} values.

Design model of Nehdi et al. (2007) is based on the genetic algorithms approach. The mean value of V_{exp}/V_{pred} is 1.23 with standard deviation of 0.27 and coefficient of variation of 22.1% for this model. Statistical analysis data of this model show that genetic algorithms approach may also be used in developing new shear design models for FRP reinforced concrete beams.

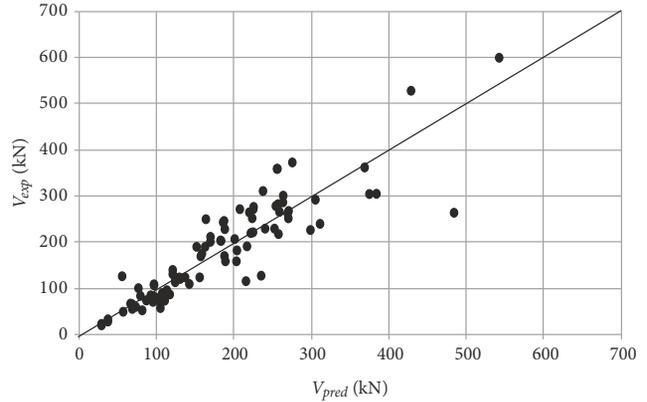


Figure 1. Experimental V_{exp} versus predicted V_{pred} shear strength by the proposed shear design model

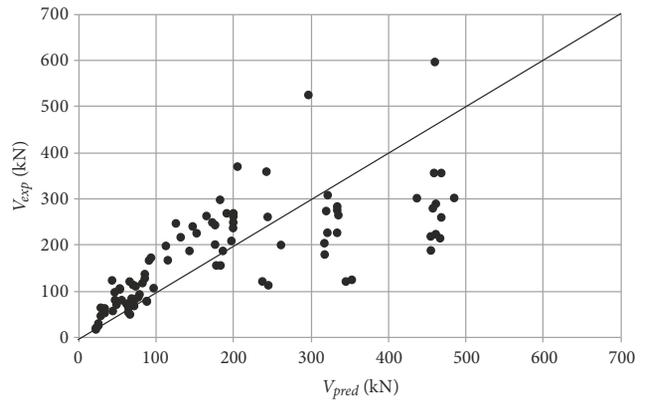


Figure 2. Experimental V_{exp} versus predicted V_{pred} shear strength by the ACI 440.1R-06 shear design model

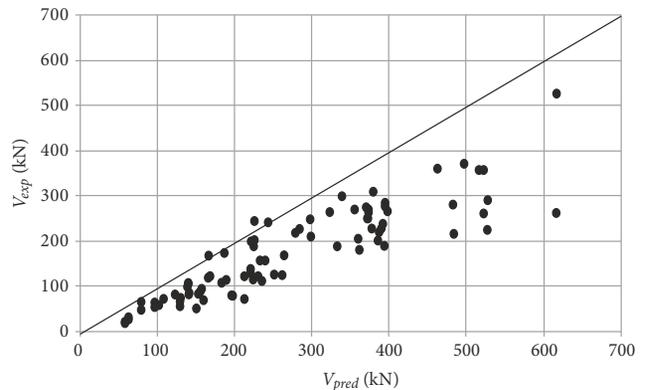


Figure 3. Experimental V_{exp} versus predicted V_{pred} shear strength by the CNR-DT 203/2006 shear design model

Table 1. Database of tested members with FRP stirrups

Author	Specimens		Geometrical properties							Concrete			Flexural reinforcement						Shear reinforcement						V_{exp} , kN
	ID	Test	b, mm	h, mm	d, mm	L, m	L_p , m	a/d	f_c , MPa	F	A_p , cm ²	ρ_p , %	E_p , GPa	f_{fp} , MPa	F	A_{fp} , mm ²	s, mm	ρ_{fp} , %	E_{fp} , GPa	f_{fmax} , MPa					
Nagasaka, Fukuyama, and Tanigaki (1993)	AC0560M	ST	250	300	253	0.6	3.0	1.19	28.9	A	12.0	1.90	56	1295	C	100	80	0.50	112	903	246.2				
	AC1060M	ST	250	300	253	0.6	3.0	1.19	34.0	A	12.0	1.90	56	1295	C	100	40	1.00	112	903	311.0				
	AC1560M	ST	250	300	253	0.6	3.0	1.19	32.9	A	12.0	1.90	56	1295	C	100	27	1.48	112	903	359.0				
	AC0590M	ST	250	300	253	0.9	3.3	1.78	28.9	A	12.0	1.90	56	1295	C	100	80	0.50	112	903	204.0				
	AC1090M	ST	250	300	253	0.9	3.3	1.78	28.9	A	12.0	1.90	56	1295	C	100	40	1.00	112	903	276.6				
	AC1590M	ST	250	300	253	0.9	3.3	1.78	28.9	A	12.0	1.90	56	1295	C	100	27	1.48	112	903	282.5				
	AC0512M	ST	250	300	253	1.2	3.6	2.37	32.9	A	12.0	1.90	56	1295	C	100	80	0.50	112	903	158.9				
	AC1012M	ST	250	300	253	1.2	3.6	2.37	32.9	A	12.0	1.90	56	1295	C	100	40	1.00	112	903	229.6				
	AA0590M	ST	250	300	253	0.9	3.3	1.78	33.5	A	12.0	1.90	56	1295	A	100	80	0.50	61	824	201.1				
	AA1090M	ST	250	300	253	0.9	3.3	1.78	34.7	A	12.0	1.90	56	1295	A	100	40	1.00	61	824	271.7				
	AH0590M	ST	250	300	253	0.9	3.3	1.78	33.5	A	12.0	1.90	56	1295	G	100	80	0.50	44	481	169.7				
	AH1090M	ST	250	300	253	0.9	3.3	1.78	33.5	A	12.0	1.90	56	1295	G	100	40	1.00	44	481	243.3				
	AG0590M	ST	250	300	253	0.9	3.3	1.78	33.5	A	12.0	1.90	56	1296	G	100	80	0.50	46	608	175.6				
	AG1090M	ST	250	300	253	0.9	3.3	1.78	33.5	A	12.0	1.90	56	1297	G	100	40	1.00	46	608	228.6				
	AC1090L	ST	250	300	253	0.9	3.3	1.78	23.5	A	12.0	1.90	56	1296	C	100	40	1.00	112	903	207.0				
	AC1590L	ST	250	300	253	0.9	3.3	1.78	22.6	A	12.0	1.90	56	1297	C	100	27	1.48	112	903	221.7				
	AC1012L	ST	250	300	253	1.2	3.6	2.37	24.3	A	12.0	1.90	56	1295	C	100	40	1.00	112	903	182.5				
	AC1512L	ST	250	300	253	1.2	3.6	2.37	23.0	A	12.0	1.90	56	1295	C	100	27	1.48	112	903	191.3				
	AA1090L	ST	250	300	253	0.9	3.3	1.78	22.6	A	12.0	1.90	56	1295	A	100	40	1.00	61	824	190.3				
	AA1590L	ST	250	300	253	0.9	3.3	1.78	22.6	A	12.0	1.90	56	1295	A	100	27	1.48	61	824	203.1				
	AH1090L	ST	250	300	253	0.9	3.3	1.78	23.5	A	12.0	1.90	56	1295	G	100	40	1.00	44	481	190.3				
	AH1590L	ST	250	300	253	0.9	3.3	1.78	23.5	A	12.0	1.90	56	1295	G	100	27	1.48	44	481	211.9				
	AC1590M'	ST	250	300	253	0.9	3.3	1.78	39.5	A	12.0	1.90	56	1295	C	100	27	1.48	112	903	292.3				
	AC1512M'	ST	250	300	253	1.2	3.6	2.37	39.2	C	12.0	1.90	56	1295	C	100	27	1.48	112	903	226.6				
FF1-20	4PB	150	300	250	3.0	3.3	3.0	36.2	C	2.06	0.55	94	1308	C	36	200	0.12	94	1308	60.3					
FF2-10	4PB	150	300	250	3.0	3.3	3.0	33.1	C	4.12	1.10	94	1308	C	36	100	0.24	94	1308	90.3					
FF1-10	4PB	150	300	250	3.0	3.3	3.0	38.3	C	2.06	0.55	94	1308	C	36	100	0.24	94	1308	85.3					
FF3-10	4PB	150	300	250	3.0	3.3	3.0	31.3	C	5.22	1.39	94	1308	C	36	100	0.24	94	1308	96.3					
FF2-20	4PB	150	300	250	3.0	3.3	3.0	35.0	C	3.96	1.06	94	1308	C	36	200	0.12	94	1308	74.1					
FF4-10	4PB	150	300	250	3.0	3.3	3.0	30.5	C	7.92	2.11	94	1308	C	36	100	0.24	94	1308	120.8					
FF4-10	4PB	150	300	250	3.0	3.3	3.0	30.5	C	7.92	2.11	94	1308	C	36	130	0.18	94	1308	87.3					
FF4-16	4PB	150	300	250	3.0	3.3	3.0	31.3	C	7.92	2.11	94	1308	C	36	160	0.15	94	1308	76.3					
FF4-20	4PB	150	300	250	3.0	3.3	3.0	34.9	C	7.92	2.11	94	1308	C	36	200	0.12	94	1308	83.8					
AA1060M	ST	250	300	253	0.6	3.0	1.19	37.7	A	10.8	1.71	61	1167	A	92	35	1.05	61	822	271.7					
AA1060M-C	ST	250	300	253	0.6	3.0	1.19	37.3	A	10.8	1.71	61	1167	A	92	35	1.05	61	822	252.1					
AA1090M	ST	250	300	253	0.9	3.3	1.78	37.7	A	10.8	1.71	61	1167	A	92	35	1.05	61	822	264.9					
AC0512M	ST	250	300	253	1.2	3.6	2.37	32.9	A	10.8	1.71	61	1167	A	92	70	0.51	113	903	158.9					
AC1060M-C	ST	250	300	253	0.6	3.0	1.78	37.3	A	10.8	1.90	61	1167	C	90	35	1.03	113	903	267.8					
AC1060M	ST	250	300	253	0.6	3.0	1.19	37.7	A	10.8	1.71	61	1167	C	90	35	1.03	113	903	278.6					
AC1090M	ST	250	300	253	0.9	3.3	1.78	37.7	A	10.8	1.71	61	1167	C	90	35	1.03	113	903	286.5					
AC1012M	ST	250	300	253	1.2	3.6	1.78	32.9	A	10.8	1.90	61	1167	C	90	35	1.03	113	903	229.6					
AC1560M	ST	250	300	253	0.6	3.0	1.19	32.9	A	10.8	1.71	61	1167	C	90	24	1.50	113	903	359.0					
AC1590L	ST	250	300	253	0.9	3.3	1.78	28.9	A	10.8	1.71	61	1167	C	90	24	1.50	113	903	217.8					
AC1512M'	ST	250	300	253	1.2	3.6	2.37	32.9	A	10.8	1.71	61	1167	C	90	24	1.50	113	903	262.9					

Maruyama and Zhao (1994)

Okamoto, Nagasaka, and Tanigaki (1994)

Author	Specimens		Geometrical properties						Concrete		Flexural reinforcement				Shear reinforcement				V_{exp} , kN		
	ID	Test	b , mm	h , mm	d , mm	L_r , m	L_o , m	a/d	f_c , MPa	F	A_p , cm ²	ρ_p , %	E_p , GPa	f_{pw} , MPa	F	A_{pw} , mm ²	s , mm	ρ_{pw} , %		E_{fpw} , GPa	f_{fpw} , MPa
Nakamura and Higai (1995)	GG05-10	3PB	200	300	250	1.5	-	3.00	35.4	G	8.04	1.61	29	751	G	70	100	0.35	31	828	84.1
	GG10-10	3PB	200	300	250	1.5	-	3.00	33.4	G	8.04	1.61	29	751	G	70	100	0.35	31	828	100.8
	GG05-20	3PB	200	300	250	1.5	-	3.00	35.2	G	8.04	1.61	29	751	G	70	200	0.18	31	828	56.0
	GG10-20	3PB	200	300	250	1.5	-	3.00	35.2	G	8.04	1.61	29	751	G	70	200	0.18	31	828	66.0
	#10	4PB	150	300	250	1.8	2.6	3.00	34.3	C	11.36	3.03	105	1124	G	57	90	0.42	39	1100	113.7
Zhao, Maruyama, and Suzuki (1995)	#14	4PB	150	300	250	1.8	2.6	3.00	34.3	C	11.36	3.03	105	1124	C	57	90	0.42	10	1300	126.6
	#16	4PB	150	300	250	1.8	2.6	3.00	34.3	C	8.52	2.27	105	1124	G	57	90	0.42	39	1100	116.9
	#18	4PB	150	300	250	1.8	-	2.00	34.3	C	5.68	1.51	105	1124	G	57	90	0.42	39	1100	124.0
	#19	4PB	150	300	250	2.0	-	4.00	34.3	C	5.68	1.51	105	1124	G	57	90	0.42	39	1100	73.8
	#25	4PB	150	300	250	1.8	-	2.50	34.0	C	3.90	1.04	100	1200	G	77	120	0.43	30	600	109.2
Maruyama and Zhao (1996)	#26	4PB	150	300	250	1.8	-	2.50	34.0	C	3.90	1.04	100	1200	G	77	120	0.43	30	600	107.0
	#27	4PB	150	300	250	1.8	-	2.50	34.0	C	3.90	1.04	100	1200	G	77	60	0.86	30	600	140.2
	#28	4PB	150	300	250	1.8	-	2.50	34.0	C	3.90	1.04	100	1200	G	77	60	0.86	30	600	131.0
	#30	4PB	300	550	500	2.5	-	2.50	29.5	C	15.60	1.04	100	1200	G	308	240	0.43	30	600	372.8
	#32	4PB	450	800	750	3.75	-	2.50	29.5	C	35.10	1.04	100	1200	G	692	360	0.43	30	600	599.3
Vijay, Kumar, and GangRao (1996)	S2	4PB	150	300	265	2.5	3.0	1.89	44.8	G	5.67	1.43	54	655	G	127	102	0.83	142	655	127.8
	S3	4PB	150	300	265	2.5	3.0	1.89	44.8	G	5.67	1.43	54	655	G	127	152	0.56	142	655	116.0
	S5	4PB	150	300	265	2.5	3.0	1.89	31.0	G	2.65	0.67	54	655	G	127	102	0.83	142	655	124.2
	S6	4PB	150	300	265	2.5	3.0	1.89	31.0	G	2.65	0.67	54	655	G	127	152	0.56	142	655	124.3
	B1-1	4PB	200	360	320	2.2	2.4	3.13	35.5	G	8.50	1.33	42	764	G	63	150	0.21	43	565	68.5
	B1-2	4PB	200	360	320	2.2	2.4	3.13	35.5	G	8.50	1.33	42	764	G	63	150	0.21	43	565	57.8
Alsayed (1998)	B2-1	4PB	200	360	320	1.68	1.8	2.28	35.7	G	7.94	1.24	42	764	G	63	80	0.40	43	565	109.8
	GB 11	4PB	150	250	210	2.3	2.5	3.65	39.8	G	4.29	1.36	45	1000	G	40	153	0.17	45	1000	49.8
Duranovic, Pilakoutas, and Waldron (1997)	GB 12	3PB	150	250	210	2.3	2.5	2.44	39.8	G	4.29	1.36	45	1000	G	40	153	0.17	45	1000	67.4
	CC-3	4PB	135	560	470	7.0	-	3.19	50.0	C	7.95	1.25	137	2200	C	77	157	0.36	137	1730	305.0
Shehata et al. (2000)	CG-3	4PB	135	560	470	7.0	-	3.19	50.0	C	7.95	1.25	137	2200	C	226	157	1.07	41	640	304.5
	BM I	3PB	178	330	279	1.5	2.4	2.69	24.1	G	11.42	2.30	40	717	G	142	152	0.52	40	717	81.8
Alkhrdaji et al. (2001)	BM II	3PB	178	330	279	1.5	2.4	2.69	24.1	G	11.42	2.30	40	717	G	142	203	0.39	40	717	71.2
	BM V	3PB	178	330	279	1.5	2.4	2.69	25.2	G	11.42	2.30	40	717	G	142	152	0.52	40	717	81.0
	BM VI	3PB	178	330	279	1.5	2.4	2.69	25.2	G	5.93	1.19	40	717	G	142	203	0.39	40	717	52.9
	Q-C-1L	4PB	300	545	441	3.96	5.0	3.02	48.3	G	48.23	3.65	63	1000	G	226	300	0.25	31	322	252.0
	Q-C-1R	3PB	300	545	441	2.66	5.0	3.02	48.3	G	48.23	3.65	63	1000	G	226	140	0.54	31	322	362.0
Niewels (2008)	Q-C-2R	3PB	300	545	441	3.96	5.0	3.02	43.3	G	48.23	3.65	63	1000	G	226	200	0.38	31	322	240.0
	Q-A-3R	3PB	300	500	412	3.90	5.0	3.16	43.3	G	40.19	3.25	44	480	G	127	140	0.30	49	524	301.0
	Q-A-4L	4PB	300	500	412	3.90	5.0	3.16	46.8	G	40.19	3.25	44	480	G	127	300	0.14	49	524	220.0
	Q-A-4R	3PB	300	500	412	3.90	5.0	3.16	46.8	G	40.19	3.25	44	480	G	127	180	0.24	49	524	266.0
	Q-A-5R	3PB	300	500	404	4.50	5.0	3.71	29.1	G	48.23	3.98	63	1000	G	100	300	0.11	52	1000	250.0
Ascione, Mancusi, and Spadea (2010)	I.1	4PB	150	200	170	2.0	2.3	4.12	20.0	G	1.57	0.62	46	970	G	57	133	0.28	46	970	20.5
	I.2	4PB	150	200	170	2.0	2.3	4.12	20.0	G	1.57	0.62	46	970	G	57	133	0.28	46	970	23.5
	II.1	4PB	150	200	170	2.0	2.3	4.12	20.0	G	3.93	1.54	46	970	G	57	133	0.28	46	970	28.6
	II.2	4PB	150	200	170	2.0	2.3	4.12	20.0	G	3.93	1.54	46	970	G	57	133	0.28	46	970	33.4
Bentz et al. (2010)	L05-1	4PB	450	1000	937	7.1	8.0	3.26	46.0	G	21.52	0.51	37	397	G	169	400	0.09	41	760	264.0
	L20-1	4PB	450	1000	857	7.1	8.0	3.56	36.0	G	86.08	2.23	37	397	G	169	400	0.09	41	760	1000
	M20-1	4PB	450	500	405	7.1	8.0	7.53	35.0	G	43.04	2.36	37	397	G	169	400	0.09	41	760	308.0

Note: ID – specimen identification; test – type of test performed; F – type of FRP reinforcement; ST – shear test; 4PB – four-point bending; 3PB – three-point bending.

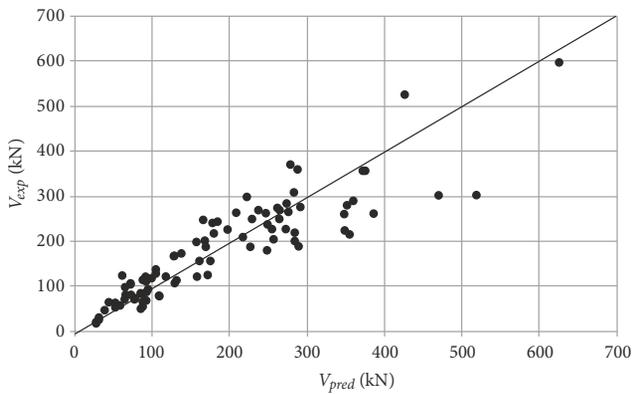


Figure 4. Experimental V_{exp} versus predicted V_{pred} shear strength by the CSA S806-12 shear design model

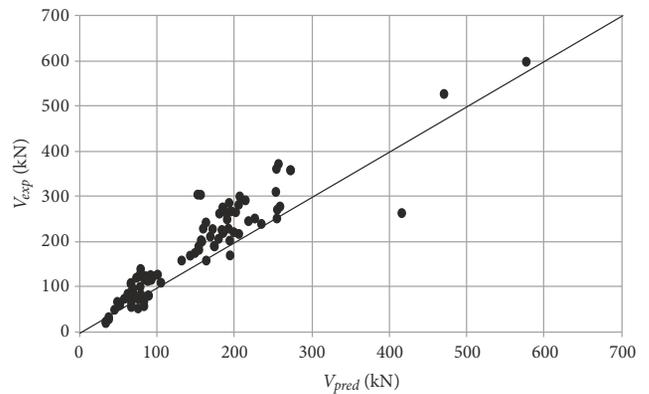


Figure 7. Experimental V_{exp} versus predicted V_{pred} shear strength by the Nehdi et al. (2007) shear design model

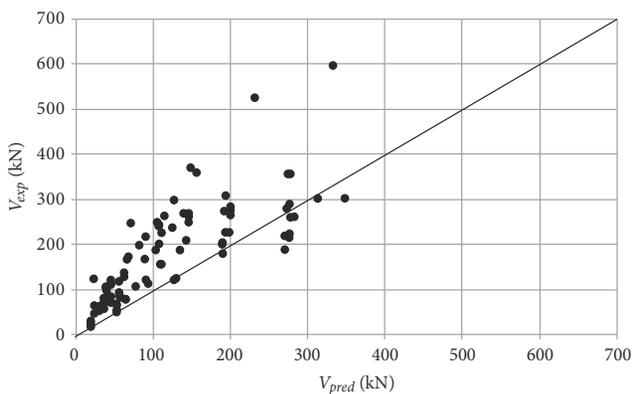


Figure 5. Experimental V_{exp} versus predicted V_{pred} shear strength by the CSA S6-14 shear design model

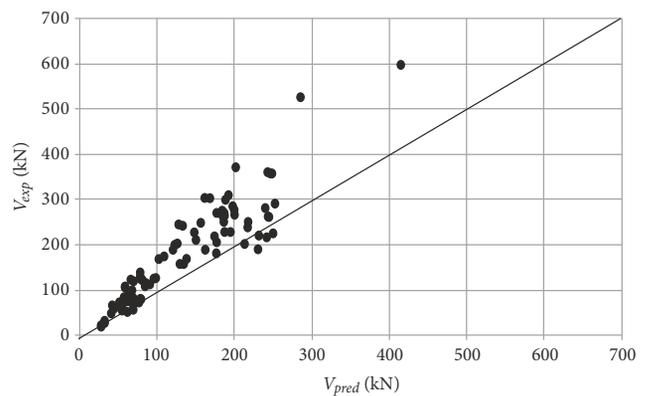


Figure 8. Experimental V_{exp} versus predicted V_{pred} shear strength by the Oller et al. (2015) shear design model

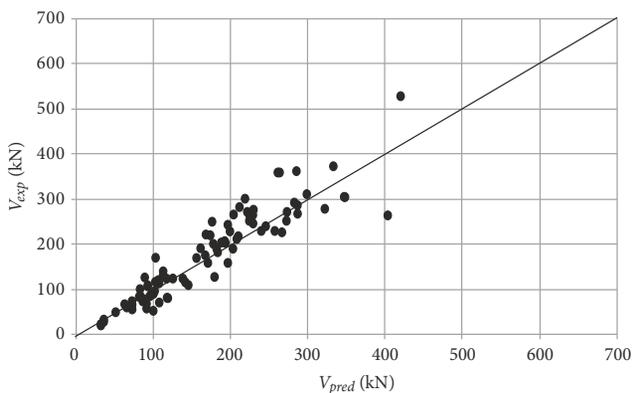


Figure 6. Experimental V_{exp} versus predicted V_{pred} shear strength by the Hegger et al. (2009) shear design model

Table 2. The comparison of experimental and predicted shear strength for beams reinforced with FRP reinforcement

Design model	V_{exp} / V_{pred}		
	Mean	Standard deviation	Coefficient of variation (%)
Proposed model	0.98	0.26	26.0
ACI 440.1R-06	1.24	0.53	42.3
CNR-DT 203/2006	0.64	0.16	25.3
CSA S806-12	1.05	0.27	26.0
CSA S6-14	1.79	0.75	41.8
Hegger et al. (2009)	1.03	0.21	20.6
Nehdi et al. (2007)	1.23	0.27	22.1
Oller et al. (2015)	1.36	0.30	21.7

Additional analysis of shear design models of Valivonis et al. (2015), CSA S806-12 and Hegger et al. (2009) was performed, because it was determined that these models are the most accurate. Correlation between ratio of experimental shear strength V_{exp} and predicted shear strength V_{pred} and different FRP reinforced concrete beam parameters was considered during this analysis. Compressive strength of concrete f_c and transverse reinforcement ratio by its modulus of elasticity $E_{fw} \cdot \rho_{fw}$ was chosen as parameters for this analysis.

Figures 9–11 shows the ratio V_{exp} / V_{pred} in relation to the compressive concrete strength f_c and to the transverse reinforcement ratio by its modulus of elasticity $E_{fw} \cdot \rho_{fw}$. For all three design models, the dispersion of V_{exp} / V_{pred} values is smaller for lower values (between 20 and 25 MPa) of concrete compressive strength f_c . In addition, all three examined models perform in a similar manner in terms of correlation between V_{exp} / V_{pred} and f_c .

Figures 10–11 presents the correlation between V_{exp} / V_{pred} and $E_{fw} \cdot \rho_{fw}$ for CSA S806-12 and Hegger et al.

(2009) shear design models. It can be seen that the dispersion of V_{exp}/V_{pred} values decreases with increasing values of $E_{fw} \cdot \rho_{fw}$. However for the proposed design model by Valivonis et al. (2015), it can be seen that dispersion not only does not decrease, but also slightly increases with increasing values of $E_{fw} \cdot \rho_{fw}$ (see Figure 9).

Conclusions

This paper presents an assessment of shear design models for FRP reinforced concrete beams available in literature. New shear design model is presented and included

in comparative analysis. The following conclusions can be drawn from the present study:

- 1) the analysis of available shear design models showed that most of the models are based on a theory that shear capacity of FRP reinforced concrete beam consists of concrete shear resistance V_c and FRP transverse reinforcement shear resistance V_f . Statistical analysis of predicted shear strength and experimental results show that this theory may be appropriate for shear design models of FRP reinforced concrete beams;
- 2) shear design model by Valivonis et al. (2015) was suggested for calculating shear capacity of FRP reinforced

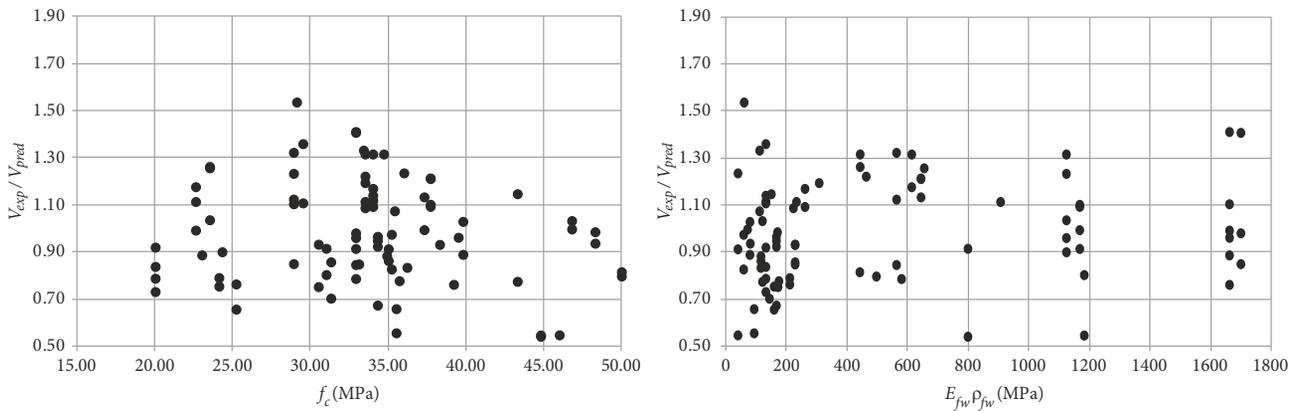


Figure 9. Correlation between V_{exp}/V_{pred} and f_c and $E_{fw} \cdot \rho_{fw}$ according to proposed design model

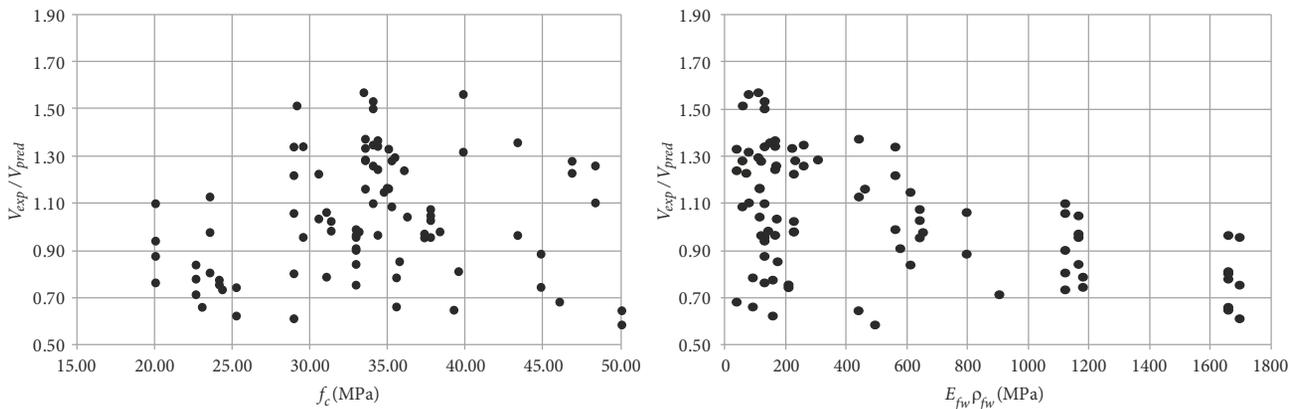


Figure 10. Correlation between V_{exp}/V_{pred} and f_c and $E_{fw} \cdot \rho_{fw}$ according to CSA S806-12 design model

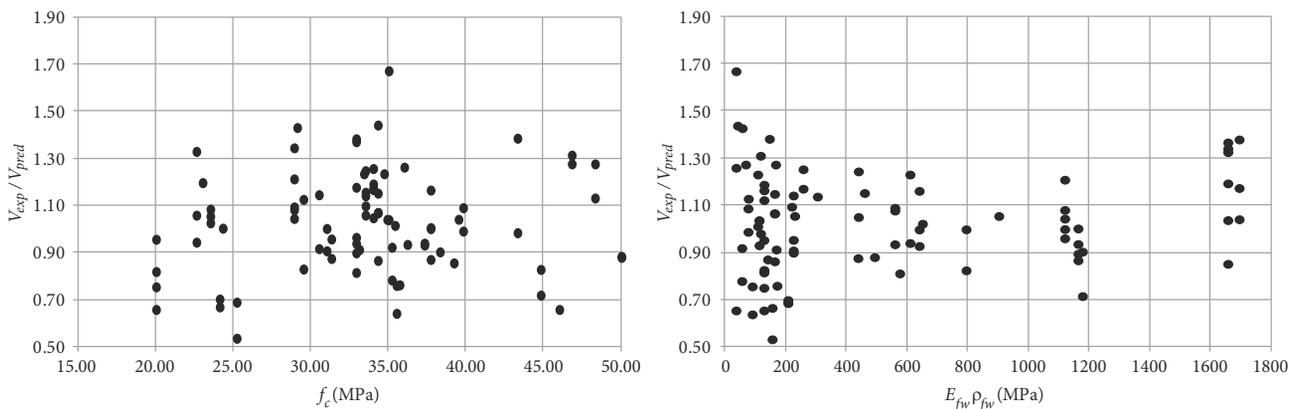


Figure 11. Correlation between V_{exp}/V_{pred} and f_c and $E_{fw} \cdot \rho_{fw}$ according to Hegger et al. (2009) design model

concrete beams with FRP stirrups. Different mechanical properties of different types of FRP reinforcement are taken into account in this shear design model;

- 3) proposed shear design model by Valivonis et al. (2015) have been applied to predict the shear capacity of 88 specimens reinforced with FRP reinforcement. The results obtained by the proposed method are very good (mean value of V_{exp}/V_{pred} is 0.98, standard deviation is 0.26, coefficient of variation is 26.0%)
- 4) correlation analysis of three most accurate design models was performed. It was observed that for all three design models, the dispersion of V_{exp}/V_{pred} values is smaller for lower values of compressive strength of concrete f_c . But for the Valivonis et al. (2015) design model, the dispersion of V_{exp}/V_{pred} values increases with increasing values of $E_{fw}\rho_{fw}$

Notation

a – shear span, mm;
 a_0 – critical projection of shear cracking zone, mm;
 A_f – area of FRP longitudinal reinforcement, mm²;
 A_{fw} – area of FRP shear reinforcement, mm²;
 b_w – beam width, mm;
 c – depth of compression zone at cracked transformed section, mm;
 d – effective depth, mm;
 d_b – diameter of the FRP bar in bent portion, mm;
 d_{max} – maximum aggregate size, mm;
 E_c – modulus of elasticity of concrete, GPa;
 E_f – modulus of elasticity of longitudinal FRP reinforcement, GPa;
 E_{fw} – modulus of elasticity of FRP shear reinforcement, GPa;
 E_s – modulus of elasticity of steel reinforcement, GPa;
 f_c – compressive strength of concrete, MPa;
 f_{cr} – cracking strength of concrete, MPa;
 f_{ct} – tensile strength of concrete, MPa;
 f_{fb} – tensile strength of FRP bent bar, MPa;
 f_{fr} – reduced tensile strength of the FRP reinforcement, MPa;
 f_{fw} – stress level in the FRP shear reinforcement at ultimate state, MPa;
 f_{fwu} – tensile strength of FRP shear reinforcement, MPa;
 G_f – fracture energy of concrete;
 h – beam height, mm;
 k – coefficient, which accounts the decreasing depth of neutral axis;
 L – span length, mm;
 L_t – overall length of the member, mm;
 M – bending moment, kNm;
 n_f – modular ratio of modulus of elasticity of FRP reinforcement and concrete;

r_b – bending radius of FRP bar, mm;
 s – spacing of shear reinforcement, mm;
 s_{ze} – effective crack spacing for members without stirrups, mm;
 V – shear load, kN;
 V_c – concrete contribution to shear capacity, kN;
 V_{cc} – shear resisted by the un-cracked concrete zone, kN;
 v_{cc} – dimensionless form of shear resisted by the un-cracked concrete zone;
 V_{exp} – experimental shear strength, kN;
 V_f – FRP transverse reinforcement contribution to shear capacity, kN;
 v_{fw} – shear strength caused by web reinforcement in the structural member's linear meter, N/mm;
 V_{pred} – predicted shear strength, kN;
 V_t – shear resisted by transverse reinforcement crossing the diagonal critical shear crack, kN;
 v_t – dimensionless form of shear resisted by transverse reinforcement crossing the diagonal critical shear crack;
 V_u – shear capacity of FRP reinforced concrete beam, kN;
 $V_{u,max}$ – concrete contribution corresponding to shear failure due to crushing of the web, kN;
 V_w – shear resisted by tensile stresses transferred along the crack, kN;
 v_w – dimensionless form of shear resisted by tensile stresses transferred along the crack;
 z – lever arm of internal forces, mm;
 α_{cw} – coefficient taking into account the state of the stress in the compression chord;
 $\gamma_{f,\Phi}$ – partial factor to account for the bending effect;
 ε_{fwu} – ultimate strain of the transverse FRP stirrups;
 $\varepsilon_{fw,m}$ – mean strain at the stirrups crossing the crack;
 ε_x – longitudinal strain at midheight of the cross section;
 ζ – coefficient which accounts the size effect on the shear failure;
 θ – angle between the concrete compression strut and the beam axis perpendicular to the shear force, °;
 λ – factor accounting for concrete density;
 v_l – strength reduction factor for concrete cracked in shear;
 ξ – relative neutral axis depth;
 ρ_f – longitudinal FRP reinforcement ratio;
 ρ_{fw} – FRP shear reinforcement ratio;
 τ – shear stress, MPa;
 $\Phi_{c2}, \Phi_{c3}, \Phi_{c4}$ – coefficients which estimate concrete's properties;
 ϕ_f – coefficient which estimate the influence of FRP flexural reinforcement for the concrete shear resistance;
 ϕ_c – resistance factor for concrete;
 ϕ_f – resistance factor for FRP reinforcement.

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