



## THE RIDDLE OF VOLATILITY CLUSTERS

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**Abstract.** In this financial engineering research we evaluate if observed non-normalities in the market price distributions are caused mainly by a volatility clustering or also by another non-clustering mechanism. Such findings allow us to assess according to which rules the market price is actually developing or even make conclusions about market price directional forecasting chances, based on the realistic financial processes which we assign to the clustering and non-clustering mechanisms.

In the research we suggest certain methodology how to recognize these processes behind the market price development. We apply the method to the European government bonds market and for the comparison also to 1 day periods of S&P 500 Index development, with respect to the different time periods.

Based on the findings we confirm the combination of both the volatility clustering and the non-clustering processes to be active inside 1 day periods and to be responsible for measured non-normalities. We also find significant non-clustering mechanism in 30 and 60 minute periods in case of European government bonds.

**Keywords:** volatility clustering, departures from normality, bond market, Euro-Bund Futures, S&P500, directional and volatility dependence, feedbacks.

**JEL Classification:** G1, G10, G17.

### Introduction

The main contribution of this financial engineering study is to resolve a general question: “Are the departures from normality caused by a volatility clustering or also by another non-clustering mechanism distributing the market price in a non-normal way?” If we find certain situations when empirically measured departures are not caused by volatility clustering we have to logically conclude that these non-normalities must be caused by some non-clustering processes. According to the empirical observations and also simulations we recommend to assign these potential non-clustering processes to the real existing feedback mechanisms which are based on the directional dependency and which will be also discussed later in the text. Such processes allow us to improve directional forecasting, which we cannot basically conclude in the case of volatility clustering process, because the clustering itself

can be also caused by the pure volatility effects without the directional dependency, and in addition: if such mechanism is also hidden (Stádník 2013a) the whole situation is then applicable to the future profit making and such findings do have a certain practical value. A solution of this question is also important for a general assessment of the market functionality, depending on the mechanisms responsible for the departures. In this research we try to answer the above formulated question for Euro Bund Futures, which directly affect the European government bond market, and we study the price distribution inside 1, 5, 10, 30, 60 minute periods and also within 1 day time series as we expect specific economic processes which are dominating inside the short time periods, processes which are significant for the longer periods and we also observe processes which are common for all the periods. In addition and also for the comparison we try to answer the same question for S&P500 index day price development.

## 1. Literature review

The volatility clustering is nowadays considered to be the main cause of the leptokurtic departures and the clusters itself are usually considered to be caused by a pure volatility dependency effects. The pure volatility dependence process is denoted as the process in which price direction is always independent of the past but the volatility is dependent. Such a process does not allow directional forecasting and it is closely connected to the size of price steps in the given time period. There are more theories of basic research in the area of volatility dependence. For example the Gaussian mixture distribution. Gaussian mixture has an acceptable interpretation: financial market occurs in two regimes with high and low volatility. We can model many non-normal distributions which characteristic depend on the probability of both regimes and their parameters. If the regimes have a Markov law of motion, the mixture is then a hidden Markov model (Baum, Petrie 1966), which is also known as the Markov regime switching model. We find many extensions of the Markov switching model (Krolzig 1997; etc.). Other famous works in this area were done by Bollerslev (1986) GARCH process; Engle (1995) ARCH process. Some new research in the area of volatility dependence was done by Witzany (2013) or Roch (2011). While GARCH, ARCH and other volatility models propose statistical constructions based on volatility clustering in financial time series, they do not provide any financial explanation. The financial explanation of volatility clustering is quite difficult. The simplest possible financial clustering mechanism is just the switching of the market between periods of high and low activity or clustering of economic news. The other idea was the competition between more trading strategies but the simulation does not allow to confirm that the mechanism is responsible for volatility clustering (Cont 2005). Some economic works contain examples where switching of economic agents between two behavioral patterns leads to large volatility. Volatility clustering should also arise from the switching of market participants between fundamentalist and chartist behavior (Lux, Marchesi 2000). Chart traders evaluate their investments using historical development, whereas fundamentalists evaluate their investment opportunity according to the difference between the market price and the fundamental valuation. According to the Lux-Marchesi model the market price development follows the Gaussian random walk until the moment when some chart traders using certain techniques surpass a certain threshold value and at this moment a volatility outbreak occurs. According to Cont 2005, the origin of volatility clustering can also be caused by threshold response of investors to news arrivals. Other new research connected to the volatility clustering were done by Jianga, Lia, Caia (2008) or Tsenga Jie-Jun, Sai-Ping Lia (2011).

Instead of the volatility dependency effects we are able to explain non-normalities using pure directional dependency effects. This way considers the price development direction

to be dependent on the past and allows certain forecasting chances in comparison to the volatility dependency. There are many case studies based on the directional dependency but comprehensive modeling of the departures from normality in this way is not so frequent. For example the commonly used technical trading rules are based on a market price direction forecasting according to the past. We can consider Technical Analysis to be the prediction tool, but its benefit is still under discussion. We meet many other interesting detailed works or case studies in the area like Henriksson, Merton (1981); Anatolyev, Gerko (2005); Diviš, Teplý (2005); Primbs, Rathinam (2009); Gontis, Ruseckas, Kononovičius (2010); Lux (2011); Džikevičius, Vetrov (2012); Černohorská, Teplý, Vrábel (2012); Janda, Svarovska (2010). Price direction development dependence also takes place in the basic feedback process according to the behavioral finance concept where upward trend is more likely to be followed by another upward movement (Schiller 2003) or in other research as for example momentum studies (Pesaran, Timmermann 1995; Stankevičienė, Gembickaja 2012), short term trend trading strategy in futures market based on chart pattern recognition (Masteika, Rutkauskas 2012) or in the development of the conception of sustainable return investment decisions strategy in capital and money markets (Rutkauskas *et al.* 2008). We have to mention also the work of Larrain 1991, which states that long term memory exists inside the financial market, other similar works of Hsieh (1991), Peters (1989, 1991, and 1994) which focus mainly on measurement of probability diversions from normality.

It is important for our research that the directional dependency way is able to explain the departures without the clustering mechanisms. For example feedbacks system according to the Dynamic Financial Market Model (Stádník 2011) is able to cause sharpness and fat tails in the distribution. Feedbacks increase the value of probability of next price step up or down direction (from 50/50 for the pure symmetrical random walk to for example 51/49) depending on the previous development. The idea of feedback processes is based on the empirical observations that traders, investors and other market participants not only watch present or historical data but according to them they are also placing buy or sell orders and thus influence future development. Feedback which keeps the movement in a certain direction is described in the model as a trend stabilizer feedback. For example traders participating in “momentum trading” try to find instruments that are moving significantly in one direction and in order to realize financial profit on the movement they basically prolong short-term trends. The other important feedback is a price inertia feedback which is pushing the market price back to a certain level and which is resulting from “level trading” where traders believe the price will return to the level which was set after the last economic news of high importance for example.

A special case is volatility clustering which could be well explained using the directional dependency effects like the spring oscillation mechanism (Stádník 2013b) when feedbacks may cooperate and under certain conditions cause volatility clusters as the final result. This is the case when we observe volatility clusters which are not caused by volatility dependence but by directional dependence behind.

## 2. Methodology

To make the decision between the clustering or non-clustering mechanism responsible for the departures from normality in the price distributions we have to, first of all, assess the impact of both the mechanisms on the character of the price distribution and its departures. The general clustering mechanism causes significant autocorrelation in volatility data series and possibly the departures from normality in the distribution but we have to mention at this point also an artificial case of observing volatility clusters with the resulting Gaussian distribution as it is simulated in the Figure 14 in the appendix. Typical non-clustering mechanisms like the price inertia feedback distribute the price to the initial (level) value and contribute to the sharpness in the distribution. On the other hand the trend stabilizer feedback contributes to the fat tails. In such cases the resulting price distribution is non-normal but the leptokurtic one and the volatility series is without the volatility clusters. To support our ideas about this impact of the feedbacks on the price distribution we have made the simulation (Fig. 15, appendix). In this simulation we simulate the price inertia and the trend stabilizer according to the Dynamic Financial Market Model. The simulation is without any volatility clustering. We can see in the figure that the volatility autocorrelation (0.0236) is insignificant but the value of acuteness (1.665) is significantly high. For the assessment of the price inertia action we have defined *acuteness* (Eq.1) as the ratio of histogram maximum value in the measured distribution over the maximum value of an adequate normal distribution:

$$acuteness = \frac{Max_{measured}}{Max_{normal}} \cdot \quad (1)$$

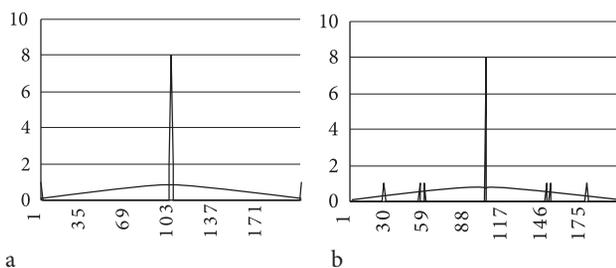


Fig. 1. Distributions (a) and (b) are with the same acuteness but a different kurtosis: 5.285 and 0.096 (source: own research)

The value of acuteness for normal distribution is 1. The value of kurtosis is not a useful quantitative pointer of the sharpness especially in this case when the price inertia is active separately. The case of the same sharpness and the different values of kurtosis are demonstrated in the Figures 1a and 1b.

Based on the previous we can logically conclude into the main methodology steps:

If there is no significant autocorrelation in the volatility data series and the price distribution exhibits certain acuteness then the departures are not caused by the volatility clusters but by some non-clustering mechanism which could for example be the price inertia feedback action.

If there is significant volatility autocorrelation and certain acuteness then the departures in the price distribution may be caused by a clustering mechanism in cooperation with a non-clustering mechanism. In such a case we have to decide if the departures are caused only by the clusters or also by the coexistence of both effects. To answer such a question we suggest the filtering of volatility clusters thus separating from data series the continuous parts without the clusters. We continue filtering until the autocorrelation of volatility time series is insignificant but we also cannot destruct the series (continuous parts without clusters must be left). Autocorrelation is measured on the absolute values of volatility series. After the filtering we are allowed to study the price distribution of the data series without the clusters and also inside the clusters separately. If the price distribution without the clusters is a non-normal one it means the non-clustering mechanism causing the departures is present. In addition to that if the value of kurtosis or acuteness of such distribution is lower than of the original distribution with the clusters we can conclude on coexistence of both the clustering and non-clustering mechanisms in the original distribution. In case that it is not possible to eliminate volatility clusters without the destruction of data series (we cannot separate continuous time periods without clusters) we cannot be sure if the departures are caused only by the volatility clustering or also by non-clustering effects. If we for example eliminate volatility clusters from one day volatility data series of certain investment instrument (stock, bond, etc.) which performs one day non-normal price distribution and if filtered price series is also non-normally distributed, we conclude that there must be present some non-clustering mechanism like for example price inertia feedback (Stádník 2012) distributing the price towards to the initial value and causing departures in the distribution. Such feedback is the typical directional dependency process which allows better directional forecasting (Stádník 2013a).

We apply the suggested methodology to European bond futures which directly affects prices of appropriate government bonds (mainly 10 years maturities), traded on EUREX exchange, contract name: Euro-Bund Futures and also on S&P500 Index. All the data time series in the research have

been downloaded from Reuters system and for the elimination of clusters we have used special software which detects continuous periods of lower and higher volatility.

### 3. Findings in Euro-Bund Futures 1 min, 5 min and 10 min price distributions

In case of 1, 5 and 10 minute price volatility data series (Figs 2a, 3, 4a) we were not successful in eliminating volatility clusters (to decrease the level of autocorrelation without the destruction of the appropriate time series).

This is why in the case of 1, 5 and 10 minute price development we cannot be sure about reliable conclusions. The departures in the price distributions (Figs 2b and 4b) are probably caused by certain clustering mechanisms (autocorrelations: 0.201, 0.22, 0.183) but we are not able to make any conclusion on non-clustering mechanism based on the measurement of volatility data series in this case. The solution could be reached by the direct market observation and according to the market participants' behavior.

### 4. Findings in Euro-Bund Futures 30 min, 60 min price distributions

In the 30 and 60 minute price volatility series (Figs 5a, 6a) the volatility has low autocorrelation (0.108 and 0.0966) but the price distributions (Figs 5b, 6b) perform the high acuteness (1.760, 1.765) and also kurtosis.

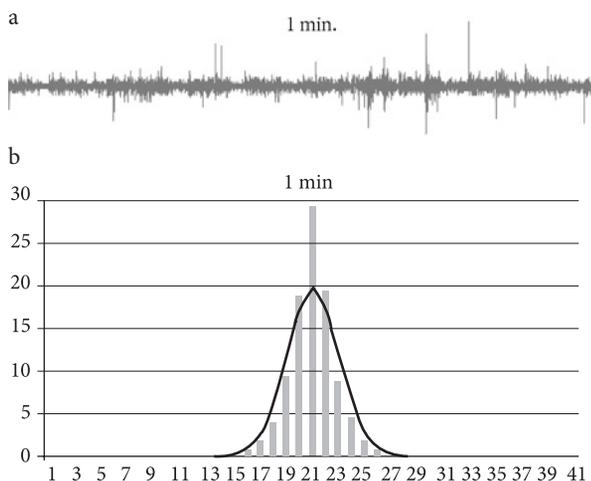


Fig. 2. 1 minute volatility series (a) and price distribution (b) of Euro-Bund Futures, volatility autocor.: 0.201, average value: 0.0000578, skewness: -0.156, kurtosis: 29.369, acuteness: 1.479, data: 2013 (source: own research)

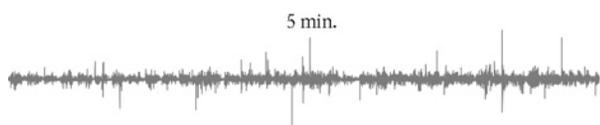


Fig. 3. 5 minutes volatility, volatility autocor.: 0,229, data: 2013 (source: own research)

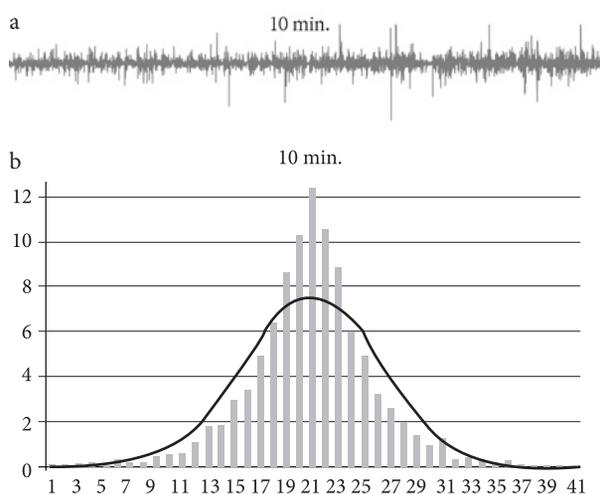


Fig. 4. 10 minutes volatility series (a) and price distribution (b) of Euro-Bund Futures, volatility autocor.: 0,18373, average value: -0.000615, skewness: -0.0852, kurtosis: 9.049, acuteness: 1.631, data: 2013 (source: own research)

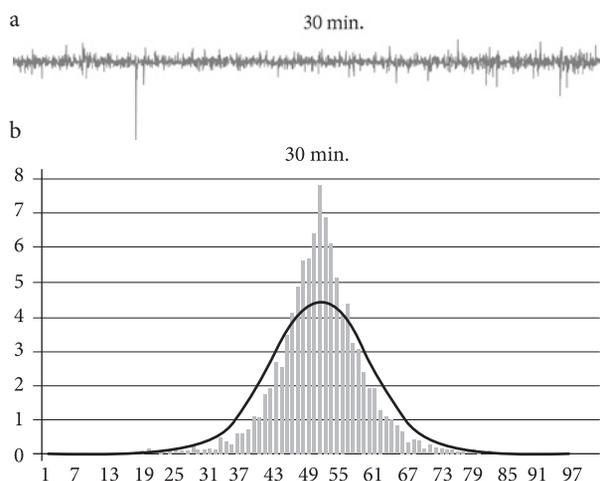


Fig. 5. 30 minutes volatility series (a) and price distribution (b) of Euro-Bund Futures, volatility autocor.: 0.10835, average value: -0.000904, skewness: -4.237, kurtosis: 84.038, acuteness: 1.760, data: 2013 (source: own research)

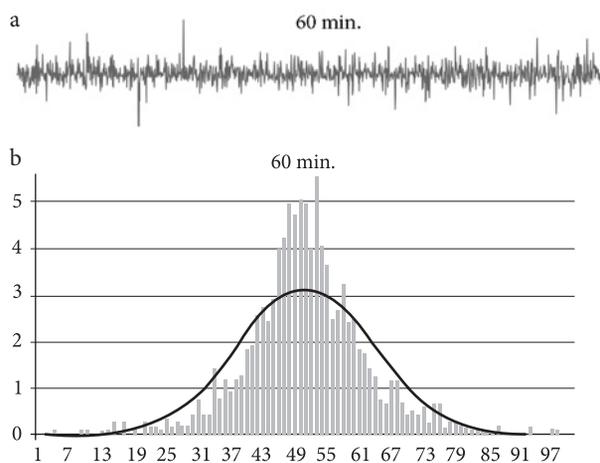


Fig. 6. 60 minutes volatility series (a) and price distribution (b) of Euro-Bund Futures, volatility autocor.: 0.0966, average value: 0.000264, skewness: -2.136, kurtosis: 34.878, acuteness: 1.765, data: 2013 (source: own research)

In this case we can conclude on the directional dependency effects mainly responsible for the departures in the price distributions.

### 5. Findings in Euro-Bund Futures 1 day price distribution

From 1 day price volatility series (Fig. 7a) we successfully eliminate the volatility clusters (Fig. 8a) thus decreasing

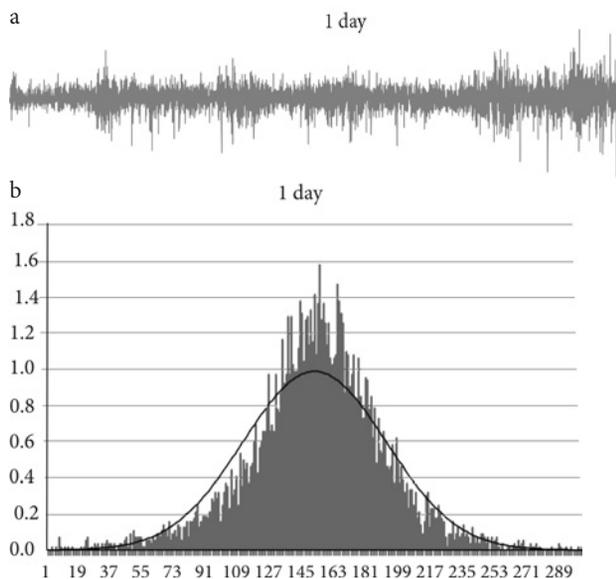


Fig. 7. 1 day volatility series (a) and price distribution (b) of Euro-Bund Futures, volatility autocor. 0.138, average value: 0.0107, skewness:  $-0.294$ , kurtosis: 3.265, acuteness: 1.593, data: 1990–2013 (source: own research)

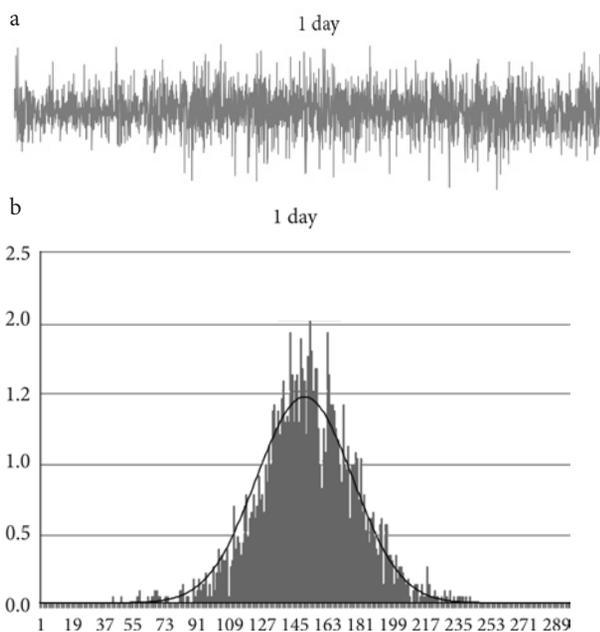


Fig. 8. 1 day volatility series (a) and price distribution (b) of Euro-Bund Futures without VOLATILITY CLUSTERS, volatility autocor.:  $-0.00502$ , average value: 0.02, skewness:  $-0.06136$ , kurtosis: 0.618, acuteness: 1.369, data: 1990–2013 (source: own research)

the volatility autocorrelation to an insignificant level (from 0.138 to  $-0.00502$ ). There is volatility data series of an independent random walk (autocorrelation 0.0120) in the Figure 13 in the appendix for a comparison. We may conclude that the price distribution which does not involve the clusters (Fig. 8b) also has significant acuteness and therefore there is present a non-clustering mechanism responsible for measured non-normalities in the price distributions.

Also inside the volatility clusters (Fig. 9a) where the volatility autocorrelation is insignificant (0.0185) but the price distribution (Fig. 9b) has significant acuteness we confirm non-clustering mechanisms. As the value of kurtosis of the original distribution (Fig. 7b) is higher than in the cases of the price distributions without the clusters and inside the clusters we conclude on coexistence of both the clustering and the non-clustering mechanisms responsible for the departures in the original price distribution.

### 6. Findings in S&P500 1 day return distribution

For the comparison we try to eliminate clusters from S&P500 return volatility series (Fig. 10a). In this case we have been successful in eliminating the volatility clusters (Fig. 11a) and thus reduce volatility autocorrelation (from 0.22 to 0.0248). Based on that we can measure that the price distribution which does not involve the clusters (Fig. 11b) has significant acuteness (1.795) and therefore there is also present a non-clustering mechanism which causes the departures from normality.

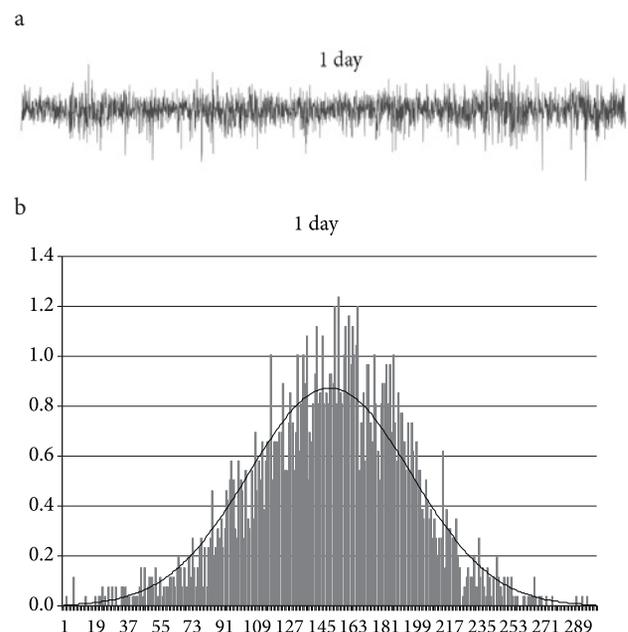


Fig. 9. 1 day volatility series (a) and price distribution (b) of Euro-Bund Futures inside VOLATILITY CLUSTERS, volatility autocor.: 0.0185, average value: 0, skewness:  $-0.383$ , kurtosis: 1.4006, acuteness: 1.416, data: 1990–2013 (source: own research)

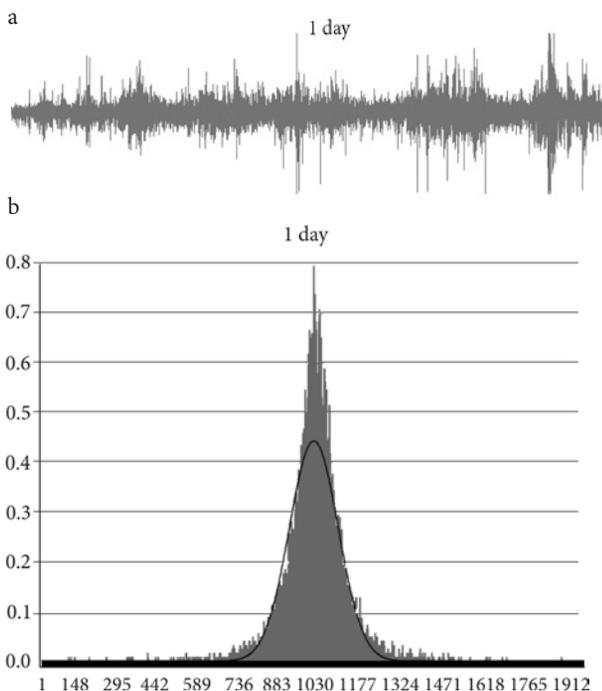


Fig. 10. 1 day volatility series (a) and return distribution (b) of S&P500, volatility autocor.: 0.22, average value: 0.0294, skewness: 0.936, kurtosis: 25.421, acuteness: 1.795, data: 1963–2013 (source: own research)

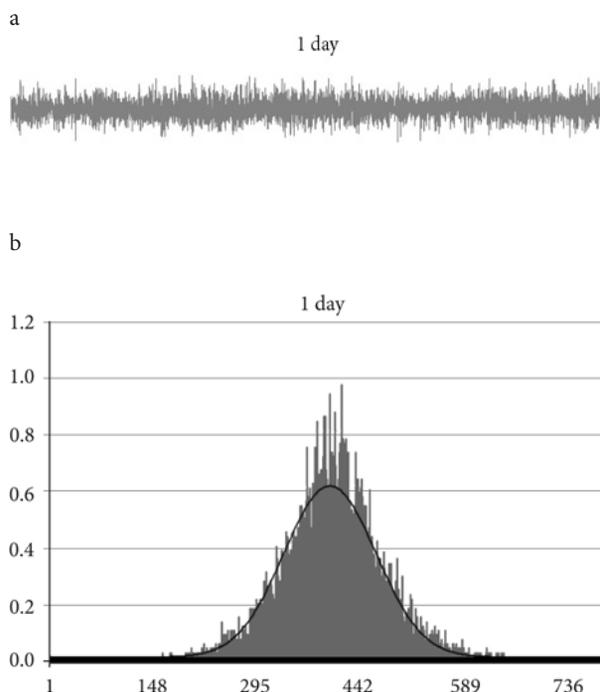


Fig. 11. 1 day volatility series (a) and return distribution of S&P500 (b) without VOLATILITY CLUSTERS, volatility autocor.: 0.0248, average value: 0.0357, skewness: kurtosis 0.0355, acuteness: 1.589, data: 1963–2013 (source: own research)

As the value of kurtosis of the original distribution with the clusters is higher we can also confirm the coexistence of the directional and volatility dependency processes responsible for the departures in the original price distribution as is the case of the Euro-Bund Futures contract. We also confirm a significant non-clustering mechanism causing the departures inside the clusters (Fig. 12a), because the value of autocorrelation is insignificant (0.045) but the acuteness (Fig. 12b) is significantly high (1.858).

### 7. Main findings summary

For the short time period series of Euro Bund Futures (1, 5, 10 minutes) we were not successful in confirming a non-clustering mechanism according to the suggested methodology and we conclude that the volatility clustering is probably the key factor causing the departures inside these high frequency distributions.

For 30 and 60 minute price distributions we recognize that the volatility autocorrelation is low and due to the significant departures in the price distribution we consider a non-clustering mechanism to be the key reason for the departures from normality.

For the daily distributions we find the coexistence of the clustering and non-clustering mechanisms. We successfully eliminate the volatility clusters from the development and we recognize that the filtered development is also distributed in a non-normal way. Also the price development inside

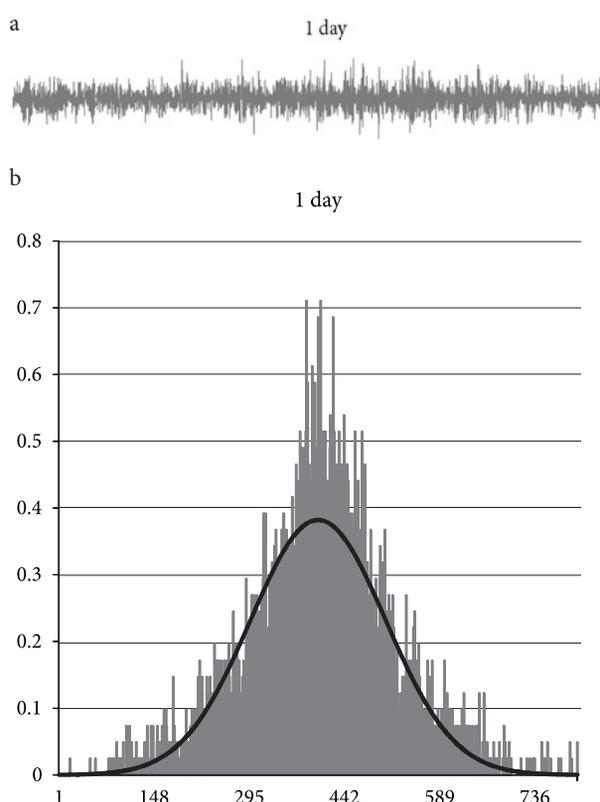


Fig. 12. 1 day volatility series (a) and price distribution (b) of S&P500 inside VOLATILITY CLUSTERS, volatility autocorrelation: 0.045, average value: 0.00629, skewness: 0.127, kurtosis 1.0181, acuteness: 1.858, data: 1963–2013 (source: own research)

the clusters is non-normal. The data set in the case of S&P 500 daily development indicates exactly the same result.

### Conclusions and the scientific discussion

In this financial research we propose a certain methodology for the recognition between the clustering and non-clustering processes being responsible for the departures from normality in the price distributions. The methodology is applicable to the worldwide financial investment instruments. From the volatility time series we basically suggest the filtering of volatility clusters and then study the price distributions without the clusters and also inside the clusters separately to make the final conclusions on the existence of certain non-clustering mechanisms distributing the price in a non-normal way. We also define certain quantitative pointer (acuteness) as the measure of expected non-clustering mechanism causing the departures which is the price inertia feedback resulting from the mentioned level trading technique.

In the study we find quite different results with respect to the different time periods. These distinctions could be connected to the various style of trading techniques dominating within certain time periods. We can state that the findings generally support the assumption that the volatility clustering is not the main or the only reason for the departures from normality in the price distributions, but there is also some non-clustering mechanism cooperating, which also causes the departures. From the financial point of view we recommend the mentioned price inertia feedback to be assigned to this non-clustering process. The existence of this feedback is also supported by the direct empirical observations, by the statistical research (Stádník 2012) and by the simulation according to the Figure 15 in the appendix (discussed in the “Methodology” chapter). Such feedback is the typical directional dependency process which is connected to the better directional forecasting (Stádník 2013a) but its practical value is still under the discussion.

In addition we also suspect this feedback to be the reason for the measured non-normalities inside the separated volatility clusters while the clustering itself could be caused by for example the clustering of economic news or trading activities.

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## APPENDIX

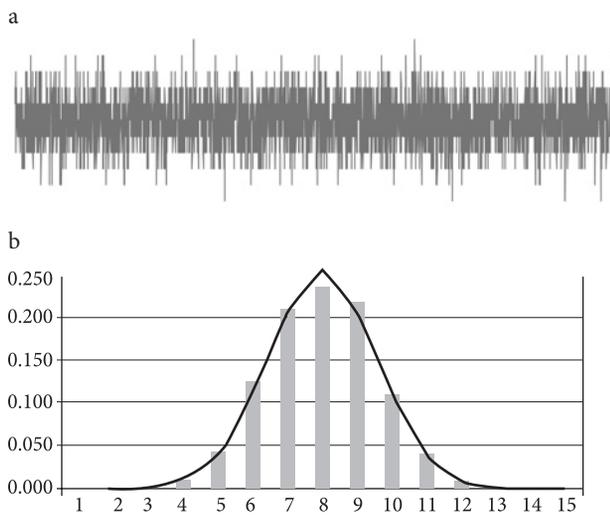


Fig. 13. Example of pure symmetric random walk volatility series (a), autocorrelation: 0.012 and price distribution (b) with average value:  $-0.0356$ , skewness:  $-0.0092$ , kurtosis:  $-0.252$  (source: own research)

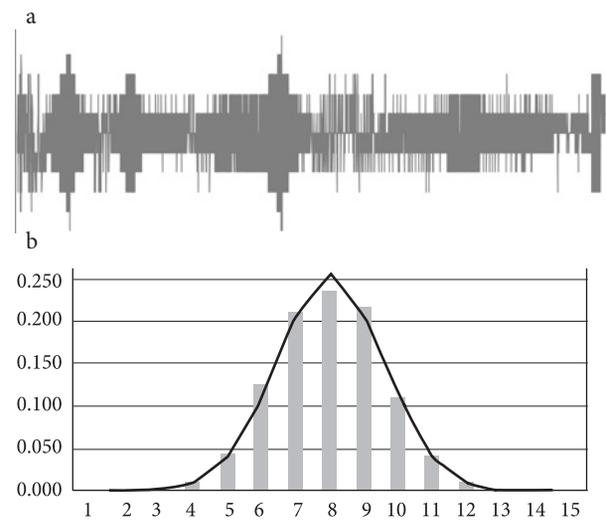


Fig. 14. Artificial example of volatility clustering with Gaussian distribution, volatility autocorrelation:  $-0.563$ , skewness:  $-0.0092$ , kurtosis:  $-0.252$  (source: own research)

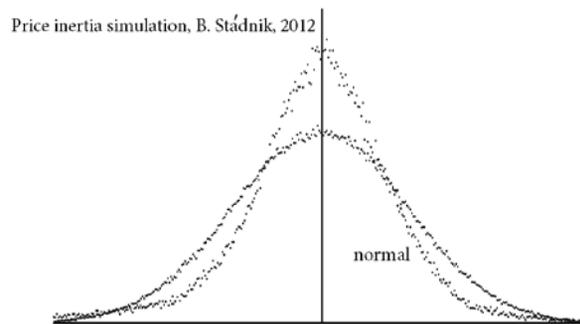


Fig. 15. 1 day returns distribution of S&P500 (b) SIMULATIONS USING FEEDBACKS (WITHOUT VOLATILITY CLUSTERING), volatility autocor.: 0.0236 (a), skewness:  $-1.057$ , kurtosis: 5.259, acuteness: 1.665 (source: own research)

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