ISSN 1648-0627 print
ISSN 1822-4202 online
VERSLAS: TEORIJA IR PRAKTIKA
BUSINESS: THEORY AND PRACTICE
http://www.vtu.lt/leidiniai; http://www.vtu.lt/editions
2006, Vol VII, No 2, 73–80

THE USE OF MONTE CARLO SIMULATION TECHNIQUE TO SUPPORT INVESTMENT DECISIONS

Rima Tamošiūnienė, Tomas Petravičius

Vilnius Gediminas Technical University, Saulėtekio al. 11, LT-10223 Vilnius, Lithuania
E-mail: rimtam@vv.vtu.l

Received 5 January 2006; accepted 27 February 2006

Abstract. In this paper the methodology and uses of Monte Carlo simulation technique in the evaluation of investment projects are presented to analyse and assess the risk. The first part presents the theoretical background of Monte Carlo analysis in various process stages. The second part examines the interpretation of the results generated by the application of project formulation, financial and risk analysis including investment decision criteria and measures of risk based on the expected NPV value concept. The final part draws some conclusions regarding the usefulness and limitations of Monte Carlo analysis in investment appraisal.

Keywords: investment project, risk analysis, net present value, Monte Carlo simulation technique.

MONTE KARLO IMITACINIO MODELIAVIMO METODO TAIKYMAS INVESTICINIAMS SPRENDIMAMS PAREMTI

Rima Tamošiūnienė, Tomas Petravičius

Vilniaus Gedimino technikos universitetas, Saulėtekio al. 11, LT-10223 Vilnius, Lietuva
El. paštas rimtam@vv.vtu.lt

Įteikta 2006-01-05; priimta 2006-02-27

Santrauka. Šiame straipsnyje pateiktos Monte Karlo metodo taikymo galimybės, siekiant išanalizuoti ir įvertinti investicinių projektų riziką. Pirmoje straipsnio dalyje analizuojami teoriniai Monte Karlo imitacinio modeliavimo metodo aspektai, aprašomi šio metodo taikymo investicinių projektų rizikai vertinti etapai. Antroje straipsnio dalyje šis metodas pritaikytas realaus investicinio projekto rizikai vertinti. Pateiktas galutinis sprendimas dėl investavimo į projektą, paremtas ne tik finansinės analizės, bet ir rizikos analizės, išreikštos tikėtinąja grynosios dabartinės vertės koncepcija, rezultatais. Straipsnis apibendrintas pateikiant Monte Karlo metodo taikymo investiciniams sprendimams priimti privalumus ir trūkumus.

Raktažodžiai: investiciniai projektai, rizikos analizė, grynoji dabartinė vertė, Monte Karlo imitacinio modeliavimo metodas.

1. Introduction

In all economic situations there is a risk of failure of the realization of an investment project or a business plan. It is impossible to eliminate potential objective reasons that lead to undesirable development of events, and as a result, the deviation from a chosen aim. However, it is always possible to find the way to obtain the selected strategic goal; the way which would offer a balance between the expected profit and the threat of a loss conforming to some compromising level of risk.

On purpose to set what methods could be used to reduce risk or its negative impact on the project, foremost we need to ascertain various factors of risk and to evaluate its significance; i. e. we must perform the analysis of risk [1].

The main purpose of risk analysis is obtaining all the necessary data for the potential partners of the project in order to make final investment decisions about the selection of methods, hedging against possible financial loss.

The main purpose of this paper is to present the methodology and uses of Monte Carlo simulation technique in the evaluation of investment projects to analyse and assess project risk.

The following research methods were used in the paper: literature analysis, Monte Carlo simulation technique and other.

2. Theoretical background of analysis

Risk analysis using a probabilistic simulation principle is based on Monte Carlo simulation technique by which the main variables are projected in order to estimate the impact of risk on the project results. It is a technique by which a mathematical model is introduced to a number of simulation runs. During the simulation process, successive scenarios are built up using input values for the project's key uncertain variables which are selected from multi-value probability distributions [2].

The simulation is controlled so that the random selection of values from the specified probability distributions does not violate the existence of known or suspected correlation relationships among the project variables. The results are collected and analyzed statistically so as to go at probability distribution of the potential outcomes of the project and to estimate various measures of project risk [3].

The risk analysis process can be broken down into the following stages as shown in Fig 1.

2.1. Forecasting model

The first stage of risk analysis application is the requirement for a model, capable of predicting correctly if fed with the correct data. This involves the creation of a forecasting model, which defines the mathematical relationships between numerical variables that relate to the forecasts of the future. It is a set of formulae that process a number of input variables to arrive at a result. A good model is such which includes all the relevant variables (and excludes all non-relevant ones) and determines the correct relationships between them.

2.2. Risk variables

Analysing risk variables we should answer these questions:

- Which variables or their inputs in the plan are uncertain and critical to the viability of the project?
- What do we know about these uncertainties?

A risk variable is defined as one which is critical to the viability of the project in the sense that small deviation from its projected value is both probable and potentially damaging to the project worth. The best way to select risk variables is to use variables, which are described in a sensitivity and uncertainty analysis.

A sensitivity analysis is used to help identify the key variables to which a project is the most sensitive and to take actions to ensure that unfavourable shifts in these variables do not occur. It involves recalculating of the project

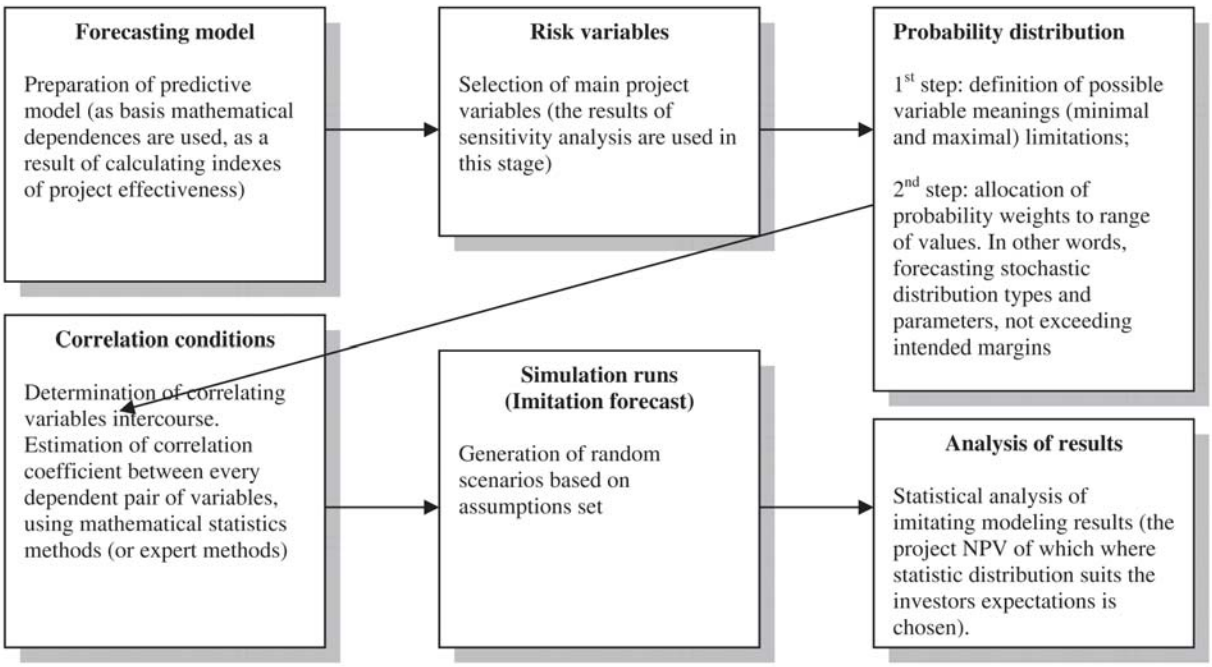

Fig 1. Risk analysis process by Monte-Carlo method [2]

results for different values of major variables where they are varied one at a time.

An uncertainty analysis is the attainment of some understanding of the type and magnitude of uncertainty encompassing the variables to be tested, and using it to select risk variables [4]. For example, it may be found that small deviation in the purchase price of given technology in year 0 is very significant for the project return. The likelihood is, however, that even such small deviation taking place may be extremely slim, if the supplier is contractually obliged and bound by guarantees to supply at the agreed price. The risk associated with this variable is therefore insignificant even though the project result is very sensitive to it. Conversely, a project variable with high uncertainty should not be included in the probabilistic analysis unless its impact on the project result, within the expected margins of uncertainty, is significant. The reason for including only the most crucial variables in risk analysis application is twofold [5]:

First, the greater the number of probability distributions employed in a random simulation, the higher the likelihood of generating inconsistent scenarios because of the difficulty of setting and monitoring relationships for correlated variables.

Second, the cost needed to define accurate probability distributions and correlation conditions for many variables with a small possible impact on the result is likely to outweigh any benefit to be derived. Hence, rather than extending the breadth of analysis to cover a larger number of project variables, it is more productive to focus attention and available resources on adding more depth to the assumptions regarding few most sensitive and uncertain variables in a project. In our simple appraisal model there are two risk variables. The administration wages, because it is expected to be lower or bigger than in the base case, and the cost of materials per unit, because the price of materials depends on market conditions at the time of purchase and vary substantially.

2.3. Probability distributions

After we identified risk variables we have to choose the best fit distributions for them, so the questions follow:

- What would be the best way to describe these uncertainties?
- What values are considered to be the highest and the lowest possible for each variable, what is the known range of values?
- Which distributions describe each variable?
- What distribution describes each variable the best?

Defining uncertainty

Although the future is "uncertain", we can still foresee the outcome of future events. There are many factors that govern our ability to forecast accurately a future event. These relate to the complexity of the system determining the outcome of a variable and the sources of uncertainty it depends on. In defining the uncertainty a given project variable one should widen the uncertainty margins to account for the lack of sufficient data or the inherent errors contained in the base data used in making the prediction. While it is almost impossible to forecast accurately the actual value that a variable may assume sometime in the future, it should be quite possible to include the true value within the limits of sufficiently wide probability distribution. The analyst should make use of the available data and expert opinion to define a range of values and probabilities that are capable to capture the outcome of the future event. The preparation of probability distribution for the selected project variable involves setting up a range of values and allocating probability weight to it.

Setting range limits

The level of variation possible for each identified risk variable is specified through the setting of limits (minimum and maximum values). Thus, a range of possible values for each risk variable is defined which sets boundaries around the value that a projected variable may assume.

When data are available, the definition of range limits for project variables is a simple process of curing the data to select probability distribution. For example, looking at historical observations of an event it is possible to organize the information in the form of frequency distribution. This may be derived by grouping the number of occurrences of each outcome at consecutive value intervals. The probability distribution in such case is the frequency distribution itself with frequencies expressed in relative rather than absolute terms (values ranging from 0 to 1 where the total sum must be equal to 1). It is usually necessary to rely on judgment and subjective factors determining the most likely values of a project appraisal variable. In such a situation the method suggested is to survey the opinion of experts [6].

The analyst should attempt to gather responses to the question "what values could be the highest and the lowest possible for a given risk variable?" If the probability distribution could be attached to the set range of values, it is one which concentrates probability towards the middle values of the range (for example the normal probability distribution), it may be better to opt for the widest range limits. If, on the other hand, the probability distribution, which is used, is one that allocates probability evenly across the range limits (for instance the uniform probability distribution) then the most likely or even one of more narrow range limits considered may be more appropriate.

The analyst should be able to understand and justify the choices made. It should be apparent, however, that the decision on the definition of a range of values is not independent on the decision regarding the allocation of probability.

Allocating probability

Each value within the defined range of limits has an equal chance of occurrence. Probability distributions are used to regulate the likelihood of the selection of values within the defined ranges.

The allocation of probability weight to values within the minimum and maximum range limits involves the selection of a suitable probability distribution profile or the specific attachment of probability weight to values (or intervals within the range).

Probability distributions are used to express quantitatively the beliefs and expectations of experts regarding the outcome of a particular future event. People who have this expertise are usually in a position to judge which one of these devices best expresses their knowledge about the subject. We can distinguish between two basic categories of probability distributions.

The First category, when there are various types of symmetrical distributions. For example, the normal, uniform and student „t" probability distributions allocate probability symmetrically across the defined range, but with varying degrees of concentration towards the middle values. The variability profile of many project variables can usually be adequately described through the use of one such symmetrical distribution. Symmetrical distributions are more appropriate in the situations for which the final outcome of the projected variable is likely to be determined by the interplay of equally important counteracting forces on both sides of the range limits defined.

The Second category of probability distributions is the step and skewed distributions. With step distribution one can define range intervals giving each its own probability weight in a step-like manner. The step distribution is particularly useful if expert opinion is abundant. It is more suitable in the situations where one-sided rigidities exist in the system that determines the outcome of the projected variable. Such situation may arise where an extreme value within the defined range is the most likely outcome.

2.4. Correlated variables

Correlation is important in Monte Carlo analysis, so we should answer these questions:

- Do correlations exist among any of the selected risk variables?
- How to diminish these correlations and (or) limit their expected dependency characteristics?

Identifying and attaching appropriate probability distributions to risk variables is fundamental in the risk analysis application. Having completed these two steps with the aid of a reliable computer programme it is technically possible to advance to the simulation stage in which a computer builds up a number of project scenarios based on random input values generated from the specified probability distributions. However, proceeding straight to simulation would be correct only if no significant correlations exist among any of the selected risk variables [7].

Two or more variables are said to be correlated, if they tend to vary together in a systematic manner. It is not uncommon to have such relationships in a set of risk variables. The existence of correlated variables among the designated risk variables can, however, distort the results of risk analysis. To give an example, suppose that a market price and quantity are both included as risk variables in the risk analysis application. It is reasonable to expect some negative covariance between them (that is, when the price is high, the quantity is more likely to assume low value and vice versa). Without restricting the random generation of values from the corresponding probability distributions defined for the two variables, it is almost sure that some of the scenarios generated would not conform to this expectation of the analyst who would result in unrealistic scenarios where price and quantity are either high or both low.

The existence of a number of inconsistent scenarios in a sample of simulation runs means that the results of risk analysis will be off target. Before proceeding to the simulation runs stage, it is essential to consider whether such relationships exist among the defined risk variables and, where necessary, to provide such constraints to the model that the possibility of generating scenarios that violate these correlations is diminished. In effect, setting of correlation conditions restricts the random selection of values for correlated variables so that it is confined within the direction and limits of their expected dependency characteristics.

2.5. Simulation runs

The simulation runs stage is a part of the risk analysis process in which a computer takes over. Once all the assumptions, including correlation conditions, have been set, it only remains to process the model (each recalculation is one run) until enough results are gathered to make up a representative sample of the near infinite number of combinations possible. During simulation the values of the "risk variables" are selected randomly within the specified ranges and in accordance with the set probability distributions and correlation conditions. The results of the model (that is the net present value of the project) are thus computed and stored following each run. Each run generates a different result because the input values for the risk variables are selected randomly from their assigned probability distributions. The result of each run is calculated and stored away for statistical analysis.

2.6. Analysis of results

The final stage in the risk analysis process is the analysis and interpretation of the results collected during the simulation runs stage.

- What is the degree of probability of the project results that is above or below the value we are interested in?
- Which of these coefficients (expected value, cost of uncertainty, expected loss ratio, variation) do we need to calculate?
- What decisions can we make?

Degree of probability being above or below any given value Hence, the probability of the project results being below a certain value is simply the number of results having a lower value times the probability weight of one run. By sorting the data in an ascending order it becomes possible to plot the cumulative probability distribution of all possible results. Through this, one can observe the degree of probability that may be expected for the result of the project being above or below any given value. Project risk is thus portrayed in the position and shape of the cumulative probability distribution of project returns.

Expected value. The expected value statistic summarizes the information contained within probability distribution. It is a weighted average of the values of all probable outcomes.

Cost of uncertainty. The cost of uncertainty is a useful concept that helps determine the maximum amount of money one should be prepared to pay to obtain information in order to reduce project uncertainty. This may be defined as the expected value of the possible gains foregone following decision to reject a project, or the expected value of the losses that may be incurred following the decision to accept a project [2].

Expected loss ratio. The expected loss ratio (*el*) is a measure indicating the magnitude of the expected loss relative to the project's overall expected NPV. This is expressed in the formula absolute value of expected loss divided by the sum of expected gain and absolute value of expected loss [2]:

$$el = \frac{|\text{Expected Loss}|}{\text{Expected Gain} + |\text{Expected Loss}|}. \qquad (1)$$

It can vary from 0, meaning no expected loss, to 1, which means no expected gain.

Coefficient of variation. The coefficient of variation is also a useful summary measure of project risk. It is the standard deviation of the projected returns divided by the expected value. Assuming the positive expected value, the lower the coefficient of variation, the less the project risk.

Conditions of limited liability. The extent of maximum loss possible under conditions of limited liability is usually defined by the legal agreements entered into by the various parties involved in a project [5]. Looking at the investment in terms of the present value the equity holders cannot lose more than the present value of their equity capital, the debt holders can only lose the present value of their loan capital, the creditors the present value of the extended credit and so on.

3. The case example

Uncertainty in project planning and risk analysis is widely recognized in a lot of industries and various programs exist to help optimize planning, an evaluation process and minimize the associated risks. „Invest Pro 2.36“ program is used to describe and formulate a project, identify sensitive key parameters using Sensitivity analysis, and run Monte Carlo simulation technique to estimate the impact of risk on the projected results. Invest Pro allows analysing Monte Carlo data in two ways:

- Data and statistics analysis for each variable (needed to ensure that selected distributions are correct and fit data);
- Data and statistics analysis of project results (the purpose is to calculate coefficients and support decisions).

Using Sensitivity analysis these key variables which are uncertain and critical to the viability of the project were identified: administration wages, direct material cost and the forecast price of network switchers. From past projects experience and "historical" knowledge possible ranges for different key variables were defined and distribution functions were chosen. Direct material costs are described with Student's "t" distribution, administration wages and selling price of products with Normal distributions. Project lifetime is five years, finance rate is ten percent. The number of simulation runs is 1000, range number is 19.

The parameters are as follows:

1. Direct material costs: first product: student t(2), shift – 14, second product: student t(3), shift – 6.
2. Administration wages: 1 year – N(235800, 2000); 2 year – N(235800, 4000); 3 year – N(235800, 5000); 4 year – N(235800, 7000); 5 year – N(235800, 9000);
3. Selling price of products: the first product – N(40,1); the second product – N(20,1);

Note: Shift is any X value that is applied, if the input data exceed the range of the fitted distribution. Shifted distribution is expressed as shift, where the distribution will have the shift amount added or subtracted from it.

After correlation conditions analysis it was estimated that direct material costs and a selling price of products correlate, so positive covariance between them can be expected. The correlation coefficient is 0,55. It is too low to obtain correlation conditions but exists, so the best way is to limit their expected dependency characteristics by excluding one of them (direct material costs or a selling price of products) from the analysis. The selling price of products was excluded, so two groups of key variables left.

Simulation was run and statistics of results is shown below (Fig 2–5, Table 1, 2):

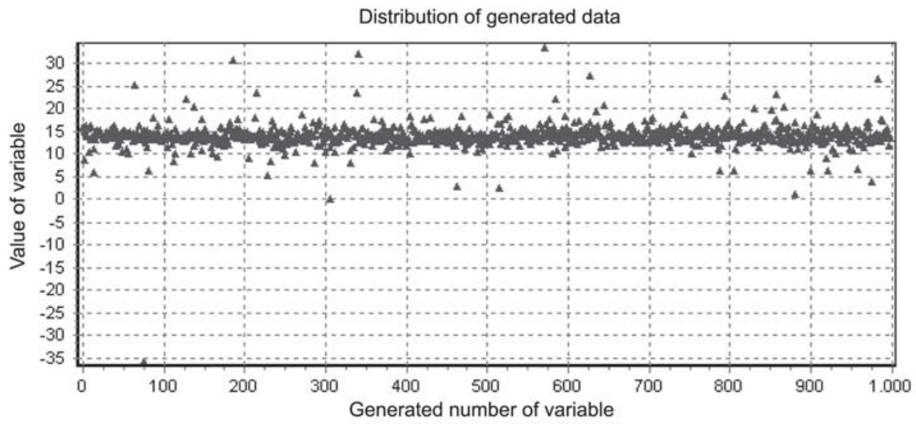

Fig 2. Distribution of generated data (the first product is shown as an example)

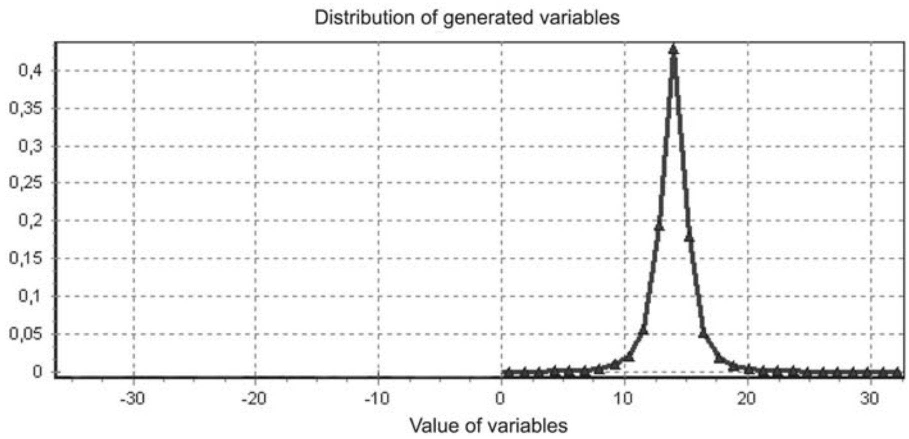

Fig 3. Distribution of generated variable (the first product is shown as an example)

Table 1. Statistics of key variables

Nr.	Direct material costs	Direct material costs	Administration wages	Administration wages	Administration wages	Administration wages	Administration wages
	Student "t" distrib. 1 product	Student "t" distrib. 2 product	Normal distrib. 1 year	Normal distrib. 2 year	Normal distrib. 3 year	Normal distrib. 4 year	Normal distrib. 5 year
Mean	13,93302	6,07127	235793,8	236002,7	235852,1	235883,6	235755,3
Standard Error	0,078987	0,047383	63,4405	128,4878	158,3503	217,2401	279,1799
Median	13,89225	6,06705	235811	235920,5	235975,2	236169,5	235636,9
Mode	14,8046	5,849	No	No	No	No	No
Std. Deviation	2,497797	1,49838	2006,164	4063,141	5007,477	6869,736	8828,444
Variance	6,238991	2,245144	4024694	16509115	25074829	47193277	77941426
Kurtosis	15,96812	4,50427	2,8651	2,8603	2,7334	3,0071	2,9747
Skewness	0,485684	0,419351	-0,058	0,0096	-0,0656	0,0097	-0,0463
Range	33,3483	15,6156	12598,33	23761,99	30592,08	48007,17	57614,5
Minimum	0,1102	0,0114	229796,5	224357,3	220383,7	212460,1	206353,9
Maximum	33,4585	15,627	242394,9	248119,3	250975,8	260467,3	263968,4
Sum	13933,02	6071,27	2,36E+08	2,36E+08	2,36E+08	2,36E+08	2,36E+08
Count	1000	1000	1000	1000	1000	1000	1000
Confidence level (95 %)	0,155	0,092982	124,3682	251,8865	310,4288	425,8759	547,3022
Shift	14	6	0	0	0	0	0

Table 2. Statistics of project results (NPV)

Nr.	For project without external sponsorship	For project with external sponsorship
Mean	113179,1	90964,1
Standard Error	4360,227	4230,615
Median	109015,5	86725,9
Mode	No	No
Std. Deviation	137882,5	133783,8
Variance	1,9E+10	1,79E+10
Kurtosis	19,1945	19,1643
Skewness	1,229	1,2191
Qtr1	52476,02	32083,6
Qtr3	173926,4	150169,5
IQR	121450,4	118085,9
Range	2063465	2003705
Minimum	-671334	–673273
Maximum	1392131	1330432
Sum	1,13E+08	90964102
Count	1000	1000
Confidence level (95 %)	8547,755	8293,665
Shift	0	0

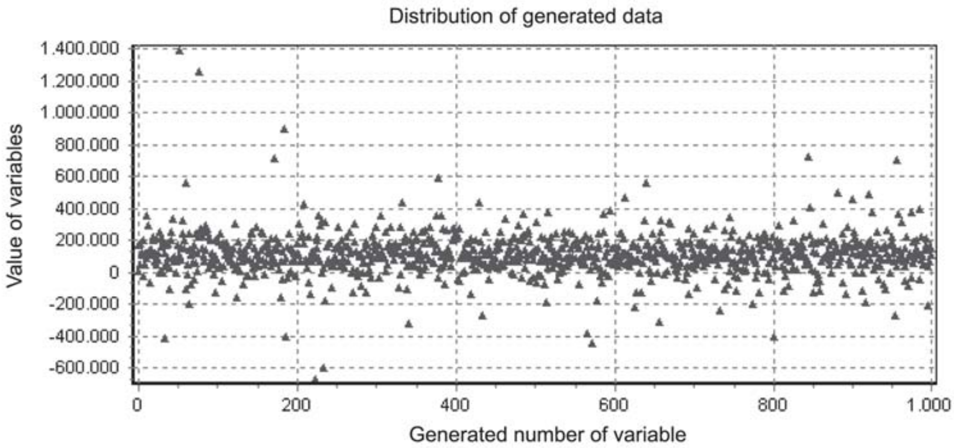

Fig 4. Distribution of project NPV data shown as an example (for project without external sponsorship)

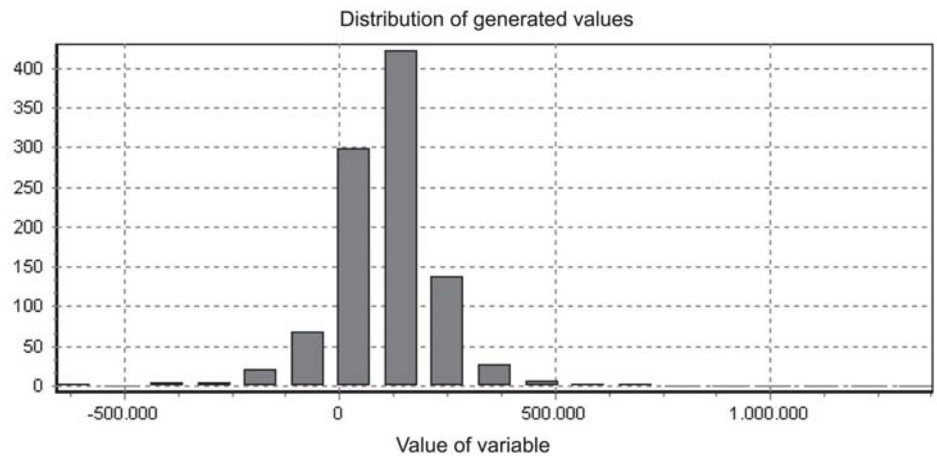

Fig 5. Distribution of project NPV shown as an example (for project without external sponsorship)

The data of NPV were fitted with „@Risk.4.5“ in order to define distribution. The ranking of the fit was based on Chi-Squared, Kolmogorov-Smirnov and Anderson-Darling statistics. The best fit distribution for project without external sponsorship is Logistic (112079, 64388). P – Value (observed significance level) for tests is: Chi-Squared – 0,0293; Kolmogorov-Smirnov – < 0,005; Anderson-Darling – <0,01.

The critical values of the fit statistics at 0,1 level of significance are: Chi-Squared – 37,9159; Kolmogorov-Smirnov – 0,6598; Anderson-Darling – 0,0226.

The critical values of the fit statistics at the 0,01 level of significance are: Chi-Squared – 48,2782; Kolmogorov-Smirnov – 0,9058; Anderson-Darling – 0,0283.

The critical values of the fit statistics at 0,005 level of significance are: Chi-Squared – 50,9934; Kolmogorov-Smirnov – 1,0097; Anderson-Darling – N/A.

The hypothesis that NPV data for project without external sponsorship fit Logistic distribution is more based on Chi-Squared test, other tests show, that likelihood is very small. The probability that NPV > 0 for project without external sponsorship is 85%. Required NPV is more than 100000 euro. The probability, that NPV > 100000 is 54,6%. The probability, that NPV < 0 for project without external sponsorship is about 15%.

4. Conclusions

Monte Carlo simulation technique is a useful tool in project appraisal. Using the technique analysts can get the following specific advantages for risk analysis:

1. It increases potentiality to make decisions on marginal projects. A project which NPV is small may still be accepted following risk analysis, if a satisfactory return is greater than the probability of making an unacceptable loss.

2. It benefits the reformulation of projects to suit the positions and requirements of the investor. A project may be redesigned to take account for the particular risk predispositions of the investor.

3. It helps to reduce project evaluation favours through eliminating the need to rely only on conservative estimates and reflects the analyst's risk expectations and aptitudes.

4. Analysis increases the communication between the analyst and the decision maker. The execution of risk analysis in project appraisal involves the collection of information which to a large part reflects the acquired knowledge and expertise of top executives in an organisation.

5. It provides all necessary information bases to ease the management of risk among various parties involved in a project. Once various sources of risk have been assessed, project risk may be contractually allocated to those parties who are best able to bear and manage it.

6. Finally, the analyst should be careful :

- he has to identify the major correlated variables and provide the analysis of the impact of such correlations in the simulation;
- the accuracy of his predictions has to be as good as the predictive capacity of the used model.

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Rima TAMOŠIŪNIENĖ. Doctor of Science, associate professor of Vilnius Gediminas Technical University, Department of Financial Engineering. Researches interests: preparation and management of business and investment projects, risk analysis and management of various projects, financial analysis and management.

Tomas PETRAVIČIUS. Doctoral student of Vilnius Gediminas Technical University, Department of Financial Engineering. Researches interests: risk analysis and management, investment analysis, management of business and investment projects.