ISSN 2669-2481 / eISSN 2669-249X

VILNIUS TECH
Vilnius Gediminas Technical University

BUSINESS, MANAGEMENT AND ECONOMICS ENGINEERING

2025
Volume 23
Issue 1
Pages 164–188

https://doi.org/10.3846/bmee.2025.22695

APPLYING THE MEAN-VARIANCE FRAMEWORK: PORTFOLIO OPTIMIZATION AND COMPARATIVE PERFORMANCE ANALYSIS IN THE EMERGING COLOMBIAN CAPITAL MARKET

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Article History:
- received 16 November 2024
- accepted 25 February 2025

Abstract. *Purpose* – this paper adopts the mean-variance approach in optimizing portfolios within the Colombian capital market, a setting full of complications such as lack of liquidity and market concentration. It delivers actionable messages for emerging market stakeholders and formulates guidance aimed at enhancing risk-adjusted returns and informing portfolio management in markets with similar structural and economic conditions.

Research methodology – a bi-objective mean-variance model has been used for analyzing the stock prices of 17 stocks on a weekly basis from 2009–2024. Annual rebalancing has made the portfolio responsive to changes in the market, considering the Sharpe ratio as the benchmark to assess risk-adjusted performance.

Findings – optimized portfolios in Colombia outperformed traditional investment funds by realizing better returns while having a balanced risk. Surely, this shows that the model is able to be flexible and react to changes in fluctuation, capture sectoral opportunities, and perform amazingly in a dynamic market.

Research limitations – focusing on adaptability and real-time rebalancing in this work can establish a basis on which future research will operate, refining optimization strategies that incorporate advanced risk measures such as CVaR.

Practical implications – the results present an effective and flexible tool for investors to optimize their portfolios in respect of risk diversification and sustainable returns, considering liquidity constraints and market turmoil.

Originality/Value – this research connects theory and practice and demonstrates the flexibility of the mean-variance model in emerging economies. It emphasizes novelty in portfolio optimization solutions and further development of strategies in sophisticated financial conditions.

Keywords: portfolio selection, diversification, efficient frontier, mean-variance, Colombia stock market.

JEL Classification: G1.

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1. Introduction

The capital market is the institutional facility through which an economy mobilizes and allocates resources, risk, and information to transform savings into productive investment. The very idea behind a stock exchange is that it offers one platform where demanders of financial assets meet their suppliers in facilitating resource flow from savers or agents with surplus liquidity to investors in the productive sector (González-Bueno, 2019; Onyshchenko, 2021; Thomas et al., 2023). However, for Colombia, it is well below the performance of the stock

exchanges of other emergent and developed countries. For example, the capitalization of the Colombian stock market as a percentage of GDP reached 76% at its peak in 2010; however, since then it has suffered a strong contraction to 31% of GDP in 2018 and to 22.7% in 2022, which reflects the depth and liquidity lost during this period (Asobancaria, 2023; Malagón González & Verdugo Rodríguez, 2023). In advanced economies like the United States and Japan, it exceeds 100% of GDP, while the Colombian stock market is underdeveloped in terms of growth and depth (Rigobón & Rodríguez Ospino, 2023; Rudenok et al., 2021).

On a regional level, Colombia faces the same challenges, especially compared to such markets as Brazil and Chile, whose stock market capitalizations reached 43.4% and 93.3% of GDP, correspondingly, in 2017, while that of Colombia was only 37.4% (Blikhar et al., 2022; Verdugo Rodríguez, 2023). Besides that, the number of quoted companies has gone down from 72 in 2013 to 63 in 2023, where results of low participation and diversification of the Colombian stock market show up in indicative points (Asobancaria, 2023). This means that the Colombian stock exchange is in danger of being classified as a frontier market according to the MSCI index, which raises concerns regarding liquidity, financial risk, and economic stability. It will raise the cost of capital, provoke foreign investment withdrawals, dilute the value of the shares of local companies, and consequently hurt the economy in general terms, as established (Asobancaria, 2023; Verdugo Rodríguez, 2023). Consequently, it is crucial for Colombia to enact structural reforms that enhance liquidity and bolster investor confidence, thereby aiding in the prevention of this reclassification and its negative economic repercussions.

As earlier pointed out, through the capital market, entities with excess liquidity are capable of diversifying their risk through investment. In this respect, portfolio selection forms an important aspect of the investor, as it not only apportions money in various classes of assets for purposes of risk diversification but also ensures that expected terminal wealth reaches a certain threshold. In fact, the very first mathematical formulation of the portfolio selection problem – the seminal and pioneering mean-variance model –was pioneered by (Markowitz, 1952). Indeed, within this model, Markowitz modeled the return of an asset as a random variable, describing the return and risk of the asset as the expected value and variance of the random variable correspondingly, and found the optimal portfolio through quadratic programming (Markowitz, 1959).

Markowitz's model was a breakthrough on two counts. The first is that it showed how the return, and attendant risk, of a portfolio could be calculated analytically from an evaluation of asset returns and relationships among assets. The second is that it framed financial decision-making within the context of an optimization problem in which an investor chooses a portfolio exhibiting least variance from among other portfolios yielding a pre-specified target return, excluding thereby all inefficient portfolios showing greater degrees of risk (Boyd et al., 2024; Fabozzi et al., 2007; Sexauer & Siegel, 2024).The propositions made by this framework have remained, up to today, at the core of many approaches dealing with asset allocation, investment analysis and valuation, risk analysis, capital budgeting, and decision making under uncertainty (Boyd et al., 2024; Zopounidis et al., 2014).

Based on the foregoing information, the following question is developed for the present research: Considering the liquidity and diversification conditions of the current Colombian stock market, what methods should Colombian investors use in the optimal distribution of

their capital to create a portfolio that maximizes diversification, reduces risk, and increases returns? It is a bi-objective mean-variance model conducted on a sample composed of 17 firms listed in the Colombian Stock Exchange, including some of the most relevant industries for the financial and energy sectors and consumption goods. This methodology seeks to assess the practical relevance of the mean-variance framework in the context of an emerging market like Colombia, where market dynamics are substantially in contrast with those observables in developed economies. It is aimed at systematically selecting those companies that have shown steady market performance and sufficient liquidity to meet investor expectations using weekly closing prices from June 2009 to June 2024. The dual objective is the maximization of return with reduced involved risk, measured through portfolio variance. This strategy has proved very efficient in market adjustments due to price variation, liquidity changes, and sector-specific trends by periodic optimization on nine rebalancing dates ranging from June 2014 to June 2023. The comparison between the obtained rebalanced portfolio and some real investment funds underlines the possibility of the model finding, with a very high degree of accuracy, those combinations that provide an optimal risk-adjusted return. The Sharpe ratio as an integral metric underpins the importance of incorporating risk management in optimization so that the outcomes from the application are in tandem with the investor's objectives both in emerging and developed markets. Current research links theoretical leads to their practical implementation by overcoming certain obstacles that are most relevant to the development of efficient capital markets in the Colombian market, such as low liquidity, market concentration, and economic fluctuation. The mean-variance model thus proves to be effective for optimization of the risk-return trade-off in a dynamic developing financial environment and hence provides practically useful insights for an investor to go through intricacies in emerging market conditions.

The rest of the paper is organized as follows. Section 2 describes the literature review concerning portfolio selection theory. Section 3 introduces the Markowitz mean-variance model. Section 4 describes the database of the stock market in Colombia. In Section 5, the Markowitz model is applied, and the most important results of that application are summarized. Finally, the last section of the manuscript sums up the most important results developed in this paper.

2. Literature review

The portfolio selection theory constitutes a fundamental pillar in the analysis of financial markets and strategic decision-making (Arribas et al., 2019, 2021; Boyd et al., 2024; García et al., 2015; Goetzmann, 2023). Markowitz's (1952) article Portfolio Selection represents a pivotal moment that transformed this field. This innovative theory addressed the following question: How should an investor allocate capital among various investment options? Markowitz's groundbreaking approach included measuring an asset's return and its associated risk using statistical measurements, concretely analyzing the mean and the variance. He also recommended an efficient analysis that would allow investors to carefully weigh return and risk when allocating capital (García et al., 2022; Oliver-Muncharaz, 2019; Ramírez Garrido, 2023; Rincon & González-Bueno, 2016).

The essence of portfolio selection theory maintains that sound decision making depends in large measure on a delicate balance between return and risk; it has come to shape financial philosophy in some fundamental ways because of two fundamental reasons: Firstly, it introduced for the first time the idea that an adequate assessment of portfolio performance and risk is possible through an analysis of returns from securities and their dependencies. Central to this proposal is portfolio diversification, wherein it is postulated that the risk associated with a portfolio emanates from the interrelations that exist between their assets, rather than from the individual risk characteristics of each asset (Fabozzi et al., 2007; Guerard, 2023; Kolm et al., 2014; Mascareñas, 2015). This is in sharp contrast to the traditional valuation approaches applied by Graham and Dodd (1934) in their search for undervalued assets by primarily focusing on discrete asset valuation through balance sheet and income statement analysis. Moreover, diversification can be depicted graphically so as to visually present the efficient frontier in determining those combinations of assets that minimize the overall risk of the portfolio within the risk-return paradigm (Boyd et al., 2024; Fabozzi et al., 2007). Quadratic programming is an important tool that portfolio managers use to optimize returns while minimizing their risk factors in managing large numbers of assets, with complex constraints such as short sales being imposed (Gasparavičius & Grigutis, 2024). Second, Markowitz redefined investment decision-making as an optimization problem. He put forth that out of the set of portfolios that satisfy a given return requirement, investors ought to choose a portfolio with the lowest variance. The portfolios of higher variance were termed "inefficient" and thus riskier (Boyd et al., 2024; Fabozzi et al., 2007). This pioneering framework really revolutionized the manner at which investors manage and think about risk and return in their investment programmers.

The portfolio optimization model, when first developed and firmly set within a dual risk-return or mean-variance paradigm, did not receive significant acceptance. Over time, though, basic concepts of this model have been embraced by both the scientific and financial worlds and provide the underlying base for today's methods of asset allocation, investment analysis, risk management, capital budgeting, and decision making under uncertainty (Goetzmann, 2023; Petukhina et al., 2024; Zopounidis et al., 2014). This became one of the revolutionary inventions in finance because, in 1990, Harry Markowitz, Merton Miller and William Sharpe were given the Nobel Prize for fundamental contributions to financial economics, Markowitz was given for his portfolio selection theory development, Sharpe for his work on asset pricing theory and the invention of the Capital Asset Pricing Model known as CAPM while Miller was given for his contribution to the enhancement of corporate finance theory (Sexauer & Siegel, 2024).

Portfolio managers employ tight evaluative criteria in filtering what is essential from that which is less important and separating the relevant from the irrelevant information (Azmi, 2013). The criteria include but are not limited to liquidity, asset type, geographical dispersion, macro and micro economic factors, market forces, among others (Azmi & Tamiz, 2010; González-Bueno, 2019). Selection of such criteria would depend on the conditions and the investors' profiles. There are two criteria that would always apply for all types of investors: the mean (expected return) and the variance of return (risk). According to Markowitz (1952), the beliefs or predictions about the assets have to satisfy the same probability laws that

the random variables do. Accordingly, the expected value of a portfolio is calculated as a weighted average of the expected values of the constituent asset, whereas the variance of a portfolio return is expressed as a different function of variances and covariances along with the corresponding weights of the constituent assets in the portfolio (Boyd et al., 2024; García et al., 2015; Guerard, 2023; Gupta et al., 2014). Thus, an investor needs to deal with risk and return together to facilitate the optimal capital allocation among competing alternatives considering their dependencies (Kolm et al., 2014).

According to Markowitz's portfolio selection theory, portfolio selection must not be strictly based on historic performance but on some rational forecast of its future performance (Lleo & MacLean, 2025). Beliefs based on back-tracking return suppose the historic average is the best estimator of future returns and that the variance of return reflects the uncertainty of returns in the future (González-Bueno, 2019; Gupta et al., 2014). Investment is also made on the principle of investment "Don't put all your eggs in one basket" (Wuttke et al., 2023). From a technical perspective, the above saying describes the gains made through diversification. In this respect, Markowitz (1952), through the mean-variance model, was the first to illustrate the gain made through diversification, particularly the reduction in risk by spreading one's equity into various financial securities. The process of diversification makes its theoretical basis on the return-risk profile, that is the covariance or correlation between unlike assets (Guerard et al., 2024). However, when all those assets have one return-risk characteristic, then more assets is not necessarily a smart choice. In other words, even with a small probability of any of these assets failing to perform well, it is not a wise thing to invest in highly correlated assets only. This is because, at the sight of failure on the part of one, it is likely to poorly affect the others due to its being highly correlated, thus affecting the overall performance of the portfolio. Consequently, a well-constructed portfolio consists of assets with return-risk characteristics that exhibit greater dissimilarity than similarity (Boyd et al., 2024; Fabozzi et al., 2007; Gupta et al., 2014).

It is a common view that the returns of assets are not normally distributed. The majority of asset returns have fat or heavy tails among other properties (Bianchi et al., 2023; Boudt et al., 2015; Fama, 1965a, 1965b; Jansen et al., 2000; Mandelbrot, 1963a, 1963b). These heavy tails are a result of the higher kurtosis that exists within the empirical distribution relative to expectations under a theoretical normal distribution serving to increase the probability of extreme values and lowering the probability of moderate values (Bianchi et al., 2023). For a normal distribution, the value for kurtosis is 3, which means it is mesokurtic. For a real-world distribution, it is called platykurtic if it has a kurtosis less than 3 and leptokurtic or fat tailed if its values exceed this threshold (Ramírez-Sánchez, 2004). Other investigations indicate that the variances of returns for some assets are not finite, meaning they are infinite and thus do not exist (Fabozzi et al., 2007). Similarly, studies have shown that if asset returns are generated by a stationary Pareto distribution, the importance of diversification as an economic activity becomes less relevant (Fama, 1965a; González-Bueno, 2019). This situation brings up a number of questions about the efficiency of diversification. Nevertheless, almost all research shares one common ground: a level of diversification is possible in financial markets (Bilbao-Terol et al., 2013; Campbell et al., 2001; Grubel, 1968; Gupta et al., 2020; Oliver, 2021). Not only is the concept of diversification intuitive, but also very powerful, repeatedly proving

itself correct in a myriad of financial situations. A considerable portion of the innovations in finance are focused either on practicing diversification or developing novel methods for improving the estimation of variances and covariances. This effort tends to produce higher precision in diversification strategies and risk assessment. Nevertheless, the total risk of a portfolio exceeds its standard deviation. A portfolio with relatively lower expected standard deviation may still perform badly due to other important dimensions of risk, which have to be taken seriously in formulating an investment policy (Fabozzi et al., 2006).

Finally, it is important to stress that, within the portfolio selection theory, asset returns are considered random variables as a result of a stochastic analysis based on historical data. The method is fully reasonable since historical data reflect certain specific characteristics of those returns. However, given the complex nature of the stock market, returns often depend on other influences such as the firm's short-term situation, interplay between the demand and supply in the market, relevant news, political factors, and other influential parameters which help predict returns (Huang, 2008, 2009, 2017; Jalota et al., 2023; Ruiz et al., 2019; Yue & Wang, 2017). Although historical data can provide useful information, they are beset with certain limitations in that they are not fully representative of the intricacy of such variables. This estimation of driving factors, because of problems in acquiring realistic data and due to measurement errors, often involves vagueness inherently, together with subjectivity in human perceptions, which seriously affects practical application (Huang, 2006; Huang & Di, 2016; Huang & Wang, 2021; Yue & Wang, 2017). Therefore, according to many experts, randomness cannot be thought about independently as one source of uncertainty in the real world (García et al., 2020a; Gupta et al., 2013; Huang & Wang, 2021). The above researchers concur on the inclusion of expert information to increase the accuracy of such forecasts (Ruiz et al., 2019; Yue & Wang, 2017). In recent years, the employment of fuzzy set theory (Zadeh, 1965) has become a very popular approach in combining expert information and addressing the non-statistical uncertainty inherent in the prediction of asset returns. This fact has given rise to many asset portfolio selection frameworks considering returns as fuzzy variables (Abdolbaghi Ataabadi et al., 2023; Garcia et al., 2020b; García et al., 2022; Jalota et al., 2023; Mandal et al., 2024).

3. Methodology

The methodology applies the mean-variance model on a data set from Colombian capital market, taking into consideration the constraints fitted for emerging markets analyzing the dynamics of the risk-return. Within this approach, it copes with liquidity constraints and market concentration to assure robustness in portfolio optimization that would be applicable in conditions of similar markets.

3.1. Data description

The Colombian stock market is more formally known as the Bolsa de Valores de Colombia, and it is the principal equity exchange in Colombia. Since its establishment in the early 20th century, the BVC has gained importance among Latin American financial markets. It is said to be very resilient and flexible with regard to national economic ups and downs. Into this

diversity of investment opportunities, ranging from local to international investors, it contributes to economic growth in Colombia while offering access to a wide range of financial instruments to stakeholders.

The bi-objective mean-variance framework addresses a specific issue, specifically: On June 2, 2014 an investor aspires to select a portfolio from the Colombian stock exchange with the intention of allocating their total wealth across n equities that provide returns on the assets, thereby maximizing anticipated returns while concurrently minimizing associated investment risks. In improving the portfolio performances so that better outcomes over the 10-year period may be realized, it is assumed that the investor makes nine rebalances of the portfolios during the period based on the price behavior and liquidity of the stocks traded on this market. The dates of rebalancing are: June 1, 2015, June 1, 2016, June 1, 2017, June 1, 2018, June 3, 2019, June 1, 2020, June 1, 2021, June 1, 2022, and June 1, 2023.

The reason behind using annual rebalancing rather than quarterly or semiannual changes is strongly underlined from a financial and pragmatic point of view. Annual rebalancing is congruent with the financial reporting cycle, a period in which firms publish elaborate performance metrics and macroeconomic indicators attain a state of stability, hence facilitating the integration of major structural alterations into portfolio modifications. A key implication of this approach is the avoidance of over-trading that would create inefficiencies and enhance the transaction cost in terms of bid-ask spreads and commissions. In Colombia, this will minimize the cost from over-trading, given the existence of more important liquidity constraints and trading frictions, while keeping the portfolio near the efficient frontier. It gives a meaningful identification of trend, volatility, and correlations in prices, with a view to making changes considering new covariance structures. It strikes a balance between theoretical precision and realistic implementability, so the portfolio shall optimally react to changes in the market with the view to maximize expected returns and to maintain optimum risk-return profiles for the investment horizon.

The Markowitz mean-variance model has been applied using the closing prices of every week from 01 June 2009 to 30 June 2024. The dataset contains the closing prices of the following 17 companies: Banco de Bogotá S.A. (Bogota), Banco Davivienda S.A. Preferred (PfDavvnda), Bancolombia S.A. Common (Bcolombia), Bancolombia S.A. Preferred (PfBColom), Celsia S.A. (Celsia), Cementos Argos S.A. Common (Cemargos), Cementos Argos S.A. Preferred (PfCemargos), Corporación Financiera Colombiana S.A. Common (CorfiColcf), Ecopetrol S.A. (Ecopetrol), Grupo Argos S.A. Common (GrupoArgos), Grupo Argos S.A. Preferred (PfGrupoarg), Grupo Aval S.A. (GrupoAval), Grupo de Inversiones Suramericana S.A. Common (Gruposura), Grupo de Inversiones Suramericana S.A. Preferred (PfGrupsura), Grupo Energía Bogotá S.A. Common (Geb), Grupo Nutresa S.A. Common (Nutresa), and Interconexion Electrica S.A. Common (Isa). These 17 selected firms belong to several sectors: 1 belongs to the Consumer Sector, represented by Nutresa; 2 to the Energy Sector, represented by ISA and Celsia; 6 to the Financial Sector, composed of GrupoAval, Banco de Bogotá, Bancolombia, PfBColom, Corficolcf, and GrupoSura; 1 to the Hydrocarbon Sector, represented by Ecopetrol; and 7 to the Construction and Infrastructure Sector, composed of Cemargos, PfCemargos, GrupoArgos, PfGrupoarg, Grupo Energía Bogotá, Bcolombia, and GrupoSura.

The sampling about the elements that will be analyzed, to which the bi-objective mean-variance model will be applied, is done according to the following:

i) Only the companies that have continuously maintained at least 90% market representation throughout all the study periods, viz., Period I (06/02/2009 to 05/30/2014), Period II (06/01/2010 to 05/29/2015), Period III (06/01/2011 to 05/31/2016), Period IV (06/01/2012 to 05/31/2017), Period V (06/03/2013 to 05/31/2018), Period VI (06/02/2014 to 05/31/2019), Period VII (06/01/2015 to 05/29/2020), Period VIII (06/01/2016 to 05/28/2021), Period IX (06/01/2017 to 05/31/2022), and Period X (06/01/2018 to 05/31/2023), are included. In other words, the firms which are not continuously traded throughout each of these periods are excluded from the analysis.

ii) On the other hand, according to the literature (Arenas-Parra et al., 2001; Gupta et al., 2008, 2011; Jalota et al., 2017a, 2017b), investors prefer those portfolios that can be liquidated not only at higher expected values but also provide more security in the liquidation values, i.e., more liquid portfolios. Searching for this approach, only those companies with an average monthly volume traded over the average monthly volume of the stock markets in which they participate during the study period have been chosen.

3.2. Mean-variance model

The expected portfolio return is the weighted average of the returns of the component stocks. These weights are typically derived largely from analysis of the history of the returns. These pieces fit together in the expression.

$$E\left(R_p\right)=\sum_{i=1}^{n} w_i E\left(R_i\right); \tag{1}$$

$$\sum_{i=1}^{n} w_i = 100\%, \tag{2}$$

where: $E\left(R_p\right)$ – Expected portfolio return; n – Number of stocks that constitute the portfolio; w_i – Proportion or weight of stock "i" in the portfolio; $E\left(R_i\right)$ – Expected return of stock "i".

Risk is viewed as the potential dispersion of the actual performance of an investment from its expected outcome. The risk of common stocks is twofold: The first one is Systematic risk; it consists of non-diversifiable, market or common risk. Secondly, there is Unsystematic risk consisting of diversifiable, specific, or idiosyncratic risk. Portfolio selection theory posits that both kinds of risk exist for all portfolios. The risk of a diversified portfolio, measured by the standard deviation of performance is:

$$\sigma_p^2 = \sum_{i=1}^{n} . \sum_{j=1}^{n} w_i w_j \sigma_{ij}; \tag{3}$$

$$\sigma_p = \sqrt{\sigma_p^2}, \tag{4}$$

where: σ_p^2 – Variance of the portfolio; σ_p – Risk of the portfolio; w_i – Proportion/eight of Stock "i" in the Portfolio; w_j – Proportion/Weight of Stock "j" in the Portfolio; σ_{ij} = Covariance between security *i* and *j*.

There are two ways to determine the securities of a portfolio. First, to select securities in a manner that would maximize the return for a particular amount of risk or choose securities in such a way that the lowest volatility is attained for a particular return.

2.2.1. The highest possible return of a portfolio for a given amount of risk

$$Max\, E\left(R_p\right) = \sum_{i=1}^{n}.\sum_{j=1}^{n}\omega_i\omega_j. \tag{5}$$

Subject to:

$$(1) = \sigma_i^2 = \sum_{i=1}^{n}.\sum_{j=1}^{n}w_i w_j\sigma_{ij}\,. \tag{6}$$

Limitation of portfolio risk: with the use of point 5 and 6, try to reach the highest profitability in the portfolio with predetermined level of risk.

$$(2) = \sum_{i=1}^{n}w_i = 100\%\,. \tag{7}$$

The corresponding budget constraint would be that all the available money should be invested leaving no residual cash behind.

$$(3) = w_i \geq 0\,. \tag{8}$$

This constraint considers short positions prohibited, which means an investor cannot take up a short sell position in the portfolio. In other words, such a no-short-selling position eliminates unlimited downside risk because the portfolio remains stable due to this strategy. Further, this approach will be developed in a more conservative and feasible direction toward the problematics of portfolio construction in developing markets like Colombia, where restrictions on liquidity and shallow markets increase the volatility associated with selling short.

$$(4) = w_i \leq 30\%\,. \tag{9}$$

This is a constraint imposed to avoid having more than a single 30% position in any one security. It considers the threshold most widely accepted within the literature on finance for achieving an appropriate balance in diversification with efficiency in the portfolio (García et al., 2019, 2022). Avoid having any holding dominate portfolio variance, σ2p, and maintain the balance in the efficient frontier risk-return tradeoff.

2.2.2. The lowest possible risk with realization of a desirable profit level

$$Min\,\sigma_i^2 = \sum_{i=1}^{n}.\sum_{j=1}^{n}w_i w_j\sigma_{ij}\,. \tag{10}$$

Subject to:

$$(1) = E\left(R_p\right) = \sum_{i=1}^{n}w_i E\left(R_i\right). \tag{11}$$

This will minimize the risk of the portfolio targeting a stipulated level of return through the implementation of steps 10 and 11.

$$(2) = \sum_{i=1}^{n} w_i = 100\% .$$

The corresponding budget constraint would be that all the available money should be invested leaving no residual cash behind.

$$(3) = w_i \geq 0 .$$

The constraint sign forbids the short position, which means the investors are not allowed to operate any kind of short selling activities within a portfolio.

$$(4) = w_i \leq 30\% .$$

The limit set here is that no more than 30% of the total investment available will be made in one stock. This kind of constraint tries to prevent undue exposure to any single stock and thereby encourages diversification across a number of stocks.

4. Results

This Section first shows the behavior of chosen 17 companies listed in the Colombian Stock Market by historical performance, both in their cumulative returns and their risk profile, from June 1, 2014 to June 28, 2024. The present research will lay the basic framework with which the dynamics of each one of the assets take place, the general trends which operate within the Colombian market, identify which are the most and least profitable stocks, those with higher and lower risk, and stability.

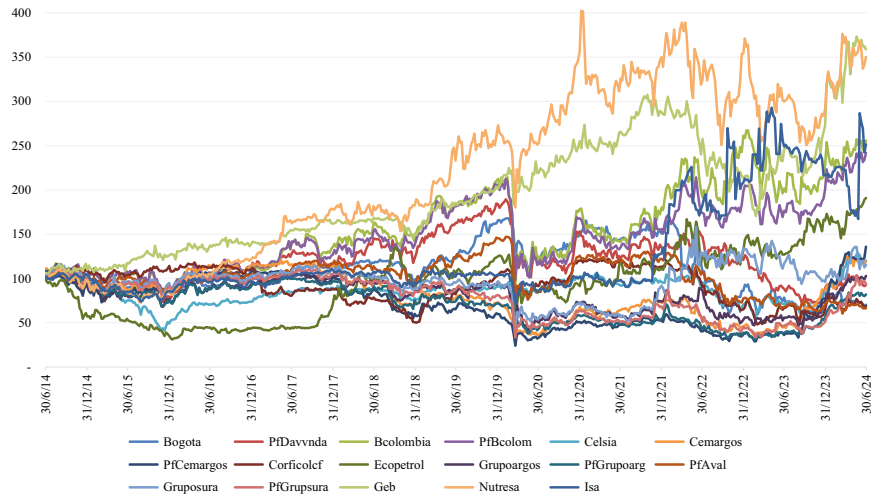

Figure 1. Performance of the 17 selected stocks in the Mean-variance model between June 1st, 2014, and June 28th, 2024 (source: Economatica, n.d.)

Figure 1 shows the development of the 17 selected stocks that make up the mean-variance model between June 1, 2014 and June 28, 2024, starting from an index base of 100. This was conducted with a view to establishing how each of these assets would develop comparatively in a ten-year timeframe, given the variation in returns and stability that might characterize the different assets in the Colombian stock market. Some of them showed impressive cumulative growth during the period under consideration. Outstanding, among them, is Geb with 358.69 and Nutresa with 350.13, which means very strong increases confirming both as leaders in the energies and consumptions sectors, respectively. These results showed a strong appreciation in those sectors, because of the greater demand for energy service and consumer goods along with good expansion and sustainability strategies. Other companies showed positive growth but were different in level: Bcolombia with 255.74, its preferred stock PfBcolom with 241.86, showing a robust increase reflecting the strength and stability of one of the biggest financial institutions in the country.

In contrast to above view, the rest of the financial companies showed less performance, like Bogota with 70.31 and PfAval with 66.58, which implies an operating or strategic issue in their environment. This inconsistency within the financial sector further expounds on the heterogeneity of the performance of companies due to their different positions they assumed to hold within the market, risk exposure, and management decisions. Besides that, Isa with 251.24 and Ecopetrol with 190.82, reflecting the strength of infrastructure and hydrocarbon companies. In spite of the volatility of the international energy market, the companies took advantage of good conjunctures and continued performing positively. Finally, some companies show up with more stable behavior, like Gruposura with 115.18 and Grupoargos with 102.89, which are the figures with average rises. These results are indicative of a conservative oriented management, which would attract risk-averse investors seeking exposure to traditional sectors that prioritize consistency over volatility.

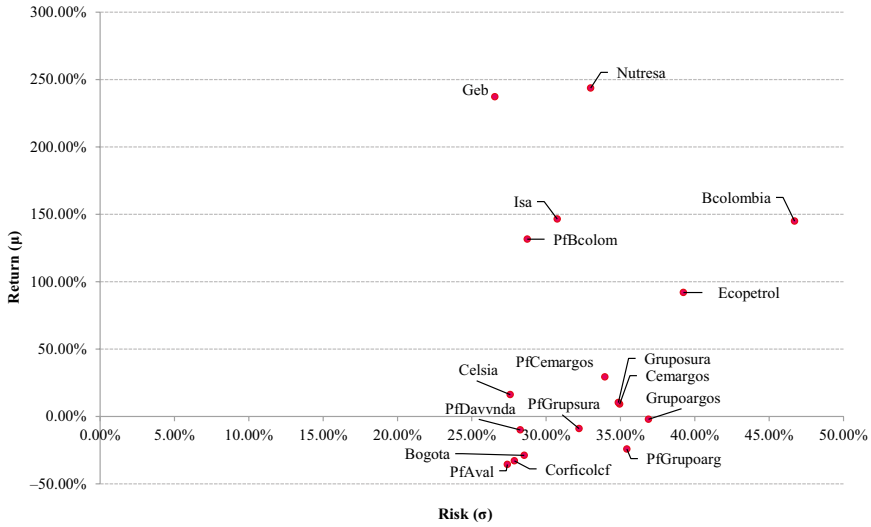

Figure 2. Return and Risk of the 17 selected stocks in the Mean-variance model between June 1st, 2014, and June 28th, 2024 (source: Economatica, n.d.)

Figure 2 presents the annualized average weekly return, μ, and annualized risk, σ, of the 17 selected companies in the Colombian market during the analysis period. Through this chart, and supported by the mean-variance model, it is possible to perceive the behavior of each company regarding accumulated returns and their corresponding volatility, which allows for the identification of those that are the most and least profitable, as well as those that are the riskiest and most stable in the Colombian market. Geb and Gruponutresa have proved to be among the most profitable companies, showing annualized returns of 237.18% and 243.63%, respectively. Both companies were at the forefront of the market, considering the study period. Geb in the energy sector and Gruponutresa in the consumer sector appear to have attracted the attention of investors with good expansion strategies and fostering sustainability in consideration of strong demand for the products and services of both groups. Meanwhile, there is also significant growth with Bcolombia at 144.89% annualized return, and with its preferred stock PfBcolom at 131.61%, which places it among the most profitable companies in the financial sector. In contrast, some companies presented less appealing performance, as their cumulative returns are negative, showing losses in terms of price. To this list, Bogota is added with an annualized return of –28.77%; PfGrupoarg, with –24.19%; Corficolcf, with –32.96%; and PfAval, with –35.60%. These results tend to reflect that these companies have been subject to certain specific challenges that negatively impacted their market performance. In each of these cases, accumulated losses may relate to internal or external factors that have affected their ability to generate shareholder value over the last decade.

About risk, Figure 2 presents the firms that show the biggest volatility. The highest ranking goes to Bcolombia, in first position, since it has an annualized standard deviation of returns of 46.72%, it has high volatility regarding the oscillation of its stock price. The next companies are Gruposura, with 36.22%; Ecopetrol, with 39.24%; and PfGrupoarg, with 35.43%, since these results also mean high levels of risk. The level of volatility is high because of industry-specific causes added to the vulnerability of companies to variations in world prices, in particular for Ecopetrol in the hydrocarbon sector. On the other hand, some companies have shown their less volatile nature, which demonstrates a better level of sustainability in the market. Other companies with low annualized standard deviation in their returns include PfDavvnda with 28.26%, Corficolcf with 27.87%, Celsia at 27.59%, and Geb with 26.55%. This reflects more conservative management or at least relatively stable demand for their shares. Such low volatility makes the firms very appealing to risk-averse investors with a focus on capital preservation strategies.

Following the preliminary analysis, the bi-objective mean-variance model is applied to address the portfolio selection problem within the Colombian stock market. This model focuses on the decision-making process of an investor who, starting on June 2, 2014, aims to construct a portfolio that maximizes expected return E(Rp) while minimizing associated risk, measured by portfolio variance $\sigma 2$. In view of the dynamics of asset prices and liquidity, the model will determine the efficient frontier based on the first period, which runs from June 2, 2014, to May 31, 2019.

This analysis of the first period was done using Economática and its portfolio optimization module, where the identification of the efficient frontier was obtained. The latter represents all the combinations of assets that offer the maximum return for a certain level of risk or the

minimum risk for a specific return. This first portfolio on that frontier was optimally chosen by maximizing the Sharpe Ratio, one of the most popular financial measures for evaluation of risk-adjusted returns, defined as:

$$Sharpe\,Ratio = \frac{E\left(R_p\right) - R_f}{\sigma_p}, \tag{12}$$

where R_f is the free-risk rate proxied by one-year Colombian government bonds. This measure integrates return and risk into one variable; this is the base to select the best portfolio in terms of risk return from the efficient set. The optimal portfolio will be at the tangency between the capital market line and the efficient frontier and represents the best trade-off between expected return and volatility.

In such a dynamic market and one in evolution, like that of Colombia, marked during this period from 2014 to 2023 by important fluctuations in prices and variations in the liquidity of capital, as well as great sensitivity to conditions at both local and global levels, periodic portfolio rebalancing becomes indispensable to maintaining efficiency. Hydrocarbon price volatility, year-by-year economic slowdowns, and episodes of political uncertainty marked investor sentiment throughout this period regarding the Colombian stock market (Ramírez Garrido, 2023; Rigobón & Rodríguez Ospino, 2023; Verdugo Rodríguez, 2023). To capture such dynamics, annual rebalancing was made for the nine following dates: June 1, 2015; June 1, 2016; June 1, 2017; June 1, 2018; June 3, 2019; June 1, 2020; June 1, 2021; June 1, 2022; and June 1, 2023.

All these windows recalculate the frontier with new information about asset prices and returns to keep the optimization current in changing market conditions. For every period, it maximizes the Sharpe Ratio so that the portfolio which gave the best risk-adjusted return could be identified. The iterative process allowed strategic changes to better fit the portfolio in an appropriate manner for huge disruptions, such as the collapse in the price of oil in 2020 and the succeeding economic recovery. Moreover, the readjustment of portfolios has captured changes in market volatility and dynamically evolving sector matters, including the different behaviors of the financial, energy, and consumer sectors throughout the period in question. These periodic recalibrations pinpoint how the model, by being proactive with the challenges and opportunities which come subtly in the market, maintains the efficiency of the portfolios. Incorporating these annual modifications, on their part, would preserve the risk-return profile expected and maximize opportunities about using market dynamics. This way, rebalancing would be one of the indispensable practices in achieving efficient portfolio management within the emerging market of Colombia.

Annual rebalancing introduces practical, real-life portfolio management practices that may shift with market developments and prevent drifts in the sought-after risk-return profile, thus allowing seized opportunities for the best performance of portfolios over time to emerge. Results from this methodology are the original portfolio along with nine rebalanced portfolios, whose configuration and key metrics are summarized in Table 1. These are the efficient solutions for each time interval and hence serve as an excellent backbone for decision-making in dynamic and complex financial markets.

The analysis of the results in Table 1 forms not only an appropriate application of the mean-variance model but also provides greater insight into how the dynamics of portfolio management have changed over time for an emerging market like Colombia. Each of the portfolio compositions, going from the initial allocation on June 2, 2014, over nine annual rebalancings, are indicative of the sophisticated accommodation of the changes in market conditions, economic shocks, and sector-specific developments.

The first portfolio configuration was more focused on those assets perceived to be more stable or successful in the Colombian capital market. Basically, this reflects the typical character of the market for Colombian equity, which has high issuer concentration. Such a composition gives rise to an unequal distribution of available stocks and limits diversification opportunities (Asobancaria, 2023; González-Bueno, 2019; Rincon & González-Bueno, 2016). However, over time, and through annual rebalancings, weights on assets change gradually, reflecting the fluid nature of the market in terms of new adoption, economic cycles, and changing expectations. One tangible example of this flexibility could be understood from the fact that, for Rebalanced Portfolio 3, made in the year 2017, the weights for preferred shares of PfDavvnda and Bcolombia went up to 17.50% and 29.10%, respectively. This could well be due to enhanced performance expectations or a reevaluation of the perceived risk for those assets in light of the underlying economic conditions and metrics describing performance. These changes underpin the flexibility of the mean-variance model in modeling and then responding to local economic events and great sectoral changes. The idea is that the efficient frontier should not be static but a dynamic framework that necessarily must evolve with the market. It must ensure the highest expected returns for each period; at the same time, re-balancing the risk profile of the portfolio is done on an active basis to adapt it to the changing realities of the Colombian market.

Table 1. Composition and rebalancing of original and optimised portfolio using the mean-variance model 2014–2023 (source: Economatica, n.d.)

	I.P.	R.P.1	R.P.2	R.P.3	R.P.4	R.P.5	R.P.6	R.P.7	R.P.8	R.P.9
Bogota	30.0%	30.0%	2.3%	23.4%	7.8%	0.2%	10.2%	16.4%	0.0%	0.0%
PfDavvnda	1.3%	0.0%	0.0%	17.5%	24.1%	2.6%	0.0%	0.0%	0.0%	0.0%
Bcolombia	8.0%	0.0%	0.0%	0.0%	8.1%	16.5%	0.0%	0.0%	0.0%	0.0%
PfBcolom	0.0%	0.0%	0.0%	0.0%	0.0%	20.7%	0.0%	0.0%	0.0%	2.6%
Cemargos	0.0%	30.0%	30.0%	29.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
PfCemargos	30.0%	0.0%	30.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Corficolcf	30.0%	30.0%	30.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Ecopetrol	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	29.8%	23.6%	29.4%	0.2%
Gruposura	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	7.2%
Geb	0.7%	10.0%	7.7%	30.0%	30.0%	30.0%	30.0%	30.0%	30.0%	30.0%
Nutresa	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	17.6%	30.0%
Isa	0.0%	0.0%	0.0%	0.0%	30.0%	30.0%	30.0%	30.0%	23.0%	30.0%

Note: I.P.: Initial Portfolio; R.P.: Rebalanced Portfolio.

The reallocation for Rebalanced Portfolio 6, made in the year 2020, should be representative. During this time, Ecopetrol and Gruponutresa reached a percentage of investment of 30% each, where strategic positioning toward those economic sectors was favorable within the market at that time. In fact, during that time, the recovery of the price of hydrocarbons favored Ecopetrol, while sustained growth in demand for consumer goods favored Gruponutresa. These decisions underline how meaningful portfolio composition adaptation to macroeconomic signals and sectoral resilience is, especially in an emerging market like Colombia. In this paper, the mean-variance model is not used only for risk optimization but also as a strategic mechanism, with which to leverage interdependencies and performance trends across industries to capitalize on sector strengths.

In the last periods, in the Rebalanced Portfolio 9 for 2023, a clearer diversification is appreciated in the composition of the holdings of Geb, Gruponutresa, and Isa, with 30% each. Regarding this, it might be a capital-preservation approach that maximizes stability in probable economic turbulence or market volatility at the end of the period analyzed. It shows how this model could enforce some form of balance in tackling risk diversification with consistency in portfolio performance by allowing less concentration in one sector or asset class. It will ensure that prudent financial management is observed during the end of a holding period by putting uttermost importance on mitigating sectoral shocks and protecting the performances gained and the investor's interests from an unexpected rise in risk.

To underline the potential impact of this research, but also to make a real thorough comparative evaluation possible, the performance of the optimum portfolio derived from the mean-variance framework was carefully evaluated against real investment alternatives that actually exist in the Colombian capital market. An exercise that becomes relevant given that it will present the practical benefits that come about from the use of Markowitz's model in the investment process, where theory is transposed into practical reality in a real investment scenario.

Financially, conventional funds seem doomed by either poor diversification or inefficient strategy that does not effectively exploit the inter-assets correlation. This thus makes them more vulnerable to some risks or losing the opportunity provided by the sectoral trends of certain sectors, which may happen very fast, such as consumer goods or energy. While the optimized portfolio assumes the correlations to be as above, over time, it improves its Sharpe ratio and is reliably placed as the most efficient alternative in terms of the risk-return trade-off.

Figure 3 presents, in some detail, both graphically and quantitatively, how far the optimized portfolio computed under the mean-variance framework outperforms three traditional investment alternatives within the Colombian equity market. Using the base-100 method, the dynamics of an initial allocation of 100 monetary units over a decade-long period are projected forward from the beginning of 2014 to the end of 2024, outlining wide differences between the return patterns and the methods for managing risks within the analysed options.

Results are that the A Share Mutual Fund decreased about 23.19%, having closed the period at 76.81 units. Such a result shows the inefficiency of the management, either for very prudent strategies or for not properly identifying high-growth assets in a fluctuating market in Colombia. In contrast, the A Shares Investment Fund, whose cumulative return

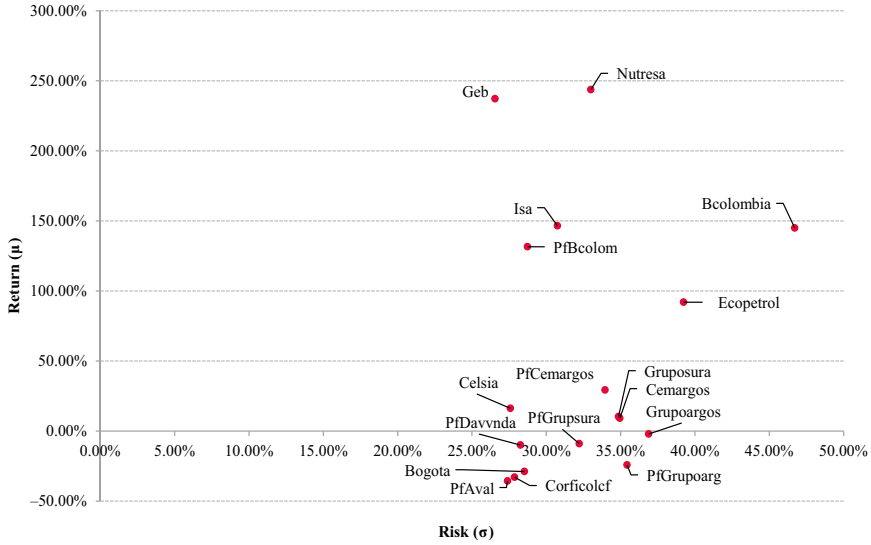

Figure 3. Comparative analysis of portfolio rebalanced versus real-world investment funds in the Colombian capital market between June 1st, 2014, and June 28th, 2024 (Source: Economatica, n.d.)

is 71.77%, and the Income-Share Mutual Fund, which has grown by 26.76%, show a better relative performance but are still far below the optimum realizable with the mean-variance approach. The rebalanced portfolio, developed and applied in a dynamic and mathematically sound framework, achieved a cumulated return commensurate of 93.96%, practically doubling the principal investment. This is not an accident but a testament to the capability of the mean-variance model to decide on a set of efficient assets that will maximize the expected return for a particular risk level or reach a minimum at an attained level of return. This rebalancing mechanism refreshes the latest price data, the changed volatility, and the correlations to the portfolio, therefore ideally updating the portfolio to adapt to a shift in market conditions.

The temporal patterns reinforce the benefits of the restructured portfolio even more. While for the benchmarked funds, there was a steady but below-par performance in the early years, the rebalanced portfolio began posting steady results from 2016 onward and continued its stable rise during the extremely volatile period between 2020 and 2022. This indicates the advantage of a system that is statistically and financially strong at its base, which acts promptly to reduce risk in periods of turmoil at either the macro or sector level. In general, mathematically speaking, the mean-variance model underlying this rebalanced portfolio has dual objectives in terms of maximization of the expected return Max $E(R_p)$ and minimization of the portfolio variance Min σ^2_p . The efficiency frontier, recalculated from time to time, would keep this portfolio at the tangent of the capital market line, meaning optimum allocation of assets as the market changes. Adaptability is especially important in emergent markets such as Colombia, where volatility and the phenomenon of issuer concentration present serious challenges to portfolio managers.

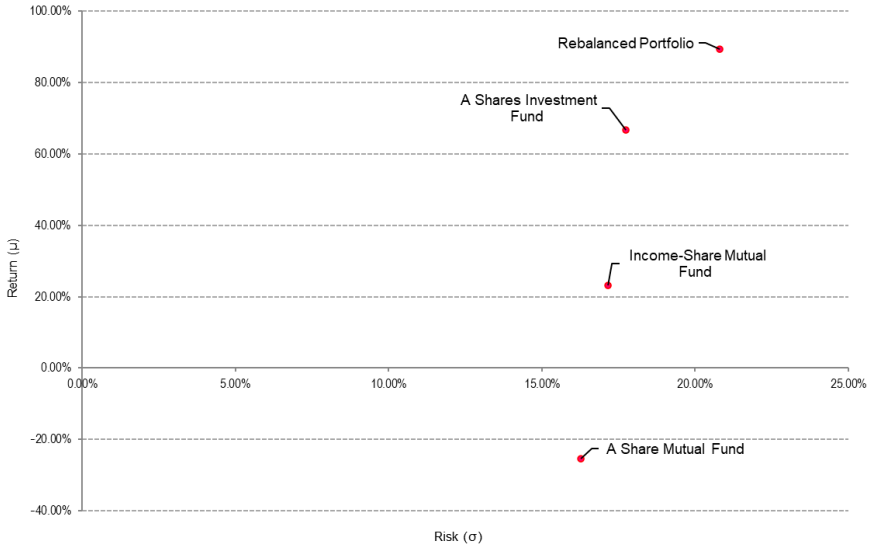

Figure 4. Return and risk of the portfolio rebalanced versus real-world investment funds in the Colombian capital market between June 1st, 2014, and June 28th, 2024 (Source: Economatica, n.d.)

Figure 4 presents the time-series behavior of the annualized return and annualized risk for the rebalanced portfolio, together with those of the three representative investment funds discussed above. The figures below show the efficiency of the rebalanced portfolio, which had an astonishing annualized return of 89.38%, but has kept the risk at 20.79%, hence proving to be the best option among the alternatives. This result thus agrees with the very basics of the mean-variance model, which defines the sets of assets that provide either maximum return for any given amount of risk or minimum risk for any given return. In this case, risk is measured by the annualized standard deviation of the returns for a week, reflecting the intrinsic volatility and unpredictability of the portfolio. The portfolio rebalanced at 20.79% would indicate that, with greater risk, the portfolio is less efficient, given that the risk is managed optimally. Instead, it realizes the importance of diversification and dynamic adjustments in keeping the portfolio at the efficient frontier.

By comparison, the A Shares Investment Fund has an annualized return of 66.73% with a risk of 17.75%, which is a good performance, but it has not outperformed the threshold set by the rebalanced portfolio in terms of risk-adjusted return. This fund represents a fair trade-off between return and risk, but it does not entail a rebalancing strategy typical of the rebalanced portfolio. This absent rebalancing most likely now binds its ability to seize the changed opportunities within the market, especially with respect to sectoral changes and macroeconomic fluctuations.

The more conservative of the two, the Income-Share Mutual Fund, represents an annualized return of 23.16% at 17.16% risk. Its relatively low return illustrates the limitations inherent in methods that prize stability over the potential for growth. While the near identical risk profile, relative to the A Shares Investment Fund, suggests poor diversification because its

focus on lower volatility assets does not translate into a comparable lessening of overall risk. This points to the missed opportunity of exploiting covariance structures that could reduce portfolio variance.

The A Share Mutual Fund has the poorest showing, with an annualized return of –25.40% at a risk level of 16.27%. This is indicative not only of the poor selection of assets but also of the lack of appropriate adaptation strategies to mitigate losses. This relative moderate volatility represents a lesser value compared to the one of the rebalanced portfolios and sets a limited exposure to high-growth potential, which, however, does not justify such extraordinary losses. It testifies to the inefficiency of static methods which do not react against changing market scenarios.

Notice that the Sharpe ratio test really highlines the strong enhancement in efficiency for the rebalanced portfolio. Undoubtedly, it enables one to materialize an excess value in terms of returns for every unit of the risk taken when compared with conventional funds, which show a poorer return-risk profile. On grounds of this systematic rebalancing methodology, weights of the portfolio are adjusted against revised covariance matrices to hedge out the idiosyncratic the financial flexibility of the rebalanced portfolio regarding market fluctuation provides a rich approach to the problems faced in emerging markets like Colombia, characterized by high issuer concentration and unstable economic contexts. With the mean-variance model, through regular recalibration of asset weights in response to changes in correlations and volatility, the portfolio can be assured of reaping growth opportunities with a successful risk containment. Thus, this higher risk-considering σp = 20.79% of the rebalanced portfolio-compared to the traditional funds reflects exposure to higher expected returns-bearing assets. However, the risk here is managed through diversification, which keeps the overall portfolio variance down despite the inclusion of assets that individually have a higher risk. Risks and keep the portfolio fully diversified across multiple sectors and asset classes.

In view of the considerations above, this result constitutes strong support for the importance of the mean-variance model, both on a sound theoretical basis and as a significant tool in practice for the field of portfolio management. Since the model allows for dynamic adjustments by periodic rebalancing, not only does the optimization of risk-return take place, but it also gives a strategic advantage in facing complexities and inefficiencies typical of the emerging market. Besides the theoretical sophistication, the model gives a concrete answer to the portfolio selection problem and is a very powerful tool for any decision-maker in Colombia to make intelligent and data-driven decisions for investments. Thus, by applying the proposed framework, the investors will efficiently control the volatility, capture the best moments of the market, and realize the optimal financial performance in harmony with their risk tolerance and return objectives. This suggests that the intrinsic flexibility of the mean-variance model will make it pertinent for institutional and individual investors who want to realize further enhancements in investment strategies within dynamic, ever-changing financial environments.

5. Discussion

The present research deals with one of the most important open issues in portfolio management, seeking to focus the investigation on those unique problems distinguishing emerging

markets, especially the case of the Colombian capital market, presents an environment with low liquidity and high concentration, creating bigger barriers to an effective investment strategy. Using the mean-variance model, adjusted for the aforementioned market limitations, it is shown how the use of annual rebalancing and limits to diversification can significantly improve risk-adjusted returns. Interestingly enough, this investigation puts forward how the sectoral dynamics, especially the energy and consumer goods sectors, may help inform portfolio strategies in structurally inefficient markets. Such findings support the development of not only portfolio optimization in practice but also create a foundation for crafting those strategies that prove to be sensitive to both macroeconomic conditions and sectoral trends in emerging economies.

The results underscore the adaptability and effectiveness of the mean-variance framework in addressing the challenges inherent in emerging markets like Colombia. The framework captures the volatile and constrained nature of these markets, offering a robust approach where traditional strategies might fail. Notably, companies such as Geb and Nutresa emerged as leaders, demonstrating the model's capacity to identify and exploit growth opportunities in energy and consumer goods sectors. Other companies like Bcolombia and its preferred shares reflected stability and substantial returns within the financial sector, showcasing the model's ability to adapt to sector-specific dynamics. On the other hand, the relative underperformance of firms like Bogotá and GrupoAval, reflects the heterogeneity of market conditions and strategic decisions at the firm level. In this respect, it is instructive to note the relative buoyancy of infrastructure-related firms like Isa and Ecopetrol against the backdrop of a crisis-racked world energy market. It was with this blending of theoretical optimization and actionable insight from such heterogeneous performances that the mean-variance framework still remained quite workable within the real-life emerging market investment practice in view-so as to help investors balance their risk-return tradeoffs while tapping into sectoral strengths.

The present paper contributes to the existing literature by offering solid quantitative evidence on portfolio optimization for emerging markets, which contrasts with the majority of existing studies that focus on developed economies. While several works have outlined diversification and efficient frontier optimizations as key mechanisms that might reduce particular risks (Bilbao-Terol et al., 2013; Campbell et al., 2001; Grubel, 1968; Gupta et al., 2020; Oliver, 2021), our research thus places these principles within the peculiarly demanding context of the Colombian market, which is low in liquidity and very concentrated. Hence, the construction of portfolios needs to respect the peculiarities of the market. Consistent with emergent patterns in less liquid markets, the most important features our results put in relief were the need to confront concentration risks and take account of local trends by sector as the basic dimensions along which the mean-variance model could be adapted in structurally constrained settings (García et al., 2019; Malagón González & Verdugo Rodríguez, 2023; Rincon & González-Bueno, 2016; Yepes-Rios et al., 2015). These gains in insight not only deepen the understanding of portfolio optimization strategies but also manifest their applicability and relevance regarding emerging market peculiarities. In this regard, our study extends the theoretical and practical scope by showing that some classical portfolio theories work well for emerging economies. By doing so, it gives a base framework that policy makers, institutional investors, and academics can use to take advantage of the nuances of underde-

veloped markets. This contribution becomes fundamental for the development of investment strategies that are theoretically adequate and, at the same time, practically adjusted to the problems that the market, for example, faces in Colombia.

These findings also underpin opportunities for policy makers to address the structural inefficiencies of the Colombian capital market, as previously discussed in the introduction and problem statement. More specifically, reforms targeted at increasing liquidity, the addition of new issues, and updating of the order-matching system will directly alleviate the constraints of low diversification and efficiency of markets. That supports the general goal of preventing the re-classification of the market as a frontier market, which would increase the cost of capital and deter foreign investment further, as discussed in earlier parts of this paper. Furthermore, improving financial literacy and encouraging innovation in securities investments are profoundly relevant measures to widen market participation, build up resilience, and increase competitiveness. This is in line with the initial focus of the study as to how meaningful investment decisions should be done to overcome such challenges as high concentration in the market and low liquidity. By so doing, the policymakers will have put a path toward sustainability to the benefit of domestic and international investors while making the financial system more resilient to external shocks. These measures are part of the broader goal in the paper to couple the theoretical perspectives with policy prescriptions so that the market can be said to be indeed on the path to stability and efficiency.

This paper acknowledges limitations, including reliance on historical data that might not accurately capture the dynamic characteristics of future market movements. Historical data are the traditional foundation for understanding asset performance, but the predictive capability residing therein is intrinsically limited in the face of rapidly changing market conditions. Moreover, the absence of sophisticated risk measures, such as CVaR or semi-variance, represents a methodological shortcoming. These would fortify the portfolio assessments with a wider angle on the risk exposure problem. Future research using the mean-variance model in other emerging economies would enable comparisons to be made and, therefore, region-specific insights into optimal investment strategy and market behaviour to be uncovered. The present study would thus be more generalized and demonstrate how portfolio optimization frameworks can be adapted across different economic regimes. These extensions would not only enhance the theoretical base of portfolio optimization but also substantially enlighten investors and policymakers about how to meet the challenges thrown up by turbulent and developing markets. Successively overcoming all these limitations and pushing ahead with the agenda will continue the progress and growth of portfolio management strategies in emerging economies.

6. Conclusions

The current research develops the understanding of portfolio optimization in the face of an emerging market, namely Colombia, in respect of the practical importance and flexibility of the bi-objective mean-variance model. Given the double objective of increasing returns and diminishing risks, the research effectively links the theoretical underpinning of the Markowitz model in application to the multi-faceted dynamic financial landscape.

The results focus on the ability to understand exactly how the model perceives the current market fluctuations in valuations of assets, liquidity, and sectoral conditions. In this regard, the frequent recalibration of the efficient frontier in concert with the process of rebalancing strategies on an annual basis reflects strengths in model flexibility as that instrument adjusts to changes in market conditions. This responsive strategy keeps investments focused more congruently on ideal risk-return profiles, considering a high concentration, low-liquidity, and economically unstable marketplace.

A comparison of the optimized portfolios with respect to real investment alternatives in the Colombian capital market will reveal the benefit of the mean-variance model in producing better risk-adjusted returns. To that end, the Sharpe Ratio proved an essential tool, further underlining the necessary importance of considering the management of risk in portfolio optimization for the successful realization of investor goals. This paper also contributes to the scant literature on portfolio management in emerging markets by addressing particular issues related to market concentration and liquidity constraints. These results confirm the theoretical efficiency and practical usability of the mean-variance model as a guide for investment decisions compatible with the changing conditions of emerging markets.

From a practical standpoint, this study provides useful guidance for both policymakers and investors in emerging markets. The encouragement of new issuers, diversification of concentration, modernization of trading platforms, and stimulation of financial innovation are ways policymakers can help make markets more liquid. Such efforts at strengthening market infrastructure would indeed provide propitious conditions not only for sustained investment growth but also for resilience.

Further research could expand this foundation by incorporating downside risk measures-semi-variance or CVaR-to offer a fuller investigation of performance of the portfolios from a range of risk viewpoints. Application of the mean-variance model to local emerging markets would also be highly beneficial in comparative terms and would expand knowledge of the suitability of the model under differing economic and financial conditions. The results from such studies would help in further understanding portfolio optimization to suit specific market conditions and thus develop more robust, widely acceptable investment strategies.

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